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Path Planning for Localization of an RF Source by Multiple UAVs on the Crammer-Rao Lower Bound

Seyyed M. Mehdi Dehghan, Mohammad Saberi Tavakkoli, Hadi Moradi
Advanced Robotics and Intelligent Systems laboratory,
Control and Intelligent Processing Center,
School of Electrical and Computer Engineering, University of Tehran
Tehran, Iran
smm.dehghan@ut.ac.ir, m_saberi@alum.sharif.edu

***Abstract*— This paper presents a path planning approach of several UAVs for Radio Frequency (RF) source localization in None Line Of Sight (NLOS) propagation condition using the Received Signal Strength Indication (RSSI). The paths are planned such that the lower bound of standard deviation of localization error, which is equivalent to the Cramer-Rao Lower Bound (CRLB) of the estimation, minimized at any time. Due to the complexity of Jacobin calculation to perform global CRLB optimization, the local values of CRLBs in the current waypoint and next probable waypoints are used in the steepest decent approach to determine the best path. Furthermore, the complexity is reduced by discretizing the space for UAVs to make the computation feasible. The effect of NLOS propagation on the RSSI measurements is simulated by a log-normal distribution and the last estimation of radio source location is used to calculate the local CRLBs. The proposed approach has been simulated and compared with the basic bio-inspired approach of going toward the sensed direction of the source. The result shows better performance than the basic approach.**

***Index Terms*— Aerial Localization, Radio Frequency Source, RSSI, Path Planning, Crammer-Rao Lower Bound (CRLB), Steepest Decent.**

I. INTRODUCTION

Aerial localization of a Radio Frequency (RF) source has various civil and military applications including the search and rescue missions [1]. The desired application in this paper is using the signal from the cell phone or other dedicated device of a person who is lost on a terrain to localize him/her. In such a situation, the ups and downs of the terrain may avoid the direct sight between the radio source and the UAV(s). Such propagation is called None Line of Sight (NLOS) propagation in which the radio signals are received indirectly. Due to the fact that one UAV may not be able to cover a large area in the needed time frame and the required accuracy to localize and find the lost target, using several UAVs to search and localize the RF source based on the received signal strengths is proposed.

Localization of the RF sources based on the different measured data include Received Signal Strength Indication (RSSI), Differential Received Signal Strength Indication (DRSSI) [2], signal strength ratio [3], Angle Of Arrival (AOA) [4-6], Time of Arrival (TOA), and Time Differential of Arrival (TDOA) [7] has been studied in many research and still efforts to improve the accuracy of localization is continuing. Aerial localization of these sources by one or more UAV has been the concerns of many papers [7-9]. Also many researches have been done in four major directions: a) optimal path planning [10-13], b) planning the optimal formation [14-15], c) cooperation of UAVs [7], and d) wide area coverage [16-18] for better localization of one or many sources.

The design of an optimal path for one or more UAVs for radio source localization is done on the basis of various criteria, such as accuracy of localization and communication constraints. The CRLB is one of these criteria that have been used in [10-12]. In these researchers, three UAVs have been used for localization that all of them own scan sensor and only one has AOA sensor. Optimized path planning for getting the maximum accuracy in the localization is done by generating way-points in the direction of maximizing the determinant of Fisher Information Matrix (FIM). Sutton et. al [13] used a spiral path for providing areal coverage and arranged a two-dimensional formation of three UAVs in the form of an equilateral triangle. Due to different turning radius of each UAV in the spiral maneuver, it is difficult to maintain the formation flight and a special algorithm is required which is emphasized in their approach. Drake et. al [14]

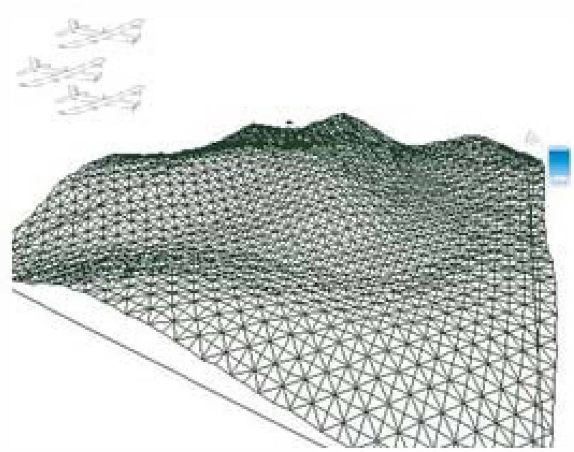

Fig. 1. Three UAVs fly over terrains searching for an RF signal

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has also studied the formation of three UAVs in two direct and circular paths. Although the direct path reduced the length of the flight for localization, however, the accuracy of localization in circular path is higher.

