# Extra Question Answers [1]

- Dear all, due to large volume of questions, there might be careless mistakes in the answer key.
- Feel free to contact me for clarification

P.S. Goodjob on putting in extra effort!
I am super proud of you guys and its my pleasure to help you guys with the extra practices!
Keep it up!

## Chapter 15.1 Ans

1. (a) Linear
   (b) Not linear
   (c) Linear
   (d) Linear
   (e) Not linear
   (f) Linear

2. Yes

3. $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = 2^1$
   $a_{21}x_1 + a_{22}x_3 + a_{23}x_3 + a_{14}x_4 = 2^2$
   $a_{31}x_1 + a_{31}x_3 + a_{33}x_3 + a_{34}x_4 = 2^3$

   $2x_1 + 4x_2 + 6x_3 + 8x_4 = 2$
   $5x_1 + 7x_2 + 9x_3 + 11x_4 = 4$
   $4x_1 + 6x_2 + 8x_3 + 10x_4 = 8$

4. $\overset{0}{a_{11}}x_1 + \overset{1}{a_{12}}x_2 + \overset{1}{a_{13}}x_3 + \overset{1}{a_{14}}x_4 = x_2 + x_3 + x_4 = b_1$

$\overset{1}{a_{21}}x_1 + \overset{0}{a_{22}}x_2 + \overset{1}{a_{23}}x_3 + \overset{1}{a_{24}}x_4 = x_1 + x_3 + x_4 = b_2$

$\overset{1}{a_{31}}x_1 + \overset{1}{a_{32}}x_2 + \overset{0}{a_{33}}x_3 + \overset{1}{a_{34}}x_4 = x_1 + x_2 + x_4 = b_3$

$\overset{1}{a_{41}}x_1 + \overset{1}{a_{42}}x_2 + \overset{1}{a_{43}}x_3 + \overset{0}{a_{44}}x_4 = x_1 + x_2 + x_3 = b_4$

$$x_1 + x_2 + x_3 + x_4 = \frac{1}{3}(b_1 + b_2 + b_3 + b_4)$$

5. a. The total quantity of each different commodities own by an individual j

(b) $a_{11} + a_{12} + \dots + a_{1n}$

The total quantity of commodity 1 owned by n no. of individuals

$a_{i1} + a_{i2} + \dots + a_{in}$

The total quantity of commodity i owned by n no. of individuals

6. $C = 0.712y + 95.05$

$y = C + x - S$

$S = 0.158(C + x) - 34.30$

$x = 93.53$

$$\begin{array}{cccccccc} x & y & & C & S & & \\ & -0.712y & + & C & & = & 95.05 \\ -x & + y & & -C & +S & = & 0 \\ -0.158x & & & -0.158C & +S & = & -34.30 \\ x & & & & & = & 93.53 \end{array}$$

$$\begin{pmatrix} 0 & -0.712 & 1 & 0 \\ -1 & 1 & -1 & 1 \\ -0.158 & 0 & -0.158 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ C \\ S \end{pmatrix} = \begin{pmatrix} 95.05 \\ 0 \\ -34.30 \\ 93.53 \end{pmatrix}$$

$x = 93.53$

$S = 0.158(C + 93.53) - 34.30$

$S = 0.158C + 14.77774 - 34.30$

$S = 0.158C - 19.52226$

$y = C + 93.53 - (0.158C - 19.52226)$

$y = 0.842C + 113.05226$ —— (1)

$C = 0.712y + 95.05$ —— (2)

sub (1) into (2)

$y = 0.842(0.712y + 95.05) + 113.05226$

$= 0.599504y + 80.0321 + 113.05226$

$= 0.599504y + 193.08436$

$y = 482.1130798 \approx 482.11$ (5sf)

$C = 0.712(482.1130798) + 95.05$

$= 438.3145128 \approx 438.31$ (5sf)

$S = 49.731$ (5sf)

$x = 93.53$

# Chapter 15.2

1.

$$\overset{1}{a_{11}} \quad \overset{0}{a_{12}} \quad \overset{0}{a_{13}}$$

$$\overset{0}{a_{21}} \quad \overset{1}{a_{22}} \quad \overset{0}{a_{23}}$$

$$\overset{0}{a_{31}} \quad \overset{0}{a_{32}} \quad \overset{1}{a_{33}}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. $A+B$

$$\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 7 & 5 \end{pmatrix}$$

$3A$

$$3\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 6 & 9 \end{pmatrix}$$

3.

$$\begin{pmatrix} (1-u)^2 & v^2 & 3 \\ v & 2u & 5 \\ 6 & 4 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 4 & u \\ v & -3v & u-v \\ 6 & v+6 & -1 \end{pmatrix}$$

$\therefore u = 3$

$2u = -3v$

$2(3) = -3v$

$\therefore v = -2$

4. $A+B$

$$\begin{pmatrix} 0 & 1 & -1 \\ 2 & 3 & 7 \end{pmatrix} + \begin{pmatrix} 1 & -1 & 5 \\ 0 & 1 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 4 \\ 2 & 4 & 16 \end{pmatrix}$$

$A-B$

$$\begin{pmatrix} 0 & 1 & -1 \\ 2 & 3 & 7 \end{pmatrix} - \begin{pmatrix} 1 & -1 & 5 \\ 0 & 1 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 & -6 \\ 2 & 2 & -2 \end{pmatrix}$$

$5A-3B$

$$5\begin{pmatrix} 0 & 1 & -1 \\ 2 & 3 & 7 \end{pmatrix} - 3\begin{pmatrix} 1 & -1 & 5 \\ 0 & 1 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 5 & -5 \\ 10 & 15 & 35 \end{pmatrix} + \begin{pmatrix} -3 & 3 & -15 \\ 0 & -3 & -27 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 8 & -20 \\ 10 & 12 & 8 \end{pmatrix}$$

# Chapter 15.3

1. (a) AB

$$\begin{pmatrix} 0 & -2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} -2 & -10 \\ -2 & 17 \end{pmatrix}$$

BA

$$\begin{pmatrix} -1 & 4 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 12 & 6 \\ 15 & 3 \end{pmatrix}$$

(b) AB

$$\begin{pmatrix} 8 & 3 & -2 \\ 1 & 0 & 4 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ 4 & 3 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} 26 & 3 \\ 6 & -22 \end{pmatrix}$$

BA

$$\begin{pmatrix} 2 & -2 \\ 4 & 3 \\ 1 & -5 \end{pmatrix}\begin{pmatrix} 8 & 3 & -2 \\ 1 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 14 & 6 & -12 \\ 35 & 12 & 4 \\ 3 & 3 & -22 \end{pmatrix}$$

