

LAPORAN PRAKTIKUM PROBABILITAS DAN STATISTIKA

TRANSFORMASI



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Judul Praktikum

Minggu/Sesi	:	IX/2
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Setoran	:	Softcopy
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Tujuan	:	1. Mengetahui tentang Transformasi Fungsi Peubah Acak

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1) Misalkan fungsi Peluang dari X berbentuk:

$$p(x) = \frac{x}{15}; x = 1, 2, 3, 4, 5$$

Hitunglah:

a) $E[(x^2+2)^2]$

b) $E[6x^2-1]$

c) M_3

d) M_3

e) $M_x(t)$

f) Berdasarkan hasil e, hitunglah $E(x)$

Jawab:

a) $E[(x^2+2)^2]$

$$(x^2+2)^2 = x^4 + 4x^2 + 4$$

$$E[(x^2+2)^2] = E[x^4 + 4x^2 + 4]$$

$$E[x^4] + 4E[x^2] + 4$$

$$E[x^2] \text{ dan } E[x^4]$$

$$E[x^2] = \sum_{x=1}^5 x^2 p(x=x) = \frac{1}{15} (1^2 + 2 \cdot 2^2 + 3 \cdot 3^2 + 4 \cdot 4^2 + 5 \cdot 5^2)$$

$$E[x^2] = \frac{1}{15} (1 + 8 + 27 + 64 + 125) = \frac{225}{15} = 15$$

$$E[x^4] = \sum_{x=1}^5 x^4 p(x=x) = \frac{1}{15} (1^4 + 2 \cdot 2^4 + 3 \cdot 3^4 + 4 \cdot 4^4 + 5 \cdot 5^4)$$

$$E[x^4] = \frac{1}{15} (1 + 32 + 243 + 1024 + 3125) = \frac{4425}{15} = 295$$

$$E[(x^2+2)^2] = 295 + 4(15) + 4 = 295 + 60 + 4 = 359$$

$$b) E[(6x^2 - 1)]$$

$$E[(6x^2 - 1)] = 6E[x^2] - 1$$

$$E[x^2] = 15$$

$$E[(6x^2 - 1)] = 6(15) - 1 = 90 - 1 = 89$$

$$c) \mu_3$$

$$E(x) = \sum_{x=1}^5 x P(x=x)$$

$$E(x) = \frac{1}{15} (1 + 2 + 3 + 4 + 5)$$

$$E(x) = \frac{1}{15} (1 + 4 + 9 + 16 + 25) = \frac{55}{15} = \frac{11}{3} \approx 3.67$$

$$\mu_3 = E[(x - \mu)^3] \cdot f(x)$$

$$= \left(1 - \frac{11}{3}\right)^3 \left(\frac{1}{15}\right) + \left(2 - \frac{11}{3}\right)^3 \left(\frac{2}{15}\right) + \left(3 - \frac{11}{3}\right)^3 \left(\frac{3}{15}\right) + \left(4 - \frac{11}{3}\right)^3 \left(\frac{4}{15}\right) + \left(5 - \frac{11}{3}\right)^3 \left(\frac{5}{15}\right)$$

$$= \left(-\frac{8}{3}\right)^3 \left(\frac{1}{15}\right) + \left(-\frac{5}{3}\right)^3 \left(\frac{2}{15}\right) + \left(-\frac{2}{3}\right)^3 \left(\frac{3}{15}\right) + \left(\frac{1}{3}\right)^3 \left(\frac{4}{15}\right) + \left(\frac{4}{3}\right)^3 \left(\frac{5}{15}\right)$$

$$= \frac{-512}{27} \left(\frac{1}{15}\right) - \frac{125}{27} \left(\frac{2}{15}\right) - \frac{8}{27} \left(\frac{3}{15}\right) + \frac{1}{27} \left(\frac{4}{15}\right) + \frac{64}{27} \left(\frac{5}{15}\right)$$

$$= \frac{-512 - 250 - 24 + 4 + 320}{405} = \frac{-462}{405} = \frac{154}{135}$$

d) μ_3

$$E(x^3) = \sum_{x=1}^5 x^3 \cdot f(x)$$

$$= (1)^3 \left(\frac{1}{15} \right) + (2)^3 \left(\frac{2}{15} \right) + (3)^3 \left(\frac{3}{15} \right) + (4)^3 \left(\frac{4}{15} \right) + (5)^3 \left(\frac{5}{15} \right)$$

$$= \frac{1}{15} + \frac{16}{15} + \frac{81}{15} + \frac{256}{15} + \frac{625}{15} = \frac{979}{15}$$

e) $M_x(t)$

$$E(e^{tx}) = \sum x e^{tx} \cdot P(x)$$

$$= e^{t1} \left(\frac{1}{15} \right) + e^{t2} \left(\frac{2}{15} \right) + e^{t3} \left(\frac{3}{15} \right) + e^{t4} \left(\frac{4}{15} \right) + e^{t5} \left(\frac{5}{15} \right)$$

$$= \frac{e^{t1}}{15} + \frac{2e^{t2}}{15} + \frac{3e^{t3}}{15} + \frac{4e^{t4}}{15} + \frac{5e^{t5}}{15}$$

