ECON 6018 TUTORIAL

- By: Teresa Ng
- Lesson One

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Essential Materials

1. Textbook

Essential Mathematics for Economic Analysis
by Knut Sydsaeter, Peter Hammond, Arne Strøm & Andres Carvajal

- Fifth Edition

2. Writing Materials

- tutorials will be doing questions

3. Contact

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  wechat: roseyjpeg —— only if extremely urgent or I have not replied your email after 3 days

Questions:

1. Will we get these notes?
- Yes, they will be sent out 2 days after lecture

2. How does tutorial help?
- help you get familiar with concepts
- majority of students who attended all my tutorial last sem got B+ and above

Any others?

1. System of Linear Equations

In Lecture:

Eg.

$$2x_1 + 3x_2 + 4x_3 = 1$$
$$3x_1 + 4x_2 + 5x_3 = 2$$
$$4x_1 + 5x_2 + 6x_3 = 3$$

- can be represented as

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\underset{3 \times 3}{A} \times \underset{3 \times 1}{x} = \underset{3 \times 1}{d}$$

coefficient matrix

Practices:

Step 1: Write all the common terms in the same column

- there may be alot of variables so this makes sure we are not confused

Step 2: Seperate the coefficients

Practice Questions:

Chpt 15.1

Q1.

$$3x_1 - 2x_2 + 6x_3 = 5$$

$$5x_1 + x_2 + 2x_3 = -2$$

Q2.

Trygve Haavelmo (1911–1999), a Norwegian Nobel prize-winning economist, devised a model of the US economy for the years 1929–1941 that is based on the following four equations:
(i) $c = 0.712y + 95.05$ (ii) $y = c + x - s$ (iii) $s = 0.158(c + x) - 34.30$ (iv) $x = 93.53$.
Here x denotes total investment, y is disposable income, s is the total saving by firms, and c is total consumption. Write the system of equations in the form (15.1.1) when the variables appear in the order x, y, s, and c.

Answer:

Q1. $$\begin{pmatrix} 3 & -2 & 6 \\ 5 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Q2. $$\begin{pmatrix} 0 & -0.712 & 1 & 0 \\ -1 & 1 & -1 & 1 \\ -0.158 & 0 & -0.158 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ c \\ s \end{pmatrix} = \begin{pmatrix} 95.05 \\ 0 \\ -34.30 \\ 93.53 \end{pmatrix}$$

Extra

How is this use in Economics?

- PhD Level Macro Economics Modelling
"Job Search Model"
- Value of being unemployed, Value of being employed } solving for wage

allow $x = \begin{bmatrix} V_u \\ V_e \\ w \end{bmatrix}$

$$V_u = (b-\kappa) + \beta[(1-p)V_u + pV_e]$$
$$V_E = w + \beta[(1-\delta)V_E + \delta V_u]$$
$$w = \phi y + (1-\phi)b$$

$$\begin{bmatrix} 1-\beta(1-p) & -\beta p & 0 \\ -\beta\delta & 1-\beta(1-\delta) & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_u \\ V_e \\ w \end{bmatrix} = \begin{bmatrix} b-\kappa \\ 0 \\ \phi y + (1-\phi)b \end{bmatrix}$$

2. Matrix Operations

In lecture: (Addition and subtraction)

Let $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ be arbitrary $m \times n$ matrices, and let α and β be real numbers. Also, let $\mathbf{0}$ denote the $m \times n$ matrix consisting only of zeros, called the *zero matrix*. Then:

(a) $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$

(b) $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

(c) $\mathbf{A} + \mathbf{0} = \mathbf{A}$

(d) $\mathbf{A} + (-\mathbf{A}) = \mathbf{0}$

(e) $(\alpha + \beta)\mathbf{A} = \alpha\mathbf{A} + \beta\mathbf{A}$

(f) $\alpha(\mathbf{A} + \mathbf{B}) = \alpha\mathbf{A} + \alpha\mathbf{B}$

Practices
Chpt 15.2
Q1.

