Optimizing Code Generation for Matrix Multiplication

DAPHNE

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Matrix Multiplication Implementation

Matrix Multiplication is Level 3 Basic Linear Algebra Subprogram [1] operation

Hardware specific hand optimization, e.g. Intel oneMKL BLAS [2]

Daphne uses BLIS [3] a “portable software framework for instantiating high-performance BLAS-like dense linear algebra libraries”

[1] Dongarra, Cruz, Hammarling, *et al.* (1990). “Algorithm 679: A set of level 3 basic linear algebra subprograms: model implementation and test programs.”

[2] Intel® (). *Accelerate Fast Math with Intel® oneAPI Math Kernel Library*.

[3] Van Zee and van de Geijn (2015). “BLIS: A Framework for Rapidly Instantiating BLAS Functionality.”

[4] Lattner, Amini, Bondhugula, *et al.* (2021). “MLIR: Scaling Compiler Infrastructure for Domain Specific Computation.”

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Alternative: Automatic *Code Generation* using MLIR [4]

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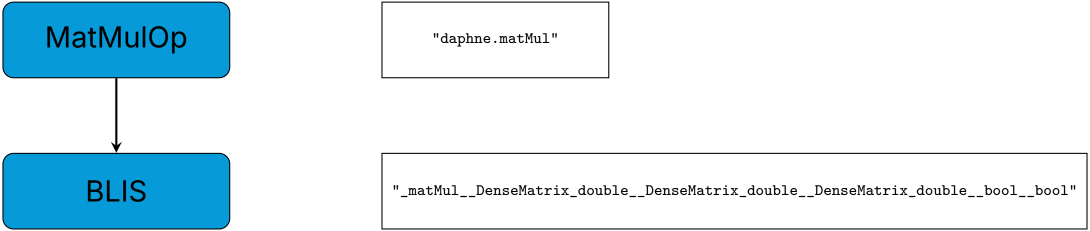
[4] Lattner, Amini, Bondhugula, *et al.* (2021). "MLIR: Scaling Compiler Infrastructure for Domain Specific Computation."

Daphne Matrix Multiplication – Kernel



Daphne Matrix Multiplication – Code Generation

Project outcomes:

- Enabled Matrix Multiplication for `ui64` value type
- Extended code generation to `f32` and `si64`, `ui64` value types

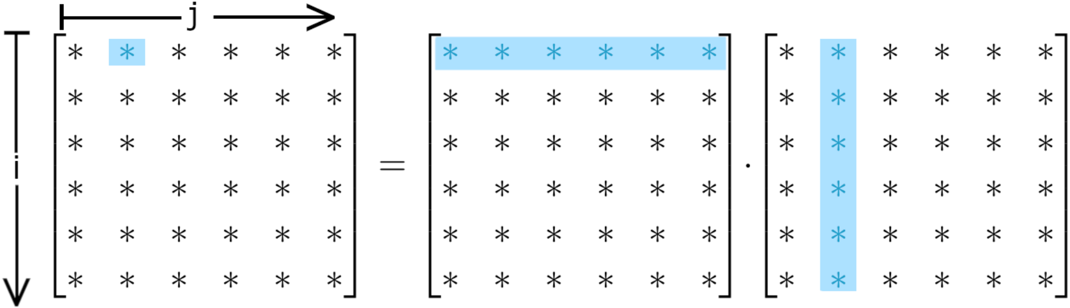
Daphne Matrix Multiplication – Code Generation

Project outcomes:

