ECON 6018
TUTORIAL 4

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- Lesson Tutorial
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Focus :

1. Partial Derivatives

2. Introduction to Implicit

3. Practices to improve speed

Chapter 11 : Partial Derivatives

If $z = f(x, y)$

$\frac{\partial z}{\partial x}$ derivative of f wrt x when y is held constant

$\frac{\partial z}{\partial y}$ derivative of wrt y when x is held constant

Let us walk through an example together!

Find $\partial z/\partial x$ and $\partial z/\partial y$ for the following functions:

(a) $z = 2x + 3y$ (b) $z = x^2 + y^3$ (c) $z = x^3y^4$ (d) $z = (x + y)^2$

Ans:

1(a) $\frac{\partial z}{\partial x} = 2$ $\quad \frac{\partial z}{\partial y} = 3$

1(b) $\frac{\partial z}{\partial x} = 2x$ $\quad \frac{\partial z}{\partial y} = 3y^2$

1(c) $\frac{\partial z}{\partial x} = 3x^2y^4$ $\quad \frac{\partial z}{\partial y} = 4x^3y^3$

Questions:

1.

Find $\partial z/\partial x$ and $\partial z/\partial y$ for the following functions:

(a) $z = x^2 + 3y^2$ (b) $z = xy$ (c) $z = 5x^4y^2 - 2xy^5$ (d) $z = e^{x+y}$

(e) $z = e^{xy}$ (f) $z = e^x/y$ (g) $z = \ln(x + y)$ (h) $z = \ln(xy)$

Ans:

2. (a) $\frac{\partial z}{\partial x} = 2x$

$\frac{\partial z}{\partial y} = 6y$

(b) $\frac{\partial z}{\partial x} = y$

$\frac{\partial z}{\partial y} = x$

(c) $\frac{\partial z}{\partial x} = 20x^3y^2 - 2y^5$

$\frac{\partial z}{\partial y} = 10x^4y - 10xy^4$

(d) $\frac{\partial z}{\partial x} = e^{x+y}$

$\frac{\partial z}{\partial y} = e^{x+y}$

(e) $\frac{\partial z}{\partial x} = ye^{xy}$

$\frac{\partial z}{\partial y} = xe^{xy}$

(f) $\frac{\partial z}{\partial x} = \frac{1}{y}(e^x)$

$= \frac{e^x}{y}$

$\frac{\partial z}{\partial y} = (e^x) \cdot -\frac{1}{y^2}$

$= -\frac{e^x}{y^2}$

(g) $\frac{\partial z}{\partial x} = \frac{1}{x+y}$

$\frac{\partial z}{\partial y} = \frac{1}{x+y}$

2.

Find $f_1'(x, y)$, $f_2'(x, y)$, and $f_{12}''(x, y)$ for the following functions:

(a) $f(x, y) = x^7 - y^7$

(b) $f(x, y) = x^5 \ln y$

(c) $f(x, y) = (x^2 - 2y^2)^5$

3.(a) $f_1'(x,y) = 7x^6$

$f_2'(x,y) = -7y^6$

$f_{12}'' = 0$

(b) $f_1'(x,y) = (\ln y)(5x^4)$

$= 5\ln y x^4$

$f_2'(x,y) = x^5\left(\frac{1}{y}\right)$

$= \frac{x^5}{y}$

$f_{12}''(x,y) = \frac{5x^4}{y}$

(c) $f_1'(x,y) = 5(x^2-2y^2)^4 . 2x$

$= 10x(x^2-2y^2)^4$

$f_2'(x,y) = 5(x^2-2y^2)^4 . 4y$

$= 20y(x^2-2y^2)^4$

$f_{12}''(x,y) = 10x . 4(x^2-2y^2)^3 . -4y$

$= -160xy(x^2-2y^2)^3$

3.

