UE20CS312 - Data Analytics - Worksheet 2b : Multiple Linear Regression

PES University

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```
###PROBLEM 1
library(tidyverse)
## -- Attaching packages ------ tidyverse 1.3.2 --
## v ggplot2 3.3.6
                     v purrr
                               0.3.4
## v tibble 3.1.8
                               1.0.9
                     v dplyr
## v tidyr
          1.2.0
                     v stringr 1.4.1
## v readr
          2.1.2
                     v forcats 0.5.2
## -- Conflicts ------ tidyverse conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()
                   masks stats::lag()
df<-read_csv('spotify.csv')</pre>
## Rows: 195 Columns: 13
## -- Column specification -----
## Delimiter: ","
## dbl (13): danceability, energy, key, loudness, mode, speechiness, acousticne...
## i Use 'spec()' to retrieve the full column specification for this data.
## i Specify the column types or set 'show_col_types = FALSE' to quiet this message.
head(df)
## # A tibble: 6 x 13
                        key loudn~2 mode speec~3 acous~4 instr~5 liven~6 valence
    danceabil~1 energy
##
          <dbl> <dbl> <dbl>
                              <dbl> <dbl>
                                           <dbl>
                                                   <dbl>
                                                           <dbl>
                                                                  dbl>
                                                                          <dbl>
          0.803 0.624
                              -6.76
## 1
                         7
                                       0 0.0477 0.451 7.34e-4 0.1
                                                                         0.628
## 2
          0.762 0.703
                             -7.95
                                       0 0.306
                                                  0.206 0
                                                                 0.0912 0.519
                         10
## 3
          0.261 0.0149
                          1 -27.5
                                       1 0.0419 0.992 8.97e-1 0.102
                                                                         0.0382
## 4
                                                                         0.582
          0.722 0.736
                          3
                             -6.99
                                       0 0.0585 0.431 1.18e-6 0.123
## 5
          0.787 0.572
                             -7.52
                                       1 0.222
                                                  0.145 0
                                                                 0.0753 0.647
          0.778 0.632
                          8
                              -6.42
                                       1 0.125
                                                  0.0404 0
                                                                 0.0912 0.827
## # ... with 3 more variables: tempo <dbl>, duration_ms <dbl>,
     time_signature <dbl>, and abbreviated variable names 1: danceability,
      2: loudness, 3: speechiness, 4: acousticness, 5: instrumentalness,
## #
    6: liveness
```

