Trading Strategies Simulation on Binary Return Model

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Introduction

We study a simple binary-return investment model and compare three strategies: - all-in reinvestment each round, - fixed stake of $1 per round, and - fixed stake of $10 per round.

In each round, investing an amount `I` yields, with equal probability (0.5 each): - up factor: return = 1.55 I - down factor: return = 0.55 I

The one-period expected return is $E[R] = 0.5 \cdot 1.55 \cdot I + 0.5 \cdot 0.55 \cdot I = 1.05 \cdot I$ (a 5% expected gain per dollar invested). We then simulate the strategies.

Investment model: helpers and quick checks

```r
# Binary-return investment draw for amount I.
# Returns a length-n vector of realized payoffs.
invest_draw <- function(investment_amount,
                        n = 1,
                        p_up = 0.5,
                        up_factor = 1.55,
                        down_factor = 0.55) {
  multipliers <- sample(c(up_factor, down_factor),
                        size = n, replace = TRUE,
                        prob = c(p_up, 1 - p_up))
  investment_amount * multipliers
}

# Single-draw wrapper mirroring the original invest(I)
invest <- function(I) {
  invest_draw(I, n = 1)
}

# Alternative vectorized way
#invest_alternative <- function(I, n = 3) {
#  Rseq <- c(1.55 * I, 0.55 * I)
#  sample(Rseq, n, replace = TRUE)
#}
#invest_alternative(100, n = 3)
```

Strategy simulators

```r
# Strategy 1: All-in reinvest each period.
# wealth[t] is wealth after t-1 rounds; wealth[1] = initial_wealth.
simulate_strategy_all_in <- function(initial_wealth,
                                     num_rounds = 200,
                                     p_up = 0.5,
```

```r
                                        up_factor = 1.55,
                                        down_factor = 0.55) {
  wealth <- numeric(num_rounds + 1)
  wealth[1] <- initial_wealth
  for (t in 2:(num_rounds + 1)) {
    multiplier <- sample(c(up_factor, down_factor), 1,
                         prob = c(p_up, 1 - p_up))
    wealth[t] <- wealth[t - 1] * multiplier
  }
  list(path = wealth, final = wealth[num_rounds + 1])
}

# Strategy 2 (generic): Invest a fixed dollar amount `stake` each period.
# Each round: withdraw `stake`, add realized payoff on that stake.
simulate_strategy_fixed_bet <- function(initial_wealth,
                                        stake,
                                        num_rounds = 200,
                                        p_up = 0.5,
                                        up_factor = 1.55,
                                        down_factor = 0.55) {
  wealth <- numeric(num_rounds)
  # After first round: start with initial_wealth - stake + payoff(stake)
  wealth[1] <- initial_wealth - stake + invest_draw(stake, 1, p_up, up_factor, down_factor)
  for (t in 2:(num_rounds)) {
    wealth[t] <- wealth[t - 1] - stake + invest_draw(stake, 1, p_up, up_factor, down_factor)
  }
  list(path = wealth, final = wealth[num_rounds])
}

# Helper: run many independent simulations and return vector of finals
replicate_finals <- function(strategy_fun, S = 500, ...) {
  finals <- numeric(S)
  for (s in 1:S) finals[s] <- strategy_fun(...)$final
  finals
}
```

One-step expectations

```r
I <- 1
E_R <- 0.5 * 1.55 * I + 0.5 * 0.55 * I
E_R
```

```
## [1] 1.05
```

```r
# Analytical expectations for T rounds
W0 <- 200
T  <- 200

# All-in: E[W_T] = W0 * (E[multiplier])^T, where multiplier in {1.55, 0.55}
E_multiplier <- 0.5 * 1.55 + 0.5 * 0.55   # = 1.05
E_WT_all_in  <- W0 * (E_multiplier ^ T)
E_WT_all_in
```

```
## [1] 3458516
```

```r
# Fixed stake s: each round adds expected net (E[R]-s) = (1.05-1)*s = 0.05*s
# So E[W_T] = W0 + 0.05 * s * T
expectation_fixed_bet <- function(W0, s, T) W0 + 0.05 * s * T
expectation_fixed_bet(W0, s = 1,  T)
```

```
## [1] 210
```

