

NCERT Mathematics Ex 9.4 Q6

EE23BTECH11059 - Tejas

Question: 1) Find the sum to n terms of $3 \times 8 + 6 \times 11 + 9 \times 14 + \dots$ **Solution:** Writing the general term of the series

$$x(n) = (3n + 3)(8 + 3n)u(n) \quad (1)$$

The sum of n terms of this progression can be given by:

$$y(n) = x(n) * u(n) \quad (2)$$

$$\Rightarrow Y(z) = X(z)U(z) \quad (3)$$

z transform of $x(n)$:

$$X(z) = \sum_{n=0}^{\infty} (3n + 3)(3n + 8)z^{-n} \quad (4)$$

$$X(z) = \sum_{n=0}^{\infty} (9n^2 + 33n + 24)z^{-n} \quad (5)$$

$$X(z) = 9z^{-1} \frac{(1 + z^{-1})}{(1 - z^{-1})^3} + \frac{33}{(1 - z^{-1})^2} + \frac{24}{(1 - z^{-1})}; |z| > 1 \quad (6)$$

Using equation (6) and equation (3) we get $Y(z)$ as:

$$Y(z) = \frac{(z)^{-1}(18 - 9z^2 + 67)}{(1 - z^{-1})^3} + \frac{(42 - 9z^{-1})}{(1 - z^{-1})^2} \quad (7)$$

$$nu(n) \xleftrightarrow{z} \frac{z^{-1}}{(1 - z^{-1})^2} \{|z| > 1\} \quad (8)$$

$$n^2u(n) \xleftrightarrow{z} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \{|z| > 1\} \quad (9)$$

$$n^3u(n) \xleftrightarrow{z} \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4} \{|z| > 1\} \quad (10)$$

$$n^4un \xleftrightarrow{z} \frac{z^{-1}(1 + 11z^{-1} + 11z^{-2} + z^{-3})}{(1 - z^{-1})^5} \{|z| > 1\} \quad (11)$$

Taking reverse z transform, using equations (8) to (11) We get $y(n)$ as:

$$y(n) = \frac{33n(n+1)}{2} + \frac{9n(n+1)(2n+1)}{6} + 24(n+1) \quad (12)$$