(c) AB

$$\begin{pmatrix} -1 & 0 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 1 \\ 0 & 2 \end{pmatrix} = \text{no solution}$$

BA

$$\begin{pmatrix} 3 & 1 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 3 & 4 \\ 4 & 8 \end{pmatrix}$$

(d) AB

$$\begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}\begin{pmatrix} 0 & -2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & -6 \\ 0 & -8 & 12 \end{pmatrix}$$

BA

$$\begin{pmatrix} 0 & -2 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix} = 16$$

**2.** $3A+2B-2C+D$

$$= 3\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} + 2\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix} - 2\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 12 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} -4 & 8 \\ 2 & -4 \end{pmatrix} + \begin{pmatrix} -4 & -6 \\ -12 & -18 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 20 \\ 5 & 2 \end{pmatrix} + \begin{pmatrix} -4 & -6 \\ -12 & -18 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 14 \\ -7 & -16 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 15 \\ -6 & -13 \end{pmatrix}$$

**3.** $A+B$

$$\begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix} + \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{pmatrix}$$

$A-B$

$$\begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix} - \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 3 & -5 \\ 1 & -2 & -3 \\ -1 & -1 & -2 \end{pmatrix}$$

$AB$

$$\begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 3 & 3 \\ 19 & -5 & 16 \\ 1 & -3 & 0 \end{pmatrix}$$

$BA$

$$\begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 9 \\ 19 & 3 & -3 \\ 8 & 1 & -3 \end{pmatrix}$$

$A(BC)$

$$\begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}\left[\begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix}\right]$$

$$= \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 14 & -4 & 10 \\ 21 & 0 & 27 \\ 11 & -4 & 13 \end{pmatrix}$$

$$= \begin{pmatrix} 23 & 8 & 25 \\ 92 & -28 & 76 \\ 4 & -8 & -4 \end{pmatrix}$$

$(AB)C$

$$\begin{pmatrix} 5 & 3 & 3 \\ 19 & -5 & 16 \\ 1 & -3 & 0 \end{pmatrix}\begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 23 & 8 & 25 \\ 92 & -28 & 76 \\ 4 & -8 & -4 \end{pmatrix}$$

4. (a) $\begin{pmatrix} 1 & 1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

5 (a) $(A-2I)C = I$

$\left[\begin{pmatrix} 2 & 2 \\ 1 & 5 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\right]C = I$

$\begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$0 + 2a_{21} = 1 \qquad a_{21} = \frac{1}{2}$

$0 + 2a_{22} = 0 \qquad a_{22} = 0$

$a_{11} + 3a_{21} = 0 \qquad a_{11} = -\frac{3}{2}$

$a_{12} + 3a_{22} = 1 \qquad a_{12} = 1$

$C = \begin{pmatrix} -\frac{3}{2} & 1 \\ \frac{1}{2} & 0 \end{pmatrix}$

(b) $\left[\begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\right]\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

D does not exist

6. $\begin{array}{cc} A & B \\ m \times n & n \times m \end{array}$ //

7. $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} a_{11}+2a_{12} & 2a_{11}+3a_{12} \\ a_{21}+2a_{22} & 2a_{21}+3a_{22} \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} a_{11}+2a_{21} & a_{12}+2a_{22} \\ 2a_{11}+3a_{21} & 2a_{12}+3a_{22} \end{pmatrix}$

$a_{11}+2a_{12} = a_{11}+2a_{21}$

$a_{12} = a_{21}$

$2a_{11}+3a_{12} = 2a_{12}+2a_{22}$

$2a_{11}+a_{12} = 2a_{22}$ $\qquad 2a_{11} = 2a_{22} - x$

$B = \begin{pmatrix} \omega - x & x \\ x & \omega \end{pmatrix}$

# Chapter 15.4

1. $A(B+C) = AB + AC$

$$\begin{pmatrix}1&2\\3&4\end{pmatrix}\left[\begin{pmatrix}2&-1&1&0\\3&-1&2&1\end{pmatrix}+\begin{pmatrix}-1&1&1&2\\-2&2&0&-1\end{pmatrix}\right] = \begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}2&-1&1&0\\3&-1&2&1\end{pmatrix} + \begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}-1&1&1&2\\-2&2&0&-1\end{pmatrix}$$

$$\begin{pmatrix}1&2\\3&4\end{pmatrix}\left[\begin{pmatrix}1&0&2&2\\1&1&2&0\end{pmatrix}\right] = \begin{pmatrix}8&-3&5&2\\18&-7&11&4\end{pmatrix} + \begin{pmatrix}-5&5&1&0\\-11&11&3&2\end{pmatrix}$$

$$\begin{pmatrix}3&2&6&2\\7&4&14&6\end{pmatrix} = \begin{pmatrix}3&2&6&2\\7&4&14&6\end{pmatrix}$$ (verified)

2.
$$(x\ y\ z)\begin{pmatrix}a&d&e\\d&b&f\\e&f&c\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$= (ax+dy+ez \quad dx+by+fz \quad ex+fy+cz)\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$= x(ax+dy+ez) + y(dx+by+fz) + z(ex+fy+cz)$$

3. $(AB)C$
$$= \left[\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\begin{pmatrix}b_{11}&b_{12}\\b_{21}&b_{22}\end{pmatrix}\right]\begin{pmatrix}c_{11}&c_{12}\\c_{21}&c_{22}\end{pmatrix}$$

$$= \begin{pmatrix}a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22}\\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22}\end{pmatrix}\begin{pmatrix}c_{11}&c_{12}\\c_{21}&c_{22}\end{pmatrix}$$

$$= \begin{pmatrix}c_{11}(a_{11}b_{11}+a_{12}b_{21})+c_{21}(a_{11}b_{12}+a_{12}b_{22}) & c_{12}(a_{11}b_{11}+a_{12}b_{21})+c_{22}(a_{11}b_{12}+a_{12}b_{22})\\ c_{11}(a_{21}b_{11}+a_{22}b_{21})+c_{21}(a_{21}b_{12}+a_{22}b_{22}) & c_{12}(a_{21}b_{11}+a_{22}b_{21})+c_{22}(a_{21}b_{12}+a_{22}b_{22})\end{pmatrix}$$