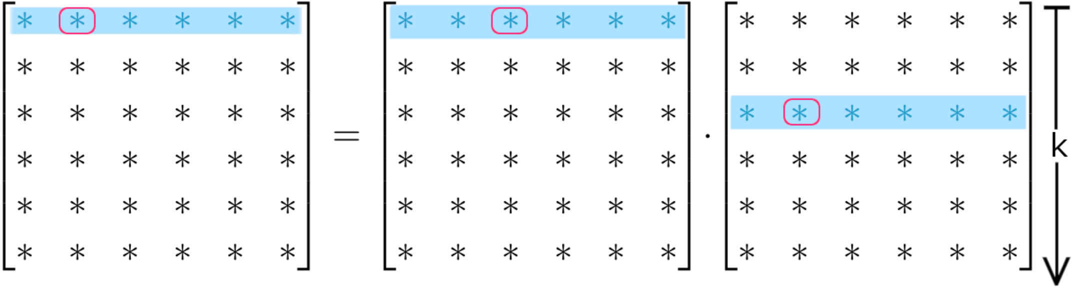
- Enabled Matrix Multiplication for `ui64` value type
- Extended code generation to `f32` and `si64`, `ui64` value types
- Enabled multiple Optimization strategies

Uday Bondhugula, “Using MLIR for High-Performance Code Gen: Part I,” bondhugula/llvm-project. (Mar. 2, 2020), [Online]. Available: `https://github.com/bondhugula/llvm-project/blob/hop/mlir/docs/HighPerfCodeGen.md` (visited on 03/01/2024)

Matrix Multiplication



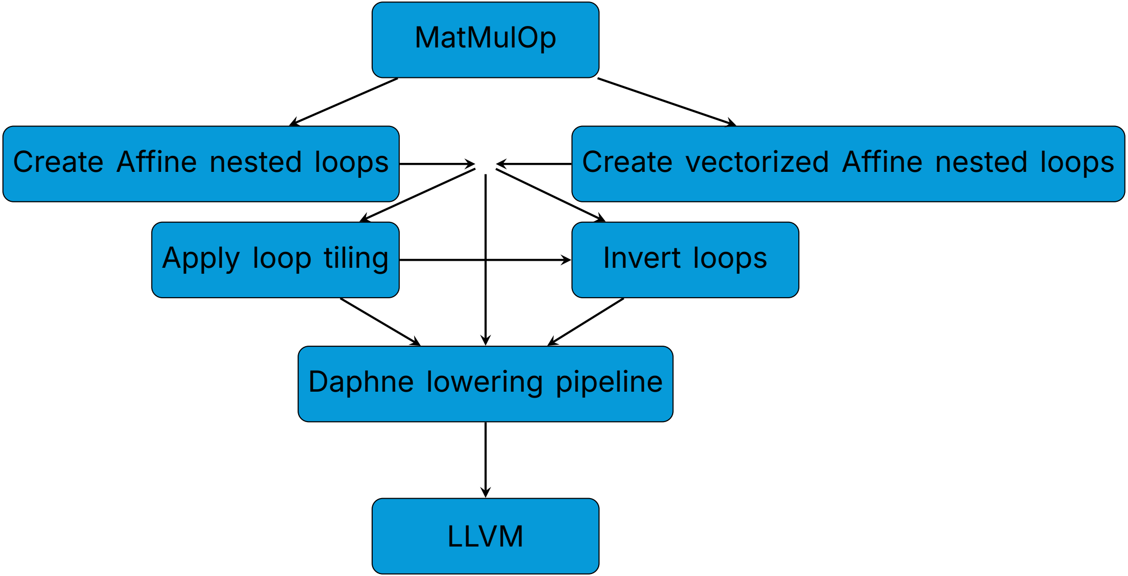
Matrix Multiplication – Inverted Loops



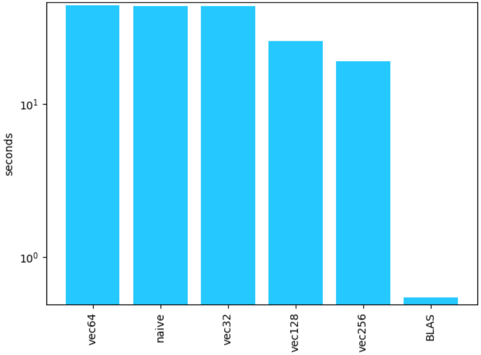
Matrix Multiplication – Tiles and Vectors

