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Runoff of the Claims Reserving Uncertainty in Non-Life Insurance: A Case Study

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Abstract: *The market-consistent value of insurance liabilities consists of the best-estimate prediction of the outstanding liabilities and a risk margin for the runoff risks of these liabilities. The aim of this case study is to compare the currently used methodology for the risk margin calculation to the new approach presented in Salzmann-Wüthrich [5]. Our findings are that the traditional risk margin calculation in general underestimates the runoff risks.*

Key words: *Best-estimate reserves, runoff risks, risk margin, one-year uncertainty, claims development result, market-consistent valuation.*

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1 Aim and Scope

The market-consistent actuarial value of insurance liability cash flows involves the following two steps: (i) we predict the outstanding liability cash flows by their best-estimates; (ii) we calculate a risk margin that quantifies the runoff uncertainties beyond these best-estimate predictions (adverse claims development). In insurance practice this risk margin is calculated by extrapolating the one-year uncertainty to the whole runoff of the liability cash flows. In Salzmann-Wüthrich [5] we present a new methodology that is based on the opposite direction. We calculate the total runoff uncertainty and then allocate this total uncertainty to the different accounting years. This latter approach is risk-based whereas the currently used methodology is not. The scope of this paper is to compare these two approaches for several non-life insurance data sets.

2 Risk Margin

There is an agreement that the balance sheet of an insurance company should be measured in a consistent way under the new solvency requirements. In particular, this means that all assets and liabilities should be measured either with market values (where available) or with market-consistent values if no market values are available. In order to obtain market-consistent values for insurance liabilities one calculates the expected present value (best-estimate value) of the future liability cash flows and then adds a risk margin to this present value. The reason for adding this risk margin is that the insurance liability cash flows are neither certain nor hedgeable, i.e. they may deviate from their best-estimate values. This uncertainty asks for adding a risk margin to the best-estimate values which is twofold, see also Section 6 in the IAA paper [2]: (i) it provides a protection beyond the best-estimate values to the insured, namely that the insurance liabilities are also covered under certain adverse claims development scenarios; (ii) it constitutes the price for risk bearing to the institution that performs the runoff of the insurance liabilities; or more economically speaking, a risk averse institution asks for an expected gain in order to be willing to take over the runoff of the insurance risks. Henceforth, we need to calculate a risk margin for the insurance liability runoff which captures the uncertainties in the liability cash flows.

Currently, neither the terminology for this risk margin nor the methodology for the calculation of the risk margin are fixed. The risk margin (RM) is sometimes also called margin over current estimates (MoCE), market-value margin (MVM), solvency capital requirement (SCR) margin, cost-of-capital (CoC) margin. The last two terminologies already denote the method with which the margin is calculated. Well-known concepts for the calculation of the risk margin are probability distortions, utility functions, risk measures, quantile approaches, etc. The method that has a lot of support in the community is the so-called CoC approach.

We describe the CoC approach for the runoff of an insurance liability cash flow. Current accounting rules prescribe that the insurance company needs to hold best-estimate reserves/provisions for the outstanding liability cash flow at any point in time. Hence, whenever new information arrives or claims payments are made, the reserves need to be adjusted according to this latest available information. If we think in business years this update of reserves is typically done at the end of each accounting year k. For the calculation of the risk margin with the CoC approach one needs to determine the risk measure ρ_k for each accounting year k protecting against possible shortfalls in the update of the reserves in this accounting year. That is, in order to run the insurance (runoff) business successfully in accounting year k the insurance company needs provide protection against possible adverse developments up to this risk measure ρ_k. In the CoC approach the company does not need to hold the risk measures ρ_k itself, but the risk margin consists of the prices for all these future risk measures ρ_k. Typically, one applies a fixed rate (CoC rate) to the risk measures ρ_k which is viewed as the price for bearing the runoff risks. Most frequently one uses 6% above the risk-free rate as CoC rate.

The main challenge in this CoC approach is the calculation of the risk measures ρ_k for all future accounting years k. If one tries to calculate these risk measures in a fully consistent way then one needs to consider multiperiod risk measures (for multiperiod cash flows). As has been highlighted for instance in Salzmann-Wüthrich [5] it turns out that these multiperiod risk measures are non-trivial mathematical objects. In most cases they can neither be handled analytically nor numerically (nested simulations) therefore various proxies are used in practice.

A very popular proxy in insurance practice is the following: calculate the risk measure for the next accounting year, say ρ_1, and then scale ρ_1 according to some volume measure. Typically the expected runoff of the reserves is used for scaling. We call this approach *proportional risk measure approach*. The proportional risk measure approach is risk-based for the next accounting year, but it is **not** risk-based for later accounting years. In Salzmann-Wüthrich [5] we propose how to improve the proportional risk measure approach. We obtain risk-based risk measures ρ_k for **all** accounting years in a particular chain ladder claims reserving model. We call this improved approach *risk-based risk measure approach*. Note that the risk-based risk measure approach is analytically tractable which makes its calculation simple and applicable in practice.

