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NCERT Mathematics Ex 9.4 Q6

EE23BTECH11059 - Tejas

Question: 1) Find the sum to n terms of $3 \times 8 + 6 \times 11 + 9 \times 14 + ...$

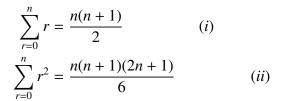
Solution:

Writing the general term of the series

$$x_n = (3r+3) \times (8+3r)$$

$$S_n = \sum_{r=0}^{n} 9r^2 + 33r + 24$$
 (1)

Using formulas for the sum of n terms (i) and sum of the squares of the n terms (ii)



Equation (1) evaluates to

$$S_n = \frac{33n(n+1)}{2} + \frac{9n(n+1)(2n+1)}{6} + 24n \quad (2)$$

z transform of x_n :

$$X(z) = \sum_{n=0}^{\infty} (3n+3)(3n+8)z^{-n}$$
 (3)

$$X(z) = \sum_{n=0}^{\infty} (9n^2 + 33n + 24)z^{-n}$$
 (4)

$$X(z) = 9z^{-1} \frac{(1+z^{-1})}{(1-z^{-1})^3} + \frac{33}{(1-z^{-1})^2} + 24\frac{1}{1-z^{-1}} \quad ; |z| > 1 \quad (5)$$

z transform of S_n :

$$S(z) = \sum_{n=0}^{\infty} \left(\frac{33n(n+1)}{2} + \frac{9n(n+1)(2n+1)}{6} + 24n \right)$$
(6)

$$S(z) = \frac{33}{2} \left(\sum_{n=0}^{\infty} n^2 z^{-n} + \sum_{n=0}^{\infty} n z^{-n} \right) + \frac{9}{6} \left(\sum_{n=0}^{\infty} n^3 z^{-n} + \sum_{n=0}^{\infty} n^2 z^{-n} + \sum_{n=0}^{\infty} n z^{-n} \right) + 24 \sum_{n=0}^{\infty} n z^{-n}$$
(7)

$$S(z) = \frac{18z^{-1} \frac{-9}{z^{-1}} + 6}{(1 - z^{-1})^3} + \frac{42 - 9z^{-1}}{(1 - z^{-1})^2}$$
(8)