Evaluate $\mathbf{A}+\mathbf{B}$ and $3\mathbf{A}$ when $\mathbf{A}=\begin{pmatrix}0 & 1\\ 2 & 3\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}1 & -1\\ 5 & 2\end{pmatrix}$.

Q2.

Evaluate $\mathbf{A}+\mathbf{B}$, $\mathbf{A}-\mathbf{B}$, and $5\mathbf{A}-3\mathbf{B}$ when $\mathbf{A}=\begin{pmatrix}0 & 1 & -1\\ 2 & 3 & 7\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}1 & -1 & 5\\ 0 & 1 & 9\end{pmatrix}$.

Ans:

Q1. A+B $\begin{pmatrix} 1 & 0 \\ 7 & 5 \end{pmatrix}$

$3A = \begin{pmatrix} 0 & 3 \\ 6 & 9 \end{pmatrix}$

Q2. A+B $\begin{pmatrix} 1 & 0 & 4 \\ 2 & 4 & 16 \end{pmatrix}$

A-B $\begin{pmatrix} -1 & 2 & -6 \\ 2 & 2 & -2 \end{pmatrix}$

5A-3B $\begin{pmatrix} -3 & 8 & -20 \\ 10 & 12 & 8 \end{pmatrix}$

In Lecture: Matrix Multiplication

$A \times B = AB$

Iff

A is $m \times n$

B is $n \times m$

Final product: $m \times m$

Eg. $\underset{2\times 1}{A = \begin{pmatrix} 0 \\ 2 \end{pmatrix}}$ $\quad \underset{1\times 2}{B = \begin{pmatrix} 1 & 2 \end{pmatrix}}$

$AB = \begin{pmatrix} 0 & 0 \\ 2 & 4 \end{pmatrix}$

Practices
Chpt 15.3
Q1.

Compute the products **AB** and **BA**, if possible, when **A** and **B** are, respectively:

(a) $\begin{pmatrix} 0 & -2 \\ 3 & 1 \end{pmatrix}$ and $\begin{pmatrix} -1 & 4 \\ 1 & 5 \end{pmatrix}$

(b) $\begin{pmatrix} 8 & 3 & -2 \\ 1 & 0 & 4 \end{pmatrix}$ and $\begin{pmatrix} 2 & -2 \\ 4 & 3 \\ 1 & -5 \end{pmatrix}$

Answer

Q1. (a)

$AB \quad \begin{pmatrix} -2 & -10 \\ -2 & 17 \end{pmatrix}$

$BA \quad \begin{pmatrix} 12 & 6 \\ 15 & 3 \end{pmatrix}$

(b)

$AB \quad \begin{pmatrix} 26 & 3 \\ 6 & -22 \end{pmatrix}$

$BA \quad \begin{pmatrix} 14 & 6 & -12 \\ 35 & 12 & 4 \\ 3 & 3 & -22 \end{pmatrix}$

In Lecture: Rules for Matrix Multiplication

$$(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$$

$$\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$$

$$(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$$

$$(\alpha\mathbf{A})\mathbf{B} = \mathbf{A}(\alpha\mathbf{B}) = \alpha(\mathbf{AB})$$

Power matrix — square matrix

$A^2 = A \times A$

$A^n = \underbrace{AA \ldots A}_{n \text{ times}}$

Practices:

Chpt 15.4

Suppose $\mathbf{P}$ and $\mathbf{Q}$ are two $n \times n$ matrices that satisfy $\mathbf{PQ} = \mathbf{Q}^2\mathbf{P}$. Prove then that $(\mathbf{PQ})^2 = \mathbf{Q}^6\mathbf{P}^2$.