Stachura et. al [15] addressed the problem of optimized way-point planning for localization of multiple RF sources, using UAVs, considering the limited communication range between them. Scerri et. al [16] has addressed the localization multiple RF sources and path planning of multiple UAVs together. They applied a revised version of RRT path planning with the ability of generating virtual terminal points of paths. Optimized paths for each UAV are determined by considering entropy map, the path of other agents and the information about the type of terrain. Frew et. al [17] also have considered the subject of localization of several RF sources by multiple UAVs. In this research, the controlling commands of each agent are selected by maximizing the determinant of fisher information matrix.

Calculation of CRLB has been studied a lot not only for using in path planning, but also for evaluating the lower bound of estimation error available in different approaches of localization and also according to different observations, for finding optimized formation of UAVs and selecting the best localization way-points. York et. al [18] compared Kalman estimation technique and triangulation technique for localization of RF mobile targets on the basis of task completion time and average accuracy of target localization. Bishop et. al [19] looked for the best N-tuple formation of sensors around a target in a sensory network to minimize the localization error.

To the best of our knowledge, there is no research addressing the path planning in NLOS condition, which is a very typical condition. Furthermore there is no generalization in the number of UAVs for the localization of an RF source. Consequently, in this paper, a new algorithm for the path planning of one or more UAVs in NLOS condition according to RSSI observations is reported. The path planning is done on the basis of reducing lower bound of estimation error that is described by the CRLB analytical criterion. NLOS propagation is simulated by log-normal distribution on received signal strength in watt, which is equivalent to normal distribution on the path loss in dB. The target position is estimated by least square techniques and it is assumed that the power of transmitter is known and its position is fixed.

II. LOCALIZATION

As mentioned earlier, the position of RF source assumes unknown in this paper and path planning is done on the basis of CRLB calculation in the last available estimation of target position. The estimation of target position is done according to RSSI data by the use of a combination of existing techniques in the family of least square error methods. So in this section it is briefly discussed how the localization takes place.

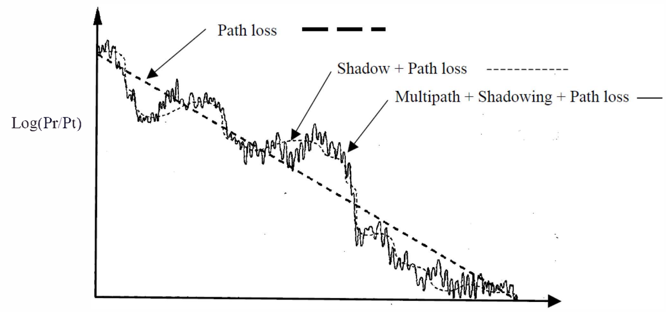

Fig. 2. Attenuation in received signal strength due to path loss, shadowing and multipath [20].

The general path loss model is used for calculating the signal attenuation due to the distance [20]:

$$PL_{dB} = PL_{d_0} + 10\lambda \log\left(d / d_0 \right) \tag{1}$$

In Eq. 1, PL_{dB} is the amount of signal strength attenuation in dB, λ is the coefficient of path loss, d is the distance of transmitter and receiver, d_0 is a reference distance, and PL_{d0} is the path loss in d_0.

When the power of transmitter is known and using RSSI measurements, the path loss can be calculated by Eq.2, and then Eq. 1 is used to compute the distance of transmitter and receiver. P_t is the transmitter signal strength and P_r is the receiver signal strength in watts.

$$PL_{dB} = 10 \log\left(P_t / P_r \right) \tag{2}$$

Propagation in NLOS condition is influenced by both shadowing and multi-path phenomena (Fig.2). The effect of multi-path on received signal strength is negligible with averaging of some consecutive samples. The effect of shadowing on the received signal strength in watt follows a log-normal distribution, which is equivalent to a Gaussian distribution (η) on the path loss in dB (Eq. 3) with σ_{sh} standard deviation (SD). The amount of this SD depends on environmental features of propagation.

