

NCERT Mathematics Ex 9.4 Q6

EE23BTECH11059 - Tejas

Question: 1) Find the sum to n terms of
 $3 \times 8 + 6 \times 11 + 9 \times 14 + \dots$

Solution:

Writing the general term of the series

$$x_n = (3r + 3) \times (8 + 3r)$$

$$S_n = \sum_{r=0}^n 9r^2 + 33r + 24 \quad (1)$$

Using formulas for the sum of n terms (i) and sum of the squares of the n terms (ii)

$$\sum_{r=0}^n r = \frac{n(n+1)}{2} \quad (i)$$

$$\sum_{r=0}^n r^2 = \frac{n(n+1)(2n+1)}{6} \quad (ii)$$

Equation (1) evaluates to

$$S_n = \frac{33n(n+1)}{2} + \frac{9n(n+1)(2n+1)}{6} + 24n$$

Z transform of x_n :

$$X(Z) = \sum_{n=0}^{\infty} (3n+3)(3n+8)Z^{-n}$$

$$X(Z) = \sum_{n=0}^{\infty} (9n^2 + 33n + 24)Z^{-n}$$

$$X(Z) = 9Z^{-1} \frac{(1+Z^{-1})}{(1-Z^{-1})^3} + \frac{33}{(1-Z^{-1})^2} + 24 \frac{1}{1-Z^{-1}} \quad ; |Z| > 1$$

Z transform of S_n :

$$S(Z) = \sum_{n=0}^{\infty} \left(\frac{33n(n+1)}{2} + \frac{9n(n+1)(2n+1)}{6} + 24n \right)$$

$$S(Z) = \frac{33}{2} \left(\sum_{n=0}^{\infty} n^2 Z^{-n} + \sum_{n=0}^{\infty} n Z^{-n} \right) + \frac{9}{6} \left(\sum_{n=0}^{\infty} n^3 Z^{-n} + \sum_{n=0}^{\infty} n^2 Z^{-n} + \sum_{n=0}^{\infty} n Z^{-n} \right) + 24 \sum_{n=0}^{\infty} n Z^{-n}$$

$$S(Z) = \frac{18Z^{-1} \frac{9}{Z^{-1}} + 6}{(1-Z^{-1})^3} + \frac{1}{(1-Z^{-1})^2} 42 - 9Z^{-1}$$