$$M_x(t) = \frac{1}{15} (e^{t1} + 2e^{t2} + 3e^{t3} + 4e^{t4} + 5e^{t5})$$

f) Berdasarkan hasil e, hitunglah $E(x)$

$$E(x) = M'_x(0)$$

$$M_x(t) = \frac{1}{15} (e^{t1} + 2e^{t2} + 3e^{t3} + 4e^{t4} + 5e^{t5})$$

$$M'_x(t) = \frac{1}{15} \left(\frac{d}{dt} (e^{t1}) + 2 \cdot \frac{d}{dt} (e^{t2}) + 3 \cdot \frac{d}{dt} (e^{t3}) + 4 \cdot \frac{d}{dt} (e^{t4}) + 5 \cdot \frac{d}{dt} (e^{t5}) \right)$$

dengan aturan eksponensial $\frac{d}{dt} (e^{tx}) = a \cdot e^{tx}$, maka:

$$M'_x(t) = \frac{1}{15} (e^t + 4e^{t2} + 9e^{t3} + 16e^{t4} + 25e^{t5})$$

substitusi $t = 0$

$$M'_x(0) = \frac{1}{15} (e^0 + 4e^0 + 9e^0 + 16e^0 + 25e^0)$$

$$= \frac{1}{15} (1 + 4 + 9 + 16 + 25) = \frac{1}{15} (55) = \frac{55}{15} = \frac{11}{3}$$

2) Misalkan fungsi densitas dari x berbentuk

$$f(x) = \begin{cases} 6x(1-x) & ; 0 < x < 1 \\ 0 & ; x \text{ lainnya} \end{cases}$$

Hitunglah:

a) $E[(3x-4)]$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x) dx$$

$$g(x) = 3x-4$$

$$E[3x-4] = \int_0^1 (3x-4) f(x) dx$$

$$f(x) = 6x(1-x)$$

$$E[3x-4] = \int_0^1 (3x-4) \cdot 6x(1-x) dx$$

$$E(x) = 6 \times \frac{1}{12} = \frac{1}{2}$$

$$\rightarrow E(3x-4)$$

$$= 3 \times \frac{1}{2} - 4 = \frac{3}{2} - 4 = -\frac{5}{2}$$

$$E(3x-4) = -\frac{5}{2}$$

b) $E[(2x^2 - x + 1)]$

$$= \int_0^1 (2x^2 - x + 1) \cdot f(x) dx$$

$$= \int_0^1 (2x^2 - x + 1) \cdot 6x(1-x) dx$$

$$= \int_0^1 (12x^3 - 6x^2 + 6x - 6x^4 + 6x^2 - 6x^2) dx$$

$$= \int_0^1 (-6x^4 + 10x^3 - 12x^2 + 6x) dx$$

$$= \left[-6 \cdot \frac{x^5}{5} + 10 \cdot \frac{x^4}{4} - 12 \cdot \frac{x^3}{3} + 6 \cdot \frac{x^2}{2} \right]_0^1$$

$$= \left(-\frac{6}{5} + \frac{10}{4} - 4 + 3 \right)$$

$$= -\frac{6}{5} + \frac{5}{2} - 4 + 3 = \frac{-8+45-40+30}{10} = \frac{29}{10}$$

c) $\mu_3 = E[x^3]$

c) μ_3

$$\mu_3 = E[(x-\mu)^3] = E[x^3] - (E[x])^3$$

$$\rightarrow E(x) = \frac{1}{2}$$

$$\rightarrow E[x^3] = \int_0^1 x^3 \cdot 6x(1-x) dx$$

$$= \int_0^1 6x^4(1-x) dx$$

$$= \int_0^1 (6x^4 - 6x^5) dx$$

$$= \left[6 \cdot \frac{x^5}{5} - 6 \cdot \frac{x^6}{6} \right]_0^1 = \frac{6}{5} - 1 = \frac{1}{5}$$

$$\rightarrow \mu_3 = E[x^3] - (E[x])^3$$

$$= \frac{1}{5} - \left(\frac{1}{2} \right)^3$$

$$= \frac{1}{5} - \frac{1}{8}$$

$$= \frac{8-5}{40}$$

$$= \frac{3}{40}$$

$$= \frac{3}{40}$$

$$d. M_3 = E[x^3]$$

$$= \frac{1}{5}$$

$$e. M_X(t) = E[e^{tx}]$$

$$= \int_{-\infty}^{\infty} e^{tx} \cdot f(x) dx$$

$$= \int_0^1 e^{tx} \cdot 6x(1-x) dx$$

$$= 6 \int_0^1 e^{tx} \cdot x(1-x) dx$$

$$M_X(t) = 6 \left(\int_0^1 e^{tx} \cdot x dx - \int_0^1 e^{tx} \cdot x^2 dx \right)$$

$$\rightarrow \int e^{tx} x dx = \frac{e^{tx}}{t} \cdot x - \int \frac{e^{tx}}{t} dx$$