The aim of this case study is to compare these two approaches. We see that in most cases the proportional risk measure approach underestimates the runoff risks. This means that one should switch to the risk-based risk measure approach whenever possible.

Organization of this paper. In the next section we describe the methodology used. In Section 4 we then study various different data sets and, finally, in Section 5 we give some conclusions and also mention the limitations of our method.

3 Applied Methodology

Our case study is based on several non-life insurance claims payment triangles. We denote cumulative payments by $C_{i,j} > 0$ where $i \in \{0,\dots,I\}$ is the accident year (origin year) and $j \in \{0,\dots,J\}$ is the development year. We assume that $J \leq I$ and that all claims are settled after development year J, that is, $C_{i,J}$ denotes the ultimate (nominal) claim amount of accident year i.

3.1 Model Assumptions

We revisit the Γ-Γ Bayes Chain Ladder Model for claims reserving presented in Salzmann-Wüthrich [5]. This model describes the development of the individual claims development factors $F_{i,j} = C_{i,j}/C_{i,j-1}$ for $j = 1,\dots,J$. The cumulative payments $C_{i,j}$ can then be rewritten in terms of these individual development factors $F_{i,j}$ as follows

$$C_{i,j} = C_{i,0} \prod_{m=1}^{j} F_{i,m}.$$

The first payment $C_{i,0} > 0$ plays the role of the initial value of the process $(C_{i,j})_{j=0,\dots,J}$ and $F_{i,j}$ are the multiplicative changes. Given the model parameters we assume that cumulative payments follow a chain ladder model.

Model 3.1 (Γ-Γ Bayes Chain Ladder Model)

(1) Conditionally, given $\Theta = (\Theta_1,\dots,\Theta_J)$,
– cumulative payments $C_{i,j}$ in different accident years i are independent;
– $C_{i,0}, F_{i,1},\dots,F_{i,J}$ are independent with (for fixed give $\sigma_j > 0$)

$$F_{i,j}\,|_{\Theta} \sim \Gamma\left(\sigma_j^{-2},\ \Theta_j\, \sigma_j^{-2}\right), \qquad \text{for } j=1,\dots,J.$$

(2) $C_{i,0}$ and Θ are independent and $C_{i,0} > 0$, almost surely.
(3) $\Theta_1,\dots,\Theta_J$ are independent with $\Theta_j \sim \Gamma\left(\gamma_j, f_i(\gamma_j - 1)\right)$ with prior parameters $f_j > 0$ and $\gamma_j > 1$.

Basically, assumption (1) in Model 3.1 says that given the model parameters Θ we have a chain ladder model with the first two moments given by

$$\mathbb{E}\,[C_{i,j}\,|\,\Theta, C_{i,0},\ldots,C_{i,j-1}] = C_{i,j-1}\,\Theta_j^{-1}, \tag{3.1}$$

$$\mathrm{Var}\,(C_{i,j}\,|\,\Theta, C_{i,0},\ldots,C_{i,j-1}) = C_{i,j-1}^2\,\sigma_j^2\,\Theta_j^{-2}. \tag{3.2}$$

That is, we have chain ladder factor $F_j = \Theta_j^{-1}$ in (3.1) and in comparison to the distribution-free chain ladder model of Mack [3] we have a different variance assumption in (3.2). This variance assumption is important in our analysis. Contrary to the distribution-free chain ladder model of Mack [3] our model satisfies the simple Bühlmann credibility model assumptions (see Section 3.1.4 in Bühlmann-Gisler [1]). This property implies that the credibility weights do not depend on the observations $C_{i,j}$ which is important for the decoupling of the risk measures.

3.2 Claims Prediction and the Claims Development Result

At time $I + k \in \{I,\ldots,I+J\}$ we have claims information

$$D_{I+k} = \{C_{i,j};\ i + j \le I + k,\ 0 \le i \le I,\ 0 \le j \le J\},$$

this describes the runoff situation after accident year I. We aim to predict the ultimate claim $C_{i,j}$ based on this information D_{I+k}. We denote the time period $(I + k - 1, I + k]$ as accounting year k for $k \in \{1,\ldots,J\}$. At time $I + k$, $k \ge 0$, the ultimate claim $C_{i,J}$ is predicted with the Bayesian predictor