Find all first- and second-order partial derivatives for the following functions:

(a) $z = 3x + 4y$

(b) $z = x^3y^2$

(c) $z = x^5 - 3x^2y + y^6$

(d) $z = x/y$

(e) $z = (x - y)/(x + y)$

(f) $z = \sqrt{x^2 + y^2}$

4.(a) $\frac{\partial z}{\partial x} = 3$

$\frac{\partial z}{\partial y} = 4$

$f''_{12}(x,y) = 0$

(b) $\frac{\partial z}{\partial x} = 3x^2y^2$

$\frac{\partial z}{\partial y} = 2x^3y$

$f''_{12}(x,y) = 6x^2y$

(c) $\frac{\partial z}{\partial x} = 5x^4 - 6xy$

$\frac{\partial z}{\partial y} = -3x^2 + 6y^5$

$f''_{12}(x,y) = -6x$

(d) $\frac{\partial z}{\partial x} = (1)(\frac{1}{y})$

$= \frac{1}{y}$

$\frac{\partial z}{\partial y} = x \cdot -\frac{1}{y^2}$

$= -\frac{x}{y^2}$

$f''_{12}(x,y) = -\frac{1}{y^2}$

(e) $\frac{\partial z}{\partial x} = 1 \cdot (\frac{1}{x+y}) + (x-y)\,(x+y)^{-2} \cdot -$

$= \frac{1}{x+y} - \frac{x-y}{(x+y)^2}$

$= \frac{\cancel{x}+y-\cancel{x}+y}{(x+y)^2}$

$= \frac{2y}{(x+y)^2}$

$\frac{\partial z}{\partial y} = -1 \cdot (\frac{1}{x+y}) + (x-y)(x+y)^{-2} \cdot -1$

$= \frac{-x-\cancel{y}-x+\cancel{y}}{(x+y)^2} = -\frac{2x}{(x+y)^2}$

$f''_{12}(x,y) = 2 \cdot (\frac{1}{(x+y)^2}) + (2y)\,(x+y)^{-3} \cdot -2$

$= \frac{2}{(x+y)^2} - \frac{4y}{(x+y)^3}$

$= \frac{2x+2y-4y}{(x+y)^3}$

$= \frac{2x-2y}{(x+y)^3}$

Application of partial derivatives

- rmbr our chain rule?

$$\frac{dy}{dx} = \frac{du}{dx} \cdot \frac{dy}{du}$$

- let's expand this

$z = f(x, y)$

$x = g(t)$

$y = h(t)$

$$z = f(g(t), h(t))$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

Let's go through an example!

$F(x,y) = x + y^2$

$x = t^2$

$y = t^3$

Question

$F(x,y) = x^p y^q$

$x = at$

$y = bt$

Introduction to Implicit (Chapter 7)

- $y = x^2$

$\frac{dy}{dx} = 2x$

notice :

$\frac{d}{dx} y = \frac{d}{dx} x^2$

$\downarrow$

$\frac{dy}{dx}$

SO : $\frac{d}{dx} y = \frac{dy}{dx}$

what is y^2 wrt. x

$\frac{d}{dx} y^2 = \frac{dy}{dx} \cdot 2y$

Example:

$x^2 y = 1$

Qns: $x - y + 3xy = 2$

Review

1. Use the chain rule to find dy/dx for the following:

(a) $y = 10u^2$ where $u = 5 - x^2$

(b) $y = \sqrt{u}$ where $u = \frac{1}{x} - 1$

2. Find the first derivatives of:

(a) $y = -7e^x$

(b) $y = e^{-3x^2}$

(c) $y = \frac{x^2}{e^x}$

(d) $y = e^x \ln(x^2 + 2)$

(e) $y = e^{5x^3}$

(f) $y = 2 - x^4 e^{-x}$

(g) $y = (e^x + x^2)^{10}$

(h) $y = \ln(\sqrt{x} + 1)$

3. Find all the first- and second-order partial derivatives for the following functions:

(a) $z = x^2 + e^{2y}$

(b) $z = y \ln x$

(c) $z = xy^2 - e^{xy}$

(d) $z = x^y$