Lab 2 Solutions - STA 360/601

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1. Problem 1

A derivation of the posterior for a beta prior and binomial likelihood (with N observations, each having n_i trials and success probability θ) is:

$$p(\theta|x_1,...,x_N) = p(x_1,...,x_N|\theta)p(\theta)/p(x_1,...,x_N)$$

$$p(\theta|x_1,...,x_N) \propto [\prod_{i=1}^{N} binom(x_i|n_i,\theta)]beta(\theta|a,b)$$

$$p(\theta|x_1,...,x_N) \sim Beta(a+\sum_{i=1}^{N} x_i, b+\sum_{i=1}^{N} n_i - \sum_{i=1}^{N} x_i)$$

We can find the posterior for a beta prior and Bernoulli likelihood as the special case when the number of trials on each observation n_i = n = 1. The posterior is:

$$p(\theta|x_1,...,x_N) \sim Beta(a+\sum_{i=1}^{N} x_i, b+N-\sum_{i=1}^{N} x_i)$$

2. Problem 2

First we note that in R, n is the number of observations, size is the number of bernoulli trials on each observation, and prob is the success probability on each trial. When we have a binomial(size=100, prob=.01) likelihood for each observation, the data can be simulated as follows:

```
set.seed(123)
obs.data.binom <- rbinom(n = 100, size = 100, prob = 0.01)
obs.data.binom
```

```
##   [1] 0 2 1 2 3 0 1 2 1 1 3 1 1 1 0 2 0 0 0 3 2 1 1 4 1 1 1 1 0 0 3 2 1 2 0
##  [36] 1 2 0 0 0 0 1 1 1 0 0 0 1 0 2 0 1 2 0 1 0 0 2 2 1 1 0 1 0 2 1 2 2 2 1
##  [71] 2 1 1 0 1 0 1 1 0 0 0 1 1 2 0 1 4 2 2 0 0 1 0 1 0 0 2 0 1 1
```

When we have a bernoulli likelihood binomial(size=1, prob=.01) for each observation, the data can be simulated as follows:

```
set.seed(123)
obs.data <- rbinom(n = 100, size = 1, prob = 0.01)
obs.data
```

```
##   [1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
##  [36] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##  [71] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```