$$\hat{C}_{i,J}^{(k)} = \mathbb{E}\,[C_{i,J}\,|\,D_{I+k}].$$

This Bayesian predictor is optimal in the sense that it minimizes the conditional L^2-distance to $C_{i,J}$ among all D_{I+k}-measurable predictors. The *claims development result* (CDR) of accident year i for the update of information $D_{I+k-1} \to D_{I+k}$ in accounting year k is given by, see Merz-Wüthrich [4],

$$\mathrm{CDR}_i(k) = \hat{C}_{i,J}^{(k-1)} - \hat{C}_{i,J}^{(k)}.$$

The CDR is a central object of interest in solvency considerations. It basically describes the runoff uncertainties of non-life insurance liability cash flows. That is, $\mathrm{CDR}_i(k)$ is the basis of the calculation of the risk measure ρ_k for accounting year k: if $\mathrm{CDR}_i(k) < 0$ we face a claims development loss in the profit & loss statement in accounting year k (and otherwise for $\mathrm{CDR}_i(k) > 0$ a gain). We have the following crucial properties: (1) the sequence $(\hat{C}_{i,J}^{(k)})_{k \ge 0}$ is a martingale; (2) the $\mathrm{CDR}_i(k)$'s, $k \ge 1$, have conditional mean 0; and (3) the $\mathrm{CDR}_i(k)$'s, $k \ge 1$, are uncorrelated. For a proof we refer to Corollary 3.6 in Salzmann-Wüthrich [5]. An important consequence of these properties is that the total prediction uncertainty measured by the prediction variance can easily be split into the different accounting years $k \geq 1$: we have for the prediction variance

$$\operatorname{Var}\left(\sum_{i=I-J+1}^{I} C_{i,J}\,\middle|\,D_I\right)=\sum_{k=1}^{J}\operatorname{Var}\left(\sum_{i=I-J+k}^{I} \mathrm{CDR}_i(k)\,\middle|\,D_I\right). \tag{3.3}$$

This is the crucial step in the analysis of the development of the claims reserving uncertainty. The left-hand side of (3.3) determines the total runoff uncertainties (long-term view) and the right-hand side of (3.3) allocates this total uncertainty to the different accounting years $k = 1,\ldots,J$. Our aim is to study this allocation.

3.3 Claims Development Result in the Γ-Γ Bayes CL Model

In Model 3.1 all the relevant terms can be calculated explicitly. We have for $I + k < i + J$ (for a proof see Proposition 3.5 in Salzmann-Wüthrich [5])

$$\hat{C}^{(k)}_{i,J} = \mathbb{E}[C_{i,J} \mid D_{I+k}] = C_{i,I-i+k} \prod_{j=I-i+k+1}^{J} \hat{f}^{(k)}_j,$$

the chain ladder factor estimators $\hat{f}^{(k)}_j$ at time $k = 0,\ldots,J$ are given by

$$\hat{f}^{(k)}_j = \mathbb{E}[\Theta_j^{-1} \mid D_{I+k}] = \frac{c_{j,k}}{\gamma_{j,k}-1},$$

and the parameters $\gamma_{j,k}$ and $c_{j,k}$ are given by

$$\gamma_{j,k} = \gamma_j + \frac{((I+k-j) \wedge I)+1}{\sigma_j^2} \quad \text{and} \quad c_{j,k} = f_j(\gamma_j-1) + \sigma_j^2 \sum_{i=0}^{(I+k-j)\wedge I} F_{i,j},$$

where we use the notation $x \wedge y = \min\{x,y\}$. Note that the chain ladder factor estimators $\hat{f}^{(k)}_j$ are a credibility weighted average between a prior estimator and the purely observation based sample mean. In Model 3.1 we cannot only calculate the ultimate claims predictors $\hat{C}^{(k)}_{i,J}$ explicitly, but also their prediction uncertainty (3.3) is obtained analytically. For i, $k \geq 1$, $I + k \leq i + J$ and $j \geq k$ we define the constants

$$\alpha_{j,k} = (I + k - j + 1 + \sigma_j^2(\gamma_j-1))^{-1},$$

$$\beta_{i,k} = (\sigma^2_{I-i+k}+1)\,\frac{\gamma_{I-i+k,k-1}-1}{\gamma_{I-i+k,k-1}-2} \prod_{m=I-i+k+1}^{J}\left[\alpha^2_{m,k}\left((\sigma_m^2+1)\,\frac{\gamma_{m,k-1}-1}{\gamma_{m,k-1}-2}-1\right)+1\right],$$

$$\delta_{i,k} = \beta_{i,k} = \left[\alpha_{I-i+k,k} + (1-\alpha_{I-i+k,k})\,(\sigma^2_{I-i+k}+1)^{-1}\frac{\gamma_{I-i+k,k-1}-2}{\gamma_{I-i+k,k-1}-1}\right].$$

