Probability and Stochastic Processes (EEE 505)

Group 3

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Question

A battery manufacturing company needs a solution to predict and manage the lifespan of its batteries. Propose a solution based on an appropriate probability distribution to estimate the lifespan of the batteries and suggest relevant parameter(s) that can elongate the lifespan of these batteries using your selected probability distribution. Demonstrate the effectiveness of your solution with a range of values and relevant plot(s).

Introduction

A Probability Distribution is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range. Probability Distribution depicts the expected outcomes of possible values for a given data-generating process. Probability Distributions come in many shapes with various characteristics defined by the **mean**, **standard deviation**, **skewness** and **kurtosis** (indicates how much data resides in the tails).

Types of Probability Distributions

1. **Discrete Probability Distributions**
   Used for random variables that take on a finite or countable number of values. Examples include:
   - **Bernoulli Distribution**: For a single trial with two possible outcomes (success or failure).
   - **Binomial Distribution**: Models the number of successes in a fixed number of independent trials.
   - **Poisson Distribution**: Represents the number of events occurring in a fixed interval of time or space.
2. **Continuous Probability Distributions**
   Used for random variables that take on an infinite number of values within a given range. Examples include:
   - **Normal (Gaussian) Distribution**: A symmetric, bell-shaped curve describing many natural phenomena.
   - **Exponential Distribution**: Used to model the time between events in a Poisson process.
   - **Uniform Distribution**: All values within a range are equally likely.

Suitable Probability Distribution

The task here is to predict and manage battery lifespan. Since battery life is inherently continuous as time measured on a continuous scale rather than in discrete intervals, **discrete probability distributions are not applicable**.

A literature review was conducted to select the most appropriate continuous probability distribution for modelling battery lifespan. The review revealed that the **Weibull**, **Log-Normal**, and **Normal** distributions are most commonly used, with the **Log-Normal** distribution performing the best. Table 1 gives an overview of the literature review. Considering the three continuous probability distribution, The Normal distribution is defined on the entire real line $(-\infty, \infty)$ meaning it assigns non-zero probabilities to negative values. In

Paper	Conclusion
Choosing the Best Lifetime Model for Commercial Lithium-Ion Batteries by Talal Mouais, Omar A. Kittaneh, M.A. Majida.[1]	Weibull, Log-Normal and Normal distribution were used in the analysis of battery lifetimes and Log-Normal performed best
Fitting lifetime distributions to interval censored cyclic-aging data of lithium-ion batteries by Marcus Johnen, Christian Schmitz, Maria Kateri, Udo Kamps.[2]	Weibull, log-Normal and inverse Gaussian distributions were used in the analysis and Log-Normal seemed to be a reasonable choice for modelling lifetimes of batteries
Recognition of battery aging variations for LiFePO4 batteries in 2nd use applications combining incremental capacity analysis and statistical approaches by Yan Jiang, Jiuchun Jiang, Caiping Zhang, Weige Zhang and Yang Gao.[3]	Weibull and Normal Distributions were sued and it was found out that Weibull distribution can be used to approximate battery module capacity distribution and Normal distribution can be used to approximate battery module resistance distribution.

Table 1: Literature Review

the context of battery lifespans, negative lifetimes are not physically meaningful. Batteries fail mostly due to multiplicative effects of temperature, discharge cycles, manufacturing variability which makes the the logarithm of the battery life tends to be normally distributed. Hence the suitable probability distribution is the **Log-Normal Distribution**.

Modelling Using Log-Normal Distribution

The **Log-Normal Distribution** is a continuous probability distribution of a random variable whose logarithm is normally distributed. This means that if you take the natural log of a variable following a log-normal distribution, the resulting values form a normal distribution. It is commonly used to model positively skewed data where values are strictly positive, such as income, stock prices, or battery lifespans. The distribution is characterized by its shape and scale parameters and is particularly useful for representing **multiplicative processes** that lead to heavy tails and rapid increases in the data.

