No.

Date . .

Nama : Agus Pranata NIM : 13323033

1) Misalkan fungsi Peluang dari X berbentuk :

$$P(x) = \frac{x}{15} \; ; x = 1, 2, 3, 4, 5$$

Hitunglah :

a) $E[(x^2+2)^2]$

b) $E[(6x^2-1)]$

c) μ_3

d) μ_3

e) $M_x(t)$

f) Berdasarkan hasil e, hitunglah $E(x)$

Jawab :

a) $E[(x^2+2)^2]$

$(x^2+2)^2 = x^4 + 4x^2 + 4$

$E[(x^2+2)^2] = E[(x^4+4x^2+4)]$

$E[x^4] + 4E[x^2] + 4$

$E[x^2]$ dan $E[x^4]$

$$E[x^2] = \sum_{x=1}^{5} x^2 \, P(X=x) = \frac{1}{15}\left(1^2 + 2\cdot 2^2 + 3\cdot 3^2 + 4\cdot 4^2 + 5\cdot 5^2\right)$$

$$E[x^2] = \frac{1}{15}(1+8+27+64+125) = \frac{225}{15} = 15$$

$$E[x^4] = \sum_{x=1}^{5} x^4 \, P(X=x) = \frac{1}{15}\left(1^4 + 2\cdot 2^4 + 3\cdot 3^4 + 4\cdot 4^4 + 5\cdot 5^4\right)$$

$$E[x^4] = \frac{1}{15}(1+32+243+1024+3125) = \frac{4425}{15} = 295$$

$$E[(x^2+2)^2] = 295 + 4(15) + 4 = 295 + 60 + 4 = 359$$

No.

Date . .

b) $E[(6x^2-1)]$

$E[(6x^2-1)] = 6[x^2]-1$

$E[x^2] = 15$

$E[(6x^2-1)] = 6(15)-1 = 90-1 = 89$

c) μ_3

$E(x) = \sum_{x=1}^{5} x \; P(x=x)$

$E(x) = \frac{1}{15}(1+2\cdot2+3\cdot3+4\cdot4+5\cdot5)$

$E(x) = \frac{1}{15}(1+4+9+16+25) = \frac{55}{15} = \frac{11}{3} = 3{,}67$

$\mu_3 = E[(x-\mu)^3] \cdot f(x)$

$= \left(1-\frac{11}{3}\right)^3\left(\frac{1}{15}\right) + \left(2-\frac{11}{3}\right)^3\left(\frac{2}{15}\right) + \left(3-\frac{11}{3}\right)^3\left(\frac{3}{15}\right) + \left(4-\frac{11}{3}\right)^3\left(\frac{4}{15}\right) + \left(5-\frac{11}{3}\right)^3\left(\frac{5}{15}\right)$

$= \left(-\frac{8}{3}\right)^3\left(\frac{1}{15}\right) + \left(-\frac{5}{3}\right)^3\left(\frac{2}{15}\right) + \left(-\frac{2}{3}\right)^3\left(\frac{3}{15}\right) + \left(\frac{1}{3}\right)^3\left(\frac{4}{15}\right) + \left(\frac{4}{3}\right)^3\left(\frac{5}{15}\right)$

$= -\frac{512}{27}\left(\frac{1}{15}\right) - \frac{125}{27}\left(\frac{2}{15}\right) - \frac{8}{27}\left(\frac{3}{15}\right) + \frac{1}{27}\left(\frac{4}{15}\right) + \frac{64}{27}\left(\frac{5}{15}\right)$

$= \frac{-512-250-24+4+320}{405} = \frac{-462}{405} = \frac{154}{135}$

No.

Date

d) μ_3

$$E(x^3) = \sum_{x=1}^{5} x^3 \cdot f(x)$$

$$= (1)^3\left(\frac{1}{15}\right) + (2)^3\left(\frac{2}{15}\right) + (3)^3\left(\frac{3}{15}\right) + (4)^3\left(\frac{4}{15}\right) + (5)^3\left(\frac{5}{15}\right)$$

$$= \frac{1}{15} + \frac{16}{15} + \frac{81}{15} + \frac{256}{15} + \frac{625}{15} = \frac{979}{15}$$

e) $M_x(t)$

$$E(e^{tx}) = \sum_x e^{tx} \cdot P(x)$$

$$= e^{t1}\left(\tfrac{1}{15}\right) + e^{t2}\left(\tfrac{2}{15}\right) + e^{t3}\left(\tfrac{3}{15}\right) + e^{t4}\left(\tfrac{4}{15}\right) + e^{t5}\left(\tfrac{5}{15}\right)$$

$$= \frac{e^{t1}}{15} + \frac{2e^{t2}}{15} + \frac{3e^{t3}}{15} + \frac{4e^{t4}}{15} + \frac{5e^{t5}}{15}$$

$$M_x(t) = \frac{1}{15}\left(e^{t1} + 2e^{t2} + 3e^{t3} + 4e^{t4} + 5e^{t5}\right)$$