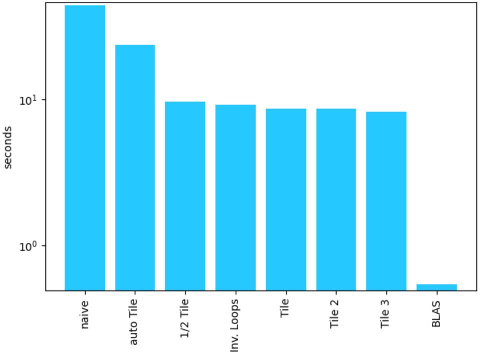
$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} = \begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \cdot \begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$

Daphne Matrix Multiplication – Code Generation



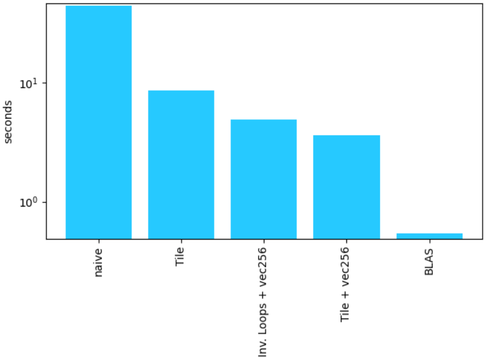
Optimizations enabled





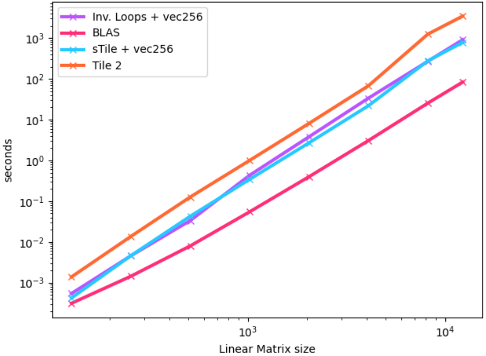
Execution times of dense 2048×2048 double precision Matrix multiplication with different vector instruction sizes (left) and tiling strategies (right) enabled.

Optimizations enabled – Tiling and Vectorization



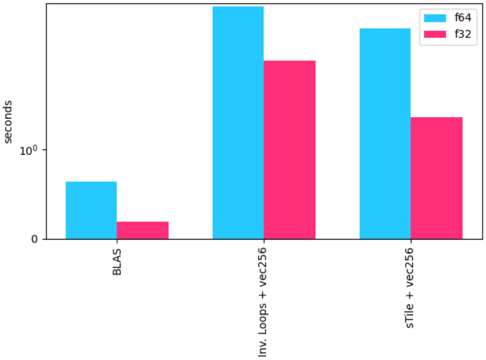
Execution times of dense 2048×2048 double precision Matrix multiplication with different optimizations.

Scaling behaviour



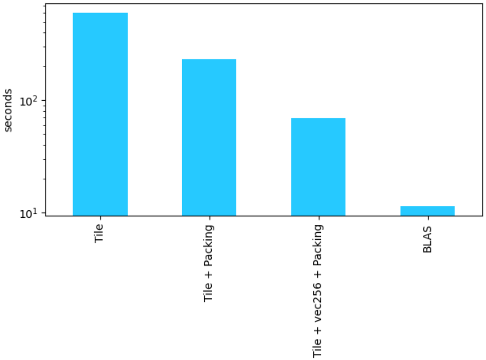
Execution times of dense double precision Matrix multiplications.

Single precision Matrix Multiplication



Execution times of 2048×2048 dense single and double precision Matrix multiplications.

Further improvement – Packing



Effect of packing, enabled through `affineDataCopyGeneratePass`, on 8192×8192 single precision Matrix multiplication executed with `mlir-cpu-runner`.

Bibliography

[1] J. J. Dongarra, J. D. Cruz, S. Hammarling, and I. S. Duff, “Algorithm 679: A set of level 3 basic linear algebra subprograms: Model implementation and test programs,” *ACM Trans. Math. Softw.*, vol. 16, no. 1, pp. 18–28, Mar. 1990, ISSN: 0098-3500. DOI: `10.1145/77626.77627`. [Online]. Available: `https://doi.org/10.1145/77626.77627`.