For any μ and $\sigma > 0$,

$$f_X(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right), x > 0 \tag{1}$$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - Q\left(\frac{\ln(x)-\mu}{\sigma}\right) & x \geq 0 \end{cases} \tag{2}$$

$$\mu_X = \exp\left(\mu + \frac{\sigma^2}{2}\right), \sigma^2 = \left[\exp(\sigma^2 - 1)\right] \exp(2\mu + \sigma^2) \tag{3}$$

Using the **Maximum Likelihood Estimation (MLE)** which is a method of estimating the parameters of an assumed probability distribution, given some observed data. The MLE of the Log-Normal distribution is given by [4]:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} \ln x_i \tag{4}$$

$$\hat{\sigma^2} = \frac{1}{2n} \sum_{i=1}^{n} (\ln x_i - \hat{\mu^2})^2 \tag{5}$$

An Example using the Log-Normal Distribution

The Dataset used in this example represents the Remaining Useful life of a battery after after a certain cycle index (number of charge cycle). It was gotten from Kaggle[5]. Part of the data is shown below:

Cycle Index	Remaining Useful Life (RUL)
1	1112
2	1111
3	1110
4	1109
6	1107
...	...
1108	4
1109	3
1110	2
1111	1
1112	0

Solution

Using the Cycle Index as the Random Variable, the parameters (μ and σ^2) of the probability distribution function can be determined by Maximum Likelihood

Estimation. Therefore,

$$\begin{aligned}\hat{\mu} &= \frac{1}{n}\sum_{i\,=\,1}^{n} \ln x_i \\ &= 6.014 \\ \hat{\sigma^2} &= \frac{1}{2n}\sum_{i\,=\,1}^{n} (\ln x_i - \hat{\mu^2})^2 \\ &= 0.977\end{aligned}$$

Therefore, the Probability Function is given as:

$$f_X(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right)$$
$$f_X(x) = \frac{1}{0.988x\sqrt{2\pi}} \exp\left(-\frac{(\ln(x)-6.014)^2}{1.954}\right)$$

The plots below shows the Probability Density function(PDF), Cumulative Density Function(CDF) and Reliability (1-CDF) over a cycle index. From the Reliability Plot, it shows that increase in the cycle index(number of charge cycle) of the battery decrease the battery lifespans. The full solution can be found in the notebook