$$= \begin{pmatrix}c_{11}a_{11}b_{11}+c_{11}a_{12}b_{21}+c_{21}a_{11}b_{12}+c_{21}a_{12}b_{22} & c_{12}a_{11}b_{11}+c_{12}a_{12}b_{21}+c_{22}a_{11}b_{12}+c_{22}a_{12}b_{22}\\ c_{11}a_{21}b_{11}+c_{11}a_{22}b_{21}+c_{21}a_{21}b_{12}+c_{21}a_{22}b_{22} & c_{12}a_{21}b_{11}+c_{12}a_{22}b_{21}+c_{22}a_{21}b_{12}+c_{22}a_{22}b_{22}\end{pmatrix}$$

$$A(BC) = \begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\left[\begin{pmatrix}b_{11}&b_{12}\\b_{21}&b_{22}\end{pmatrix}\begin{pmatrix}c_{11}&c_{12}\\c_{21}&c_{22}\end{pmatrix}\right]$$

$$= \begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\begin{pmatrix}b_{11}c_{11}+b_{12}c_{21} & b_{11}c_{12}+b_{12}c_{22}\\ b_{21}c_{11}+b_{22}c_{21} & b_{21}c_{12}+b_{22}c_{22}\end{pmatrix}$$

$$= \begin{pmatrix}a_{11}(b_{11}c_{11}+b_{12}c_{21})+a_{12}(b_{21}c_{11}+b_{22}c_{21}) & a_{11}(b_{11}c_{12}+b_{12}c_{22})+a_{12}(b_{21}c_{12}+b_{22}c_{22})\\ a_{21}(b_{11}c_{11}+b_{12}c_{21})+a_{22}(b_{11}c_{11}+b_{22}c_{21}) & a_{21}(b_{11}c_{12}+b_{12}c_{22})+a_{22}(b_{21}c_{12}+b_{22}c_{22})\end{pmatrix}$$

$= A(BC)$ (verified)

4(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5 & 3 & 1 \\ 2 & 0 & 9 \\ 1 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 3 & 1 \\ 2 & 0 & 9 \\ 1 & 3 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & -3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -3 \end{pmatrix}$

5 (i) $(A+B)(A-B) = A^2 - AB + BA - B^2 \neq A^2 - B^2$ unless $AB = BA$

(ii) $(A-B)(A-B) = A^2 - AB - BA + B^2 \neq A^2 - 2AB + B^2$ unless $AB = BA$

6.(a) $\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$

$= A^2$

$= A$ (shown)

(b) $AB = A = AA$

$B = A$

$BA = B$ when $B = A$

$BB = B$

$B^2 = B$ (shown)

(c) $A^n = A$

when $n=1$

$A = A$

Induction

$A^k = A$

$A^{k+1} = A^k A = AA = A$ (shown)

8. (a) $A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$= \begin{pmatrix} a^2 + bc & ab+bd \\ ac+cd & +bc+d^2 \end{pmatrix}$

$(a+d)\begin{pmatrix} a & b \\ c & d \end{pmatrix} - (ad-bc)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} a^2+ad & ab+bd \\ ac+cd & +ad+d^2 \end{pmatrix} - \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$

$= \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$

$= A^2$ (Proven)

(b) $\begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix} = 0$

$a^2+bc=0$

let $a=1$ $\quad d=-1$

$1+bc=0$ $\quad bc=-1$ $\quad b$ or $c \neq 0$

$b=1$ $\quad c=-1$

$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$ //

# Chapter 15.5

1. $A = \begin{pmatrix} 3 & 5 & 8 & 3 \\ -1 & 2 & 6 & 2 \end{pmatrix}$

$A' = \begin{pmatrix} 3 & -1 \\ 5 & 2 \\ 8 & 6 \\ 3 & 2 \end{pmatrix}$

$B = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix}$

$B' = \begin{pmatrix} 0 & 1 & -1 & 2 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 5 & 0 & -1 \end{pmatrix}$

$C' = \begin{pmatrix} 1 \\ 5 \\ 0 \\ -1 \end{pmatrix}$

2. $A' = \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}$

$B' = \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix}$

$(A+B)' = \left( \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix} \right)'$

$= \begin{pmatrix} 3 & 1 \\ 4 & 7 \end{pmatrix}'$

$= \begin{pmatrix} 3 & 4 \\ 1 & 7 \end{pmatrix}$

$(\alpha A)' = \left( -2 \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix} \right)'$

$= \begin{pmatrix} -6 & 2 \\ -4 & -10 \end{pmatrix}$

$AB = \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix}$

$= \begin{pmatrix} 4 & 10 \\ 10 & 8 \end{pmatrix}$

$(AB)' = \begin{pmatrix} 4 & 10 \\ 10 & 8 \end{pmatrix}$

$B'A' = \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}$

$= \begin{pmatrix} 4 & 10 \\ 10 & 8 \end{pmatrix}$

$A'B' = \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix}$

$= \begin{pmatrix} -2 & 4 \\ 10 & 14 \end{pmatrix}$

$(A')' = A$

$(A+B)' = A' + B'$

$(\alpha A)' = \alpha A'$

$(AB)' = B'A'$

3. $A' = \begin{pmatrix} 3 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 0 \end{pmatrix}$

$= A$ (shown)

$B' = \begin{pmatrix} 0 & 4 & 8 \\ 0 & 0 & 13 \\ 8 & 13 & 0 \end{pmatrix}$

$= B$ (shown)

4. $\begin{pmatrix} a & a^2-1 & -3 \\ a+1 & 2 & a^2+4 \\ -3 & 4a & -1 \end{pmatrix} = \begin{pmatrix} a & a+1 & -3 \\ a^2-1 & 2 & 4a \\ -3 & a^2+4 & -1 \end{pmatrix}$

$a^2-1=a+1$ $\quad a^2-a-2=0$ $\quad (a-2)(a+1)=0$ $\quad a=2$ or $a=-1$ (rej)

$a^2+4=4a$ $\quad a^2-4a+4=0$ $\quad (a-2)^2=0$ $\quad a=2$

$\therefore a=2$

5. No

6. $(A_1A_2A_3)' = (A_3)'(A_1A_2)'$ $\qquad (A')' = A$

$= A_3' A_2' A_1'$ $\qquad (A+B)' = A' + B'$

$(\alpha A)' = \alpha A'$

$(AB)' = B'A'$

7. (a) $P' = \begin{pmatrix} \lambda & \lambda & 0 \\ 0 & 0 & 1 \\ \lambda & -\lambda & 0 \end{pmatrix}$

$$P'P = \begin{pmatrix} \lambda & \lambda & 0 \\ 0 & 0 & 1 \\ \lambda & -\lambda & 0 \end{pmatrix}\begin{pmatrix} \lambda & 0 & \lambda \\ \lambda & 0 & -\lambda \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2\lambda^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2\lambda^2 \end{pmatrix}$$ when $\lambda = \pm\frac{1}{\sqrt{2}}$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ (shown)

