SEQUENCE AND SERIES

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Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
Solution:

Variable	Description	Value
$x(1)$	Second term of AP	14
$x(2)$	Third term of AP	18
$x(0)$	First term of AP	$2x(1) - x(2) = 10$
d	Common difference of AP $(x(2) - x(1))$	4
$x(n)$	n^{th} term of sequence	$(4n + 10)u(n)$

TABLE 0
INPUT PARAMETERS

For an AP,

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{1}$$

$$\implies X(z) = \frac{10}{(1 - z^{-1})} + \frac{4z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \tag{2}$$

$$y(n) = x(n) * u(n) \tag{3}$$

$$Y(z) = X(z)\, U(z) \tag{4}$$

$$Y(z) = \frac{10}{(1 - z^{-1})^2} + \frac{4z^{-1}}{(1 - z^{-1})^3} \tag{5}$$

$$\implies Y(z) = \frac{(-6z^{-1} + 10)}{(1 - z^{-1})^3}, |z| > 1 \tag{6}$$

Using Contour Integration to find the inverse Z-transform,

$$y(50) = \frac{1}{2\pi j} \oint_C Y(z)\, z^{49}\, dz \tag{7}$$

$$= \frac{1}{2\pi j} \oint_C \frac{(-6z^{-1} + 10)z^{49}}{(1 - z^{-1})^3}\, dz \tag{8}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z - a)^m f(z)\right) \tag{9}$$

$$\implies R = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \frac{(-6z^{-1} + 10)z^{52}}{(z - 1)^3}\right) \tag{10}$$

$$\implies R = \frac{1}{2} \lim_{z \to 1} \frac{d^2}{dz^2} (10z^{52} - 6z^{51}) \tag{11}$$

$$\implies R = 5610 \tag{12}$$

$$\therefore y(50) = 5610 \tag{13}$$

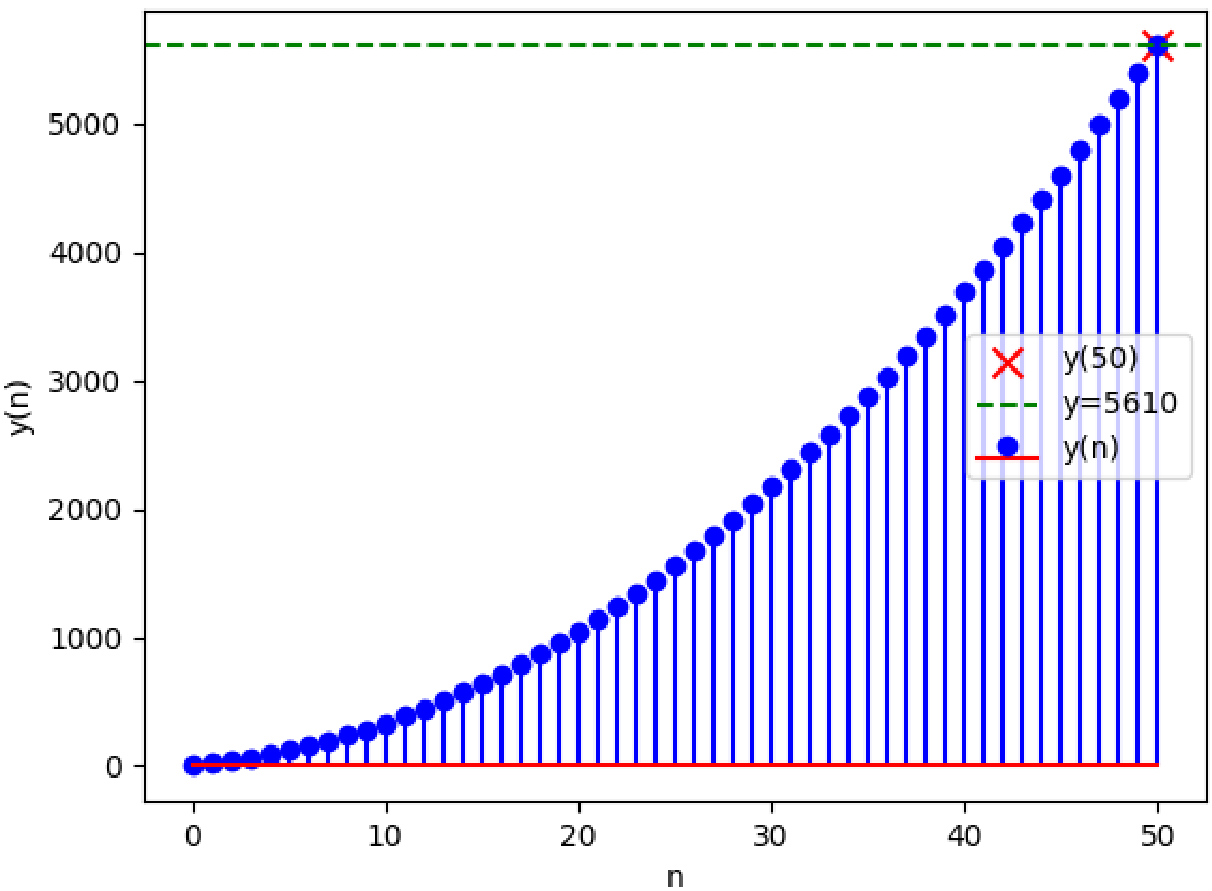

Fig. 0. Analysis vs Simulation