f) Berdasarkan hasil e, hitunglah $E(x)$

$$E(x) = M'_x(0)$$

$$M_x(t) = \frac{1}{15}\left(e^{t1} + 2e^{t2} + 3e^{t3} + 4e^{t4} + 5e^{t5}\right)$$

$$M'_x(t) = \frac{1}{15}\left(\frac{d}{dt}(e^{t}) + 2\cdot\frac{d}{dt}(e^{t2}) + 3\cdot\frac{d}{dt}(e^{t3}) + 4\cdot\frac{d}{dt}(e^{t4}) + 5\cdot\frac{d}{dt}(e^{t5})\right)$$

dengan aturan eksponensial $\frac{d}{dt}(e^{tx}) = a \cdot e^{tx}$, maka:

$$M'_x(t) = \frac{1}{15}\left(e^{t} + 4e^{t2} + 9e^{t3} + 16e^{t4} + 25e^{t5}\right)$$

substitusi $t = 0$

$$M'_x(0) = \frac{1}{15}\left(e^0 + 4e^0 + 9e^0 + 16e^0 + 25e^0\right)$$

$$= \frac{1}{15}(1 + 4 + 9 + 16 + 25) = \frac{1}{15}(55) = \frac{55}{15} = \frac{11}{3}$$

No.

Date

2) Misalkan fungsi densitas dari x berbentuk

$$f(x) = \begin{cases} 6x(1-x) & ; 0 < x < 1 \\ 0 & ; x \text{ lainnya} \end{cases}$$

Hitunglah :

a) $E[(3x-4)]$

$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$

$g(x) = 3x - 4$

$E[3x-4] = \int_0^1 (3x-4) f(x)\, dx$

$f(x) = 6x(1-x)$

$E[(3x-4)] = \int_0^1 (3x-4) \cdot 6x(1-x)\, dx$

$E(x) = 6 \times 1/12 = 1/2$

$\rightarrow E(3x-4)$

$= 3 \times 1/2 - 4 = 3/2 - 4 = -5/2$

$E(3x-4) = -5/2$

b) $E[(2x^2 - x + 1)]$

$= \int_0^1 (2x^2 - x + 1) \cdot f(x)\, dx$

$= \int_0^1 (2x^2 - x + 1) \cdot 6x(1-x)\, dx$

$= \int_0^1 (12x^3 - 6x^2 + 6x - 6x^4 + 6x^3 - 6x^2)\, dx$

$= \int_0^1 (-6x^4 + 18x^3 - 12x^2 + 6x)\, dx$

$= \left[-6 \cdot \frac{x^5}{5} + 18 \cdot \frac{x^4}{4} - 12 \cdot \frac{x^3}{3} + 6 \cdot \frac{x^2}{2}\right]_0^1$

$= \left(-\frac{6}{5} + \frac{18}{4} - 4 + 3\right)$

$= -\frac{6}{5} + \frac{9}{2} - 4 + 3 = \frac{-12 + 45 - 40 + 30}{10} = \frac{29}{10}$

c) μ_3

$\mu_3 = E[(x-\mu)^3] = E[x^3] - (E[x])^3$

$\cdot)\ E(x) = \frac{1}{2}$

$\cdot)\ E[x^3] = \int_0^1 x^3 \cdot 6x(1-x) \cdot dx$

$= \int_0^1 6x^4(1-x)\, dx$

$= \int_0^1 (6x^4 - 6x^5)\, dx$

$= \left[6 \cdot \frac{x^5}{5} - 6 \cdot \frac{x^6}{6}\right]_0^1 = \frac{6}{5} - 1 = \frac{1}{5}$

$\cdot)\ \mu_3 = E[x^3] - (E[x])^3$

$= \frac{1}{5} - \left(\frac{1}{2}\right)^3$

$= \frac{1}{5} - \frac{1}{8}$

$= \frac{8-5}{40}$

$= \frac{3}{40}$

No.

Date . .

d. $\mu_3 = E[x^3]$

$= \frac{1}{5}$

e. $M_x(t) = E[e^{tx}]$

$= \int_{-\infty}^{\infty} e^{tx} \cdot f(x)\, dx$

$= \int_0^1 e^{tx} \cdot 6x(1-x)\, dx$

$= 6 \int_0^1 e^{tx} \cdot x(1-x)\, dx$

$M_x(t) = 6 \left(\int_0^1 e^{tx} \cdot x\, dx - \int_0^1 e^{tx} \cdot x^2\, dx \right)$

$\Rightarrow \int e^{tx} x\, dx = \frac{e^{tx}}{t} \cdot x - \int \frac{e^{tx}}{t}\, dx$

$= \frac{e}{t} x - \frac{1}{t^2} e^{tx} + C$

$= \left[\frac{e^t}{t} - \frac{1}{t^2} e^t \right] - \left[0 - \frac{1}{t^2} \cdot 1 \right]$