(b) $$\begin{pmatrix} p & -q \\ q & p \end{pmatrix}\begin{pmatrix} p & q \\ -q & p \end{pmatrix} = \begin{pmatrix} p^2+q^2 & pq-pq \\ qp-pq & q^2+p^2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

8 (a) $$T.S = \begin{pmatrix} p & q & 0 \\ \frac{1}{2}p & \frac{1}{2} & \frac{1}{2}q \\ 0 & p & q \end{pmatrix}\begin{pmatrix} p^2 & 2pq & q^2 \\ p^2 & 2pq & q^2 \\ p^2 & 2pq & q^2 \end{pmatrix}$$

$$= \begin{pmatrix} \underset{p^2(p+q)}{p^3+p^2q} & \underset{2pq(p+q)}{2p^2q+2pq^2} & \underset{q^2(p+q)}{pq^2+q^3} \\ \frac{1}{2}p^3+\frac{1}{2}p^2+\frac{1}{2}qp & p^2q+pq+pq^2 & \frac{1}{2}pq^2+\frac{1}{2}q^2+\frac{1}{2}q^3 \\ \underset{p^2(p+q)}{p^3+p^2q} & 2p^2q+2pq^2 & pq^2+q^3 \end{pmatrix}$$

# Review Exercises

1. (a) $A = \begin{pmatrix} \overset{2}{a_{11}} & \overset{3}{a_{12}} & \overset{4}{a_{13}} \\ \overset{3}{a_{21}} & \overset{4}{a_{22}} & \overset{5}{a_{23}} \end{pmatrix}$

$= \begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

(b) $A = \begin{pmatrix} \overset{1}{a_{11}} & \overset{-1}{a_{12}} & \overset{1}{a_{13}} \\ \overset{-1}{a_{21}} & \overset{1}{a_{22}} & \overset{-1}{a_{23}} \end{pmatrix}$

$= \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{pmatrix}$

**2** (a) $\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$

$= \begin{pmatrix} 3 & -2 \\ -2 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} - 2\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} -4 & -6 \\ -2 & -8 \end{pmatrix}$

$= \begin{pmatrix} -3 & -4 \\ -2 & -8 \end{pmatrix}$

(c) $AB = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$

$= \begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix}$

(d) $C(AB) = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 2 & -3 \end{pmatrix}$

$= \begin{pmatrix} 2 & -1 \\ 6 & -8 \end{pmatrix}$

(e) $AD = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 2 & 2 & 2 \\ 0 & 2 & 3 \end{pmatrix}$

(f) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ = no sol$^n$

(g) $2\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} - 3\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ -2 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -6 \\ -3 & 3 \end{pmatrix} = \begin{pmatrix} 7 & -6 \\ -5 & 5 \end{pmatrix}$

(h) $(A-B)' = \begin{pmatrix} 3 & -2 \\ -2 & 2 \end{pmatrix}' = \begin{pmatrix} 3 & -2 \\ -2 & 2 \end{pmatrix}$

(i) $(C'A')B' = \left[\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 4 & -1 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} -6 & 5 \\ -4 & 5 \end{pmatrix}$

(j) $C'(A'B') = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\left[\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}\right]$

$= \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -4 & 3 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} -6 & 5 \\ -4 & 5 \end{pmatrix}$

(k) $\begin{pmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$ = no sol$^n$

(l) $\begin{pmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 5 \\ 4 & 10 & 13 \\ 5 & 13 & 9 \end{pmatrix}$

3

(a) $$\begin{pmatrix} 2 & -5 \\ 5 & 8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

(b) $$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 2 & 4 \\ 1 & 4 & 8 & 0 \\ 2 & 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

(c) $$\begin{pmatrix} a-1 & 3 & -2 \\ a & 2 & -1 \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$$

4. $$A+B = \begin{pmatrix} 0 & 1 & -2 \\ 3 & 4 & 5 \\ -6 & 7 & 15 \end{pmatrix} + \begin{pmatrix} 0 & -5 & 3 \\ 5 & 2 & -1 \\ -4 & 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -4 & 1 \\ 8 & 6 & 4 \\ -10 & 9 & 15 \end{pmatrix}$$

$$A-B = \begin{pmatrix} 0 & 1 & -2 \\ 3 & 4 & 5 \\ -6 & 7 & 15 \end{pmatrix} - \begin{pmatrix} 0 & -5 & 3 \\ 5 & 2 & -1 \\ -4 & 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 6 & -5 \\ -2 & 2 & 6 \\ -2 & 5 & 15 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 1 & -2 \\ 3 & 4 & 5 \\ -6 & 7 & 15 \end{pmatrix} \begin{pmatrix} 0 & -5 & 3 \\ 5 & 2 & -1 \\ -4 & 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 13 & -2 & -1 \\ 0 & 3 & 5 \\ -25 & 74 & -25 \end{pmatrix}$$

$$BA = \begin{pmatrix} 0 & -5 & 3 \\ 5 & 2 & -1 \\ -4 & 2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & -2 \\ 3 & 4 & 5 \\ -6 & 7 & 15 \end{pmatrix}$$

$$= \begin{pmatrix} -33 & 1 & 20 \\ 12 & 6 & -15 \\ 6 & 2 & 18 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 0 & 1 & -2 \\ 3 & 4 & 5 \\ -6 & 7 & 15 \end{pmatrix} \left[ \begin{pmatrix} 0 & -5 & 3 \\ 5 & 2 & -1 \\ -4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 6 & -2 & -3 \\ 2 & 0 & 1 \\ 0 & 5 & 7 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 0 & 1 & -2 \\ 3 & 4 & 5 \\ -6 & 7 & 18 \end{pmatrix} \begin{pmatrix} -10 & 15 & 16 \\ 34 & -15 & -20 \\ -20 & 8 & 14 \end{pmatrix}$$

$$= \begin{pmatrix} 74 & -81 & -48 \\ 6 & 25 & 38 \\ -2 & -75 & -26 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 13 & -2 & -1 \\ 0 & 3 & 5 \\ -75 & 74 & -25 \end{pmatrix} \begin{pmatrix} 6 & -2 & -3 \\ 2 & 0 & 1 \\ 0 & 5 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 74 & -31 & -48 \\ 6 & 25 & 38 \\ -2 & -75 & -26 \end{pmatrix}$$

