NCERT Mathematics Ex 9.4 Q6

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Question: 1) Find the sum to n terms of $3 \times 8 + 6 \times 11 + 9 \times 14 + ...$
Solution:
Writing the general term of the series

$$x_n = (3r + 3) \times (8 + 3r)$$

$$S_n = \sum_{r=0}^{n} 9r^2 + 33r + 24 \tag{1}$$

Using formulas for the sum of n terms (i) and sum of the squares of the n terms (ii)

$$\sum_{r=0}^{n} r = \frac{n(n+1)}{2} \tag{i}$$

$$\sum_{r=0}^{n} r^2 = \frac{n(n+1)(2n+1)}{6} \tag{ii}$$

Equation (1) evaluates to

$$S_n = \frac{33n(n+1)}{2} + \frac{9n(n+1)(2n+1)}{6} + 24n \tag{2}$$

z transform of x_n:

$$X(z) = \sum_{n=0}^{\infty} (3n+3)(3n+8)z^{-n} \tag{3}$$

$$X(z) = \sum_{n=0}^{\infty} (9n^2 + 33n + 24)z^{-n} \tag{4}$$

$$X(z) = 9z^{-1}\frac{(1+z^{-1})}{(1-z^{-1})^3} + \frac{33}{(1-z^{-1})^2} + 24\frac{1}{1-z^{-1}} \quad ; |z| > 1 \tag{5}$$

z transform of S_n:

$$S(z) = \sum_{n=0}^{\infty} \left(\frac{33n(n+1)}{2} + \frac{9n(n+1)(2n+1)}{6} + 24n) \right) \tag{6}$$

$$S(z) = \frac{33}{2}\left(\sum_{n=0}^{\infty} n^2 z^{-n} + \sum_{n=0}^{\infty} nz^{-n} \right) + \frac{9}{6}\left(\sum_{n=0}^{\infty} n^3 z^{-n} + \sum_{n=0}^{\infty} n^2 z^{-n} + \sum_{n=0}^{\infty} nz^{-n} \right) + 24\sum_{n=0}^{\infty} nz^{-n} \tag{7}$$

$$S(z) = \frac{18z^{-1}\frac{-9}{z^{-1}} + 6}{(1-z^{-1})^3} + \frac{42 - 9z^{-1}}{(1-z^{-1})^2} \tag{8}$$