```
energy
##
       danceability
                                                  key
                                                              loudness
##
                                                    0
                  0
                                   0
##
                                         acousticness instrumentalness
               mode
                         speechiness
##
                  0
                                                    0
                                   0
##
          liveness
                             valence
                                                           duration ms
                                                tempo
##
                                   0
                                                    0
                                                                     0
                  0
##
     time_signature
##
                  0
df<-as.data.frame(scale(df)) #normalizing</pre>
summary(df)
##
     danceability
                                              key
                                                              loudness
                          energy
          :-2.3390
##
   Min.
                     Min. :-2.44537
                                         Min.
                                             :-1.6097
                                                           Min.
                                                                  :-5.02359
##
   1st Qu.:-0.8040
                     1st Qu.:-0.40343
                                         1st Qu.:-1.0241
                                                           1st Qu.:-0.07362
                     Median : 0.07908
##
   Median : 0.3155
                                         Median : 0.1472
                                                           Median: 0.26293
   Mean
         : 0.0000
                     Mean : 0.00000
                                         Mean : 0.0000
                                                           Mean : 0.00000
   3rd Qu.: 0.7495
                      3rd Qu.: 0.76537
                                                           3rd Qu.: 0.55978
##
                                         3rd Qu.: 0.7328
                     Max. : 1.37476
##
   Max. : 1.4281
                                         Max.
                                                : 1.6112
                                                           Max.
                                                                  : 1.09510
                                         {\tt acousticness}
                                                          instrumentalness
##
        mode
                      speechiness
          :-1.0774
                            :-1.0062
                                               :-0.9947
                                                                 :-0.5555
   Min.
                     Min.
                                        Min.
                                                          Min.
   1st Qu.:-1.0774
                      1st Qu.:-0.7653
                                        1st Qu.:-0.8632
                                                          1st Qu.:-0.5555
##
                                                          Median :-0.5555
##
   Median : 0.9234
                     Median :-0.4381
                                        Median :-0.3307
##
   Mean : 0.0000
                     Mean : 0.0000
                                        Mean : 0.0000
                                                          Mean : 0.0000
   3rd Qu.: 0.9234
                      3rd Qu.: 0.6772
                                        3rd Qu.: 0.5764
                                                          3rd Qu.:-0.2739
##
   Max. : 0.9234
                      Max. : 3.2475
                                        Max.
                                             : 2.1071
                                                          Max.
                                                                : 2.2432
##
      liveness
                         valence
                                                           duration_ms
                                            tempo
##
   Min.
          :-1.0885
                     Min.
                            :-1.7121
                                        Min.
                                              :-2.1690
                                                          Min.
                                                                 :-1.8878
##
   1st Qu.:-0.6082
                     1st Qu.:-0.8391
                                        1st Qu.:-0.7422
                                                          1st Qu.:-0.4866
## Median :-0.4101
                     Median : 0.1172
                                        Median : 0.1357
                                                          Median :-0.1304
   Mean : 0.0000
                     Mean : 0.0000
                                        Mean : 0.0000
                                                          Mean : 0.0000
##
   3rd Qu.: 0.2694
                      3rd Qu.: 0.8363
                                        3rd Qu.: 0.7611
                                                          3rd Qu.: 0.4014
## Max. : 4.5723
                     Max. : 1.8169
                                        Max. : 2.0990
                                                          Max. : 6.1232
  time signature
##
## Min.
          :-6.4538
  1st Qu.: 0.1932
## Median: 0.1932
## Mean : 0.0000
   3rd Qu.: 0.1932
   Max. : 2.4088
#AIC is decreasing with each attribute. Even with far fewer variables, the R2 has decreased by an insign
###PROBLEM 2
model<-lm(energy~., data=df)</pre>
summary(model)
```

colSums(is.na(df))

Call:

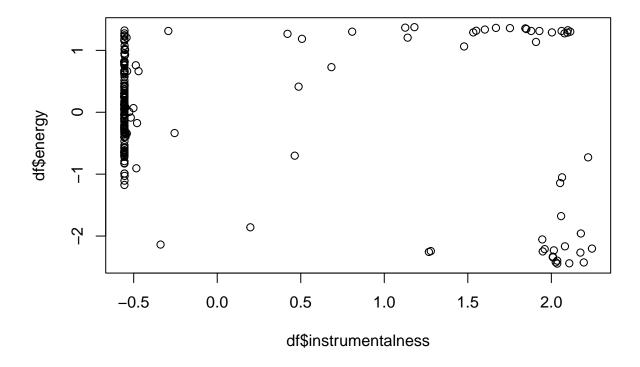
```
## lm(formula = energy ~ ., data = df)
##
## Residuals:
##
                    Median
                                  3Q
       Min
                 1Q
                                          Max
## -1.00232 -0.22889 -0.00973 0.27796 1.24597
##
## Coefficients:
##
                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                         0.000 1.00000
                    9.156e-17 2.920e-02
                   -2.751e-01 5.555e-02 -4.952 1.67e-06 ***
## danceability
                                         1.652 0.10030
## key
                   4.970e-02 3.009e-02
## loudness
                    7.015e-01 4.561e-02 15.381 < 2e-16 ***
## mode
                   -4.794e-02 3.034e-02 -1.580 0.11582
## speechiness
                                         0.670 0.50343
                    2.359e-02 3.519e-02
## acousticness
                   -3.435e-01 4.136e-02 -8.306 2.21e-14 ***
## instrumentalness 1.493e-01 5.577e-02
                                          2.677 0.00811 **
## liveness
                                         0.646 0.51880
                    2.004e-02 3.100e-02
## valence
                   2.046e-01 3.884e-02
                                          5.269 3.85e-07 ***
## tempo
                   -2.395e-02 3.295e-02 -0.727 0.46817
                   -1.865e-02 3.303e-02 -0.565 0.57298
## duration ms
## time_signature
                   2.409e-02 3.220e-02 0.748 0.45535
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 0.4077 on 182 degrees of freedom
## Multiple R-squared: 0.844, Adjusted R-squared: 0.8338
## F-statistic: 82.08 on 12 and 182 DF, p-value: < 2.2e-16
#The adjustment in the "Adjusted R Square" value in the summary output is a correction for the number o
###PROBLEM 3
library(corrplot)
## corrplot 0.92 loaded
```

