Bayes Assignment 4 of 2025 by Sean van der Merwe, UFS

In this assignment you will conduct a simulation study to compare two priors in terms of their performance on a particular task. The scenario is that a set of proportions $(x_1, \ldots, x_n)$ are measured and a decision must be made whether they are close enough to uniform or adhering to a beta distribution with parameter values away from 1. The procedure is that a beta distribution is fitted to the proportions, and if the posterior median estimates of the parameters fall in the region $(h_1 < a < h_2) \cap (h_3 < b < h4)$ where $h_1, \ldots, h_4$ are close to 1 then the conclusion is made that they are 'close enough to uniform'. If you incorrectly conclude they are close enough then the loss is h_5; while if you incorrectly conclude they are too far away then the loss is h_6. Correct conclusions make a profit of h_7.

From the history, it is known that samples come from either a $beta(1,1)$ (uniform) distribution with probability h_8, or a $beta(h_9, h_{10})$ distribution with probability $1 - h_8$. The two priors being compared are **(A)** the objective prior $\pi(a, b) \propto a^{-1}b^{-1}$ and **(B)** the subjective prior $a \sim lognormal(0, h_{11})$ and $b \sim lognormal(0, h_{12})$.

Given the hyperparameter values $(h_1, \ldots, h_{12}, n)$ next to your student number below, which prior provides the minimum risk (expected loss)? Answer this question on the basis of M simulated samples.

Student_num	h1	h2	h3	h4	h5	h6	h7
2014095653	0.96	1.08	0.96	1.09	315	442	112
2017159092	0.93	1.07	0.95	1.06	263	365	144
2017418365	0.95	1.06	0.97	1.06	388	326	151
2018006516	0.97	1.07	0.92	1.04	267	382	143
2018395968	0.94	1.03	0.96	1.08	236	424	107
2019369780	0.95	1.06	0.92	1.04	270	483	130
2020231664	0.93	1.06	0.97	1.03	295	335	104
2021603747	0.91	1.03	0.94	1.04	328	431	96
2024180487	0.97	1.11	0.94	1.07	324	400	96
2028830517	0.94	1.10	0.93	1.05	366	397	133
2021234567	0.94	1.06	0.96	1.08	324	429	152
2022345678	0.94	1.07	0.93	1.09	207	314	101
2023456789	0.95	1.04	0.93	1.04	398	480	98

Student_num	h8	h9	h10	h11	h12	n	M
2014095653	0.32	1.09	1.16	2.0	1.7	50	1300
2017159092	0.40	0.91	0.86	1.5	2.0	43	1500
2017418365	0.62	1.09	1.13	1.8	1.9	36	1500
2018006516	0.42	0.92	0.88	2.0	1.9	41	1600
2018395968	0.34	1.14	1.17	1.9	0.9	41	1500
2019369780	0.46	0.86	0.81	1.9	1.7	37	1200
2020231664	0.36	1.07	1.18	1.9	1.2	37	1300
2021603747	0.67	0.88	0.85	0.8	1.1	46	1500
2024180487	0.70	1.10	1.18	1.0	1.8	49	1300
2028830517	0.40	0.88	0.81	1.3	1.3	45	1600
2021234567	0.68	1.09	1.15	2.1	1.5	45	1100
2022345678	0.69	0.89	0.85	1.6	1.5	46	1500
2023456789	0.48	1.13	1.19	0.9	2.0	44	1600