NCERT Physics Chapter-15 Q7

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QUESTION 7:

A hospital uses an ultrasonic scanner to locate tumors in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km/s? The operating frequency of the scanner is 4.2 MHz.

SOLUTION:

Using the relation between wavelength(λ), frequency(f) and speed of sound in a medium(v)

$$c = f\lambda \tag{1}$$

$$\lambda = c/f \tag{2}$$

The general equation of a sound

Input Parameter	Value	Description
c	1.7×10^3 m/s	Speed of Wave
f	4.2×10^6 Hz	Frequency
A	1	Wave Amplitude

wave is

$$y(t) = A\sin(2\pi \text{f}t - kx) \tag{3}$$

Input Paramter	Value	Description	Formulae
A	1	Amplitude	-
f	4.2×10^6	Frequency	-
k	4.04×10^-4	Wave Number	$\frac{2\pi}{\lambda}$
x	Arbitrary	Position	-

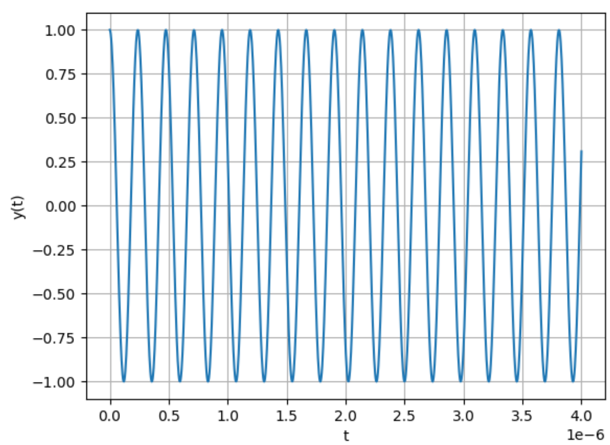

Fig. 1. Waveplot

The frequency of sound relates to the pitch of that sound, the higher the frequency, the higher the pitch and lower the frequency lower the pitch. For example, the voice frequency of males and females are different in their adulthood (Generally females have a higher frequency of voice than males) and the frequency of small kids is even more than that.

The Amplitude of sound refers to the loudness of sound i.e. intensity of the sound. The higher the amplitude higher the loudness and the lower the amplitude, the lower the loudness. The SI unit amplitude is meter (m)

K is called as the wave number, it gives an idea about how many waves are present in a unit length (here it is a meter), So the higher the wave length lower the wave number and lower the wavelength higher the wave number.

Physically, the wavelength of a sound wave can be visualized as the distance between two successive compressions (high-pressure regions) or rarefactions (low-pressure regions) in the air or any other medium through which the sound is propagating.

Physical interpretation of speed of sound

The speed of a sound wave in a medium can be interpreted in a physical manner without directly referencing its frequency and wavelength. The speed of sound is determined by the properties of the medium through which it travels, primarily its density (ρ) and elasticity.

The general formula for the speed of sound in a medium is given by:

$$\mathrm{v} = \sqrt{\frac{\gamma \cdot P}{\rho}}$$

where:

- v is the speed of sound
- γ is the adiabatic index or ratio of specific heats
- P is the pressure of the medium
- ρ is the density of the medium.

Fourier transform (Wave equation)

$$S(f) = \int_0^T A\cos(kx - 2\pi t + \phi)e^{-2\pi f t}dt$$

$$S(f) = \frac{A}{2}\left(\frac{e^{j(kx-2\pi tt+\phi)} + e^{-j(kx-2\pi t+\phi)}\Big) e^{-j2\pi t} \cdot dt}{2}\right.$$

$$S(f) = \frac{A}{2}\left\{\int_0^T e^{j(kx+\phi)}e^{-4\pi f t} \cdot dt + \int_0^T e^{-j(kx+\phi)} \cdot 1 \cdot dt\right)$$

$$S(f) = \frac{A}{2}Te^{-j(kx+\phi)}$$

$$S(f) = \frac{A}{2f}e^{-j(kx+\phi)}$$

$$S(f) = \frac{A}{2f}Cos(kx + \phi) - \frac{A}{2f}jSin(kx + \phi)$$