```
correl<-cor(df)
corrplot(correl,method='number',addCoef.col = 1, number.cex=0.6, tl.cex=0.6,col=colorRampPalette(c("red</pre>
```

	danceability	energy	key	loudness	mode	speechiness	acousticness	instrumentalness	liveness	valence	tempo	duration_ms	time_signature	— 1
danceability	1.00	0.14	-0.06	0.46	0.04	0.39	-0.23	-0.81	-0.14	0.61	0.22	-0.23	0.32	0.9
energy	0.14	1.00	0.13	0.81	-0.07	0.12	-0.77	-0.24	0.17	0.32	0.21	-0.13	0.12	0.8
key	-0.06	0.13	1.00	0.05	-0.10	-0.09	-0.07	0.00	-0.04	0.03	0.10	0.05	0.05	0.7
loudness	0.46	0.81	0.05	1.00	-0.04	0.28	-0.66	-0.54	0.08	0.36	0.27	-0.21	0.21	0.5
mode	0.04	-0.07	-0.10	-0.04	1.00	0.03	-0.03	0.08	-0.05	0.03	-0.04	-0.06	-0.11	0.4
speechiness	0.39	0.12	-0.09	0.28	0.03	1.00	-0.08	-0.34	-0.01	0.18	0.31	-0.39	0.14	0.2
acousticness	-0.23	-0.77	-0.07	-0.66	-0.03	-0.08	1.00	0.29	-0.14	-0.31	-0.26	0.14	-0.14	0.1
instrumentalness	-0.81	-0.24	0.00	-0.54	0.08	-0.34	0.29	1.00	0.06	-0.57	-0.30	0.25	-0.38	0.1 0.2
liveness	-0.14	0.17	-0.04	0.08	-0.05	-0.01	-0.14	0.06	1.00	-0.01	-0.01	-0.14	-0.14	-0.2
valence	0.61	0.32	0.03	0.36	0.03	0.18	-0.31	-0.57	-0.01	1.00	0.22	-0.11	0.20	0.4 0.5
tempo	0.22	0.21	0.10	0.27	-0.04	0.31	-0.26	-0.30	-0.01	0.22	1.00	-0.26	0.07	-0.5
duration_ms	-0.23	-0.13	0.05	-0.21	-0.06	-0.39	0.14	0.25	-0.14	-0.11	-0.26	1.00	-0.04	-0.7
time_signature	0.32	0.12	0.05	0.21					-0.14	_		-0.04		-0.8 -0.9 -1

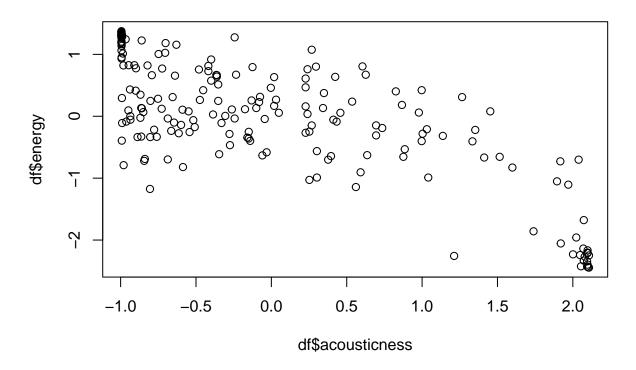
#scatter plots
plot(x=df\$instrumentalness,y=df\$energy, main="instrumentalness vs energy")

instrumentalness vs energy



plot(x=df\$acousticness,y=df\$energy, main="acousticness vs energy")

acousticness vs energy



```
reduced<-lm(energy~loudness+acousticness, data=df) #reducing
summary(reduced)</pre>
```

