CHAPTER 5
DIRECTED GRAPH

Introduction to directed graph

- Directed graphs are graphs in which the edges are one way. Such graphs are frequently more useful in various dynamical systems such as:
- Digital computer
- Flow system
- Communication system
- Transportation system

Cont.

Definition :- a digraph D is a graph consisting of two things:

i) A set V whose elements are called vertices, Points or node of D

ii) A set E whose elements are order pairs (u, v) of distinct vertices called arcs or directed edges of D.

Suppose $e = (u, v)$ is a directed edge in a digraph D

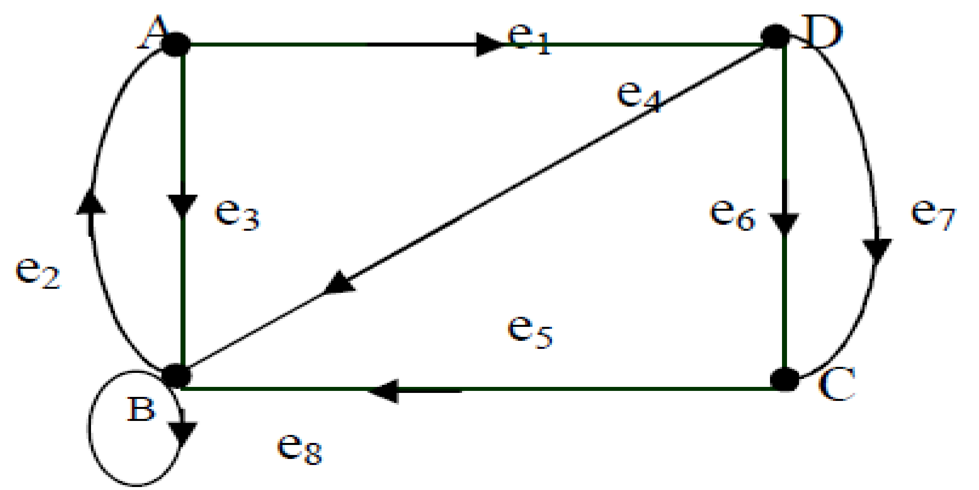
Cont.

Then the following terminologies are used.

* **e** begins at **u** and ends at **v**.
* **u** is the origin or initial point of **e** where as **v** is destination or terminal point of **e**.
* **v** is the successor of **u** and **u** is the predecessor of **v**.
* **u** is adjacent to **v** where as **v** is adjacent from **u**
* If **u = v** then **e** is a loop.

Cont.

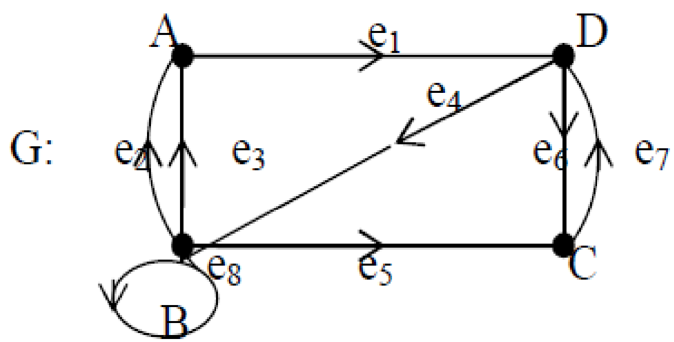
Example :-



- $e4 = (D, B) \neq (B, D)$
- $e8$ is a loop.
- e2 and e3 are parallel arcs.

Cont.

- **Degree**: Suppose G is a direct graph. The out degree of a vertex v of G, $outdeg(v)$, is the number of edges incident from v, and the in degree of v, $indeg(v)$, is the number of edges incident to v.
- **Example :-**



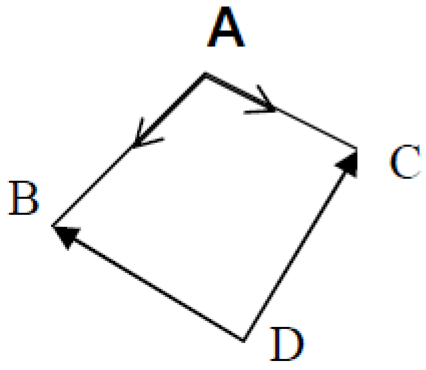
- $Outdeg(A) = 1\ indeg(A) = 2$
- $Outdeg(B) = 4, indeg(B) = 2$
- $Outdeg(C) = 1, indeg(C) = 2$
- $Outdeg\ (D) = 2, indeg(D\ ) = 2$
- $Sum\ (outdeg) = 8, sum\ (indeg) = 8$

Cont.

