# SEQUENCE AND SERIES

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Q: Find the sum to n terms of $3 \times 8 + 6 \times 11 + 9 \times 14 + ...$

**Solution:**

| Variable | Description | Value |
|----------|-------------|-------|
| $x(n)$ | $n^{th}$ term of sequence | $(3n + 3)(3n + 8)u(n)$ |

TABLE 0

INPUT PARAMETERS

Sum of $n$ terms of AP is given by

$$x(n) = (3n + 3)(3n + 8) u(n) \tag{1}$$

$$y(n) = x(n) * u(n) \tag{2}$$

$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{(1 - z^{-1})} \quad |z| > 1 \tag{3}$$

$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \tag{4}$$

$$n^2 u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \quad |z| > 1 \tag{5}$$

$$n^3 u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(1 + 4z^{-1} + z^{-2}\right)}{(1 - z^{-1})^4} \quad |z| > 1 \tag{6}$$

$$\implies X(z) = 9z^{-1}\frac{\left(1 + z^{-1}\right)}{(1 - z^{-1})^3} + \frac{33\left(z^{-1}\right)}{(1 - z^{-1})^2} + \frac{24}{(1 - z^{-1})} \quad |z| > 1 \tag{7}$$

$$Y(z) = X(z) U(z) \tag{8}$$

$$\implies Y(z) = 9z^{-1}\frac{\left(1 + z^{-1}\right)}{(1 - z^{-1})^4} + \frac{33\left(z^{-1}\right)}{(1 - z^{-1})^3} + \frac{24}{(1 - z^{-1})^2} \quad |z| > 1 \tag{9}$$

Now from (3), (4), (5), (6), (9) By using inverse Z-transform pairs,

$$y(n) = \left(\frac{9n(n + 1)(2n + 1)}{6} + \frac{33n(n + 1)}{2} + 24(n + 1)\right) u(n) \tag{10}$$

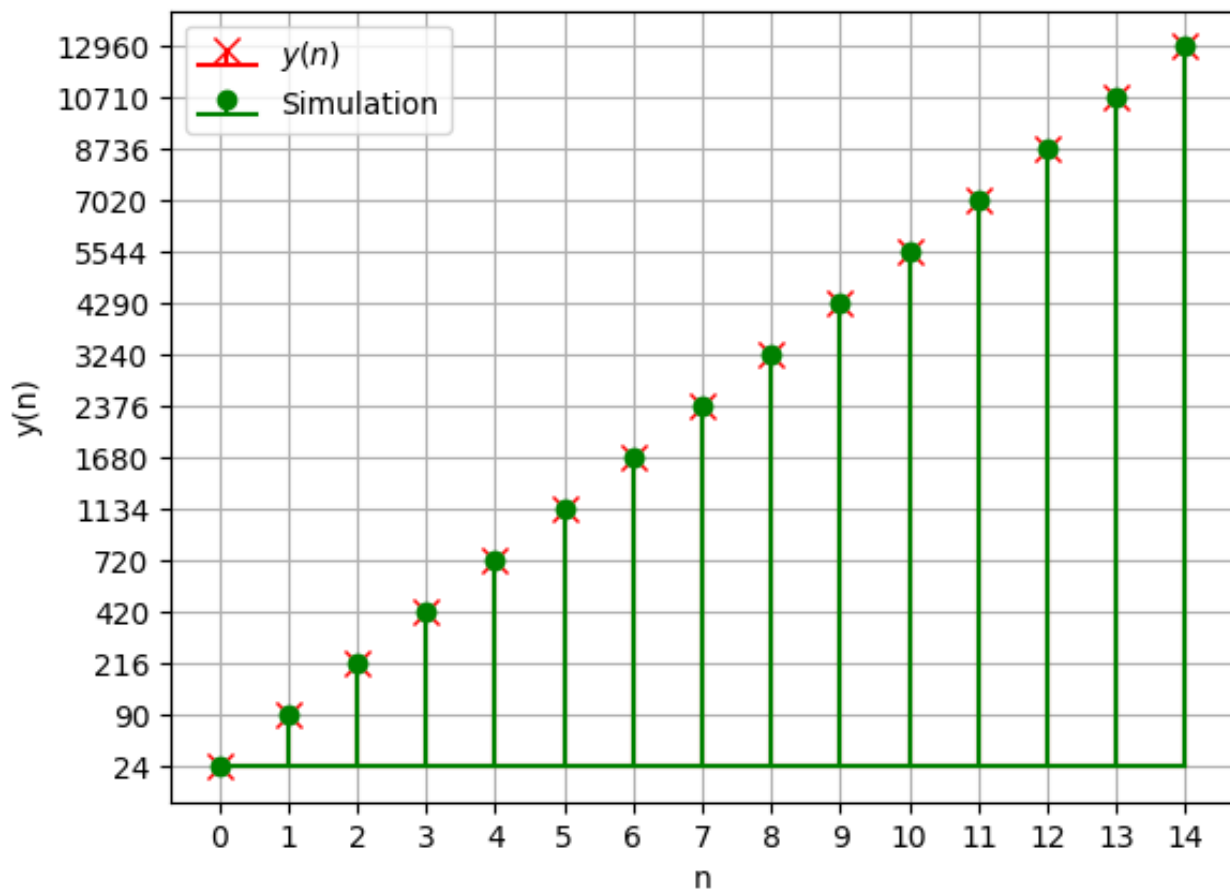$\therefore$ Sum of $n$ terms of the series whose $n^{th}$ term is given by $(3n + 3)(3n + 8) u(n)$ is $(\frac{9n(n+1)(2n+1)}{6} + \frac{33n(n+1)}{2} + 24(n + 1))u(n)$

Fig. 0.  Theory vs Simulation