```
##
## Call:
## lm(formula = energy ~ loudness + acousticness, data = df)
## Residuals:
##
                 1Q
                      Median
## -1.22073 -0.34349 0.00132 0.34870 1.12953
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                1.001e-16 3.541e-02
                                       0.000
                5.375e-01 4.753e-02 11.308 < 2e-16 ***
## loudness
## acousticness -4.152e-01 4.753e-02 -8.734 1.2e-15 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 0.4945 on 192 degrees of freedom
## Multiple R-squared: 0.758, Adjusted R-squared: 0.7555
## F-statistic: 300.7 on 2 and 192 DF, p-value: < 2.2e-16
```

p-value of the F-statistic is < 2.2e-16, which is highly significant, heaviest the predictor variable

```
###PROBLEM 4
anova(reduced, model)
## Analysis of Variance Table
## Model 1: energy ~ loudness + acousticness
## Model 2: energy ~ danceability + key + loudness + mode + speechiness +
      acousticness + instrumentalness + liveness + valence + tempo +
##
      duration_ms + time_signature
##
              RSS Df Sum of Sq
##
    Res.Df
                                         Pr(>F)
## 1
       192 46.942
## 2
       182 30.257 10
                        16.686 10.037 2.416e-13 ***
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
#HO: All coefficients removed from the full model are zero.
#A: At least one of the coefficients removed from the full model is non-zero.
###PROBLEM 5
library(olsrr)
##
## Attaching package: 'olsrr'
## The following object is masked from 'package:datasets':
##
##
      rivers
stepwise<-lm(energy~.,data=df)</pre>
summary(stepwise)
##
## Call:
## lm(formula = energy ~ ., data = df)
##
## Residuals:
##
       Min
                 1Q Median
                                   3Q
## -1.00232 -0.22889 -0.00973 0.27796 1.24597
##
## Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                    9.156e-17 2.920e-02
                                          0.000 1.00000
## danceability
                   -2.751e-01 5.555e-02 -4.952 1.67e-06 ***
                    4.970e-02 3.009e-02
                                          1.652 0.10030
## key
## loudness
                    7.015e-01 4.561e-02 15.381 < 2e-16 ***
## mode
                   -4.794e-02 3.034e-02 -1.580 0.11582
## speechiness
                    2.359e-02 3.519e-02
                                          0.670 0.50343
                -3.435e-01 4.136e-02 -8.306 2.21e-14 ***
## acousticness
## instrumentalness 1.493e-01 5.577e-02
                                           2.677 0.00811 **
## liveness
                    2.004e-02 3.100e-02
                                          0.646 0.51880
## valence
                   2.046e-01 3.884e-02
                                          5.269 3.85e-07 ***
```

-2.395e-02 3.295e-02 -0.727 0.46817

tempo

