

All-in Trading Strategies Simulation on Binary Return Model

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Introduction

We study a simple binary-return investment model and compare three strategies: - all-in reinvestment each round, - fixed stake of \$1 per round, and - fixed stake of \$10 per round.

In each round, investing an amount I yields, with equal probability (0.5 each): - up factor: return = $1.55 I$
- down factor: return = $0.55 I$

The one-period expected return is $E[R] = 0.5 \cdot 1.55 \cdot I + 0.5 \cdot 0.55 \cdot I = 1.05 \cdot I$ (a 5% expected gain per dollar invested). We then simulate the strategies.

Investment model: helpers and quick checks

```
# Binary-return investment draw for amount I.
# Returns a length-n vector of realized payoffs.
invest_draw <- function(investment_amount,
                        n = 1,
                        p_up = 0.5,
                        up_factor = 1.55,
                        down_factor = 0.55) {
  multipliers <- sample(c(up_factor, down_factor),
                       size = n, replace = TRUE,
                       prob = c(p_up, 1 - p_up))
  investment_amount * multipliers
}

# Single-draw wrapper mirroring the original invest(I)
invest <- function(I) {
  invest_draw(I, n = 1)
}

# Alternative vectorized way
#invest_alternative <- function(I, n = 3) {
#   Rseq <- c(1.55 * I, 0.55 * I)
#   sample(Rseq, n, replace = TRUE)
#}
#invest_alternative(100, n = 3)
```

Strategy simulators

```
# Strategy 1: All-in reinvest each period.
# wealth[t] is wealth after t-1 rounds; wealth[1] = initial_wealth.
simulate_strategy_all_in <- function(initial_wealth,
                                     num_rounds = 200,
                                     p_up = 0.5,
```

```

                                up_factor = 1.55,
                                down_factor = 0.55) {
wealth <- numeric(num_rounds + 1)
wealth[1] <- initial_wealth
for (t in 2:(num_rounds + 1)) {
  multiplier <- sample(c(up_factor, down_factor), 1,
                        prob = c(p_up, 1 - p_up))
  wealth[t] <- wealth[t - 1] * multiplier
}
list(path = wealth, final = wealth[num_rounds + 1])
}

# Strategy 2 (generic): Invest a fixed dollar amount `stake` each period.
# Each round: withdraw `stake`, add realized payoff on that stake.
simulate_strategy_fixed_bet <- function(initial_wealth,
                                         stake,
                                         num_rounds = 200,
                                         p_up = 0.5,
                                         up_factor = 1.55,
                                         down_factor = 0.55) {
wealth <- numeric(num_rounds)
# After first round: start with initial_wealth - stake + payoff(stake)
wealth[1] <- initial_wealth - stake + invest_draw(stake, 1, p_up, up_factor, down_factor)
for (t in 2:(num_rounds)) {
  wealth[t] <- wealth[t - 1] - stake + invest_draw(stake, 1, p_up, up_factor, down_factor)
}
list(path = wealth, final = wealth[num_rounds])
}

# Helper: run many independent simulations and return vector of finals
replicate_finals <- function(strategy_fun, S = 500, ...) {
  finals <- numeric(S)
  for (s in 1:S) finals[s] <- strategy_fun(...)$final
  finals
}

```

One-step expectations

```

I <- 1
E_R <- 0.5 * 1.55 * I + 0.5 * 0.55 * I
E_R

## [1] 1.05

# Analytical expectations for T rounds
W0 <- 200
T <- 200

# All-in:  $E[W_T] = W0 * (E[multiplier])^T$ , where multiplier in {1.55, 0.55}
E_multiplier <- 0.5 * 1.55 + 0.5 * 0.55 # = 1.05
E_WT_all_in <- W0 * (E_multiplier ^ T)
E_WT_all_in

## [1] 3458516

```

```

# Fixed stake s: each round adds expected net  $(E[R]-s) = (1.05-1)*s = 0.05*s$ 
# So  $E[W_T] = W_0 + 0.05 * s * T$ 
expectation_fixed_bet <- function(W0, s, T) W0 + 0.05 * s * T
expectation_fixed_bet(W0, s = 1, T)

```

```
## [1] 210
```

```
expectation_fixed_bet(W0, s = 10, T)
```

```
## [1] 300
```

