ECON 6018 TUTORIAL 3

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- Lesson Tutorial

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Differentiation

- the most important thing is your foundation and speed for comparative statics
- For a function to be differentiable, it has to be continuous
- welcome to my favorite Topic!

Chapt 6

Chapt 7

Chapt 11

Chapt 12

1. Derivatives and how we derive them using limits

Derivative — rate of change
- how much y change when x changes
- formula:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- Application
  maximization of utility

Questions:

Q1. Let $f(x) = 4x^2$. Show that $f(5+h) - f(5) = 40h + 4h^2$, implying that $\frac{f(5+h) - f(5)}{h} = 40 + 4h$ for $h \neq 0$. Use this result to find $f'(5)$. Compare the answer with (6.2.6).

Q2. Let $f(x) = 3x^2 + 2x - 1$.

Show that $\frac{f(x+h) - f(x)}{h} = 6x + 2 + 3h$ for $h \neq 0$, and use this result to find $f'(x)$.

Answers:

Q1. $f(x) = 4x^2$

$f(5+h) - f(5) = 40h + 4h^2$ (shown)

$$\lim_{h \to 0} \frac{40h + 4h^2}{h} = 40$$

$f'(5) = 40$

Q2. $f(x+h) = 3x^3 + 6xh + 3h^2 + 2x + 2h - 1$

$$\frac{f(x+h) - f(x)}{h} = 6x + 3h + 2$$

$h \to 0$

$f'(x) = 6x + 2$

2. Differentiation

2.1 Basic Differentiation — chpt 6.6

Given any constant a,

$$f(x) = x^a \Rightarrow f'(x) = ax^{a-1}$$

Questions

1. Compute the derivatives of the following functions:

   (a) $y = 5$ (b) $y = x^4$ (c) $y = 9x^{10}$ (d) $y = \pi^7$

2. Suppose we know $g'(x)$. Find expressions for the derivatives of the following:

   (a) $2g(x) + 3$ (b) $-\frac{1}{6}g(x) + 8$ (c) $\dfrac{g(x) - 5}{3}$

3. Find the derivatives of the following:

   (a) x^6 (b) $3x^{11}$ (c) x^{50} (d) $-4x^{-7}$

   (e) $\dfrac{x^{12}}{12}$ (f) $\dfrac{-2}{x^2}$ (g) $\dfrac{3}{\sqrt[3]{x}}$ (h) $\dfrac{-2}{x\sqrt{x}}$

1(a) 0 (b) $4x^3$ (c) $90x^9$ (d) 0

2(a) $2g'(x)$ (b) $-\frac{1}{6}g'(x)$ (c) $\frac{g'(x)}{3}$

3(a) $6x^5$ (b) $33x^{10}$ (c) $50x^{49}$ (d) $28x^{-8}$

(e) x^{11} (f) $-4x^{-3}$ (g) $-x^{-4/3}$ (h) $3x^{-5/2}$

2.2 Sum, Product, Quotient — chpt 6.7

a. Sum

$$F(x) = f(x) \pm g(x) \Rightarrow F'(x) = f'(x) \pm g'(x)$$

b. Product

$$F(x) = f(x) \cdot g(x) \Rightarrow F'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

c. Quotient

$$F(x) = \frac{f(x)}{g(x)} \Rightarrow F'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

Questions:

Q1. Differentiate w.r.t x the following functions:

(a) $x+1$ (b) $x+x^2$ (c) $3x^5+2x^4+5$

(d) $8x^4+2\sqrt{x}$ (e) $\frac{1}{2}x-\frac{3}{2}x^2+5x^3$ (f) $1-3x^7$

Q2. Differentiate w.r.t x the following functions:

(a) $\frac{3}{5}x^2-2x^7+\frac{1}{8}-\sqrt{x}$ (b) $(2x^2-1)(x^4-1)$ (c) $\left(x^5+\frac{1}{x}\right)(x^5+1)$

Q3. Differentiate w.r.t x the following functions:

(a) $\dfrac{\sqrt{x}-2}{\sqrt{x}+1}$ (b) $\dfrac{x^2-1}{x^2+1}$ (c) $\dfrac{x^2+x+1}{x^2-x+1}$