***Theorem*:** The sum of the out degrees of the vertices of the diagraph G equals the sum of the in degrees of the vertices, which equals the number of edges in G.

***Note*:** A vertex u in a diagraph with zero in degree is called a **source** and a vertex u with zero out degree is called a **sink.**

Example :-



A and D are source

B and C are sink

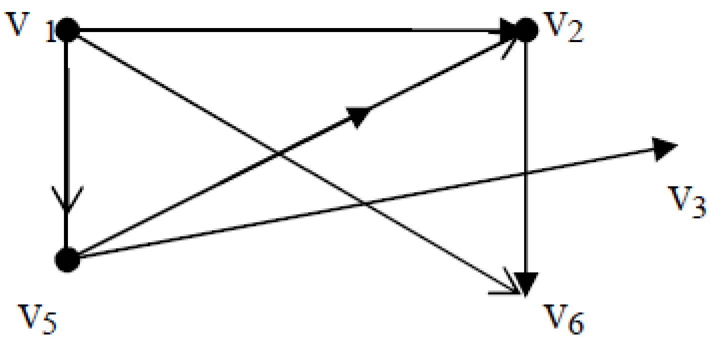
Matrix Representation of a diagraph

* Definition :- **Adjacency matrix:** The adjacency matrix A= [aij] of a diagraph is defined as a matrix with:

$$\begin{cases} 1 \text{ if number of } (vi,vj) \in E \text{ is n} \\ \quad 0 \text{ otherwise} \end{cases}$$

Example 1: Write the adjacency matrix for the following diagraph.

D_1:



$$A(D_1) = \begin{array}{c|ccccc} & V_1 & V_2 & V_3 & V_4 & V_5 \\ \hline V_1 & 0 & 1 & 0 & 1 & 1 \\ V_2 & 0 & 0 & 0 & 1 & 0 \\ V_3 & 0 & 0 & 0 & 0 & 0 \\ V_4 & 0 & 0 & 0 & 0 & 0 \\ V_5 & 0 & 1 & 1 & 0 & 0 \end{array}$$

Paths and Connectivity

Let G be a directed graph. The concept of path, simple patch, cycle and trial carry over from non-directed graphs G except that the direction of the edges must agree with the direction of the path.

Connectivity: There are three types of connectivity in a directed graph D

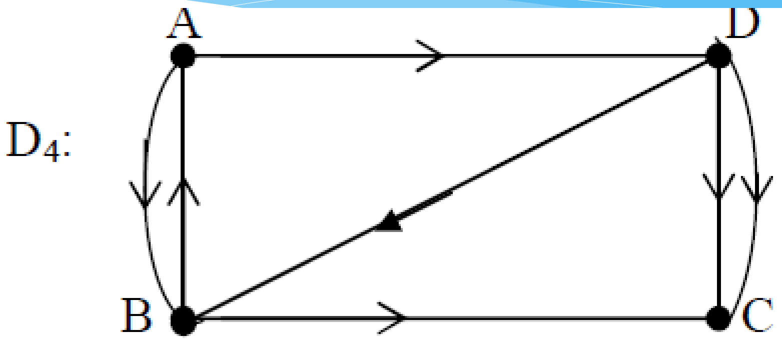
i) D is strongly connected or strong if, for any pair of vertices u and v in D, there is a path from u to v and a path from v to u (each is reach able from the other)

ii) G is unilaterally connected or unilateral if, for any pair of vertices u and v, there is a path from u to v and a path from v to u (one of them is reachable from the other).

iii) G is weakly connected or weak if its underlying graph is connected.

Cont.

Example :-Let D4 be the diagraph shown in the figure. Then describe the connectivity



Solution:-

i) D4 is weakly connected since the under lining graph is connected or D has a spanning semi-path, like ABCD.

ii) D4 is unilaterally connected since it has a spanning path, like ADBC or BADC.

iii) D4 is not strongly connected since C is a sink (i.e. every vertex is not reachable from C) or since D has not a closed spanning path.