The following theorem is crucial (see Theorems 4.2 and 4.9 in Salzmann-Wüthrich [5]):

Theorem 3.2 *In Model 3.1 we have for $I + k \le i + J < m + J$*

$$\mathrm{Var}\,(\mathrm{CDR}_i(k)\,|\,D_I) = \left(\hat{C}^{(0)}_{i,J}\right)^2 \prod_{j=1}^{k-1} \beta_{i,j}\,(\beta_{i,k}-1),$$

$$\mathrm{Cov}\,(\mathrm{CDR}_i(k),\ \mathrm{CDR}_m(k)\,|\,D_I) = \hat{C}^{(0)}_{i,J}\,\hat{C}^{(0)}_{m,J} \prod_{j=1}^{k-1} \delta_{i,j}\,(\delta_{i,k}-1).$$

Theorem 3.2 gives us all the necessary tools for studying the runoff uncertainties of non-life insurance claims reserves according to (3.3). This is the subject of the next subsection.

3.4 Calculation of the Runoff Uncertainties

The total claims reserves at time $k=0,\dots,J-1$ are given by

$$R^{(k)} = \sum_{i=I-J+k+1}^{I} \hat{C}^{(k)}_{i,J} - C_{i,I-i+k}.$$

Note that $C_{i,I-i+k}$ is only observable at time $I+k$. The expected claims reserves at time k viewed from time 0 are then given by

$$r^{(k)} = \mathbb{E}[R^{(k)}\,|\,\mathrm{D}_I] = \sum_{i=I-J+k+1}^{I} \hat{C}^{(0)}_{i,J} - \hat{C}^{(0)}_{i,I-i+k},$$

with for $i + j > I$

$$\hat{C}^{(0)}_{i,j} = \mathbb{E}[C_{i,j}\,|\,\mathrm{D}_I] = C_{i,I-i} \prod_{l=I-i+1}^{j} \hat{f}^{(0)}_l.$$

Using these expected claims reserves $r^{(k)}$ we can define an expected runoff pattern $\mathbf{w} = (w_1,\dots,w_J)$. For $k=1,\dots,J$

$$w_k = r^{(k-1)}/r^{(0)}. \tag{3.4}$$

This is the expected percentage of the total claims reserves $r^{(0)}$ needed at the beginning of accounting year k.

Proportional (P) Risk Measure Approach.

Frequently one calculates the risk measure for the first accounting year $k = 1$ and then scales the risk measures for the remaining accounting years $k > 1$ proportional to the expected runoff $\mathbf{w}$ of the claims reserves. We choose as risk measure a standard deviation loading with constant loading factor $\varphi > 0$.
For $k = 1$ we define the risk measure $\rho_1^{(\mathrm{P})}$ to be

$$\rho_1^{(\mathrm{P})} \stackrel{\text{def.}}{=} \varphi \operatorname{Var}\left(\sum_{i=I-J+1}^{I} \mathrm{CDR}_i(1) \middle| D_I \right)^{1/2},$$

and for $k > 1$, see (3.4),

$$\rho_1^{(\mathrm{P})} \stackrel{\text{def.}}{=} w_k \, \rho_1^{(\mathrm{P})}.$$

Note that this risk measure is not risk-based for accounting years $k > 1$.

Risk-Based (RB) Risk Measure Approach of Salzmann-Wüthrich [5].
Salzmann-Wüthrich [5] propose the following choices: for $k \geq 1$ choose

$$\rho_k^{(RB)} \stackrel{\text{def.}}{=} \varphi \operatorname{Var}\left(\sum_{i=I-J+k}^{I} \mathrm{CDR}_i(k) \middle| D_I \right)^{1/2}.$$

Note that this provides risk-based risk measures for all accounting years $k \geq 1$ (basically we allocate the total uncertainty (3.3) to the corresponding accounting years). Our aim is to compare the risk measures $\rho_1^{(\mathrm{P})}$ and $\rho_k^{(RB)}$ for different data sets. Note that all these risk measures can be calculated analytically with Theorem 3.2, and moreover for the first accounting year $k = 1$ we have $\rho_1^{(\mathrm{P})} = \rho_k^{(RB)}$.

4 Case Study: Non-Life Runoffs

In our case study we consider several different data sets from European and US non-life insurers and re-insurers. In the analysis we have restricted ourselves to cumulative payments data only. We have not smoothed the parameter estimates and we have not estimated tail factors (for developments beyond the observed triangles D_I).

In a first step we need to explain how we choose the prior parameters γ_j and f_j as well as the variance parameters σ_j^2 in Model 3.1.

