

SEQUENCE AND SERIES

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Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution:

Variable	Description	Value
$x(n)$	n^{th} term of sequence	$(4n + 10)u(n)$

TABLE 0

INPUT PARAMETERS

For an AP,

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (1)$$

$$\Rightarrow X(z) = \frac{10}{(1 - z^{-1})} + \frac{4z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \quad (2)$$

$$y(n) = x(n) * u(n) \quad (3)$$

$$\Rightarrow Y(z) = X(z) U(z) \quad (4)$$

$$\Rightarrow Y(z) = \frac{10}{(1 - z^{-1})^2} + \frac{4z^{-1}}{(1 - z^{-1})^3} \quad (5)$$

$$\Rightarrow Y(z) = \frac{(-6z^{-1} + 10)}{(1 - z^{-1})^3}, |z| > 1 \quad (6)$$

Using Contour Integration to find the inverse Z-transform,

$$y(50) = \frac{1}{2\pi j} \oint_C Y(z) z^{49} dz \quad (7)$$

$$= \frac{1}{2\pi j} \oint_C \frac{(-6z^{-1} + 10)z^{49}}{(1 - z^{-1})^3} dz \quad (8)$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (9)$$

$$\Rightarrow R = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{(-6z^{-1} + 10)z^{52}}{(z-1)^3} \right) \quad (10)$$

$$\Rightarrow R = \frac{1}{2} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (10z^{52} - 6z^{51}) \quad (11)$$

$$\Rightarrow R = 5610 \quad (12)$$

$$\therefore y(50) = 5610 \quad (13)$$

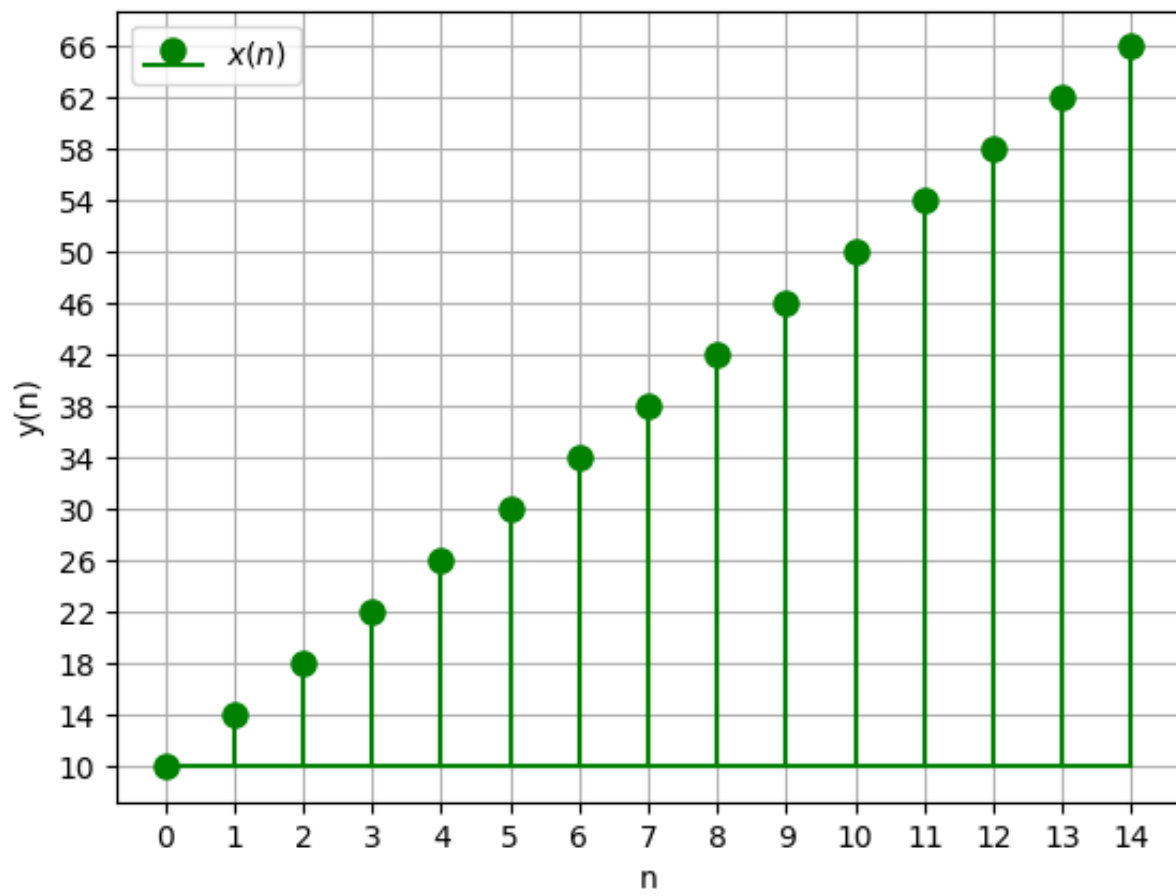


Fig. 0. Plot of $x(n)$ vs n