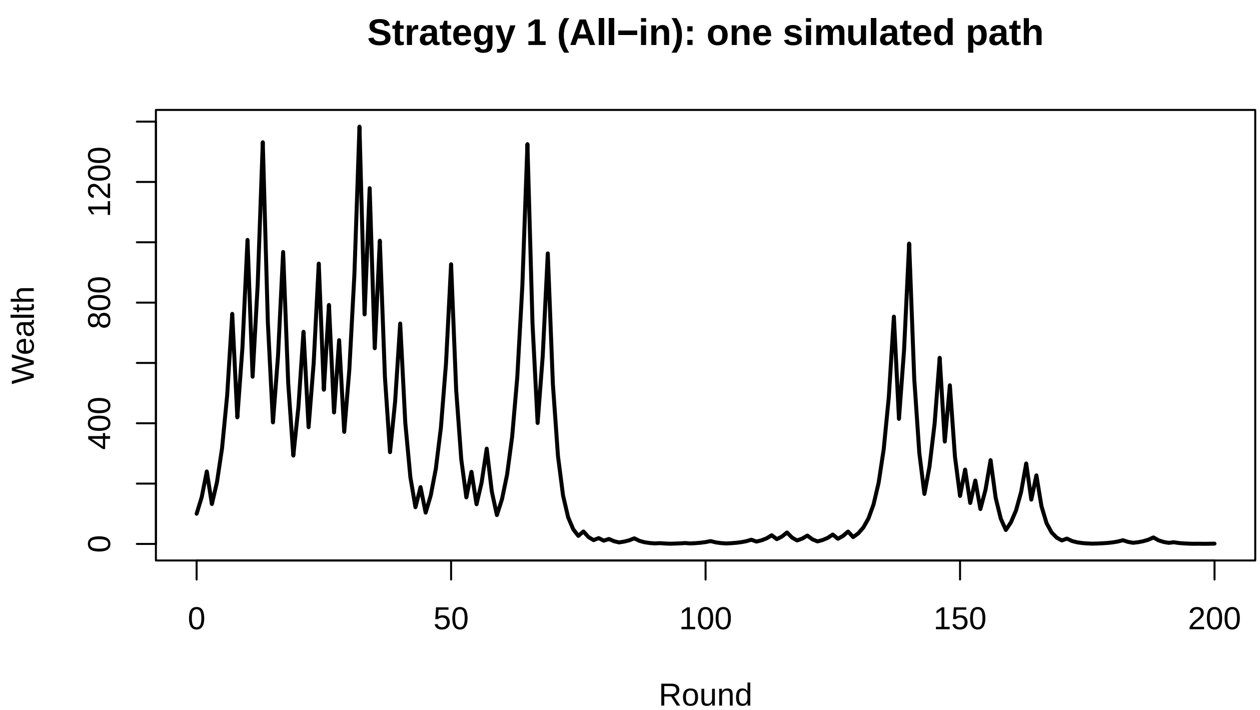
```r
expectation_fixed_bet(W0, s = 10, T)
```

```
## [1] 300
```