[2] Intel®, “Accelerate Fast Math with Intel® oneAPI Math Kernel Library,” Intel.com. (), [Online]. Available: `https://www.intel.com/content/www/us/en/developer/tools/oneapi/onemkl.html` (visited on 03/01/2024).

[3] F. G. Van Zee and R. A. van de Geijn, “BLIS: A framework for rapidly instantiating BLAS functionality,” *ACM Transactions on Mathematical Software*, vol. 41, no. 3, 14:1–14:33, Jun. 2015. [Online]. Available: `https://doi.acm.org/10.1145/2764454`.

Bibliography

[4] C. Lattner, M. Amini, U. Bondhugula, *et al.*, "MLIR: Scaling compiler infrastructure for domain specific computation," in *2021 IEEE/ACM International Symposium on Code Generation and Optimization (CGO)*, 2021, pp. 2–14. DOI: `10.1109/CGO51591.2021.9370308`.

[5] Uday Bondhugula, "Using MLIR for High-Performance Code Gen: Part I," bondhugula/llvm-project. (Mar. 2, 2020), [Online]. Available: `https://github.com/bondhugula/llvm-project/blob/hop/mlir/docs/HighPerfCodeGen.md` (visited on 03/01/2024).

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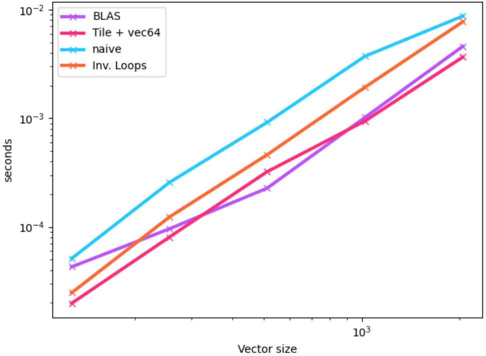
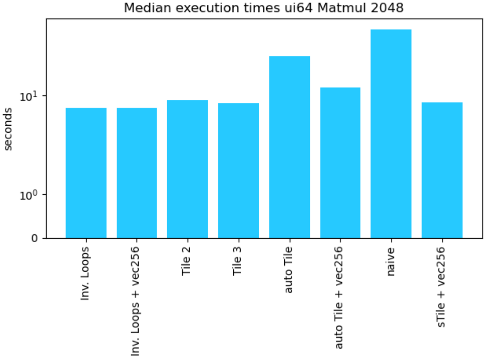
Appendix



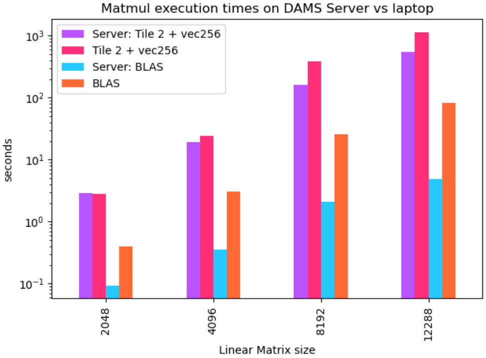
Execution times of double precision Matrix Vector product.

Appendix

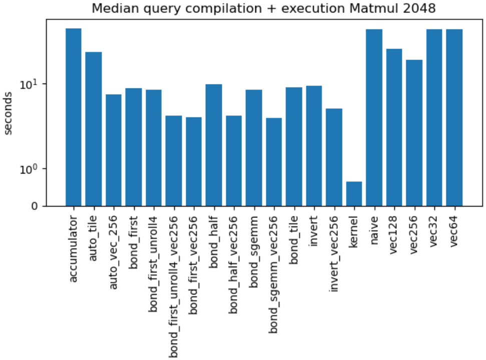


Exection times of ui64 2048×2048 Matrix multiplication. Note this is not supported by the provided Kernel implementation.

