Rotor on flexible supports

Vibration Mechanics – Academic Year 2024/2025

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Introduction

The purpose of this laboratory is to analyze the dynamic response of a mechanical system composed of a shaft with three rods mounted on flexible supports. This assembly simulates the typical behavior of rotating machinery subject to small oscillations. By modeling the system as a two degrees-of-freedom (2DOF) configuration—one translational and one rotational—we investigate the influence of distributed mass and inertia, as well as the elastic behavior of the supports.

The analysis involves both theoretical modeling using Newtonian mechanics and experimental modal analysis. The theoretical part estimates system parameters such as mass, moment of inertia, and support stiffness using data from CAD drawings and MATLAB computations. Experimental results, such as natural frequencies and damping ratios, are obtained from vibration tests performed using a shaker and accelerometers. These two approaches are then compared to assess the accuracy of the simplified physical model.

1. Free-Body Diagram of the 2DOF System

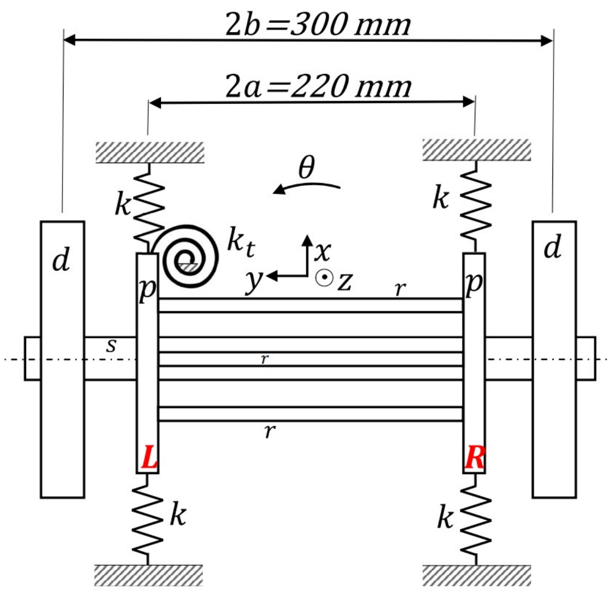

Figure 1: Mechanical schematic of the rotor-support system (2DOF model)

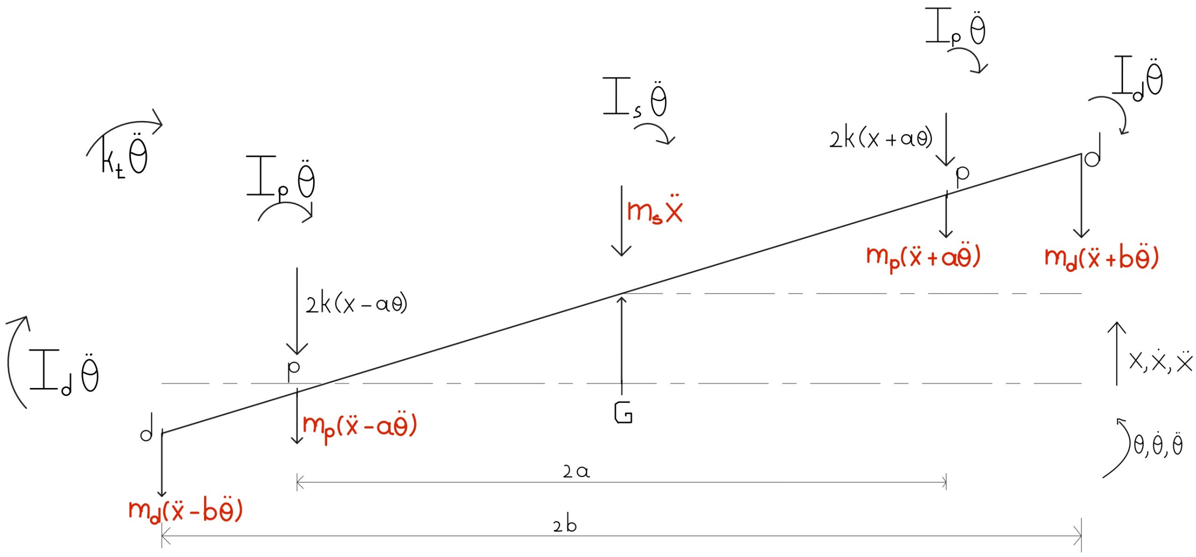

Figure 2: Free-body diagram of the 2DOF system

2. Equations of Motion

We derive the equations of motion for the system using Newton's second law and the rotational dynamics for small oscillations.

Translational Motion (Force Balance)

$$\begin{aligned} &2k(x-a\theta)+2k(x+a\theta)+m_s\ddot{x}+m_p(\ddot{x}-a\ddot{\theta})+m_d(\ddot{x}-b\ddot{\theta}) \\ &\quad+m_p(\ddot{x}+a\ddot{\theta})+m_d(\ddot{x}+b\ddot{\theta})=0 \end{aligned} \tag{1}$$

After simplification:

$$(m_s+2m_p+2m_d)\ddot{x}+4kx=0 \tag{1}$$

where m_s means the mass of the shaft + 3 rods

Rotational Motion (Moment Balance)

$$\begin{aligned} &I_s\ddot{\theta}+2I_p\ddot{\theta}+2I_d\ddot{\theta}+2ak(x+a\theta)-2ak(x-a\theta) \\ &+am_p(\ddot{x}+4\ddot{\theta})+bm_d(\ddot{x}+b\ddot{\theta})-am_p(\ddot{x}-a\ddot{\theta})-bm_d(\ddot{x}-b\ddot{\theta}) \\ &+k_t\theta=0 \end{aligned} \tag{2}$$