For $\gamma_j \to 1$ we obtain non-informative priors. That is, for $\gamma_j = 1$ we obtain the chain ladder factor estimators $\hat{f}_j^{(0)}$ at time 0 given by

$$\hat{f}_j^{(0)} = \mathbb{E}[\Theta_j^{-1} \mid D_I] = \frac{1}{I-j+1} \sum_{i=0}^{I-j} F_{i,j},$$

i.e. the non-informative prior case provides the sample mean of the individual development factors $F_{i,j}$ as chain ladder factor estimators. Moreover, for $\gamma_j = 1$ is the choice of f_j irrelevant because it does not appear in the posterior distributions. There remains the estimate of the variance parameters σ_j^2. We choose them in an empirical Bayesian way and estimate them from the data as follows: for $j \leq \min\{I-1, J\}$

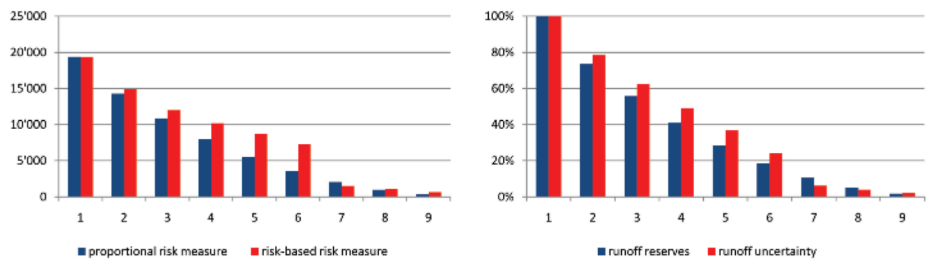
$$\hat{\sigma}_j^2 = (\hat{f}_j^{(0)})^{-2} \frac{1}{I-j} \sum_{i=0}^{I-j} (F_{i,j} - \hat{f}_j^{(0)})^2.$$

In the case $I = J$ we cannot estimate σ_J^2 because we do not have sufficient data. Therefore we set (see (3.13) in Wüthrich-Merz [6])

$$\hat{\sigma}_J^2 = \min\{\hat{\sigma}_{J-1}^4 / \hat{\sigma}_{J-2}^2, \hat{\sigma}_{J-1}^2, \hat{\sigma}_{J-2}^2\}.$$

These choices provide the figures below. We describe Figure 1 in detail and then provide line of business related comments for the other figures.

Figure 1: 10×10 triangle example from Salzmann-Wüthrich [5].



Left Panel. The figure shows the runoff of the risk measures $\rho_k^{(P)}$ and $\rho_k^{(RB)}$ (we have chosen $\varphi = 1$) as a function of the accounting years $k = 1,\ldots,9$. The blue columns (left) show the proportional risk measure approach $\rho_k^{(P)}$ and the red columns (right) show the risk-based risk measure approach $\rho_k^{(RB)}$. Both start at the same level $\rho_1^{(P)} = \rho_1^{(RB)}$, but then we see a faster decay in $\rho_k^{(P)}$ which means that the proportional risk measure approach underestimates the runoff risks in this case. For example, $\rho_6^{(P)}$ has only half of the size of $\rho_6^{(RB)}$!

Right Panel. In this figure we compare the runoff of the expected reserves $r^{(0)}$, modeled by the expected reserve runoff pattern $\mathbf{w} = (w_1,\ldots,w_J)$, see (3.4), with the runoff of the reserving uncertainties defined by $\mathbf{v} = (v_1,\ldots,v_J)$ with

$$v_k = \left[\frac{\sum_{m=k}^{J} \mathrm{Var}\left(\sum_{i=I-J+m}^{I} \mathrm{CDR}_i(m) \mid \mathrm{D}_I\right)}{\mathrm{Var}\left(\sum_{i=I-J+1}^{I} \mathrm{C}_{i,J} \mid \mathrm{D}_I\right)} \right]^{1/2},$$

see also (3.3). Not surprisingly (in view of the left panel) we see that the runoff of the reserves $\mathbf{w}$ (blue columns, left) is faster than the runoff of the uncertainties $\mathbf{v}$ (red columns, right), at least up to accounting year $k = 6$. The reason for the jump in the uncertainty from accounting year 6 to accounting year 7 lies in the fact that the variance parameter estimate $\hat{\sigma}_6^2$ is rather large (we have not smoothed the estimates).

Concluding: Figure 1 shows that the proportional risk measure approach $\rho_k^{(P)}$ underestimates the runoff risks and should be replaced by the risk-based risk measure approach $\rho_k^{(RB)}$ in this example.

Example 4.1 (Short-tailed line of business)
We start with the analysis of short-tailed lines of business. These are, for example, private and commercial property insurance, health insurance, motor hull insurance, travel insurance, etc. The runoff of these lines of business is rather fast and therefore the claims settlement is completed after a few development years.

