NCERT Mathematics Ex 9.4 Q6

EE23BTECH11059 - Tejas

Question: 1) Find the sum to n terms of
$3 \times 8 + 6 \times 11 + 9 \times 14 + ...$
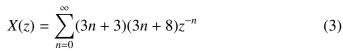
Solution:

Writing the general term of the series

$$x(n) = (3n + 3)(8 + 3n) \tag{1}$$

$$y(n) = \sum_{r=0}^{n}(3r + 3)(8 + 3r) \tag{2}$$

Stem Plot of Sn

z transform of $x(n)$:

$$X(z) = \sum_{n=0}^{\infty}(3n + 3)(3n + 8)z^{-n} \tag{3}$$

$$X(z) = \sum_{n=0}^{\infty}(9n^2 + 33n + 24)z^{-n} \tag{4}$$

$$X(z) = 9z^{-1}\frac{(1 + z^{-1})}{(1 - z^{-1})^3} + \frac{33}{(1 - z^{-1})^2} + 24\frac{1}{1 - z^{-1}} \quad ; |z| > 1 \tag{5}$$

z transform of $y(n)$:

$$y(z) = \sum_{n=0}^{\infty}\left(\frac{33n(n + 1)}{2} + \frac{9n(n + 1)(2n + 1)}{6} + 24n)\right) \tag{6}$$

$$y(z) = \frac{33}{2}\left(\sum_{n=0}^{\infty}n^2z^{-n} + \sum_{n=0}^{\infty}nz^{-n}\right) + \frac{9}{6}\left(\sum_{n=0}^{\infty}n^3z^{-n} + \sum_{n=0}^{\infty}n^2z^{-n} + \sum_{n=0}^{\infty}nz^{-n}\right) + 24\sum_{n=0}^{\infty}nz^{-n} \tag{7}$$

$$y(z) = \frac{18z^{-1}\frac{-9}{z^{-1}} + 6}{(1 - z^{-1})^3} + \frac{42 - 9z^{-1}}{(1 - z^{-1})^2} \tag{8}$$

Using Contour Integration to find the inverse Z-transform,

$$R = \frac{1}{(m - 1)!}\lim_{z \to a}\frac{d^{m-1}}{dz^{m-1}}\left((z - a)^m f(z)\right) \tag{5}$$

We get $y(n)$ as:

$$y(n) = \frac{33n(n + 1)}{2} + \frac{9n(n + 1)(2n + 1)}{6} + 24n \tag{6}$$