Strategy 1: All-in reinvestment

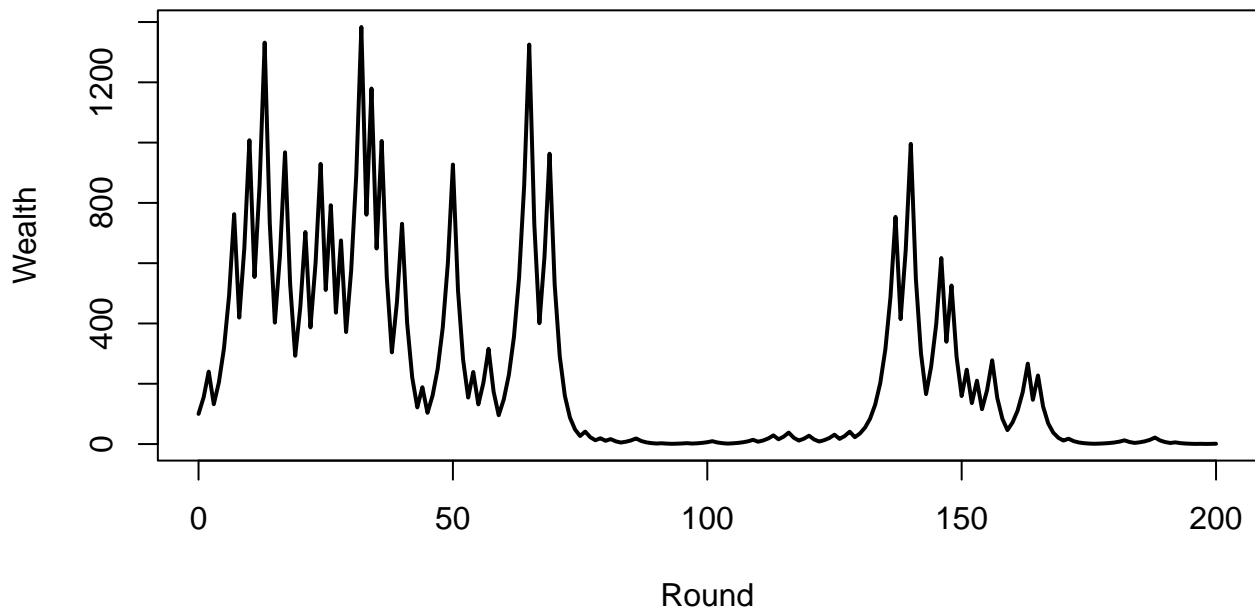
```

W0 <- 100
T <- 200

# A single sample path
one_path <- simulate_strategy_all_in(W0, T)
plot(seq(0, T), one_path$path, type = "l", lwd = 2,
     xlab = "Round", ylab = "Wealth",
     main = "Strategy 1 (All-in): one simulated path")

```

Strategy 1 (All-in): one simulated path

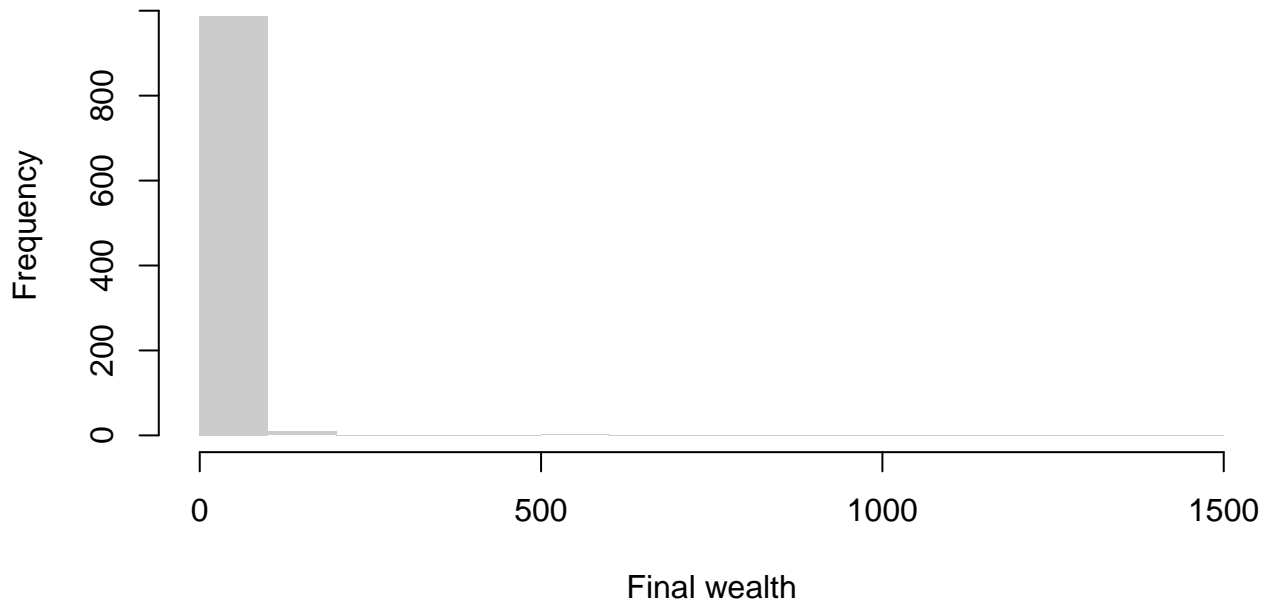


```

# Distribution of final wealth across S simulations
S <- 1000
finals_all_in <- replicate_finals(simulate_strategy_all_in, S = S, initial_wealth = W0, num_rounds = T)
hist(finals_all_in, breaks = 20, col = "gray80", border = NA,
     main = sprintf("Strategy 1 (All-in): final wealth over %d sims", S),
     xlab = "Final wealth")