Figure 2: 7×7 triangle of commercial property insurance.

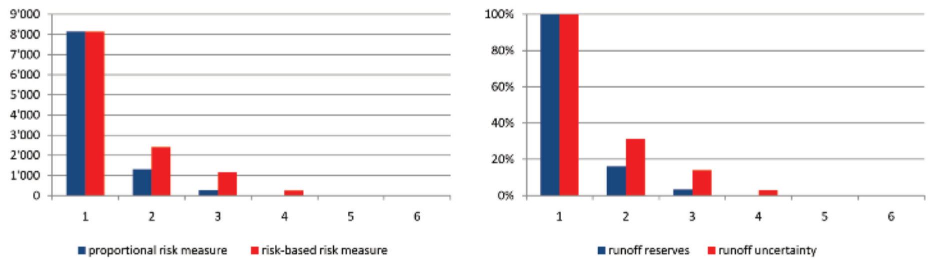
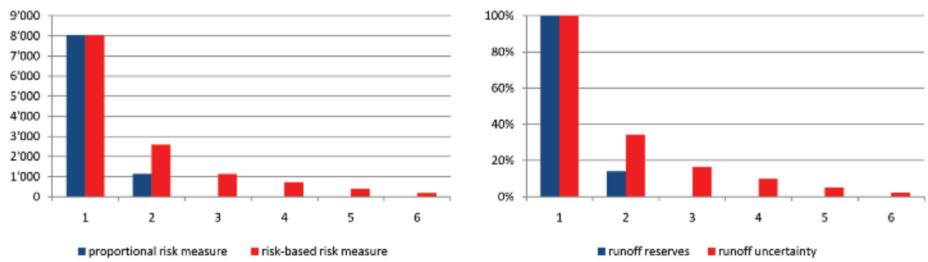


Figure 3: 7×7 triangle of European health insurance.



Figures 2-3 show typical examples. We see that after 3 development years the claims reserves are negligible. Therefore the proportional risk measure approach provides a risk measure $\rho_k^{(P)} = 0$ for $k \geq 3$ (left panel). Note however, that this underestimates risk, because clearly $\rho_k^{(RB)} > 0$ for $k \geq 3$ (left panel). The reason for $r^{(k)} = 0$ for $k \geq 3$ is that the refunding by the insured (deductibles and regresses) has about the same size as the claims payments to the insured. This means that though the claims reserves are equal to zero there is still some claims development risk. However, the right panels in Figures 2-3 also show that this risk is not too large after 3 development years.

Example 4.2 (Liability Insurance)

In this example we give private and commercial liability portfolios. Of course, these portfolios can be very diverse concerning the underlying products. This can also be seen in Figures 4-6. However, all of them have a long-tailed claims development.

Figure 4: 17×17 triangle of private liability insurance.

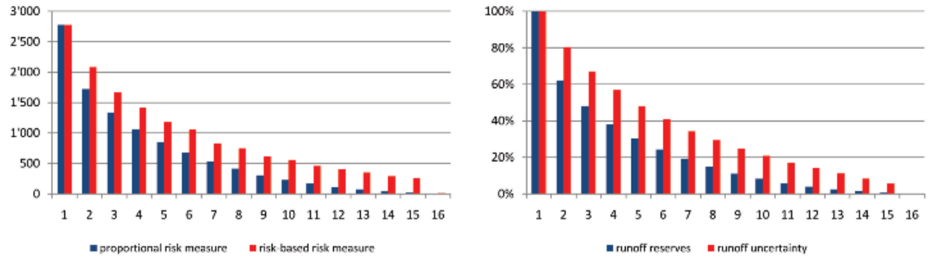


Figure 5: 22×22 triangle of commercial liability insurance.

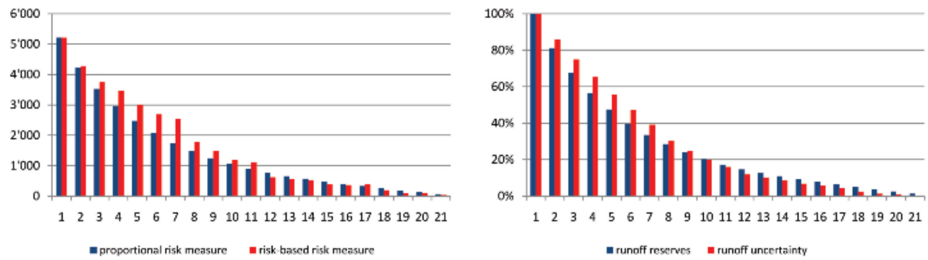
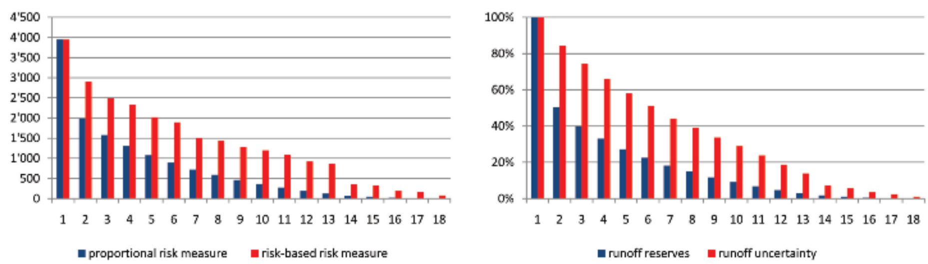