After simplification:

$$\left(I_s+2I_p+2I_d+2a^2m_p+2b^2m_d\right)\ddot{\theta}+\left(4a^2k+k_t\right)\theta=0 \tag{2}$$

where I_s means the moment of inertia of the shaft + 3 rods

3. Mass and Moment of Inertia Estimation

The following table summarizes the estimated mass and moment of inertia for each component of the system. The values were computed using MATLAB based on geometric properties extracted from CAD drawings. From the CAD drawings, we see that shaft and rod are steel, so I used density of the steel, instead for the disk and plate I used density of aluminium to calculate the masses. For the moments of inertia I used standard formulas based on the geometry of each component.

Component	**Mass (kg)**	**Moment of Inertia ($\mathbf{kg\,m^2}$)**
Disk	0.47713	$6.7096\ 10^{-4}$
Plate	0.44550	$4.4921\ 10^{-4}$
Shaft	0.86315	$2.5197\ 10^{-2}$
Rod	0.27744	$9.2481\ 10^{-4}$

Table 1: Estimated Masses and Moments of Inertia from MATLAB

Since the shaft and the 3 rods are rotating all together, we can consider them as 1 body, by summing up their mass and their moment of inertia. By doing so, we obtain:

Component	**Mass (kg)** (m_s)	**Moment of Inertia ($\mathbf{kg\,m^2}$)** (I_s)
Shaft + 3 rods	1.6955	$2.7971\ 10^{-2}$

Table 2: Mass and Moments of Inertia of shaft + 3 rods

4. Stiffness Estimation

To compute the translational stiffness in the x-direction, we model the four vertical rods as cantilever beams undergoing bending. For a single cantilever beam of rectangular cross-section, the area moment of inertia is:

$$I = \frac{bh^3}{12} = 2.16 \times 10^{-12} m^4$$

where:

- b is the width of the beam (along the axis of bending) equal to 15mm,
- h is the thickness of the beam (perpendicular to bending) equal to 1.2mm,
- L is the beam length equal to 144mm,
- E is the Young's modulus of the material equal to $2.1 \times 10^{11} Pa$.

From bending theory, the stiffness of one beam is:

$$k = \frac{12EI}{L^3} = 1823.3 N/m$$

For N identical beams in parallel (in our case, $N = 4$), the total stiffness becomes:

$$k_{\text{tot}} = \frac{12E}{L^3} \cdot \sum_{i=1}^{N} I_i = 7293.2 N/m$$

This expression represents the total translational stiffness of the structure along the x-direction due to bending of the four supporting beams.

5. Natural Frequencies

Starting from the equations of motion:

$$(m_s + 2m_p + 2m_d)\ddot{x} + 4kx = 0 \tag{1}$$

$$\left(I_s + 2I_p + 2I_d + 2a^2m_p + 2b^2m_d\right)\ddot{\theta} + \left(4a^2k + k_t\right)\theta = 0 \tag{2}$$

We can write them in the form:

$$M\ddot{x} + 4kx = 0$$

$$I\ddot{\theta} + \left(4a^2k + k_t\right)\theta = 0$$

These equations can be written in matrix form as:

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4k & 0 \\ 0 & 4a^2k + k_t \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

To compute the natural frequencies, we solve the eigenvalue problem:

$$\det\left(\mathbf{K} - \omega^2\mathbf{M}\right) = 0$$

Which gives:

$$\det\left(\begin{bmatrix} 4k - \omega^2 M & 0 \\ 0 & 4a^2k + k_t - \omega^2 I \end{bmatrix}\right) = 0$$

This results in the two characteristic equations (where M=3.5407 and I= 0.06246) :

$$4k - \omega^2 M = 0 \qquad \Rightarrow \qquad \omega_1 = \sqrt{\frac{4k}{M}} = 45 rad/s \quad \Rightarrow \quad f_1 = \frac{\omega_1}{2\pi} = \frac{45}{2\pi} \approx 7.16\,\text{Hz}$$

$$4a^2k + k_t - \omega^2 I = 0 \qquad \Rightarrow \qquad \omega_2 = \sqrt{\frac{4a^2k + k_t}{I}} = 204.7 rad/s \quad \Rightarrow \quad f_2 = \frac{\omega_2}{2\pi} \approx 32.59\,\text{Hz}$$

6.Theoretical and Experimental Results

- **Idealizations in the Theoretical Model:** The 2DOF model assumes point masses, perfectly rigid bodies, and linear elastic supports without damping. In contrast, the physical system has distributed mass, possible nonlinearities, and energy dissipation.

- **Neglect of Structural Details:** Elements such as holes, bolts, screws, and joints are ignored in the model but can influence mass and stiffness distributions significantly.

- **Variability in Material Properties:** Theoretical computations rely on nominal values for density and Young's modulus, while actual components may vary due to manufacturing tolerances and material inconsistencies.

- **Experimental Setup Effects:** The fixture used during testing may introduce additional constraints or flexibility, and the presence of sensors and cables could alter the dynamic response slightly.

- **Errors in Parameter Estimation:** Mass and inertia were estimated from CAD-based measurements assuming perfect geometry and homogeneous materials, potentially deviating from reality.

Overall, despite the assumptions and simplifications, the theoretical model provides a reasonably accurate prediction of the natural frequencies.