$$\hat{PL}_{dB} = PL_{dB} + \eta \tag{3}$$

If the equations of a system can be formed - in accordance to the parameters those should be estimated- such as Eq. 4, least square error techniques can be used for estimating the unknown parameters.

$$AX = B \tag{4}$$

For localization (the estimation of position) of RF source, the components of Eq. 4 are specified by Eq. 5 to 7 [21].

$$A = \begin{bmatrix} 2(x_1 - x_2) & 2(y_1 - y_2) \\ 2(x_1 - x_3) & 2(y_1 - y_3) \\ \vdots & \vdots \\ 2(x_1 - x_N) & 2(y_1 - y_N) \end{bmatrix} \tag{5}$$

$$B = \begin{bmatrix} d_2^2 - d_1^2 - x_2^2 - y_2^2 + x_1^2 + y_1^2 \\ d_3^2 - d_1^2 - x_3^2 - y_3^2 + x_1^2 + y_1^2 \\ \vdots \\ d_N^2 - d_1^2 - x_N^2 - y_N^2 + x_1^2 + y_1^2 \end{bmatrix} \tag{6}$$

$$\hat{X} = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} \tag{7}$$

In this situation, the parameters are estimated by Eq. 8.

$$\hat{X} = \left(A^T A\right)^{-1} A^T B \tag{8}$$

A. Localization by Recursive Least Square errors (RLS) technique

In addition to using least square error technique for initial estimation, the recursive version of least square errors technique is used after the number of RSSI measurements by UAV(s) became more than a threshold, to reduce the computational load resulting from enlargement of matrices A and B.

Estimating by the RLS method is done by Eq. 9 to 12 [22].

$$\hat{X}_i = \hat{X}_{i-1} + k\left(b - a_i^T \hat{X}_{i-1}\right) \tag{9}$$

$$k = \frac{1}{1 + a_i^T P_0 a_i} P_0 a_i \tag{10}$$

$$P_i = \left[I - k a_i^T\right] P_{i-1} \tag{11}$$

$$P_0 = \left(a_0^T a_0\right)^{-1} \tag{12}$$

$\hat{X}$ is the estimated position of target and ***a*** and ***b*** are some component of matrices explained in Eq. 5 and Eq. 6 or in other words, they include the used data in each stage of recursive estimation. In each step, $\hat{X}_{i-1}$ is the result of previous step of estimation. Matrix P in the first step, i.e. P_0 is computed by Eq. 12 and in the next steps is calculated in recursive manner by Eq. 11.

III. "Crammer-Rao Lower Bound" for RF Source Localization based on the N-tuple RSSI Reading

In an unbiased estimation, determining of accessible lower bound for the error variance of estimation is a proper criterion for comparing different estimators. Crammer-Rao lower bound is a known analytical tool for calculating this lower bound. In the other word, if $\hat{X}$ is an unbiased estimation of X then:

$$\sigma_{\hat{X}}^2 \geq CRLB(X) \tag{13}$$

CRLB is determined by Eq. 14 in which $\varphi(X)$ is called Fisher Information Matrix (FIM).

$$CRLB(X) = \left|\varphi(X)\right|^{-1} \tag{14}$$

Generally, FIM is calculated by Eq. 15 in which $\hat{\theta}$ is the observation and f is the probability density function of the observations.

$$\varphi(X) = -E\left\{\left(\frac{\partial}{\partial X}\ln f\left(\hat{\theta}\middle|X\right)\right)\left(\frac{\partial}{\partial X}\ln f\left(\hat{\theta}\middle|X\right)\right)^T\right\} \tag{15}$$

In this paper, $\hat{\theta}$ is the signal strength attenuation in dB that is shown by $\hat{PL}$.