Figure 6: 19×19 triangle of commercial liability insurance.



We see in Figures 4 and 6 a lot of claims payments in accounting year 1. This substantially reduces the claims reserves, however the underlying risk is not much reduced (right panels). This comes from the fact that we can settle a lot of small claims in the beginning but large (risky) claims stay in the portfolio and can only be settled much later and therefore the uncertainty stays in the portfolio for much longer. In Figure 5 the reserves and the uncertainty balance after 10 accounting years.

Example 4.3 (Motor Third Party Liability Insurance)
In Figures 7-10 we give runoff examples of European and US motor third party liability insurance portfolios. First of all we remark that in all cases the proportional risk measure approach underestimates the runoff risks.

Figure 7: 14×14 triangle of motor third party liability insurance.

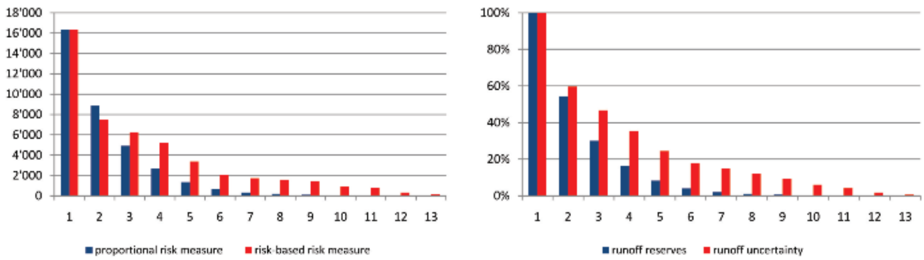
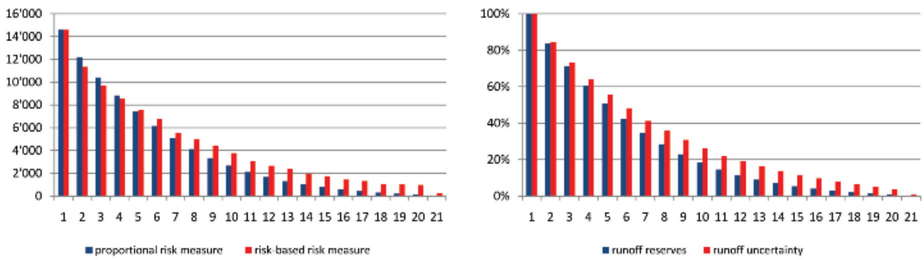


Figure 8: 22×22 triangle of motor third party liability insurance.



Figures 7 and 8 show two completely different runoff patterns (for the same line of business). The reason for this difference is that these are two runoffs in two different countries. This shows the importance of the local specifics such as jurisdiction, etc. In Figure 7 claims are settled after a view years, whereas in Figure 8 the claims reserves and the underlying uncertainties are driven by bodily injury claims whose development can take up to 20 years. This example shows that there is no global development pattern but these patterns need to be estimated locally.

Figures 9-10 give an other interresting comparison.

Figure 9: 14×14 triangle of motor third party liability insurance.

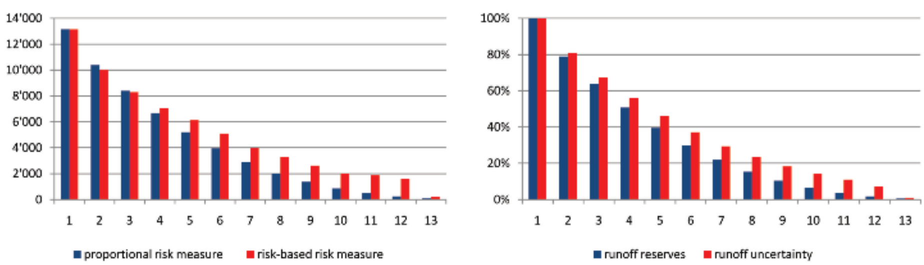
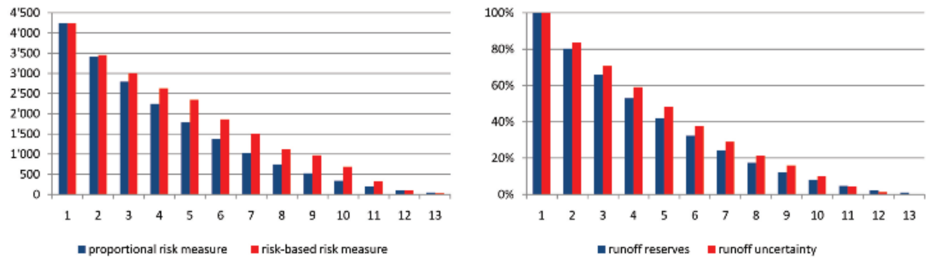