```

Strategy 1 (All-in): final wealth over 1000 sims



```
c(mean = format(mean(finals_all_in),scientific=FALSE), sd = format(sd(finals_all_in),scientific=FALSE),
  min = format(min(finals_all_in),scientific=TRUE), max = format(max(finals_all_in),scientific=FALSE),
```

```
##           mean           sd           min           max           var
##    "4.835967"    "55.17115" "5.249906e-16"    "1475.302"    "3043.855"
```

As shown in the histogram, most of the time we loss all the money by going all-in.

Strategy 2: Fixed stake each round ($s = 1$; $s = 10$)

```
W0 <- 200; T <- 200

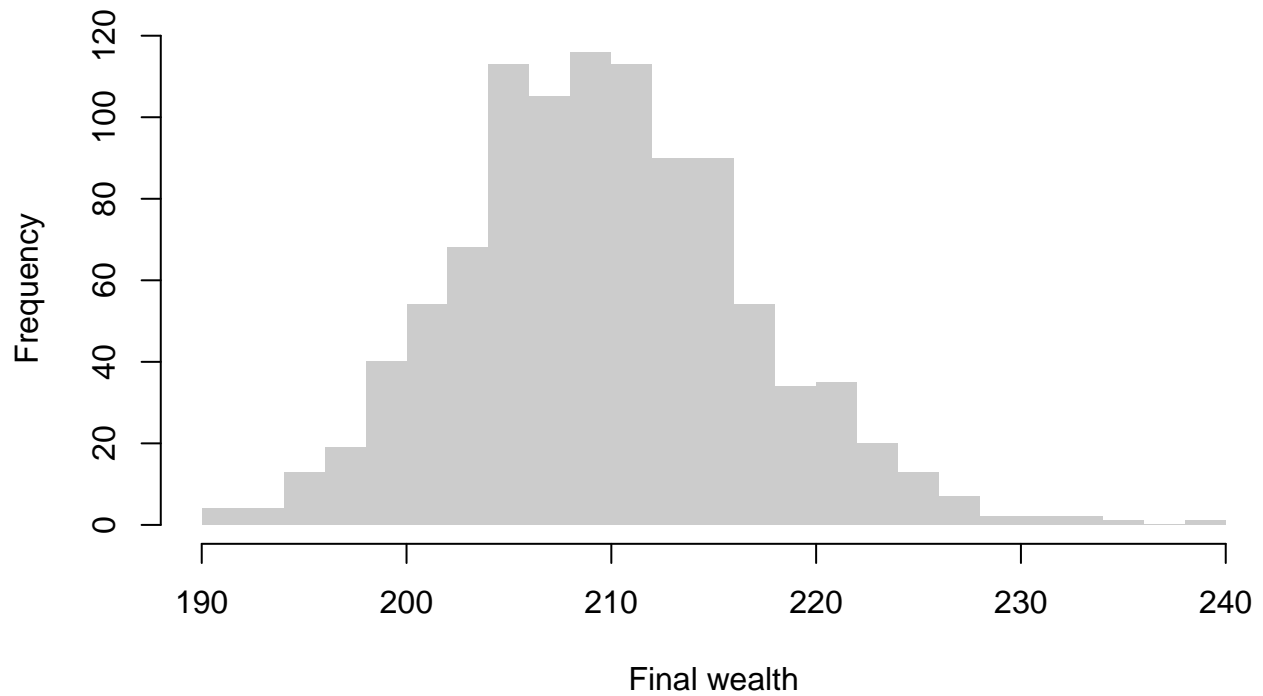
# s = 1 (expected mean 200 + 0.05*1*200 = 210, sd sqrt(200)*0.5 7.07)
s <- 1
finals_s1 <- replicate(1000, simulate_strategy_fixed_bet(W0, s, T)$final)
summary(finals_s1); range(finals_s1)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    191.0  206.0   210.0   210.2   215.0   239.0

## [1] 191 239

hist(finals_s1, breaks = 30, col = "gray80", border = NA,
  main = "Strategy 2 (Stake = $1): final wealth over 1000 sims",
  xlab = "Final wealth")
```