5. $$\begin{pmatrix} a & b \\ x & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ x & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2a+b & a+b \\ 2x & x \end{pmatrix} - \begin{pmatrix} a & b \\ 2a+x & 2b \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 4 \end{pmatrix}$$

$a+b-b=1 \quad \therefore a=1$

$2a+b-a=2$

$\therefore b=1$

$2x-2a-x=4$

$\therefore x=4+2=6$

6. (Not relevont) Sorry! Accidentally Included

# Chpt 16.1

1(a) $\begin{vmatrix} 3 & 0 \\ 2 & 6 \end{vmatrix} = 18$

(b) $\begin{vmatrix} a & a \\ b & b \end{vmatrix} = 0$

(c) $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix} = a^2+2ab+b^2 - (a^2-2ab+b^2)$

$= 4ab$

(d) $\begin{vmatrix} 3^t & 2^t \\ 3^{t-1} & 2^{t-1} \end{vmatrix} = 3^t.2^t.2^{-1} - 3^t.3^{-1}.2^t$

$= \dfrac{3^t.2^t}{6}$

2. Not relevant Sorry! Accidentally Included

3.(a) $|A| = \begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix}$

$= -6+1$

$= -5$

$|B| = \begin{vmatrix} 8 & -1 \\ 5 & -2 \end{vmatrix}$

$= -16+5$

$= -11$

$|C| = \begin{vmatrix} 3 & 8 \\ 1 & 5 \end{vmatrix}$

$= 15-8$

$= 7$

$x = \frac{11}{5}$

$y = -\frac{7}{5}$

$3x - y = 8$

$x = 5+2y$

$3(5+2y) - y = 8$

$15+5y = 8$

$5y = -7$

$y = -\frac{7}{5}$

$x = 5 - \frac{14}{5}$

$= \frac{11}{5}$

(b) $|A| = \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix}$

$= -2 - 9$

$= -11$

$|B| = \begin{vmatrix} 1 & 3 \\ 14 & -2 \end{vmatrix}$

$= -2 - 42$

$= -44$

$|C| = \begin{vmatrix} 1 & 1 \\ 3 & 14 \end{vmatrix}$

$= 14 - 3$

$= 11$

$x = \frac{44}{11} = 4$

$y = -1$

$x = 1 - 3y$

$3(1-3y) - 2y = 14$

$3 - 9y - 2y = 14$

$-11y = 11$

$y = -1$

$x = 1 - 3(-1) = 4$

(c) $|A| = \begin{vmatrix} a & -b \\ b & a \end{vmatrix}$

$= a^2 + b^2$

$|B| = \begin{vmatrix} 1 & -b \\ 2 & a \end{vmatrix}$

$= a + 2b$

$|C| = \begin{vmatrix} a & 1 \\ b & 2 \end{vmatrix}$

$= 2a - b$

$x = \frac{a+2b}{a^2+b^2}$

$y = \frac{2a-b}{a^2+b^2}$

$ax - by = 1$

$ax = 1 + by$

$x = \frac{1+by}{a}$

$b\left(\frac{1+by}{a}\right) + ay = 2$

$b(1+by) + a^2y = 2a$

$b + b^2y + a^2y = 2a$

$(a^2 + b^2)y = 2a - b$

$y = \frac{2a-b}{a^2+b^2}$

$x = \left[1 + b\left(\frac{2a-b}{a^2+b^2}\right)\right] \times \frac{1}{a}$

$= \frac{a^2+b^2+2ab-b^2}{a^2+b^2} \times \frac{1}{a}$

$= \frac{a(a+2b)}{a^2+b^2} \times \frac{1}{a}$

$= \frac{a+2b}{a^2+b^2}$

4. $tr(A) = 0$

$a+1=0$

$\therefore a=-1$

$\begin{vmatrix} -1 & 3 \\ b & 1 \end{vmatrix} = -1-3b$

$-1-3b=-10$

$-3b=-9$

$\therefore b=3$

5. $\begin{vmatrix} 2-x & 1 \\ 8 & -x \end{vmatrix} = -x(2-x)-8$

$= -2x+x^2-8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4$ or $x=-2$

6. $|AB| = \left|\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\right|$

$= \left|\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{pmatrix}\right|$

$= (a_{11}b_{11}+a_{12}b_{21})(a_{21}b_{12}+a_{22}b_{22}) - (a_{21}b_{11}+a_{22}b_{21})(a_{11}b_{12}+a_{12}b_{22})$

$= a_{11}b_{11}a_{22}b_{22} + a_{12}b_{21}a_{21}b_{12} - a_{12}b_{11}a_{21}b_{22} - a_{11}b_{21}a_{22}b_{12}$

$|A|.|B| = (a_{11}a_{22} - a_{21}a_{12}) \cdot (b_{11}b_{22} - b_{21}b_{12})$

$= a_{11}b_{11}a_{22}b_{22} + a_{12}b_{21}a_{21}b_{12} - a_{12}b_{11}a_{21}b_{22} - a_{11}b_{21}a_{22}b_{12}$

$= |AB|$ (Shown)

7. Let $A = B = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$

$|A+B| = \begin{vmatrix} 4 & 2 \\ 4 & 6 \end{vmatrix}$

$= 24 - 8 = 16$

$|A| + |B| = 8 \neq |A+B|$

8. $Y = C + I_0 + G_0$

$C = a + bY$

$-C + Y = I_0 + G_0$

$C - bY = a$

$$|A| = \begin{vmatrix} -1 & 1 \\ 1 & -b \end{vmatrix}$$

$= b - 1$

$$|B| = \begin{vmatrix} I_0 + G_0 & 1 \\ a & -b \end{vmatrix}$$

$= -b(I_0 + G_0) - a$

$$|C| = \begin{vmatrix} -1 & I_0 + G_0 \\ 1 & a \end{vmatrix}$$

$= -a - (I_0 + G_0)$

$$C = \frac{-b(I_0 + G_0) - a}{b - 1}$$

$$Y = \frac{-a - I_0 - G_0}{b - 1}$$

9. (a) Imports of country 1 comes from all the exports of country 2

Imports of country 2 comes from all the exports of country 1

(b) $Y_1 = C_1 Y_1 + A_1 + m_2 Y_2 - m_1 Y_1$

Take note of this Qns $Y_2 = C_2 Y_2 + A_2 + m_1 Y_1 - m_2 Y_2$

$Y_1 + m_1 Y_1 - C_1 Y_1 \quad - m_2 Y_2 = A_1$

$Y_2 + m_2 Y_2 - C_2 Y_2 \quad - m_1 Y_1 = A_2$

$$\begin{pmatrix} 1+m_1-C_1 & -m_2 \\ -m_1 & 1+m_2-C_2 \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1+m_1-C_1 & -m_2 \\ -m_1 & 1+m_2-C_2 \end{vmatrix}$$

