

NCERT Mathematics Ex 9.4 Q6

EE23BTECH11059 - Tejas

Question: 1) Find the sum to n terms of
 $3 \times 8 + 6 \times 11 + 9 \times 14 + \dots$

Solution:

Writing the general term of the series

$$x(n) = (3n + 3)(8 + 3n)u(n) \quad (1)$$

The sum of n terms of this progression can be given by:

$$y(n) = x(n) * u(n) \quad (2)$$

$$\Rightarrow Y(z) = X(z) U(z) \quad (3)$$

z transform of $x(n)$:

$$X(z) = \sum_{n=0}^{\infty} (3n + 3)(3n + 8)z^{-n} \quad (4)$$

$$X(z) = \sum_{n=0}^{\infty} (9n^2 + 33n + 24)z^{-n} \quad (5)$$

$$X(z) = 9z^{-1} \frac{(1 + z^{-1})}{(1 - z^{-1})^3} + \frac{33}{(1 - z^{-1})^2} + 24 \frac{1}{1 - z^{-1}}; |z| > 1 \quad (6)$$

z transform of $y(n)$:

using equation (6) and equation (3):

$$Y(z) = \frac{(18 - 9z^2 + 67)Z^{-1}}{(1 - z^{-1})^3} + \frac{(42 - 9z^{-1})}{(1 - z^{-1})^2} \quad (7)$$

Using Contour Integration to find the inverse Z -transform,

We get $y(n)$ as:

$$y(n) = \frac{33n(n+1)}{2} + \frac{9n(n+1)(2n+1)}{6} + 24n \quad (8)$$