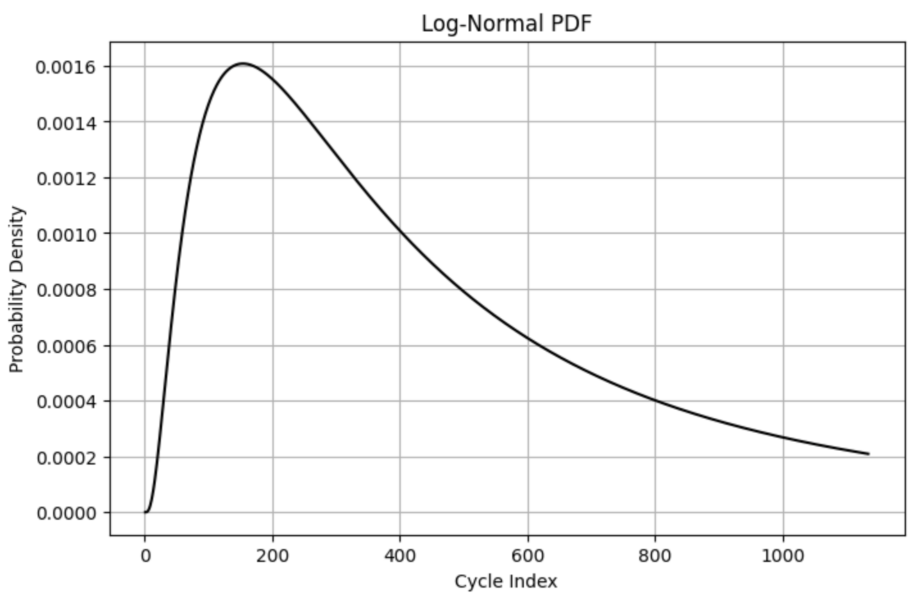

Figure 1: Probability Density Function

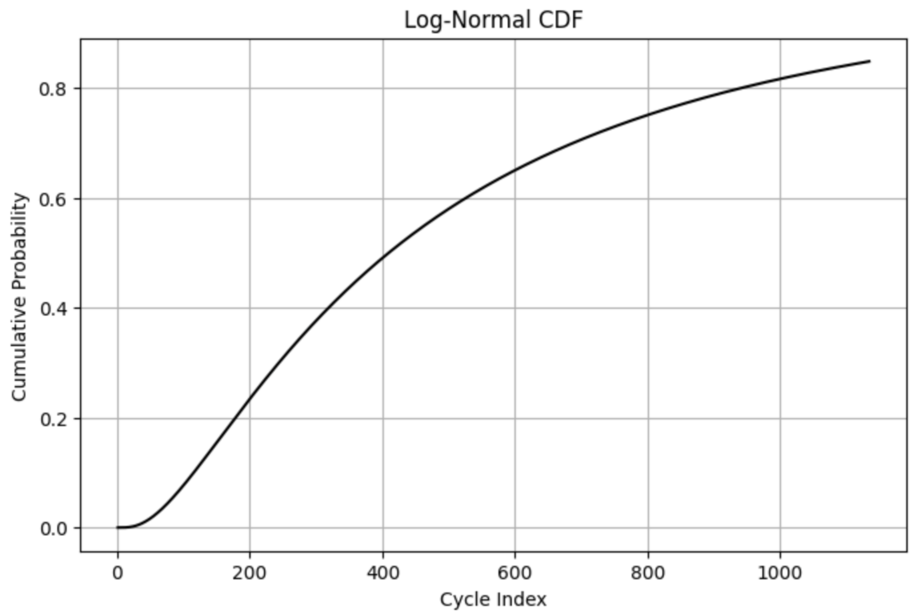

Figure 2: Cumulative Density Function

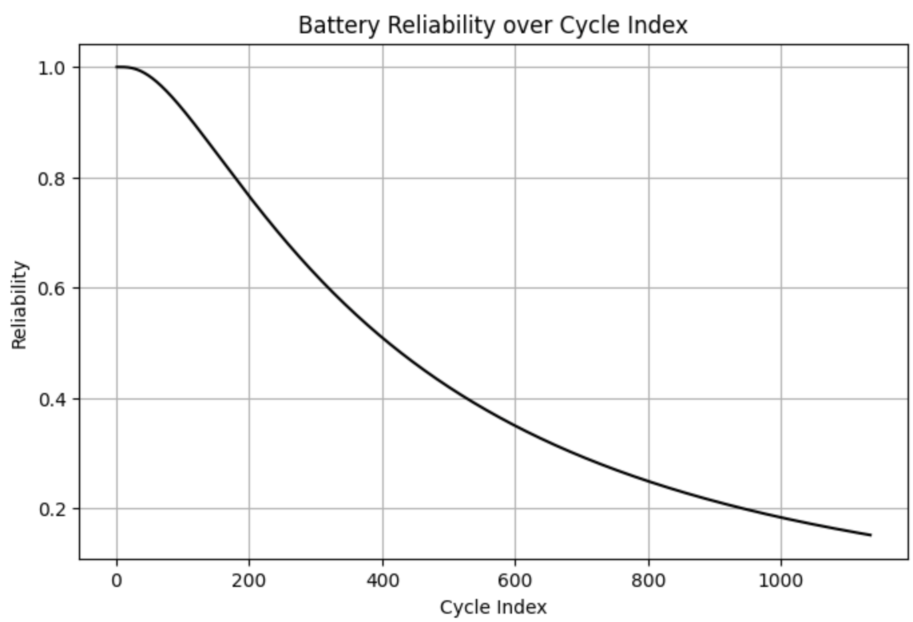

Figure 3: Reliability

References

[1] Mouais Talal, Kittaneh Omar A., and Majid M.A. Choosing the best lifetime model for commercial lithium-ion batteries. *Journal of Energy Storage*, 2021.

[2] Johnen Marcus, Schmitz Christian, Kateri Maria, and Kamps Udo. Fitting lifetime distributions to interval censored cyclic-aging data of lithium-ion batteries. *Computers & Industrial Engineering*, 2020.

[3] Jiang Yan, Jiang Jiuchun, Zhang Caiping, Zhang Weige, and Gao Yang. Recognition of battery aging variations for lifepo4 batteries in 2nd use applications combining incremental capacity analysis and statistical approaches. *Journal of Power Sources*, 2017.

[4] William Q Meeker, Luis A Escobar, and Francis G Pascual. *Statistical methods for reliability data.* John Wiley & Sons, 2022.

[5] Battery Remaining Useful Life (RUL), Kaggle Dataset.