Ans:

$$PQ = PQPQ$$
$$= Q^2 PPQ$$
$$= Q^2 P(Q^2 P)$$
$$= Q^2 PQ\ QP$$
$$= Q^2 (Q^2 P) QP$$
$$= Q^4 (PQ) P$$
$$= Q^4 (Q^2 P) P$$
$$= Q^6 P^2$$

In Lecture:

Identity Matrix

$$\mathbf{I}_n = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}_{n \times n}$$

$$\mathbf{A}\mathbf{I}_n = \mathbf{I}_n\mathbf{A} = \mathbf{A}$$

Practices:

Chpt 15.4

Qns 1.

Suppose that $\mathbf{A}$ and $\mathbf{B}$ are square matrices of order n.

(a) Prove that, in general

(i) $(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B}) \neq \mathbf{A}^2-\mathbf{B}^2$ (ii) $(\mathbf{A}-\mathbf{B})(\mathbf{A}-\mathbf{B}) \neq \mathbf{A}^2-2\mathbf{AB}+\mathbf{B}^2$

Qns 2.

A square matrix $\mathbf{A}$ is said to be *idempotent* if $\mathbf{A}^2=\mathbf{A}$.

(a) Show that the matrix $\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ is idempotent.

(b) Show that if $\mathbf{AB}=\mathbf{A}$ and $\mathbf{BA}=\mathbf{B}$, then $\mathbf{A}$ and $\mathbf{B}$ are both idempotent.

(c) Show that if $\mathbf{A}$ is idempotent, then $\mathbf{A}^n=\mathbf{A}$ for all positive integers n.

Qns 3.

. Suppose that $\mathbf{P}$ and $\mathbf{Q}$ are $n \times n$ matrices and that $\mathbf{P}^3\mathbf{Q}=\mathbf{PQ}$. Prove that $\mathbf{P}^5\mathbf{Q}=\mathbf{PQ}$.

Answers

Qns 1:

(ai) $(A+B)(A-B) = A(A-B) + B(A-B)$

$= A^2 - AB + BA - B^2$

$\neq A^2 - B^2$

(aii) $(A-B)(A-B) = A^2 - AB - BA + B^2$

$\neq A^2 - 2AB + B^2$

Q2. (a) $A^2 = A$

(b) $AB = A = AA$

$B = A$

$BA = B$

$BB = B$

$B^2 = B$ (Shown)

(c) $A^n = A$ when $n = 1$

Induction

$A^k = A$

$A^{k+1} = A^k A = AA = A$

Q3. $P^5 Q = P^2 P^3 Q$

$= P^2 (PQ)$

$= P^3 Q$

$= P$

3. Transpose

In Lecture:

$$(\mathbf{A}')' = \mathbf{A}$$

$$(\mathbf{A} + \mathbf{B})' = \mathbf{A}' + \mathbf{B}'$$

$$(\alpha\mathbf{A})' = \alpha\mathbf{A}'$$

$$(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$$

Practice Question

Chpt 15.5

Q1.

Find the transposes of $\mathbf{A} = \begin{pmatrix} 3 & 5 & 8 & 3 \\ -1 & 2 & 6 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix}$, and $\mathbf{C} = \begin{pmatrix} 1 & 5 & 0 & -1 \end{pmatrix}$.

Q2.

If $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$ are matrices for which the given products are defined, show that

$$(\mathbf{A}_1\mathbf{A}_2\mathbf{A}_3)' = \mathbf{A}_3'\mathbf{A}_2'\mathbf{A}_1'$$

Generalize to products of n matrices.

Ans:

Q1. $A' = \begin{pmatrix} 3 & -1 \\ 5 & 2 \\ 8 & 6 \\ 3 & 2 \end{pmatrix}$

$B' = \begin{pmatrix} 0 & 1 & -1 & 2 \end{pmatrix}$

$C' = \begin{pmatrix} 1 \\ 5 \\ 0 \\ -1 \end{pmatrix}$

Q2. $(A_1 A_2 A_3)' = (A_3)'(A_1 A_2)'$

$= A_3' A_2' A_1'$

$=$