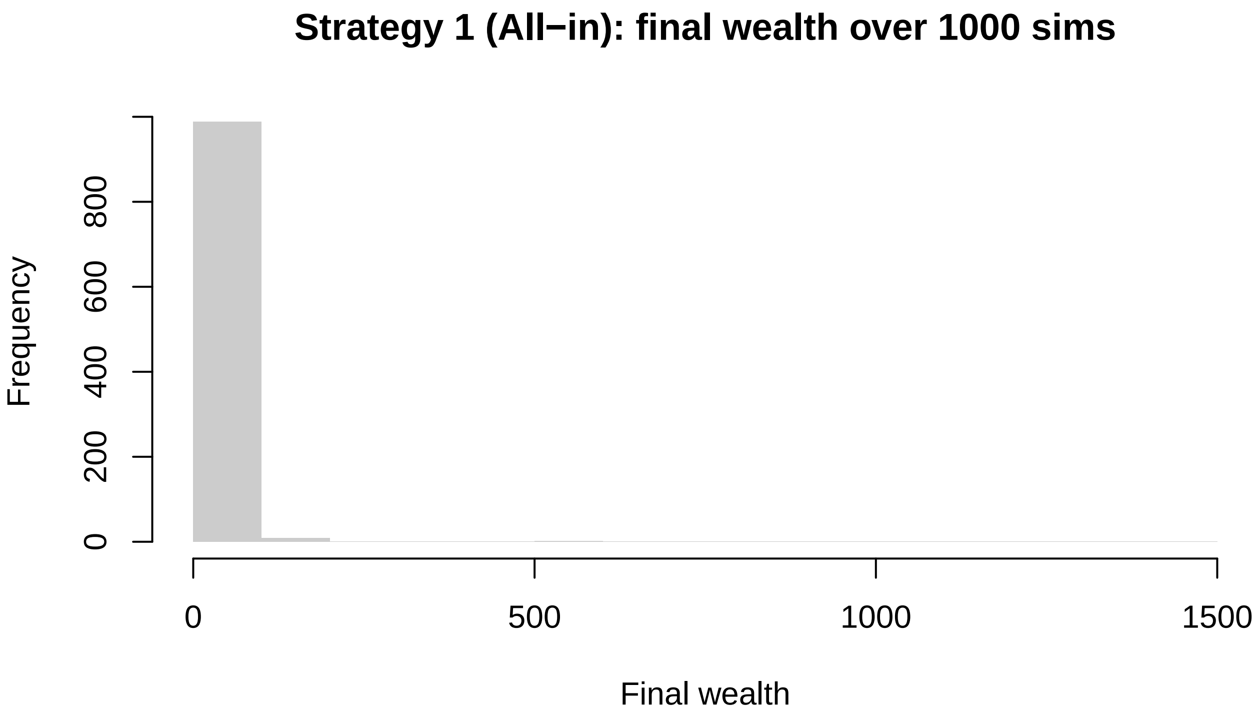
Strategy 1: All-in reinvestment

```r
W0 <- 100
T  <- 200

# A single sample path
one_path <- simulate_strategy_all_in(W0, T)
plot(seq(0, T), one_path$path, type = "l", lwd = 2,
     xlab = "Round", ylab = "Wealth",
     main = "Strategy 1 (All-in): one simulated path")
```




```r
# Distribution of final wealth across S simulations
S <- 1000
finals_all_in <- replicate_finals(simulate_strategy_all_in, S = S, initial_wealth = W0, num_rounds = T)
hist(finals_all_in, breaks = 20, col = "gray80", border = NA,
     main = sprintf("Strategy 1 (All-in): final wealth over %d sims", S),
     xlab = "Final wealth")
```




```
c(mean = format(mean(finals_all_in),scientific=FALSE), sd = format(sd(finals_all_in),scientific=FALSE),
  min = format(min(finals_all_in),scientific=TRUE), max = format(max(finals_all_in),scientific=FALSE),
```

```
##          mean            sd           min           max           var
##   "4.835967"    "55.17115" "5.249906e-16"    "1475.302"    "3043.855"
```

As shown in the histogram, most of the time we loss all the money by going all-in.