Appendix

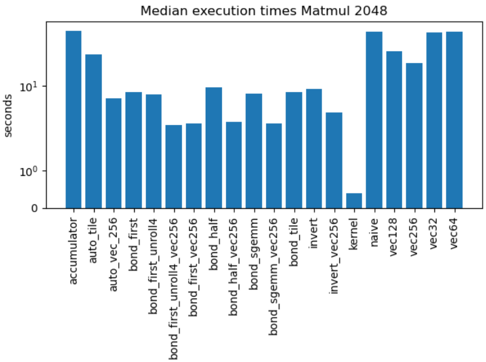

Matmul execution times on DAMS Server vs laptop
Server: Tile 2 + vec256
Tile 2 + vec256
Server: BLAS
BLAS
seconds
10^3
10^2
10^1
10^0
10^{-1}
2048
4096
8192
12288
Linear Matrix size

Appendix

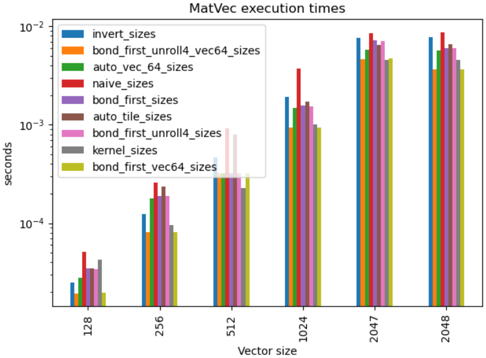


Median query compilation + execution Matmul 2048
seconds
10^1
10^0
0
accumulator
auto_tile
auto_vec_256
bond_first
bond_first_unroll4
bond_first_unroll4_vec256
bond_first_vec256
bond_half
bond_half_vec256
bond_sgemm
bond_sgemm_vec256
bond_tile
invert
invert_vec256
kernel
naive
vec128
vec256
vec32
vec64

Appendix

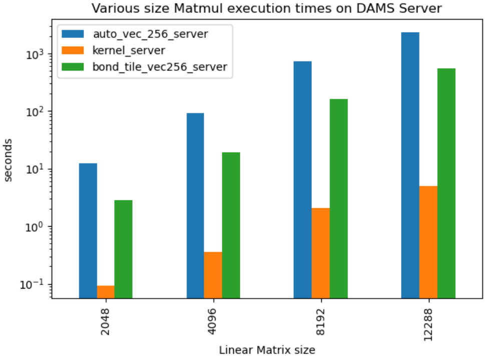

Median execution times Matmul 2048
seconds
10^1
10^0
0
accumulator
auto_tile
auto_vec_256
bond_first
bond_first_unroll4
bond_first_unroll4_vec256
bond_first_vec256
bond_half
bond_half_vec256
bond_sgemm
bond_sgemm_vec256
bond_tile
invert
invert_vec256
kernel
naive
vec128
vec256
vec32
vec64

Appendix



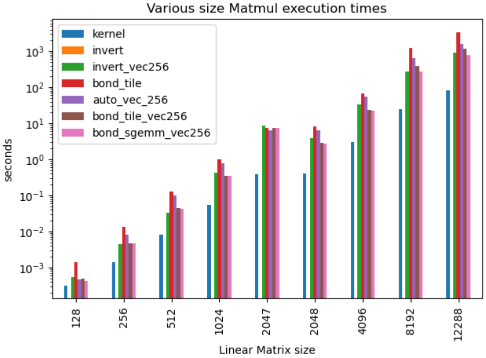
MatVec execution times
invert_sizes
bond_first_unroll4_vec64_sizes
auto_vec_64_sizes
naive_sizes
bond_first_sizes
auto_tile_sizes
bond_first_unroll4_sizes
kernel_sizes
bond_first_vec64_sizes
10^{-2}
10^{-3}
10^{-4}
seconds
128
256
512
1024
2047
2048
Vector size

Appendix



Various size Matmul execution times on DAMS Server
auto_vec_256_server
kernel_server
bond_tile_vec256_server
seconds
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
2048
4096
8192
12288
Linear Matrix size

Appendix



Various size Matmul execution times
kernel
invert
invert_vec256
bond_tile
auto_vec_256
bond_tile_vec256
bond_sgemm_vec256
seconds
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
128
256
512
1024
2047
2048
4096
8192
12288
Linear Matrix size