ECON 6018 TUTORIAL

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- Lesson Tutorial
  Date: 30/01/26

1. Inverse Matrix

From Lect:

Important things to note:

a. All Inverse Matrix are square matrix

b. The determinants must be > 0

Properties of Inverse

a. $AA^{-1} = A^{-1}A = I$

b. $Ax = d$

$x = A^{-1}(d)$

Qns:

(1) Given that A, B are $n \times n$ matrices such that $AB = 0$.
Prove that if A is invertible then B is not invertible

(2) Prove that $(AB)^{-1} = B^{-1}A^{-1}$

(3) Prove that if $BA = I$ then $BA = AB$

(2) $(AB)^{-1} = B^{-1}A^{-1}$

$(AB)(AB)^{-1} = I$

$AB\,B^{-1}A^{-1} = AIA^{-1}$

$= I$

Therefore: $(AB)^{-1} = B^{-1}A^{-1}$

(3) $BA = I$

$BAA^{-1} = IA^{-1}$

$B = A^{-1} \longrightarrow AB = AA^{-1} = I = BA$

$B^{-1}BA = B^{-1}I$ or

$A = B^{-1} \longrightarrow AB = B^{-1}B = I = BA$

2. Determinant

Qns: Determinant of order 2 (16.1)

(1)

Calculate the following determinants:

(a) $\begin{vmatrix} 3 & 0 \\ 2 & 6 \end{vmatrix}$ (b) $\begin{vmatrix} a & a \\ b & b \end{vmatrix}$ (c) $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$ (d) $\begin{vmatrix} 3^t & 2^t \\ 3^{t-1} & 2^{t-1} \end{vmatrix}$

(2)

The *trace* of a square matrix $\mathbf{A}$ is the sum of its diagonal elements, denoted by $\text{tr}(\mathbf{A})$. Given the matrix $\mathbf{A} = \begin{pmatrix} a & 3 \\ b & 1 \end{pmatrix}$, find numbers a and b such that $\text{tr}(\mathbf{A}) = 0$ and $|\mathbf{A}| = -10$.

Answer:

1(a) $\begin{vmatrix} 3 & 0 \\ 2 & 6 \end{vmatrix} = 18 - 0 = 18$

1(b) $\begin{vmatrix} a & a \\ b & b \end{vmatrix} = 0$

1(c) $\begin{vmatrix} (a+b) & (a-b) \\ (a-b) & (a+b) \end{vmatrix} = 4ab$

1(d) $\begin{vmatrix} 3^t & 2^t \\ 3^{t-1} & 2^{t-1} \end{vmatrix} = 3^t 2^t 2^{-1} - 3^t 2^t 3^{-1}$

$= \frac{1}{6}(3^t 2^t) = \frac{6^t}{6} = 6^{t-1}$

(2) $a+1=0 \quad a=-1$

$$\begin{vmatrix} -1 & 3 \\ b & 1 \end{vmatrix} = -1-3b=-10$$

$$-3b=-9$$

$$b=3$$

Qns: Determinants of Order 3 (16.2)

(1)

(a) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 0 & 0 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix}$ (c) $\begin{vmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{vmatrix}$ (d) $\begin{vmatrix} a & 0 & b \\ 0 & e & 0 \\ c & 0 & d \end{vmatrix}$

(2)

Define the matrix $\mathbf{A}_t = \begin{pmatrix} 1 & t & 0 \\ -2 & -2 & -1 \\ 0 & 1 & t \end{pmatrix}$.

(a) Calculate the determinant of $\mathbf{A}_t$, and show that it is never 0.

Ans

1(a) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 0 & 0 \end{vmatrix} = -1(-1)^3 \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix}$

$= (-2)$

$= -2$

1(b) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$

$= -2$

1(c) adf

1(d) = aed

So how does this apply to inverse ☆

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}^T$$

$\downarrow$

$Adj(A)$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ for } 2\times2$$

$|A| = 0$ no solution or infinite sol^n

$|A| > 0$, unique solution

(1) Find the sol^n to

$$
\begin{aligned}
3x_1 + x_2 &= 1 \\
x_1 - x_2 + 2x_3 &= 2 \\
2x_1 + 3x_2 - x_3 &= 3
\end{aligned}
$$

$Sol^{\underline{n}}$

Step 1: Put them in terms of $Ax = d$

$$\begin{pmatrix} 3 & 1 & 0 \\ 1 & -1 & 2 \\ 2 & 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Step 2

$$|A| = \begin{vmatrix} 3 & 1 & 0 \\ 1 & -1 & 2 \\ 2 & 3 & -1 \end{vmatrix} = -15 + 5 = -10$$

Step3

$$|B| = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 2 \\ 3 & 3 & -1 \end{vmatrix} = -3$$

$$|C| = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = 19$$

$$|D| = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & 3 & 3 \end{vmatrix} = -21$$

$$x_1 = \frac{|B|}{|A|} = 0.3 \qquad x_2 = \frac{|C|}{|A|} \qquad x_3 = \frac{|D|}{|A|}$$

$$= 1.9 \qquad = 2.1$$

(2) $Y - C = A_0$ (Chpt 16.2)

$-bY + C + bT = a$

$-tY + T = d$

Soln :

$$\begin{pmatrix} 1 & -1 & 0 \\ -b & 1 & b \\ -t & 0 & 1 \end{pmatrix} \begin{pmatrix} Y \\ C \\ T \end{pmatrix} = \begin{pmatrix} A_0 \\ a \\ d \end{pmatrix}$$

$$|A| = 1 - b - tb$$

$$|B| = A_0 + a - db$$

$$|C| = a - db + A_0 b - A_0 tb$$

$$|D| = at - bd + d + A_0 t$$

(3) Given

$$x + y - z = 1$$

$$x - y + 2z = 2$$

$$x + 2 + az = b$$

For what value of a is there a unique sol^n

Ans:

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 1 & 2 & a \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ b \end{pmatrix}$$

$$\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 1 & 2 & a \end{vmatrix} = -a-4 - (a-2) + (-1)(2+1)$$

$$= -2a - 4 + 2 - 3$$

$$= -2a-5$$

For unique

$$-2a-5 > 0$$

$$-2a > 5$$

$$a > -\frac{5}{2}$$

(4) Given

$$\begin{pmatrix} a & 1 & (a+1) \\ 1 & 2 & 1 \\ 3 & 4 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

For what value of a is there a unique sol^n