

SEQUENCE AND SERIES

EE23BTECH11059- Tejas Mehtre*

Q: Find the sum to n terms of $3 \times 8 + 6 \times 11 + 9 \times 14 + \dots$

Solution:

Variable	Description	Value
$x(n)$	n^{th} term of sequence	$(3n + 3)(3n + 8)u(n)$

TABLE 0

INPUT PARAMETERS

Sum of n terms of AP is given by

$$x(n) = (3n + 3)(3n + 8)u(n) \quad (1)$$

$$y(n) = x(n) * u(n) \quad (2)$$

$$u(n) \xleftrightarrow{Z} \frac{1}{(1 - z^{-1})} \quad |z| > 1 \quad (3)$$

$$nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (4)$$

$$n^2u(n) \xleftrightarrow{Z} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \quad |z| > 1 \quad (5)$$

$$n^3u(n) \xleftrightarrow{Z} \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4} \quad |z| > 1 \quad (6)$$

$$\Rightarrow X(z) = 9z^{-1} \frac{(1 + z^{-1})}{(1 - z^{-1})^3} + \frac{33(z^{-1})}{(1 - z^{-1})^2} + \frac{24}{(1 - z^{-1})} \quad |z| > 1 \quad (7)$$

$$Y(z) = X(z)U(z) \quad (8)$$

$$\Rightarrow Y(z) = 9z^{-1} \frac{(1 + z^{-1})}{(1 - z^{-1})^4} + \frac{33(z^{-1})}{(1 - z^{-1})^3} + \frac{24}{(1 - z^{-1})^2} \quad |z| > 1 \quad (9)$$

Now from (3), (4), (5), (6), (9) By using inverse Z-transform pairs,

$$y(n) = \left(\frac{9n(n+1)(2n+1)}{6} + \frac{33n(n+1)}{2} + 24(n+1) \right) u(n) \quad (10)$$

\therefore Sum of n terms of the series whose n^{th} term is given by $(3n + 3)(3n + 8)u(n)$ is $\left(\frac{9n(n+1)(2n+1)}{6} + \frac{33n(n+1)}{2} + 24(n+1) \right) u(n)$

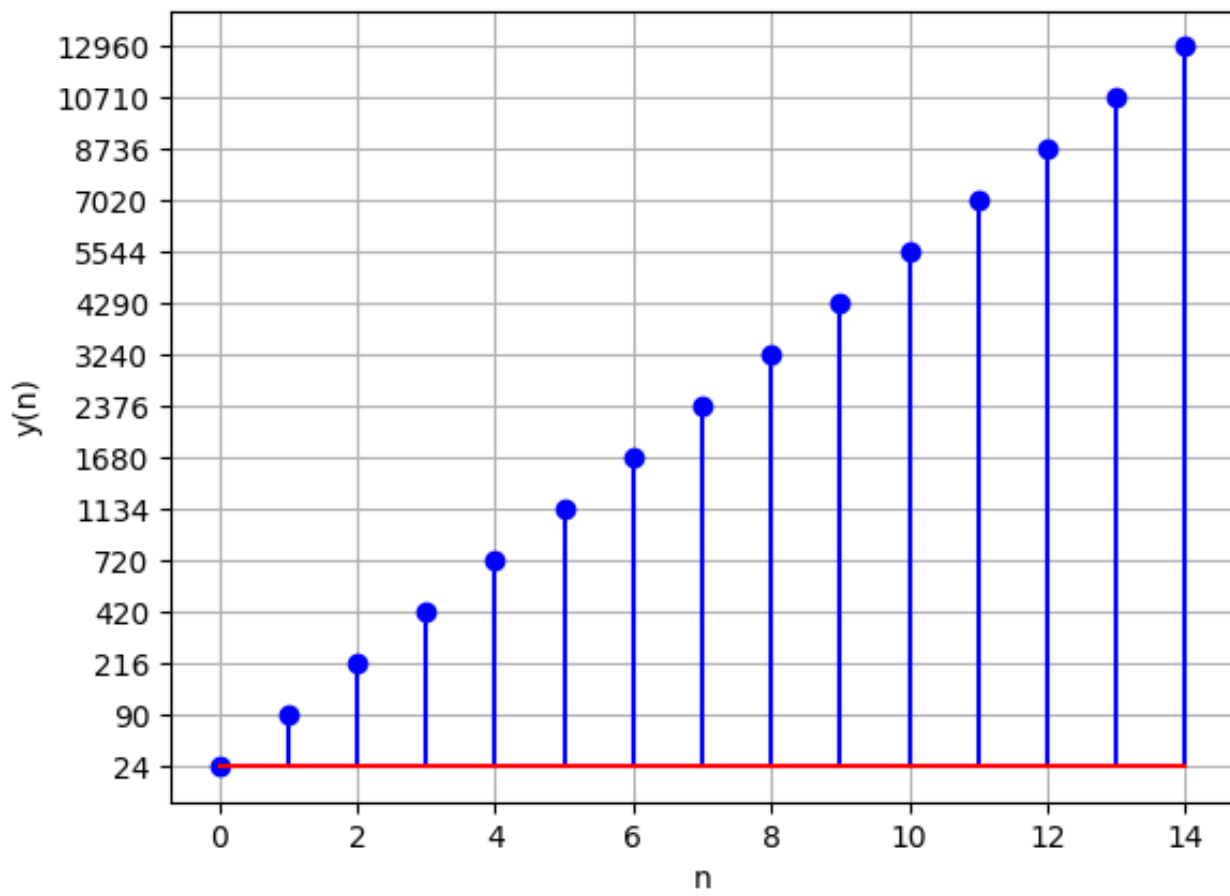


Fig. 0. Sum of n terms