NCERT Mathematics Ex 9.4 Q6

EE23BTECH11059 - Tejas

Question: 1) Find the sum to n terms of $3 \times 8 + 6 \times 11 + 9 \times 14 + ...$

Solution:

Writing the general term of the series

$$x(n) = (3n+3)(8+3n) \tag{1}$$

The sum of n terms of this progression can be given by:

$$y(n) = x(n) * u(n)$$
 (2)

$$\implies Y(z) = X(z) U(z)$$
 (3)

z transform of x(n):

$$X(z) = \sum_{n=0}^{\infty} (3n+3)(3n+8)z^{-n}$$
 (4)

$$X(z) = \sum_{n=0}^{\infty} (9n^2 + 33n + 24)z^{-n}$$
 (5)

$$X(z) = 9z^{-1} \frac{(1+z^{-1})}{(1-z^{-1})^3} + \frac{33}{(1-z^{-1})^2} + 24\frac{1}{1-z^{-1}}; |z| > 1$$
(6)

z transform of y(n): using equation (6) and equation (3):

$$Y(z) = \frac{18z^{-1}\frac{-9}{z^{-1}} + 6}{(1 - z^{-1})^3} + \frac{42 - 9z^{-1}}{(1 - z^{-1})^2}$$
(7)

Using Contour Integration to find the inverse Z-transform,

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z) \right) \tag{8}$$

We get y(n) as:

$$y(n) = \frac{33n(n+1)}{2} + \frac{9n(n+1)(2n+1)}{6} + 24n \quad (9)$$