Strategy 2: Fixed stake each round ($s = 1$; $s = 10$)

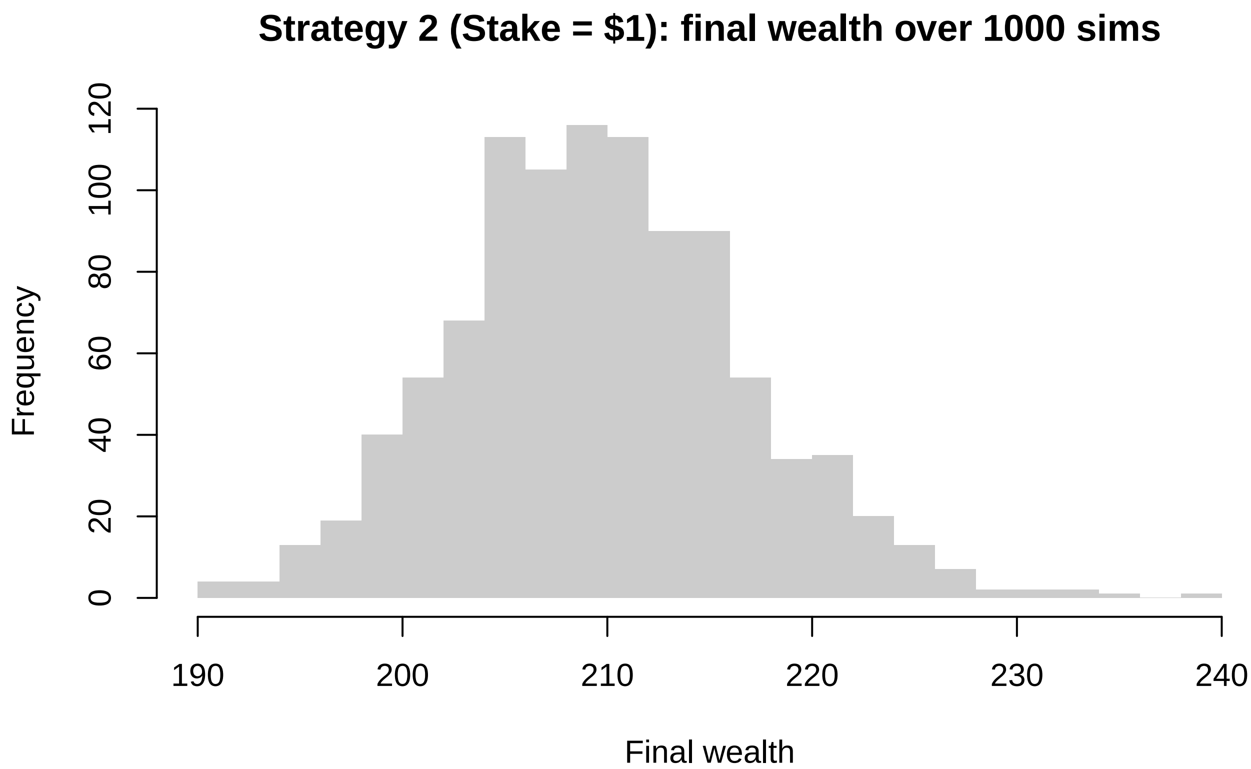
```
W0 <- 200; T <- 200

# s = 1 (expected mean  200 + 0.05*1*200 = 210, sd  sqrt(200)*0.5  7.07)
s <- 1
finals_s1 <- replicate(1000, simulate_strategy_fixed_bet(W0, s, T)$final)
summary(finals_s1); range(finals_s1)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   191.0   206.0   210.0   210.2   215.0   239.0

## [1] 191 239
```

```
hist(finals_s1, breaks = 30, col = "gray80", border = NA,
     main = "Strategy 2 (Stake = $1): final wealth over 1000 sims",
     xlab = "Final wealth")
```