```
## duration_ms     -1.865e-02     3.303e-02     -0.565     0.57298
## time_signature     2.409e-02     3.220e-02     0.748     0.45535
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4077 on 182 degrees of freedom
## Multiple R-squared: 0.844, Adjusted R-squared: 0.8338
## F-statistic: 82.08 on 12 and 182 DF, p-value: < 2.2e-16</pre>
```

ols_step_both_aic(stepwise)

##

Stepwise Summary

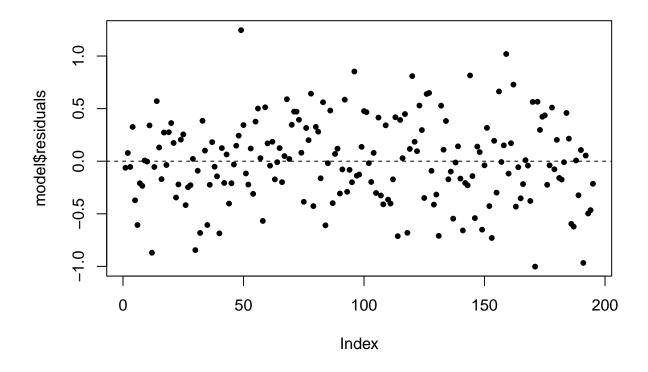
##							
##	Variable	Method	AIC	RSS	Sum Sq	R-Sq	Adj. R-Sq
##							
##	loudness	addition	346.927	65.593	128.407	0.66189	0.66014
##	acousticness	addition	283.690	46.942	147.058	0.75803	0.75551
##	danceability	addition	237.092	36.587	157.413	0.81141	0.80844
##	valence	addition	215.654	32.444	161.556	0.83276	0.82924
##	instrumentalness	addition	212.234	31.554	162.446	0.83735	0.83305
##	mode	addition	211.005	31.036	162.964	0.84002	0.83491
##	key	addition	210.607	30.657	163.343	0.84198	0.83606
##							

###PROBLEM 6

#Full model residuals

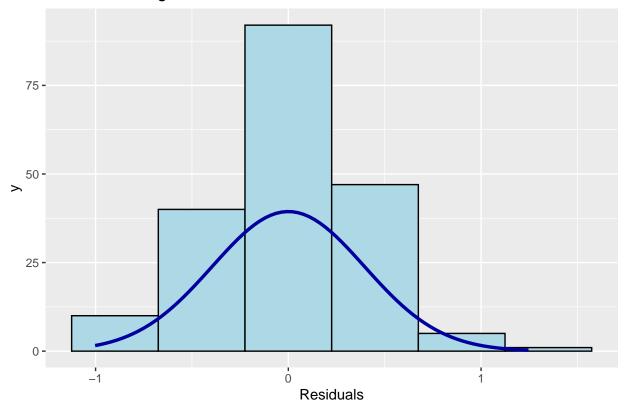
plot(model\$residuals, pch=20)

abline(h=0,lty=2)

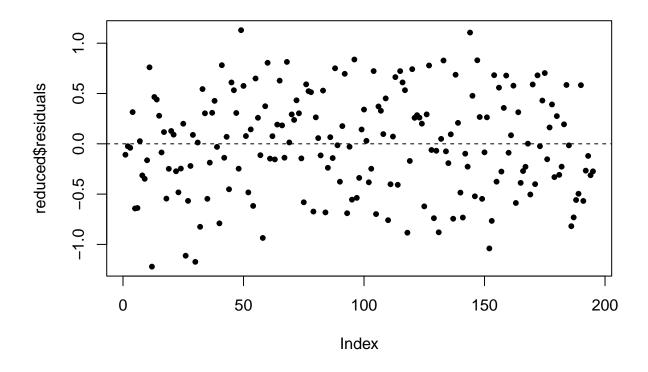


ols_plot_resid_hist(model)

Residual Histogram

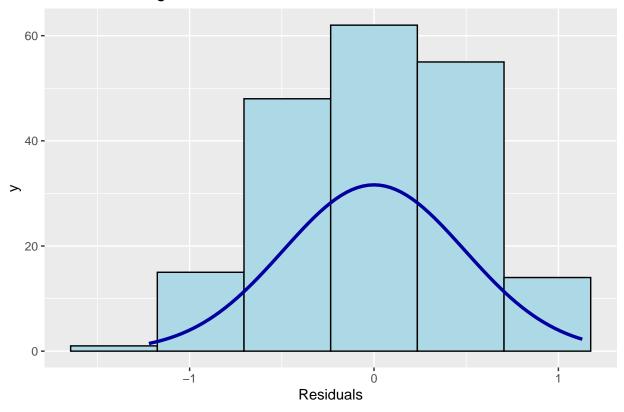


```
#Reduced mdel residuals
plot(reduced$residuals, pch=20)
abline(h=0,lty=2)
```

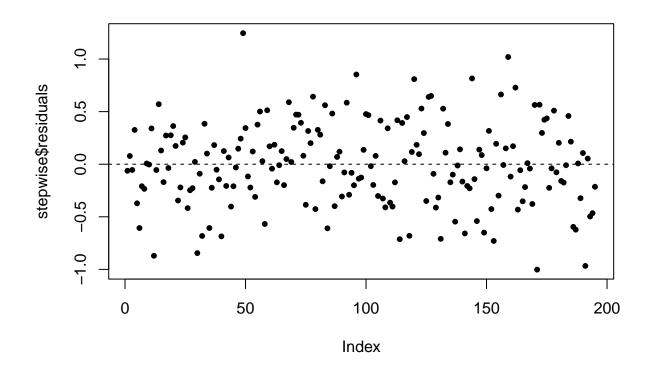


ols_plot_resid_hist(reduced)

Residual Histogram

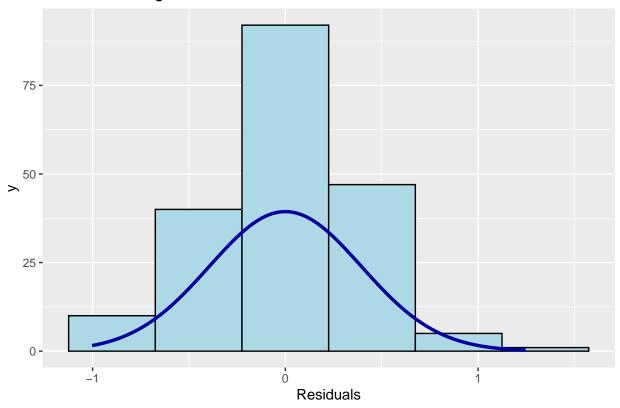