1(a) 1 (b) $1+2x$ (c) $15x^4+8x^3$

(d) $32x^3+2x^{-\frac{1}{2}}$ (e) $\frac{1}{2}-3x+15x^2$

(f) $-2bx^6$

2(a) $\frac{6}{5}x-14x^6-\frac{1}{2}x^{-\frac{1}{2}}$

(b) $(4x^3)(x^4-1)+(2x^2-1)(4x^3)$

(c) $(5x^4+x^{-2})(x^5+1)+(x^5+\frac{1}{x})(5x^4)$

3(a) $(\frac{1}{2}x^{-\frac{1}{2}})\left(\frac{1}{\sqrt{x}+1}\right)+(\sqrt{x}-2)(\sqrt{x}+1)^{-2}(\frac{1}{2}x^{-\frac{1}{2}})$

(b) $\dfrac{(2x)(x^2+1)-(x^2-1)(2x)}{(x^2+1)^2}$

(c) $\dfrac{(2x+1)(x^2-x+1)-(x^2+x+1)(2x-1)}{(x^2-x+1)^2}$

2.3 Chain Rule ☆ — chpt 6.8

If y is a differentiable function of u, and u is a differentiable function of x, then y is a differentiable function of x, and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Questions:

1. Use the chain rule (6.8.1) to find dy/dx for the following:

   (a) $y = 5u^4$, where $u = 1 + x^2$

   (b) $y = u - u^6$, where $u = 1 + 1/x$

2. Compute the following:

   (a) dY/dt, when $Y = -3(V+1)^5$ and $V = \frac{1}{3}t^3$.

   (b) dK/dt, when $K = AL^a$ and $L = bt + c$, where A, a, b, and c are positive constants.

Answer:

1(a) $\frac{dy}{dx} = 20u^3 (2x)$

$= 20(1+x^2)^3(2x)$

(b) $\frac{dy}{dx} = (1-6u^5)(-x^{-2})$

$= (1-1(1+\frac{1}{x})^5)(-x^{-2})$

2(a) $\frac{dv}{dt} = [-15(v+1)^4](t^2)$

$= [-15(\frac{1}{3}t^3+1)^4](t^2)$

2.4 Higher Order Derivatives — Chpt 6.09

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Question

Compute the second derivatives of:

(a) $y = x^5 - 3x^4 + 2$

(b) $y = (1 + x^2)^{10}$

Ans:

(a) $\frac{dy}{dx} = 5x^4 - 12x^3$

$$\frac{d^2y}{dx^2} = 20x^3 - 36x^2$$

(b) $\frac{dy}{dx} = 10(1+x^2)^9 \cdot 2x$

$= 20x\,(1+x^2)^9$

$$\frac{d^2y}{dx^2} = 20\,(1+x^2)^9 + 20x \cdot 9(1+x^2)^8 \cdot 2x$$

2.5 Exponential — Chpt 6.10

$$f(x) = e^x \implies f'(x) = e^x$$

$$f(x) = e^{g(x)} \qquad f'(x) = g'(x)e^{g(x)}$$

Questions:

Find the first and second derivatives of:

(a) $y = e^{-3x}$ (b) $y = 2e^{x^3}$ (c) $y = e^{1/x}$ (d) $y = 5e^{2x^2-3x+1}$

Ans:

(a) $-3e^{-3x}$

(b) $6x^2e^{x^3}$

(c) $-x^2\, e^{\frac{1}{x}}$

(d) $(4x+3)\cdot 5e^{2x^2-3x+1}$

2.6 Logarithmic — Chpt 6.11

$$y = \ln h(x) \Rightarrow y' = \frac{h'(x)}{h(x)}$$

Questions

Find the derivatives of:

(a) $y = x^3(\ln x)^2$ (b) $y = \dfrac{x^2}{\ln x}$ (c) $y = (\ln x)^{10}$ (d) $y = (\ln x + 3x)^2$

Ans:

(a) $3x^2(\ln x^2) + (3x^2)(2\ln x)(\frac{1}{x})$

(b) $\dfrac{2x.\ln x - \frac{1}{x}.x^2}{(\ln x)^2}$

(c) $10.(\ln x)^9.\frac{1}{x}$

(d) $2(\ln x+3x).(\frac{1}{x}+3)$