Figure 10: 14×14 triangle of motor third party liability insurance.



Figures 9-10 show two similar motor third party liability portfolios in the same country. This can be seen because their runoff pattern (blue columns, left) are very similar. The difference is that the portfolio in Figure 9 has about 5 times the size of the portfolio in Figure 10. This results in $\rho_1^{(P)}$ being about 3 times as large in Figure 9 compared to Figure 10 (we have more diversification in the larger portfolio). The right panels show that this diversification effect becomes smaller for later accident years (this is because the same class of claims has a long settlement delay and drives the risk for late development periods). In fact, for late development periods the underlying risk in Figure 9 has almost 5 times the size as the one in Figure 10 (i.e. no diversification effect is left).

Example 4.4 (Re-Insurance)

Figures 11-12 show two different re-insurance portfolios.

Figure 11: 17×17 triangle of re-insurance.

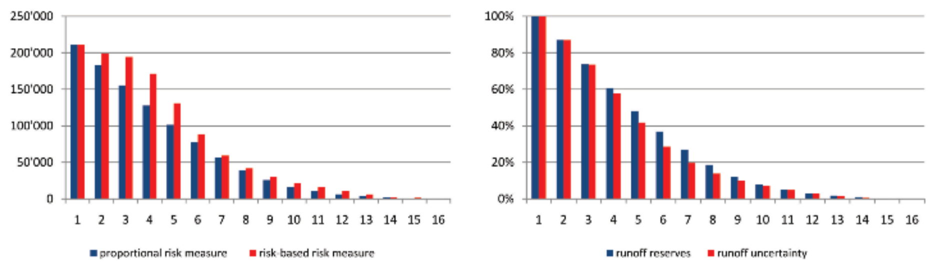
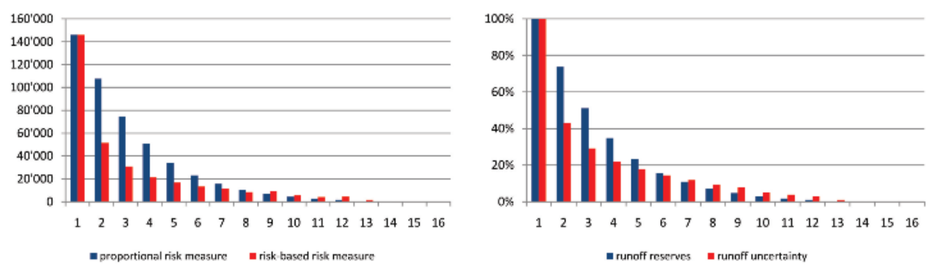


Figure 12: 17×17 triangle of re-insurance.



In our opinion the re-insurance examples in Figures 11-12 do not look reasonable, especially Figure 12 is not convincing. The reason for these rather puzzling pictures is that the Γ-Γ chain ladder model does not provide an appropriate claims reserving method for these two portfolios. In both examples we have very little payments in the first development years which results in an insufficient basis $C_{i,I-i}$ (i close to the youngest observed accident year I) for the chain ladder development. Especially, in Figure 12 we see that the uncertainty lies in the next development period (insufficient basis $C_{I,0}$) which distorts the whole picture.

5 Conclusions and Limitations

In Salzmann-Wüthrich [5] we have presented a new approach for the calculation of the risk measures ρ_k for all future accounting years $k \geq 1$. This approach is based on the idea that we allocate the total uncertainty to the corresponding accounting years. The case study has shown that this approach has a better performance than the one currently used in insurance practice.

The limitations of our approach are the following: Our analysis is based on claims payment information only. In practice, one should consider all relevant information for claims reserving. Moreover, our method is based on the chain ladder method. In the future similar studies should be done for other claims reserving methods and for other risk measures than the standard deviation risk measure. Finally, we would like to encourage research in the areas claims discounting, claims inflation and dependence modeling between different lines of business.

6 References

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