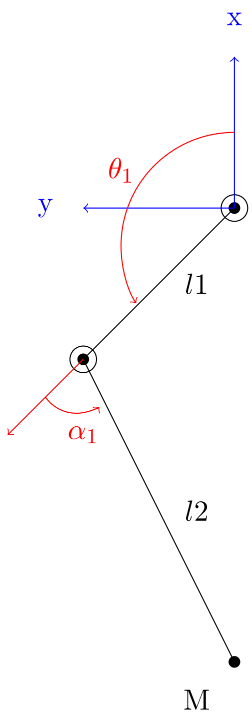
Minitaur leg kinematics

The goal here is to find the motor angles θ_1, θ_2 to give to the two motors in order for the leg end point M to reach the cartesian coordinates (x_M,y_M)
You can see on the figure below a schematic of one of the minitaur leg segment :



Now we can write :

$x_M = l_1 * cos(\theta_1) + l_2 * cos(\theta_1 + \alpha_1)$
$y_M = l_1 * sin(\theta_1) + l_2 * sin(\theta_1 + \alpha_1)$

If we take the polar coordinates of M (r, α_M) :

$r^2 = l_1^2 cos^2(\theta_1) + l_2^2 cos^2(\theta_1 + \alpha_1) + 2l_1 l_2 cos(\theta_1) cos(\theta_1 + \alpha_1) +$
$l_1^2 sin^2(\theta_1) + l_2^2 sin^2(\theta_1 + \alpha_1) + 2l_1 l_2 sin(\theta_1) sin(\theta_1 + \alpha_1)$

$r^2 = l_1^2 + l_2^2 + 2l_1 l_2 [cos(\theta_1)[cos(\theta_1)cos(\alpha_1) - sin(\theta_1)sin(\alpha_1)] + sin(\theta_1)[sin(\theta_1)cos(\alpha_1) +$
$sin(\alpha_1)cos(\theta_1)]]$

$r^2 = l_1^2 + l_2^2 + 2l_1l_2[cos^2(\theta_1) * cos(\alpha_1) + sin^2(\theta_1) * cos(\alpha_1)]$

$r^2 = l_1^2 + l_2^2 + 2l_1l_2cos(\alpha_1)$

Hence we have :

$$\boxed{\alpha_1 = acos(\frac{r^2 - l_1^2 - l_2^2}{2l_1l_2})} \tag{1}$$

Now we can find θ_1 :

$x_M = l_1 * cos(\theta_1) + l_2 * cos(\theta_1 + \alpha_1)$
$y_M = l_1 * sin(\theta_1) + l_2 * sin(\theta_1 + \alpha_1)$

$x_M = l_1 * cos(\theta_1) + l_2 * [cos(\theta_1)cos(\alpha_1) - sin(\theta_1)sin(\alpha_1)]$
$y_M = l_1 * sin(\theta_1) + l_2 * [sin(\theta_1)cos(\alpha_1) + sin(\theta_1)sin(\alpha_1)]$

$x_M = cos(\theta_1)[l_1 + l_2cos(\alpha_1)] + sin(\theta_1)[-l_2 * sin(\alpha_1)]$
$y_M = cos(\theta_1)[l_2 * sin(\alpha_1)] + sin(\theta_1)[l_1 + l_2cos(\alpha_1)]$

$x_M = r_1cos(\theta_1) - r_2sin(\theta_1)$
$y_M = r_2cos(\theta_1) + r_1sin(\theta_1)$

With :

$$\boxed{r_1 = l_1 + l_2cos(\alpha_1)} \tag{2}$$

$$\boxed{r_2 = l_2 * sin(\alpha_1)} \tag{3}$$

$r_1x_M = r_1^2cos(\theta_1) - r_1r_2sin(\theta_1)$
$r_2y_M = r_2^1cos(\theta_1) + r_1r_1sin(\theta_1)$

$r_1x_M + r_2y_M = (r_1^2 + r_2^2)cos(\theta_1)$

$$\boxed{\theta_1 = acos(\frac{r_1x_M + r_2y_M}{r_1^2 + r_2^2})} \tag{4}$$

Now for the second leg segment and by symmetry we will have the same formulas. Nevertheless the y axis will be in an opposite direction. (Indeed the motor of the second leg segment is facing the motor of the first leg segment). We will then have to use (x_M,$-y_M$) has a desired end leg position. Hence we have :

$$\boxed{\alpha_2 = \alpha_1} \tag{5}$$

$$\boxed{\theta_2 = acos(\frac{r_1 x_M - r_2 y_M}{r_1^2 + r_2^2})} \tag{6}$$