$$= (1+m_1-C_1)(1+m_2-C_2) - m_1 m_2$$

$$|B| = \begin{vmatrix} A_1 & -m_2 \\ A_2 & 1+m_2-C_2 \end{vmatrix}$$

$$= A_1(1+m_2-C_2) + A_2 m_2$$

$$|C| = \begin{vmatrix} 1+m_1-C_1 & A_1 \\ -m_1 & A_2 \end{vmatrix}$$

$$= A_2(1+m_1-C_2) + A_1 m_1$$

$$Y_1 = \frac{A_1(1+m_2-C_2)+A_2 m_2}{(1+m_1-C_1)(1+m_2-C_2)-m_1 m_2}$$

$$Y_2 = \frac{A_2(1+m_1-C_2)+A_1 m_1}{(1+m_1-C_1)(1+m_2-C_2)-m_1 m_2}$$

# Chpt16.2

1. use your normal determinants method
   of course its good to learn alternatives!

1 (a)

$$\left|\begin{array}{ccc|cc} 1 & -1 & 0 & 1 & -1 \\ 1 & 3 & 2 & 1 & 3 \\ 1 & 0 & 0 & 1 & 0 \end{array}\right.$$

$(0-2+0)-(0+0+0)=-2$

(b)

$$\left|\begin{array}{ccc|cc} 1 & -1 & 0 & 1 & -1 \\ 1 & 3 & 2 & 1 & 3 \\ 1 & 2 & 1 & 1 & 2 \end{array}\right.$$

$(3+(-2)+0)-(0+4-1)$

$1-3=-2$

(c)

$$\left|\begin{array}{ccc|cc} a & b & c & a & b \\ 0 & d & e & 0 & d \\ 0 & 0 & f & 0 & 0 \end{array}\right.$$

$(adf)-(0)=adf$

(d)

$$\left|\begin{array}{ccc|cc} a & 0 & b & a & 0 \\ 0 & e & 0 & 0 & e \\ c & 0 & d & c & 0 \end{array}\right.$$

$(aed)-0=aed$

2. $AB = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & -1 \end{pmatrix}$

$= \begin{pmatrix} -1 & -1 & -1 \\ 7 & 13 & 13 \\ 5 & 9 & 10 \end{pmatrix}$

$|A| = \left| \begin{matrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{matrix} \right| \begin{matrix} 1 & -1 \\ 1 & 3 \\ 1 & 2 \end{matrix}$

$(3+(-2)) - (4-1)$

$1-3=-2$

$|B| = \left| \begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & -1 \end{matrix} \right| \begin{matrix} 1 & 2 \\ 2 & 3 \\ 0 & 1 \end{matrix}$

$(-3+6) - (4-4) = 3$

$|A|.|B| = -6$

$|AB| = \left| \begin{matrix} -1 & -1 & -1 \\ 7 & 13 & 13 \\ 5 & 9 & 10 \end{matrix} \right| \begin{matrix} -1 & -1 \\ 7 & 13 \\ 5 & 9 \end{matrix}$

$= (-130-65-63) - (-65-117-70)$

$= -258 + 252$

$= -6 = |A||B|$ (verified)

(3) (a) $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & -1 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -6 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & -1 \end{vmatrix}$

$= 1\begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}$

$= 1(-1-(1)) - (-1)(-1-1) + (1)(-1-(-1))$

$= -2 - 2$

$= -4$

$|B| = \begin{vmatrix} 2 & -1 & 1 \\ 0 & 1 & -1 \\ -6 & -1 & -1 \end{vmatrix}$

$= 2\begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ -6 & -1 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ -6 & -1 \end{vmatrix}$

$= 2(-1-(1)) + (0-(6)) + (0-(-6))$

$= -4$

$|C| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \\ -1 & -6 & -1 \end{vmatrix}$

$= 1\begin{vmatrix} 0 & -1 \\ -6 & -1 \end{vmatrix} - 2\begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ -1 & -6 \end{vmatrix}$

$= (0-(6)) - 2(-1-1) + (6)$

$= -12 + 4$

$= -8$

$|D| = \begin{vmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ -1 & -1 & -6 \end{vmatrix}$

$= 1\begin{vmatrix} 1 & 0 \\ -1 & -6 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 0 \\ -1 & -6 \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}$

$= 1(-6) + 1|-6| + 2(-1+1)$

$= -12$

$x = \frac{|B|}{|A|} = \frac{-4}{-4} = 1 \qquad y = \frac{|C|}{|A|} = \frac{-8}{-4} = 2 \qquad z = \frac{|D|}{|A|} = \frac{-12}{-4} = 3$

3(b) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix}$

$= 1\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$

$= -1 + 1$

$= 0$

Since $x = \frac{|B|}{|A|} = 0$

$y = \frac{|C|}{|A|} = 0$

$z = \frac{|D|}{|A|} = 0$ //

8(c) $\begin{bmatrix} 1 & 3 & -2 \\ 3 & -2 & 5 \\ 2 & -5 & 3 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 14 \\ 1 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & 3 & -2 \\ 3 & -2 & 5 \\ 2 & -5 & 3 \end{vmatrix}$

$= 1\begin{vmatrix} -2 & 5 \\ -5 & 3 \end{vmatrix} - 3\begin{vmatrix} 3 & 5 \\ 2 & 3 \end{vmatrix} - 2\begin{vmatrix} 3 & -2 \\ 2 & -5 \end{vmatrix}$

$= (-6-(-25)) - 3(9-10) - 2(-15+4)$

$= 19 + 3 + 22$

$= 44$

$|B| = \begin{vmatrix} 1 & 3 & -2 \\ 14 & -2 & 5 \\ 1 & -5 & 3 \end{vmatrix}$

$= 1\begin{vmatrix} -2 & 5 \\ -5 & 3 \end{vmatrix} - 3\begin{vmatrix} 14 & 5 \\ 1 & 3 \end{vmatrix} - 2\begin{vmatrix} 14 & -2 \\ 1 & -5 \end{vmatrix}$

