# SEQUENCE AND SERIES

EE23BTECH11059- Tejas Mehtre*

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
**Solution:**

| Variable | Description | Value |
|----------|-------------|-------|
| $x(0)$ | First term of $AP$ | 10 |
| $d$ | Common difference of $AP$ | 4 |
| $x(n)$ | $n^{th}$ term of sequence | $(4n + 10)u(n)$ |

TABLE 0

INPUT PARAMETERS

For an $AP$,

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{1}$$

$$\implies X(z) = \frac{10}{(1 - z^{-1})} + \frac{4z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \tag{2}$$

$$y(n) = x(n) * u(n) \tag{3}$$

$$\implies Y(z) = X(z)U(z) \tag{4}$$

$$\implies Y(z) = \frac{10}{(1 - z^{-1})^2} + \frac{4z^{-1}}{(1 - z^{-1})^3} \tag{5}$$

$$\implies Y(z) = \frac{(-6z^{-1} + 10)}{(1 - z^{-1})^3}, |z| > 1 \tag{6}$$

Using Contour Integration to find the inverse $Z$-transform,

$$y(50) = \frac{1}{2\pi j} \oint_C Y(z) \, z^{49} \, dz \tag{7}$$

$$= \frac{1}{2\pi j} \oint_C \frac{(-6z^{-1} + 10)z^{49}}{(1 - z^{-1})^3} \, dz \tag{8}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \tag{9}$$

$$\implies R = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left( (z - 1)^3 \frac{(-6z^{-1} + 10)z^{52}}{(z - 1)^3} \right) \tag{10}$$

$$\implies R = \frac{1}{2} \lim_{z \to 1} \frac{d^2}{dz^2} (10z^{52} - 6z^{51}) \tag{11}$$
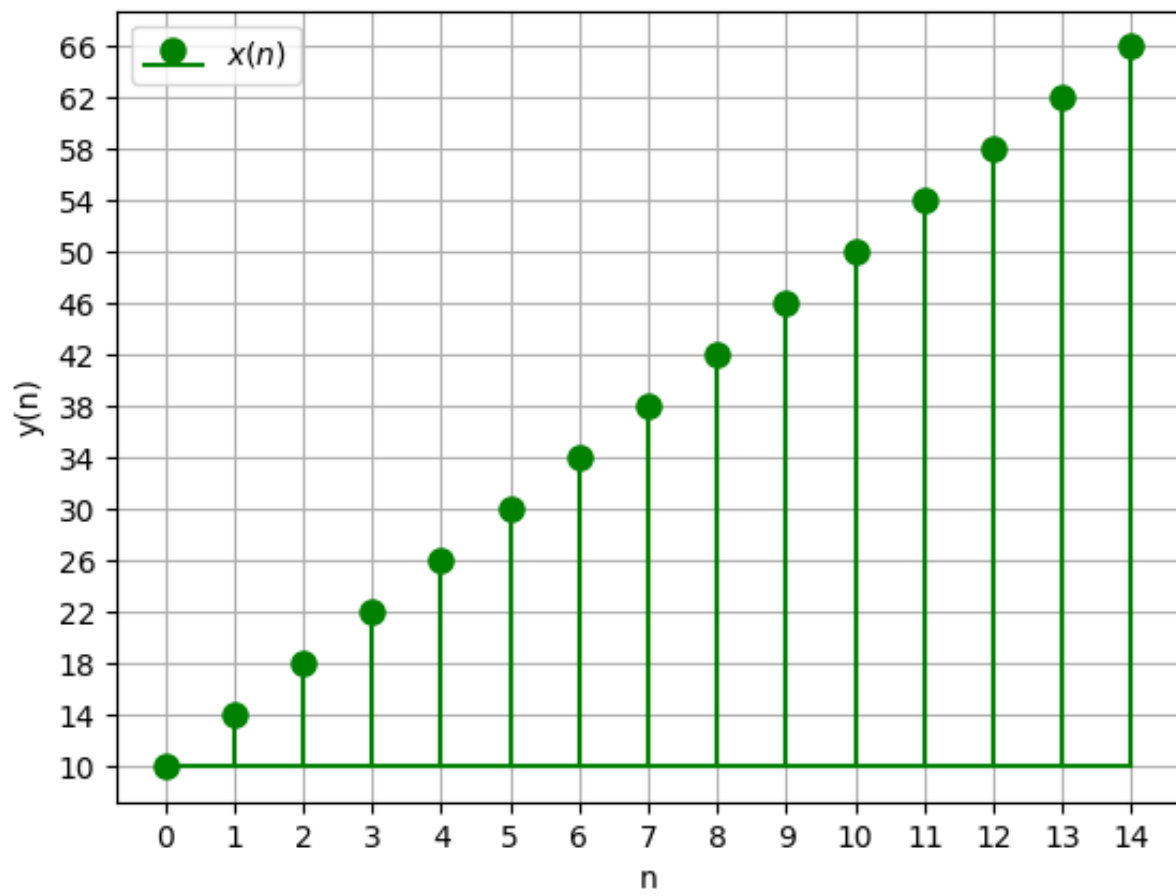
$$\implies R = 5610 \tag{12}$$

$$\therefore y(50) = 5610 \tag{13}$$

Fig. 0. Plot of x(n) *vs* n