$= \frac{e^t}{t} - \frac{e^t}{t^2} - \frac{1}{t^2} = \frac{e^t}{t} - \frac{e^t - 1}{t^2}$

$\Rightarrow \int e^{tx} \cdot x^2\, dx = \frac{e^{tx}}{t} \cdot x^2 - \int \frac{e^{tx}}{t} (2x)\, dx$

$= \frac{e^{tx}}{t} x^2 - \frac{2}{t} \int e^{tx} \cdot x\, dx$

$\Rightarrow \int_0^1 e^{tx} x^2\, dx = \left[\frac{e^{tx}}{t} x^2 - \frac{2}{t} \left(\frac{e^t}{t} - \frac{1}{t^2} e^t \right) \right]_0^1$

$= \frac{e^t}{t} - \frac{2}{t} \left(\frac{e^t}{t} - \frac{1}{t^2} \right)$

No.

Date . .

$\Rightarrow M_x(t) = 6\left(\int_0^1 e^{tx} \cdot x\, dx - \int_0^1 e^{tx} \cdot x^2\, dx\right)$

f) $E(x)$

$E(x) = M'_x(0)$

$\Rightarrow M_x(t) = 6\left(\int_0^1 e^{tx} \cdot x(1-x)\, dx\right)$

$M_x(t) = 6\int_0^1 e^{tx} \cdot 6x(1-x)\, dx$

$= 6\left(\frac{e^t}{t} - \frac{1}{t^2}(e^t - 1)\right) - 6\int_0^1 e^{tx} x^2\, dx$

$\Rightarrow E[x] = \frac{1}{2}$

$\Rightarrow M_x(0) = 6\int_0^1 1 \cdot 6x(1-x)\, dx$

$= 6 \cdot E[x] = 6 \cdot \frac{1}{2} = 3$

$\Rightarrow M'_x(t) = 6\int_0^1 x e^{tx}(1-x)\, dx$

$M'_x(0) = 6 \cdot E[x] = 3$

$\Rightarrow E(x) = M'_x(0) = \frac{1}{2}$

No.

Date . .

3) Misalkan X_1 dan X_2 adalah variabel - Variabel acak yang saling bebas, dan mempunyai PDF yang sama yaitu :

$$f(x) = \frac{x}{6}, \; x = 1,2,3$$

Carilah distribusi peluang peubah acak dengan variable acak baru $Y = X_1 + X_2$

Jawab:

$\cdot)\; f(1) = \frac{1}{6}$ $\quad \cdot)\; f(2) = \frac{2}{6} = \frac{1}{3}$ $\quad \cdot)\; f(3) = \frac{3}{6} = \frac{1}{2}$

$$P(Y = k) = \sum_{1}^{3} P(X_1 = i)\; P(X_2 = k - i)$$

$\cdot)\; P(Y=2) = P(X_1 = 1) \cdot P(X_2 = 1)$

$$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$\cdot)\; P(Y=3) = P(X_1 = 1)\; P(X_2 = 2) + P(X_1 = 2)\; P(X_2 = 1)$

$$= \left(\frac{1}{6} \cdot \frac{1}{3}\right) + \left(\frac{1}{3} \cdot \frac{1}{6}\right) = \frac{1}{18} + \frac{1}{18} = \frac{2}{18} = \frac{1}{9}$$

$\cdot)\; P(Y=4) = P(X_1 = 1)\; P(X_2 = 3) + P(X_1 = 2)\; P(X_2 = 2) + P(X_1 = 3)\; P(X_2 = 1)$

$$= \left(\frac{1}{6} \cdot \frac{1}{2}\right) + \left(\frac{1}{3} \cdot \frac{1}{3}\right) + \left(\frac{1}{2} \cdot \frac{1}{6}\right)$$

$$= \frac{1}{12} + \frac{1}{9} + \frac{1}{12} = \frac{3}{36} + \frac{4}{36} + \frac{3}{36} = \frac{10}{36} = \frac{5}{18}$$

•) $P(Y=5) = P(X_1=2)\,P(X_2=3) + P(X_1=3)\,P(X_2=2)$

$$= \left(\frac{1}{3}\cdot\frac{1}{2}\right) + \left(\frac{1}{2}\cdot\frac{1}{3}\right) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

•) $P(Y=6) = P(X_1=3)\,P(X_2=3)$

$$= \left(\frac{1}{2}\cdot\frac{1}{2}\right) = \frac{1}{4}$$

Maka, $P(y) = \begin{cases} 1/36 & Y=2 \\ 1/9 & Y=3 \\ 5/18 & Y=4 \\ 1/3 & Y=5 \\ 1/4 & Y=6 \\ 0 & \text{Lainnya} \end{cases}$

Pembuktian : $\frac{1}{36} + \frac{1}{9} + \frac{5}{18} + \frac{1}{3} + \frac{1}{4} = 1$