$= (-6+25) - 3(42-5) - 2(-70+2)$

$= (19) - 111 + 136$

$= 44$

$|C| = \begin{vmatrix} 1 & 1 & -2 \\ 3 & 14 & 5 \\ 2 & 1 & 3 \end{vmatrix}$

$= 1\begin{vmatrix} 14 & 5 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 3 & 5 \\ 2 & 3 \end{vmatrix} - 2\begin{vmatrix} 3 & 14 \\ 2 & 1 \end{vmatrix}$

$= 1(42-5) - 1(9-10) - 2(3-28)$

$= 37 + 1 + 50$

$= 88$

$|D| = \begin{vmatrix} 1 & 3 & 1 \\ 3 & -2 & 14 \\ 2 & -5 & 1 \end{vmatrix}$

$= 1\begin{vmatrix} -2 & 14 \\ -5 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & 14 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & -2 \\ 2 & -5 \end{vmatrix}$

$= 1(-2+70) - 3(3-28) + (-15+4)$

$= 1(68) - 3(-25) + (-11)$

$= 68 + 75 - 11$

$= 132$

$x = \frac{|B|}{|A|} = \frac{44}{44} = 1 \quad y = \frac{|C|}{|A|} = \frac{88}{44} = 2 \quad z = \frac{|D|}{|A|} = \frac{132}{44} = 3$

4. $\left|\begin{array}{ccc|cc} 1+a & 1 & 1 & 1+a & 1 \\ 1 & 1+b & 1 & 1 & 1+b \\ 1 & 1 & 1+c & 1 & 1 \end{array}\right.$

$=((1+a)(1+b)(1+c) + 1 + 1) - ((1)(1+b)(1) + (1+a) + (1+c))$

$= (1+b+a+ab)(1+c) + 2 - (1+b) - (1+a) - (1+c)$

$= \cancel{1} + \cancel{b} + \cancel{a} + ab + \cancel{c} + cb + ca + abc + \cancel{2} - \cancel{1} - \cancel{b} - \cancel{1} - \cancel{a} - \cancel{1} - \cancel{c}$

$= abc + ab + ac + bc$ (Shown)

5. $tr(A) = a + (-1) + b$

$a+b-1=0$

$a+b=1 \qquad a=1-b$

$\left|\begin{array}{ccc|cc} a & 1 & 0 & a & 1 \\ 0 & -1 & a & 0 & -1 \\ -b & 0 & b & -b & 0 \end{array}\right.$

$-ab-ab=12$

$-2ab=12$

$ab=-6$

$(1-b)(b)=-6$

$b-b^2=-6$

$b^2-b-6=0$

$(b-3)(b+2)=0$

$b=3$ or $b=-2$

$a=1-3 \qquad a=1-(-2)$

$=-2 \qquad =3$

6. $\left|\begin{matrix} 1-x & 2 & 2 \\ 2 & 1-x & 2 \\ 2 & 2 & 1-x \end{matrix}\right|\begin{matrix} 1-x & 2 \\ 2 & 1-x \\ 2 & 2 \end{matrix}$

$(1-x)^3 + 8 + 8 - 4(1-x) - 4(1-x) - 4(1-x)$

$(1-x)^3 + 16 \quad - 12(1-x)$

$(1-x)^3 + 16 - 12(1-x) = 0$

$(1-2x+x^2)(1-x) + 16 - 12 + 12x = 0$

$1 - x - 2x + 2x^2 + x^2 - x^3 + 4 + 12x = 0$

$1 - 3x + 3x^2 - x^3 + 4 + 12x = 0$

$5 + 9x + 3x^2 - x^3 = 0$ (learn how to solve w/o calc)

$x = 5$ or $x = -1$

7.(a) $\begin{pmatrix} 1 & t & 0 \\ -2 & -2 & -1 \\ 0 & 1 & t \end{pmatrix}\begin{matrix} 1 & t \\ -2 & -2 \\ 0 & 1 \end{matrix}$

$|A_t| = (-2t) - ((-1)(-2t^2))$

$-2t - 2t^2$

$t^2 \neq 0$

Thereby, $-2t - 2t^2 \neq 0$, thereby $|A_t| \neq 0$

(b) $A_t^2 = \begin{pmatrix} 1 & t & 0 \\ -2 & -2 & -1 \\ 0 & 1 & t \end{pmatrix}\begin{pmatrix} 1 & t & 0 \\ -2 & -2 & -1 \\ 0 & 1 & t \end{pmatrix}$

$= \begin{pmatrix} 1-2t & t-2t & -t \\ 2 & -2t+3 & 2-t \\ -2 & -2+t & -1+t^2 \end{pmatrix}$

$(A_t)^3 = \begin{pmatrix} 1-2t & t-2t & -t \\ 2 & -2t+3 & 2-t \\ -2 & -2+t & -1+t^2 \end{pmatrix}\begin{pmatrix} 1 & t & 0 \\ -2 & -2 & -1 \\ 0 & 1 & t \end{pmatrix}$

$= \begin{pmatrix} 1-2t-2(t-2t) & t(1-2t)-2(t-2t)-t & -(t-2t)-t(t) \\ 2-2(-2t+3) & 2t-2(-2t+3)+(-2+t) & -(2+t)+t(2-t) \\ -2(1)-2(-2+t) & -2t-2(-2+t)+(-1+t^2) & -(-2+t)+t(-1+t^2) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$-2 - 2(-2+t) = 0$

$-2 + 4 - 2t = 0 \qquad 2 - 2t = 0 \qquad -2t = -2 \qquad t = 1 /\!/$

8. $Y = C + A_0$ $\qquad C = a + b(Y - T)$ $\qquad T = d + tY$

$Y - C = A_0$ $\qquad C = a + bY - bT$ $\qquad -tY + T = d$

$-bY + C + bT = a$

$$Y - C = A_0 \quad \text{——(1)}$$

$$-bY + C + bT = a \quad \text{——(2)}$$

$$-tY + T = d \quad \text{——(3)}$$

$$C = Y - A_0$$

$$-bY + Y - A_0 + b(d + tY) = a$$

$$-bY + Y - A_0 + bd + tbY = a$$

$$Y(1 - b + tb) = a + A_0 - bd$$

$$Y = \frac{a + A_0 - bd}{1 - b + tb}$$

$$C = \frac{a + A_0 - bd}{1 - b + tb} - A_0$$

$$= \frac{a + A_0 - bd - A_0 + bA_0 - A_0 tb}{1 - b + tb}$$

$$= \frac{a - bd + bA_0 - A_0 tb}{1 - b + tb}$$

$$T = d + t\left(\frac{a + A_0 - bd}{1 - b + tb}\right)$$

$$= d + \frac{at + A_0 t - tbd}{1 - b + tb}$$

$$= \frac{at + A_0 t + d - db}{1 - b + tb}$$

(b) $Y - C = A_0$ —— (1)