If the probability density function follows a Gaussian distribution, calculation of FIM is done easily by Eq. 16 in which J is Jacobin matrix and Σ is covariance matrix of measurement noises.

$$\varphi(X) = J^T \Sigma^{-1} J \tag{16}$$

Assuming η_i as the noises of path loss measurements (through measuring RSSI), the measured path loss can be modeled by Eq. 17. In Eq. 17, N is the number of measurements via N UAVs or in N different way-points of one UAV. The measurement noises in all way-points and various UAVs are assumed similar, independent of each other and time invariant with σ_{sh} standard deviation.

$$\hat{PL}_i = PL_i + \eta_i \;\; ; \;\; i = 1:N \tag{17}$$

The measuring error is calculated by Eq. 18 in which X is the position of target which has to be estimated.

$$e(X) = \hat{PL}_i - PL_i(X) \tag{18}$$

According to Eq. 18, Jacobin matrix can be presented as Eq. 19 that each component of it can be calculated by Eq. 20.

$$J = \left.\frac{\partial e(X)}{\partial X}\right|_{X=P} = \left[\frac{\partial e_1(X)}{\partial X}, \frac{\partial e_2(X)}{\partial X}, \ldots, \frac{\partial e_N(X)}{\partial X}\right]^T \tag{19}$$

$$\left.\frac{\partial e_i(X)}{\partial X}\right|_{X=P} = -10\lambda \frac{\partial}{\partial X}\log\left(\left\|P_T - P_i\right\|_2\right) \tag{20}$$

In Eq. 20, $\|P_T - P_i\|_2$ is the distance of UAV form the target. If this equation is written according to y_i and x_i, and its derivation is calculated relative to these components, the Eq. 21 will be generated.

$$\left.\frac{\partial e_i(X)}{\partial X}\right|_{X=P} = \frac{10\lambda}{(ln10)(d_i)^2}\begin{bmatrix} x_p - x_i \\ y_p - y_i \end{bmatrix} \tag{21}$$

Using Eq. 16, Eq. 19 and Eq. 21, FIM can be calculated by Eq. 22.

$$\varphi(X) = \left(\frac{10\lambda}{ln10}\right)^2 \begin{bmatrix} \sum_{i=1}^{N}\frac{(x_p - x_i)^2}{\sigma_i^2 d_i^4} & \sum_{i=1}^{N}\frac{(x_p - x_i)(y_p - y_i)}{\sigma_i^2 d_i^4} \\ \sum_{i=1}^{N}\frac{(x_p - x_i)(y_p - y_i)}{\sigma_i^2 d_i^4} & \sum_{i=1}^{N}\frac{(y_p - y_i)^2}{\sigma_i^2 d_i^4} \end{bmatrix} \tag{22}$$

According to the mentioned criterion for path planning, the determinant of FIM should be maximized or in the other word the CRLB should be minimized.

IV. PATH PLANNING BY STEEPEST DECENT APPROACH ON CRLBS

One of the aims of path planning in an RF source localization problem via one or more UAVs is attaining to the lowest estimation error. As mentioned, the lowest estimation error in an unbiased estimation such as desired problem is described by the lowest variance of estimation errors, which is computed by Cramer-Rao Lower Bound (CRLB). Therefore, the paths of UAVs should minimize the CRLB of the estimation in each instant (each way-point). Minimizing of CRLB requires Jacobin calculation of Eq. 22; due to the complexity of Jacobin calculation in the optimization of the CRLB, the local values of CRLBs are used in the steepest decent approach.

If the location of all UAVs in current time is shown by the vector π_i, the best place for them in the next instant is:

$$\pi_{i+1} = \pi_i - \varepsilon \frac{\partial \varphi(X)}{\partial G_p} \tag{23}$$

In this equation, ε is the step size and G_P is the vector of geometric variables those can be the components of UAVs position in Cartesian or polar coordinate system. In general, π_i includes the position of N UAVs shown by Eq. 24.

$$\begin{cases} \pi_i = [P_1; P_2; \ldots; P_N]^T \\ P_i = [x_i; y_i]^T \end{cases} \tag{24}$$

The problem of this technique is the Jacobin calculation of FIM according to several parameters. For removing this problem, first-order approximation for derivation and local path planning can be used. For this purpose, a set of admissible moves is generated based on the UAVs current positions, the direction of their velocities and possible displacement in each instant; then the CRLBs of all new positions of UAVs is calculated. Based on these calculated values and the current value of CRLB of estimation, the next position of UAVs will be selected according to the highest difference.

