NCERT Mathematics Ex 9.4 Q6

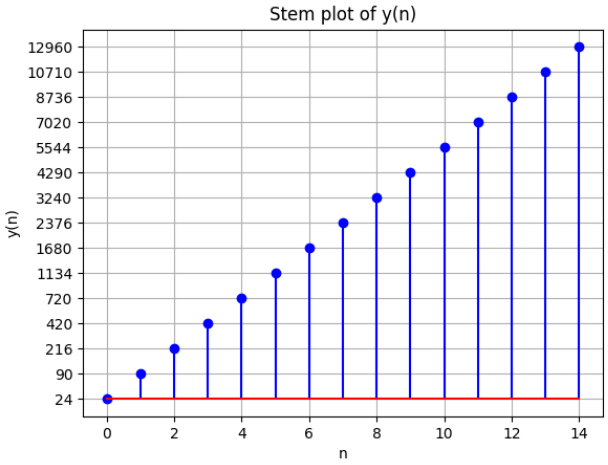
EE23BTECH11059 - Tejas

Question: 1) Find the sum to n terms of $3 \times 8 + 6 \times 11 + 9 \times 14 + ...$

Solution:

Writing the general term of the series

$$x(n) = (3n + 3)(8 + 3n) \tag{2.15}$$



The sum of n terms of this progression can be given by:

$$y(n) = x(n) * u(n) \tag{2.16}$$

$$\implies Y(z) = X(z)\,U(z) \tag{2.17}$$

z transform of $x(n)$:

$$X(z) = \sum_{n=0}^{\infty} (3n + 3)(3n + 8)z^{-n} \tag{2.18}$$

$$X(z) = \sum_{n=0}^{\infty} (9n^2 + 33n + 24)z^{-n} \tag{2.19}$$

$$X(z) = 9z^{-1}\frac{(1 + z^{-1})}{(1 - z^{-1})^3} + \frac{33}{(1 - z^{-1})^2} + 24\frac{1}{1 - z^{-1}}; |z| > 1 \tag{2.20}$$

z transform of $y(n)$:
using equation (2.20) and equation (2.17):

$$Y(z) = \frac{(18 - 9z^2 + 67)Z^{-1}}{(1 - z^{-1})^3} + \frac{(42 - 9z^{-1})}{(1 - z^{-1})^2} \tag{2.21}$$

Using (.2.6) to find the inverse Z-transform,

We get $y(n)$ as:

$$y(n) = \frac{33n(n + 1)}{2} + \frac{9n(n + 1)(2n + 1)}{6} + 24n \tag{2.22}$$