$$= \frac{e}{t} x - \frac{1}{t^2} e^{tx} + C$$

$$= \left[\frac{e^t}{t} - \frac{1}{t^2} e^t \right] - \left[0 - \frac{1}{t^2} \cdot 1 \right]$$

$$= \frac{e^t}{t} - \frac{e^t}{t^2} - \frac{1}{t^2} = \frac{e^t}{t} - \frac{e^t - 1}{t^2}$$

$$\rightarrow \int e^{tx} \cdot x^2 dx = \frac{e^{tx}}{t} \cdot x^2 - \int \frac{e^{tx}}{t} (2x) dx$$

$$= \frac{e^{tx}}{t} x^2 - \frac{2}{t} \int e^{tx} \cdot x dx$$

$$\rightarrow \int_0^1 e^{tx} x^2 dx = \left[\frac{e^{tx}}{t} x^2 - \frac{2}{t} \left(\frac{e^t}{t} - \frac{1}{t^2} e^t \right) \right]_0^1$$

$$= \frac{e^t}{t} - \frac{2}{t} \left(\frac{e^t}{t} - \frac{1}{t^2} \right)$$

$$\rightarrow M_x(t) = 6 \left(\int_0^1 e^{tx} \cdot x \, dx - \int_0^1 e^{tx} \cdot x^2 \, dx \right)$$

f) $E(x)$

$$E(x) = M'_x(0)$$

$$\rightarrow M_x(t) = 6 \left(\int_0^1 e^{tx} \cdot x(1-x) \, dx \right)$$

$$M_x(t) = 6 \int_0^1 e^{tx} \cdot 6x(1-x) \, dx$$

$$= 6 \left(\frac{e^t}{t} - \frac{1}{t^2} (e^t - 1) \right) - 6 \int_0^1 e^{tx} x^2 \, dx$$

$$\rightarrow E[x] = \frac{1}{2}$$

$$\rightarrow M_x(0) = 6 \int_0^1 1 \cdot 6x(1-x) \, dx$$

$$= 6 \cdot E[x] = 6 \cdot \frac{1}{2} = 3$$

$$\rightarrow M'_x(t) = 6 \int_0^1 x e^{tx} (1-x) \, dx$$

$$M'_x(0) = 6 \cdot E[x] = 3$$

$$\rightarrow E(x) = M'_x(0) = \frac{1}{2}$$

3) Misalkan X_1 dan X_2 adalah variabel-variabel acak yang saling bebas, dan mempunyai PDF yang sama yaitu:

$$f(x) = \frac{x}{6}, x = 1, 2, 3$$

Cari distribusi peluang peubah acak dengan variabel acak baru

$$Y = X_1 + X_2$$

Jawab:

$$\rightarrow f(1) = \frac{1}{6}$$

$$\rightarrow f(2) = \frac{2}{6} = \frac{1}{3}$$

$$\rightarrow f(3) = \frac{3}{6} = \frac{1}{2}$$

$$P(Y=k) = \sum_{i=1}^3 P(X_1=i) P(X_2=k-i)$$

$$\rightarrow P(Y=2) = P(X_1=1) \cdot P(X_2=1)$$

$$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$\rightarrow P(Y=3) = P(X_1=1) P(X_2=2) + P(X_1=2) P(X_2=1)$$

$$= \left(\frac{1}{6} \cdot \frac{1}{3} \right) + \left(\frac{1}{3} \cdot \frac{1}{6} \right) = \frac{1}{18} + \frac{1}{18} = \frac{2}{18} = \frac{1}{9}$$

$$\rightarrow P(Y=4) = P(X_1=1) P(X_2=3) + P(X_1=2) P(X_2=2) + P(X_1=3) P(X_2=1)$$

$$= \left(\frac{1}{6} \cdot \frac{1}{2} \right) + \left(\frac{1}{3} \cdot \frac{1}{3} \right) + \left(\frac{1}{2} \cdot \frac{1}{6} \right)$$

$$= \frac{1}{12} + \frac{1}{9} + \frac{1}{12} = \frac{3}{36} + \frac{4}{36} + \frac{3}{36} = \frac{10}{36} = \frac{5}{18}$$

$$\begin{aligned} \cdot) P(Y=5) &= P(X_1=2) P(X_2=3) + P(X_1=3) P(X_2=2) \\ &= \left(\frac{1}{3} \cdot \frac{1}{2} \right) + \left(\frac{1}{2} \cdot \frac{1}{3} \right) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \cdot) P(Y=6) &= P(X_1=3) P(X_2=3) \\ &= \left(\frac{1}{2} \cdot \frac{1}{2} \right) = \frac{1}{4} \end{aligned}$$

$$\text{Maka, } P(Y) = \begin{cases} \frac{1}{36} & Y=2 \\ \frac{1}{9} & Y=3 \\ \frac{5}{18} & Y=4 \\ \frac{1}{3} & Y=5 \\ \frac{1}{4} & Y=6 \\ 0 & \text{lainnya} \end{cases}$$

$$\text{Pembuktian: } \frac{1}{36} + \frac{1}{9} + \frac{5}{18} + \frac{1}{3} + \frac{1}{4} = 1$$