If the norm of the velocity of all N UAVs is equal to $|v_{UAV}|$ and only the direction of the velocity is variable, the best place in next step for UAVs will be calculated by Eq. 25:

$$\pi_{i+1} = \pi_i + V_{best} dt \tag{25}$$

In Eq. 25, dt is the time step and V_{best} is the best velocity vector for all UAVs from the viewpoint of highest difference in CRLBs (Eq. 26).

$$\begin{cases} V_{best} = [v_{UAV\,1}; v_{UAV\,2}, \ldots, v_{UAV\,N}]^T \\ v_{UAV\,i} = [v_{ix}; v_{iy}]^T \end{cases} \tag{26}$$

After estimating the target position in each step, for determining the V_{best}, different velocities for UAVs (in term of direction) are considered and the CRLBs in new positions of UAVs are calculated. From all possible direction of velocity vectors, the ones that cause the highest decrease of CRLB are selected. Then, UAVs go one step further in the selected direction and this process will be repeated until reaching the target.

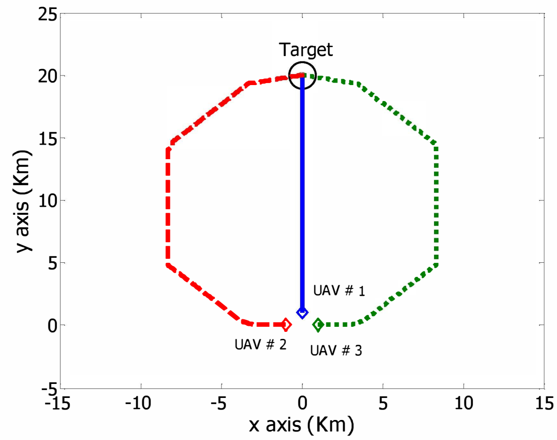

Fig. 3. Optimized paths generated by CRLB minimization approach when the position of RF source is known.

Calculation of CRLB in path planning for N UAVs, just based on their current position is the same in calculation with one UAV which uses N way-points. Obviously, this similar calculation causes different paths.

To evaluate this path planner, another technique is introduced in which UAVs in each instant move toward decreasing the distance from the estimated place of target. This scenario is described by Eq. 27:

$$P_{i+1} = P_i + U_{(P_T - P_i)} |v_{UAV}| dt \tag{27}$$

In Eq.27, $U_{(P_T - P_i)}$ is the unit vector in the direction of UAVs to the target.

V. SIMULATION

The proposed approach has been simulated for NLOS condition for three UAVs. The UAVs are assumed to fly at the speed of 150 km/h and λ is equal to 4, d_0=10 meter and PL_{d0}=12.1 dB. Both the basic approach, i.e. the bio-inspired approach of moving directly toward the source, and the proposed approach have been implemented to better show the performance of the proposed approach.

In the first step, it is assumed that at each instant the target

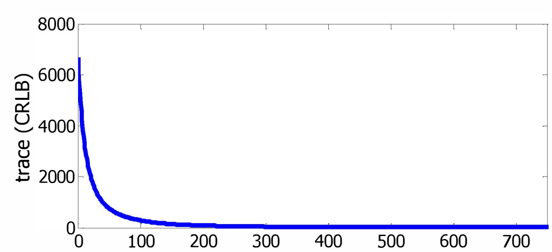

Fig. 4. The trend of CRLB decreasing in CRLB minimization approach when the position of RF source is known.

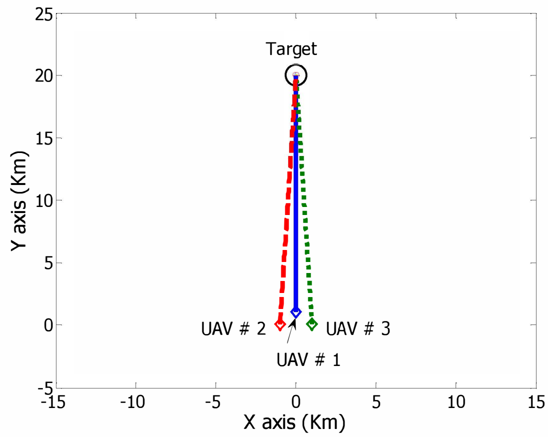

Fig. 5. The generated path by the basic bio-inspired approach when the position of RF source is known.