```
#Stepwise model residuals
plot(stepwise$residuals, pch=20)
abline(h=0,lty=2)
```



ols_plot_resid_hist(stepwise)

Residual Histogram

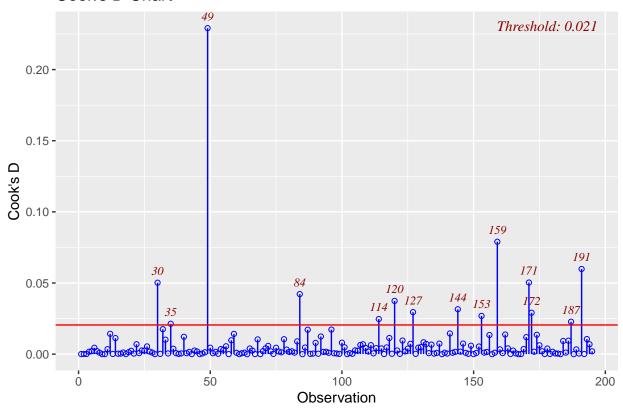


#high density of points close to the origin and a low density of points away from the origin. Hence, th
###PROBLEM 7
ols_vif_tol(model)

```
##
             Variables Tolerance
## 1
          danceability 0.2776703 3.601393
## 2
                   key 0.9467671 1.056226
## 3
              loudness 0.4119898 2.427245
## 4
                  mode 0.9308390 1.074300
## 5
           speechiness 0.6921660 1.444740
## 6
          acousticness 0.5009458 1.996224
## 7
      instrumentalness 0.2755568 3.629016
## 8
              liveness 0.8914397 1.121781
## 9
               valence 0.5680642 1.760364
## 10
                 tempo 0.7892957 1.266952
## 11
           duration_ms 0.7855373 1.273014
## 12
        time_signature 0.8262918 1.210226
```

cookdgraph<-ols_plot_cooksd_chart(model)</pre>

Cook's D Chart



```
cooksD<-cooks.distance(model)</pre>
n<-nrow(df)
influential<-cooksD[(cooksD>4/n)]
head(influential)
##
                                   49
                                              84
                                                         114
## 0.05022185 0.02132929 0.22910565 0.04220288 0.02466182 0.03740761
names_of_influential<-names(influential)</pre>
outliers<-df[names_of_influential,]</pre>
noOutliers<-df %>% anti_join(outliers)
## Joining, by = c("danceability", "energy", "key", "loudness", "mode",
## "speechiness", "acousticness", "instrumentalness", "liveness", "valence",
## "tempo", "duration_ms", "time_signature")
newmod<-lm(energy~., data=noOutliers)</pre>
summary(newmod)
##
## Call:
## lm(formula = energy ~ ., data = noOutliers)
## Residuals:
```

```
1Q
             Median
                      3Q
## -0.76364 -0.20836 0.01581 0.23506 0.95145
##
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
## (Intercept)
            ## danceability
            ## key
## loudness
            0.838411 0.045399 18.468 < 2e-16 ***
            -0.012666 0.026559 -0.477 0.634036
## mode
## speechiness
            ## acousticness
## instrumentalness 0.199483 0.051442 3.878 0.000151 ***
## liveness
             ## valence
             ## tempo
## duration_ms
            ## time_signature
             0.036680
                    0.028430
                          1.290 0.198761
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## Residual standard error: 0.337 on 168 degrees of freedom
## Multiple R-squared: 0.8778, Adjusted R-squared: 0.8691
## F-statistic: 100.6 on 12 and 168 DF, p-value: < 2.2e-16
```

 $\#The\ fit\ improves\ once\ the\ outliers\ are\ removed$