Strategy 2 (Stake = \$1): final wealth over 1000 sims



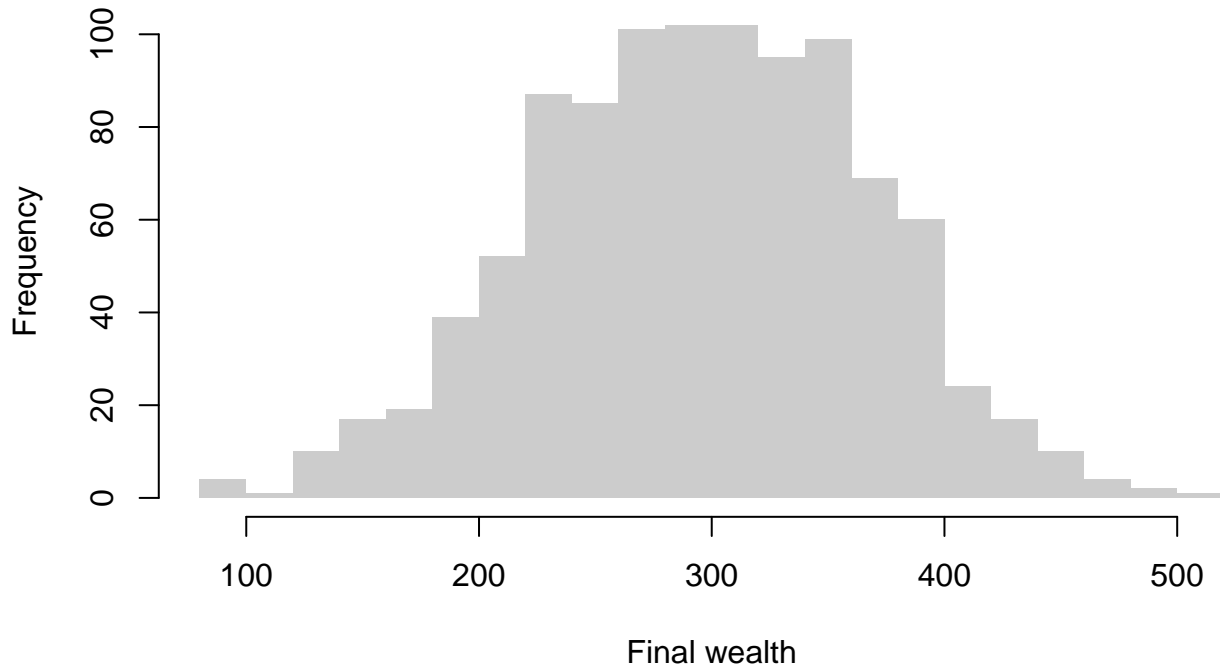
```
# s = 10 (expected mean 300, sd sqrt(200)*0.5*10 70.7)
s <- 10
finals_s10 <- replicate(1000, simulate_strategy_fixed_bet(W0, s, T)$final)
summary(finals_s10); range(finals_s10)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      90.0  250.0   300.0   300.8  360.0   510.0

## [1]  90 510
```

```
hist(finals_s10, breaks = 20, col = "gray80", border = NA,
     main = "Strategy 2 (Stake = $10): final wealth over 1000 sims",
     xlab = "Final wealth")
```

Strategy 2 (Stake = \$10): final wealth over 1000 sims



As shown in the histogram, the Final wealth is more stable about 210 for $s=1$ and 300 for $s=10$

Conclusion

- The one-period expected payoff is $E[R] = 1.05 \cdot I$ (a 5% gain per dollar).
- For all-in reinvestment over T rounds, $E[W_T] = W_0 \cdot 1.05^T$, but simulation tells us that the long-run return is likely to be 0. The difference in realized paths can be highly volatile due to compounding randomness (geometric mean < arithmetic mean).
- Fixed-stake strategies concentrate outcomes: with stake s , $E[W_T] = W_0 + 0.05 \cdot s \cdot T$. Simulations align with these expectations and show much lower variance than all-in.