$-bY + C + bT = a$ —— (2)

$-tY + T = d$ —— (3)

$$\begin{pmatrix} 1 & -1 & 0 \\ -b & 1 & b \\ -t & 0 & 1 \end{pmatrix}\begin{pmatrix} Y \\ C \\ T \end{pmatrix} = \begin{pmatrix} A_0 \\ a \\ d \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 0 \\ -b & 1 & b \\ -t & 0 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & b \\ 0 & 1 \end{vmatrix} - (-1)\begin{vmatrix} -b & b \\ -t & 1 \end{vmatrix}$$

$$= 1(1) + 1(-b - tb)$$

$$= 1 - b - tb$$

$$|B| = \begin{vmatrix} A_0 & -1 & 0 \\ a & 1 & b \\ d & 0 & 1 \end{vmatrix}$$

$$= A_0\begin{vmatrix} 1 & b \\ 0 & 1 \end{vmatrix} - (-1)\begin{vmatrix} a & b \\ d & 1 \end{vmatrix}$$

$$= A_0(1) + (1)(a - db)$$

$$= A_0 + a - db$$

$$|C| = \begin{vmatrix} 1 & A_0 & 0 \\ -b & a & b \\ -t & d & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} a & b \\ d & 1 \end{vmatrix} - A_0\begin{vmatrix} -b & b \\ -t & 1 \end{vmatrix}$$

$$= (a - db) - A_0(-b + tb)$$

$$= a - db + A_0 b - A_0 tb$$

$$|D| = \begin{vmatrix} 1 & -1 & A_0 \\ -b & 1 & a \\ -t & 0 & d \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & a \\ 0 & d \end{vmatrix} - (-1)\begin{vmatrix} -b & a \\ -t & d \end{vmatrix} + A_0\begin{vmatrix} -b & 1 \\ -t & 0 \end{vmatrix}$$

$$= d + (-bd + ta) + A_0 t$$

$$= at - bd + d + A_0 t$$

$$Y = \frac{|B|}{|A|} = \frac{A_0 + a - db}{1 - b - tb}$$

$$C = \frac{|C|}{|A|} = \frac{a - db + A_0 b - A_0 tb}{1 - b - tb}$$

$$T = \frac{|D|}{|A|} = \frac{at - bd + d + A_0 t}{1 - b - tb}$$

# Chapter 16.6

7. $A = PDP^{-1}$

$A = (PDP^{-1})(PDP^{-1})$

$= PDIPDP^{-1}$

$= PD^2P^{-1}$ (Shown)

# Chapter 16.9*

5. $x_1 = 0.1x_1 + 0.2x_2 + 0.1x_3 + 85$

$x_2 = 0.3x_1 + 0.2x_2 + 0.2x_3 + 95$

$x_3 = 0.2x_1 + 0.2x_2 + 0.1x_1 + 20$

$0.9x_1 - 0.2x_2 - 0.1x_3 = 85$

$-0.3x_1 + 0.8x_2 - 0.2x_3 = 95$

$-0.2x_1 - 0.2x_2 + 0.9x_1 = 20$

$$|A| = \begin{vmatrix} 0.9 & -0.2 & -0.1 \\ -0.3 & 0.8 & -0.2 \\ -0.2 & -0.2 & 0.9 \end{vmatrix} = 0.9\begin{vmatrix} 0.8 & -0.2 \\ -0.2 & 0.9 \end{vmatrix} + 0.2\begin{vmatrix} -0.3 & -0.2 \\ -0.2 & 0.9 \end{vmatrix} - 0.1\begin{vmatrix} -0.3 & 0.8 \\ -0.2 & -0.2 \end{vmatrix}$$

$$= \frac{153}{250} - \frac{31}{500} - \frac{11}{500}$$

$$= \frac{66}{125}$$

$$|B| = \begin{vmatrix} 85 & -0.2 & -0.1 \\ 95 & 0.8 & -0.2 \\ 20 & -0.2 & 0.9 \end{vmatrix} = 85\begin{vmatrix} 0.8 & -0.2 \\ -0.2 & 0.9 \end{vmatrix} + 0.2\begin{vmatrix} 95 & -0.2 \\ 20 & 0.9 \end{vmatrix} - 0.1\begin{vmatrix} 95 & 0.8 \\ 20 & -0.2 \end{vmatrix}$$

$$= 85\left(\frac{17}{25}\right) + 0.2\left(\frac{179}{2}\right) - 0.1(-35)$$

$$= \frac{396}{5}$$

$$|C| = \begin{vmatrix} 0.9 & 85 & -0.1 \\ -0.3 & 95 & -0.2 \\ -0.2 & 20 & 0.9 \end{vmatrix} = 0.9\begin{vmatrix} 95 & -0.2 \\ 20 & 0.9 \end{vmatrix} - 85\begin{vmatrix} -0.3 & -0.2 \\ -0.2 & 0.9 \end{vmatrix} - 0.1\begin{vmatrix} -0.3 & 95 \\ -0.2 & 20 \end{vmatrix}$$

$$= 0.9\left(171/2 + 4\right) - 85\left(-\frac{27}{100} - \frac{1}{25}\right) - 0.1\left(-6 + 19\right)$$

$$= \frac{1611}{20} + \frac{527}{20} - \frac{13}{10}$$

$$= \frac{528}{5}$$

$$|D| = \begin{vmatrix} 0.9 & -0.2 & 85 \\ -0.3 & 0.8 & 95 \\ -0.2 & -0.2 & 20 \end{vmatrix} = 0.9\begin{vmatrix} 0.8 & 95 \\ -0.2 & 20 \end{vmatrix} + 0.2\begin{vmatrix} -0.3 & 95 \\ -0.2 & 20 \end{vmatrix} + 85\begin{vmatrix} -0.3 & 0.8 \\ -0.2 & -0.2 \end{vmatrix}$$

$$= 0.9\left(16 + 19\right) + 0.2\left(-6 + 19\right) + 85\left(\frac{3}{50} + \frac{4}{25}\right)$$

$$= \frac{63}{2} + \frac{13}{5} + \frac{187}{10}$$

$$= \frac{264}{5}$$

$$x_1 = \frac{396}{5} \div \frac{66}{125} = 150$$

$$x_2 = \frac{528}{5} \div \frac{66}{125} = 200$$

$$x_3 = \frac{264}{5} \div \frac{66}{125} = 100 \qquad \text{(verified)}$$