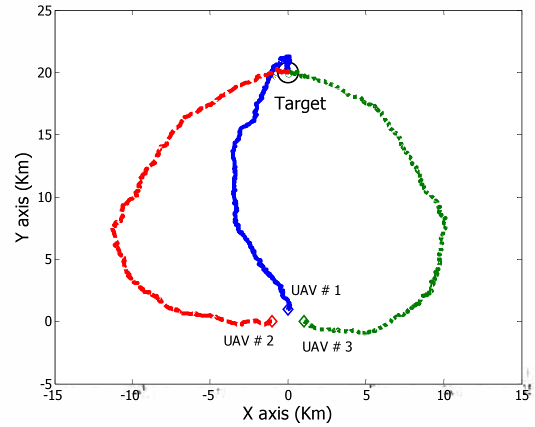

Fig.7. Optimized paths generated by CRLB minimization approach based on the estimated position of RF source.

location is accurately known. Figure 3 is a view of three UAVs that start to move from their initial position and plan their optimized path, after receiving the first signal from the target, to decrease the CRLB. This figure shows that the UAVs initially are not on the path toward the target, rather they move away from each other for better localization. After more than half-way through, the UAVs on the left and right gradually approach the goal and eventually reach the target. Figure 4 shows the CRLB at each step of the path. It can be seen that the CRLB decreases sharply as it is expected.

If the accurate position of the target is given, the generated path by the direct approach is as illustrated in Fig. 5. In this figure, it can be seen that the path length is too short in comparison to the previous approach, but based on Fig. 6 the decreasing trend of CRLB is not as good as precedent paths and CRLB in the first approach is always lower than the CRLB in the second one.

To show the effect of not having the accurate position of the source, the last estimated position of the source is used and the CRLB is calculated (Fig. 7). In such a situation, the paths of UAVs wouldn't have the previous arrangement because of imprecise localization. The generated paths based on the basic bio-inspired approach are shown in Fig. 8. As it can be seen, the generated path is not anymore direct, short, and continues. This is due to the fact that the target location is no accurately known.

According to Fig. 8, the generated paths by the basic bio-inspired approach are not suitable for tracking by UAVs due to their discontinuities. In contrary, the generated path by the CRLB based approach (Fig. 7) is usable by UAVs although there are some slight fluctuations.

VI. Conclusion and Future Work

In this paper, a new approach for multi-UAV path planning for RF source localization in NLOS condition is proposed. The obtained results indicate that the steepest descent approach in the direction of minimizing the CRLB of target pose estimation is effective for path planning.

In the presented results, for using just current RSSI observation in each instant, at least three UAVs are needed for localization; therefore path planning was done for these numbers

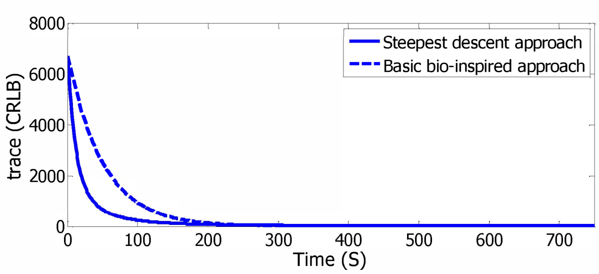

Fig.6. The trend of CRLB decreasing in two path planning approaches when the position of RF source is known.

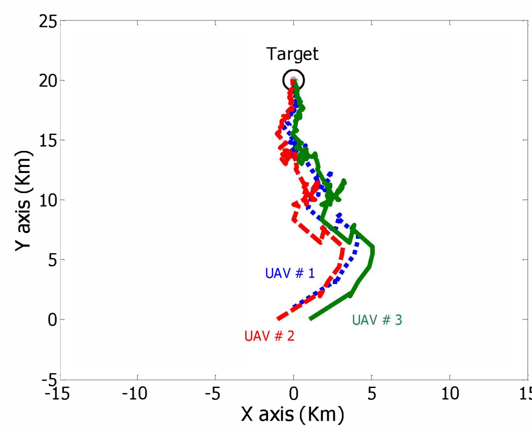

Fig.8. The generated path by the basic bio-inspired approach based on the estimated position of RF source.

of UAVs. A dual approach to the proposed multi-UAV approach is to use a single UAV with several measurements at different locations. Consequently, the proposed approach can be used for path planning of a single UAV.

One of the future researches is to limit the set of admissible moves of UAVs to access continuous motion and removing the constraint of equal norm of velocity vectors of all UAVs. The other important case is the path planning in a condition in which the power of the transmitter is unknown. Finally, the time criterion should be also considered in future studies to make sure that the UAV(s) can find the source in the given time limit.

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