Question 1

On a la série de paiements suivante:

t	0	1	2	3	4	5	6
$\$$		X	X	$2X$	X	X	$2X$

On peut voir le schéma suivant:

t	0	1	2	3	4	5	6
$\$_1$		X	X	X	X	X	X
$\$_2$				X			X

On a donc l'équation suivante:

$$PV = Xa_{\overline{6}|i} + Xa_{\overline{2}|j} \qquad \text{où} \quad j = (1+i)^3 - 1$$

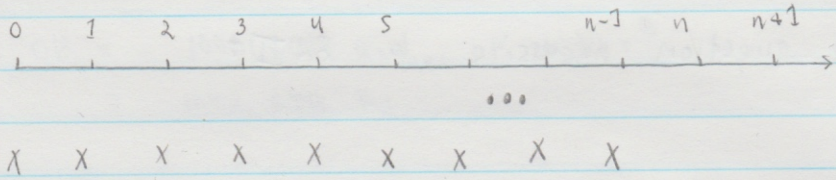
On aurait aussi pu passer par les séries géométriques:

$$\begin{aligned} PV = & \; Xv + Xv^2 + Xv^3 + Xv^4 + Xv^5 + Xv^6 \\ & + Xv^3 + Xv^6 \end{aligned}$$

$$\begin{aligned} & = X(v + v^2 + v^3 + v^4 + v^5 + v^6) \\ & + X(v^3 + v^6) \end{aligned}$$

$$= Xa_{\overline{6}|i} + Xa_{\overline{2}|j} \qquad \text{où} \quad j = (1+i)^3 - 1$$

Question 2



$$
\begin{aligned}
VA_{@n} &= X(1+i)^n + X(1+i)^{n-1} + \ldots + X(1+i) \\
&= X(1+i)^n \left[1 + (1+i)^{-1} + \ldots + (1+i)^{1-n} \right] \\
&= X(1+i)^n \left[1 + v + \ldots + v^{n-1} \right] \\
&= X(1+i)^n \sum_{k=0}^{n-1} v^k \\
&= X(1+i)^n \left[\frac{1 - v^n}{1-v} \right] \\
&= X \left[\frac{(1+i)^n - 1}{d} \right] \\
&= X \, \ddot{s}_{\overline{n}|i}
\end{aligned}
$$

Question 3

t	0	1	2	3	4	5	6	7	8	9	10
\$			0,5			1					

$$\therefore \quad A(5) = A(2)\, a(2,5)$$

$$1 = 0{,}5 \exp\left(\int_2^5 a\,(1+t)^{-1}\,dt \right)$$

$$2 = \exp \int_3^6 a u^{-1}\,du \qquad u = 1+t$$

$$2 = \exp\left(a \ln(u) \Big|_3^6 \right)$$

$$2 = \exp\left(a \cdot \ln(6) - \ln(3) \right)$$

$$2 = \exp\left(a \ln(6/3) \right)$$

$$2 = \exp\left(\ln(2)^a \right)$$

$$1 = a$$

$$\therefore \quad A(10) = \int_1^X a(t,X)\,dt \cdot a(X,10)$$

$$= \int_1^X \left[\exp \int_t^X (1+u)^{-1}\,du \right] dt \cdot \exp\left(\int_X^{10} (1+u)^{-1}\,du \right)$$

$$= \int_1^X e^{\int_t^X (1+u)^{-1}\,du}\, e^{\int_X^{10} (1+u)^{-1}\,du}\, dt$$

$$= \int_1^X e^{\int_t^{10} (1+u)^{-1}\,du}\, dt$$

$$= \int_1^x e^{\int_{t+1}^{11} v^{-1} dv} \, dt$$ $v = 1+u$
$du = du$

$$= \int_1^x \exp[\ln(v) \,|_{t+1}^{11}] \, dt$$

$$= \int_1^x \exp(\ln(^{11}/_{t+1})) \, dt$$

$$= \int_1^x \frac{11}{t+1} \, dt$$

$$= \int_2^{x+1} 11 u^{-1} \, du$$ $u = t+1$
$du = dt$

$$= 11 \ln(u) \,\Big|_2^{x+1}$$

$$= 11 \ln\left(\frac{x+1}{2}\right)$$

$$\therefore \quad 10 = 11 \ln\left(\frac{x+1}{2}\right)$$

$$2e^{10/11} - 1 = X$$

$$3,464\,130\,169 = X$$

Question 4

Nous sommes ici en présence d'une annuité continue. Traçons le schéma de la situation:

Schéma

$$
\begin{array}{lcccccc}
t & 0 & 1 & 2 & 3 & 4 & 5 \\
\$ & \longleftarrow & & ? & & \longrightarrow &
\end{array}
$$

$$\therefore \quad 100 = \int_x^5 h(t)\, e^{\int_t^5 \delta(u)\,du}\, dt$$

$$100 = \int_x^5 10(1+t)\, \exp\left[\int_t^5 2(1+u)^{-1}\, du\right] dt$$

$$100 = \int_x^5 10(1+t)\, \exp\left[\, 2 \ln(1+u)\Big|_t^5 \right] dt$$

$$100 = \int_x^5 10(1+t)\, \exp\left[\ln\left(\tfrac{6}{1+t}\right)^2\right] dt$$

$$100 = \int_x^5 10(1+t)\left(\frac{6}{1+t}\right)^2 dt$$

$$100 = \int_x^5 \frac{360}{1+t}\, dt$$

$$100 = 360 \ln(1+t) \Big|_x^5$$

$$100 = 360 \ln\left(\tfrac{6}{1+x}\right)$$

$$e^{5/18} = \tfrac{6}{1+x}$$

$$x = \tfrac{6}{e^{5/18}} - 1 = 3{,}544\ 790\ 770$$