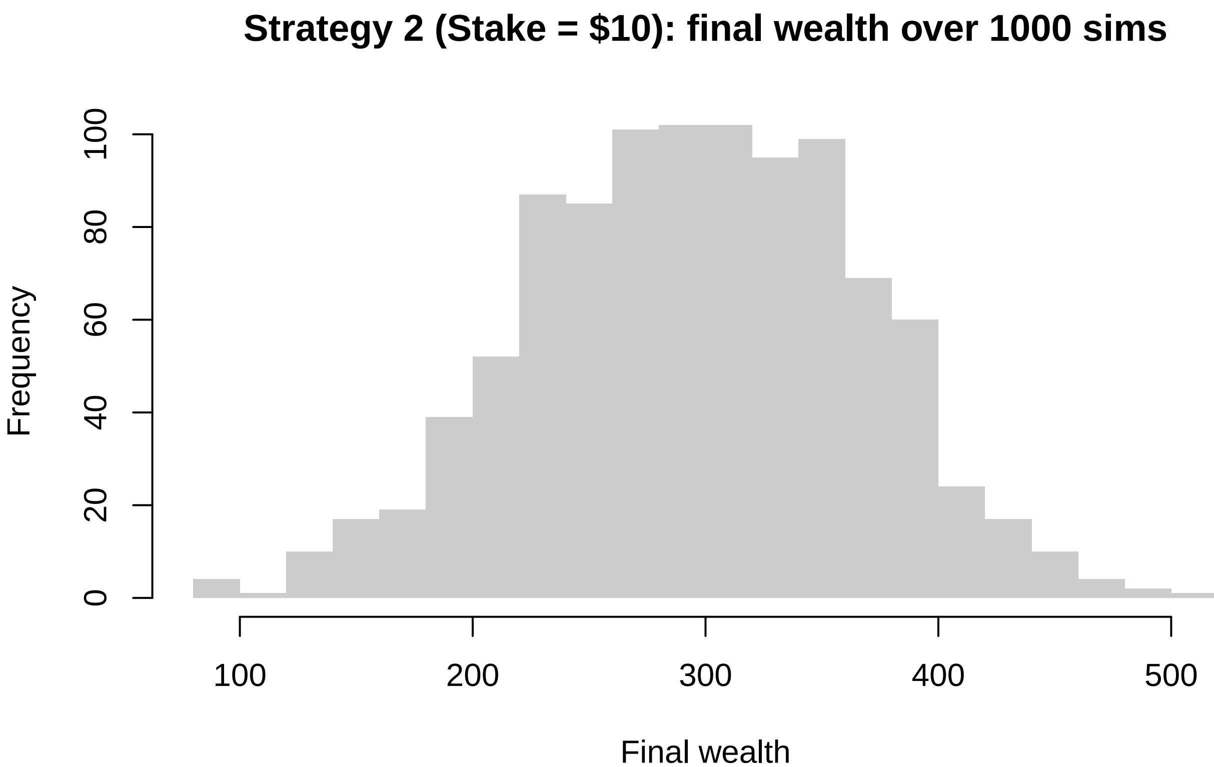



```r
# s = 10 (expected mean  300, sd  sqrt(200)*0.5*10  70.7)
s <- 10
finals_s10 <- replicate(1000, simulate_strategy_fixed_bet(W0, s, T)$final)
summary(finals_s10); range(finals_s10)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    90.0   250.0   300.0   300.8   360.0   510.0
```

```
## [1]  90 510
```

```r
hist(finals_s10, breaks = 20, col = "gray80", border = NA,
     main = "Strategy 2 (Stake = $10): final wealth over 1000 sims",
     xlab = "Final wealth")
```



As shown in the histogram, the Final wealth is more stable about 210 for s=1 and 300 for s=10

Conclusion

- The one-period expected payoff is $E[R] = 1.05 \cdot I$ (a 5% gain per dollar).
- For all-in reinvestment over T rounds, $E[W\_T] = W0 \cdot 1.05\hat{}T$, but simulation tells us that the long-run return is likely to be 0. The difference in realized paths can be highly volatile due to compounding randomness (geometric mean < arithmetic mean).
- Fixed-stake strategies concentrate outcomes: with stake s, $E[W\_T] = W0 + 0.05 \cdot s \cdot T$. Simulations align with these expectations and show much lower variance than all-in.