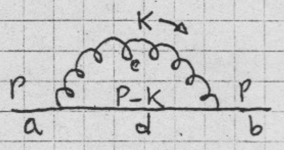
ECUACIONES DEL GRUPO DE RENORMALIZACION PARA LAS CONSTANTES GAUGE DEL MODELO ESTANDAR

1. Cálculo de diagramas a un loop

En esta sección calcularemos los diagramas a un loop para una teoría gauge no abeliana con simetría gauge SU(N).

1.1 Diagrama de Autoenergía ($d \to 4$, es la dimensión del espacio tiempo)



$$= -i\Sigma^{ab}(p) = -g^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\gamma_\mu\frac{1}{\not p-\not k-m}\gamma_\nu\frac{g^{\mu\nu}}{k^2}(T^c)_{ad}(T^c)_{db}$$

Esto es simplemente $(T^cT^c)_{ab}$ multiplicado por la correspondiente expresión de autoenergía en QED.

$$\Sigma^{ab}(p) = (T^cT^c)_{ab}\left[-ig^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\gamma_\mu\frac{1}{\not p-\not k-m}\gamma_\nu\frac{g^{\mu\nu}}{k^2}\right] \equiv (T^cT^c)_{ab}\,\Sigma(p)$$

$$\Sigma(p) = -ig^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{\gamma_\mu(\not p-\not k+m)\gamma^\mu}{[(p-k)^2-m^2]\,k^2}$$

El procedimiento para evaluar la integral es el siguiente:

a) Introducir los parametros de Feynman.
b) Hacer un cambio de variable en k de manera que no queden productos cruzados con otros momentums. Por ejemplo: $k' = k + pz$
c) Usar las formulas dadas en el apendice A para realizar las integrales. Realmente, los terminos lineales en k' integran a cero, debido a la ausencia de terminos cruzados.
d) Finalmente los términos divergentes son extraidos usando expansiones en $\varepsilon = 4-d$.

a) $$\frac{1}{ab} = \int_0^1\frac{dz}{[az+b(1-z)]^2}$$

escogemos:

$$a = (p-k)^2 - m^2 \qquad b = k^2$$
$$az + b(1-z) = (p-k)^2z - m^2z + k^2(1-z)$$

$$\Sigma(p) = -i\mu^{4-d}g^2\int_0^1 dz\int\frac{d^dk}{(2\pi)^d}\frac{\gamma_\mu(\not p-\not k+m)\gamma^\mu}{[\underbrace{(p-k)^2z}_{2kpz}-m^2z+k^2(1-z)]^2}$$

b) $k' = k - pz$ ó $k = k' + pz$

$$\begin{aligned}(p-k)^2z + k^2(1-z) &= (p-k'-pz)^2z + (k'+pz)^2(1-z)\\ &= (p-(k'+pz))^2z + (k'+pz)^2 - (k'+pz)^2z\\ &= p^2z - 2(k'+pz)pz + \cancel{(k'+pz)^2z} + (k'+pz)^2 - \cancel{(k'+pz)^2z}\\ &= p^2z - \cancel{2k'pz} - 2p^2z^2 + k'^2 + \cancel{2k'pz} + p^2z^2\\ &= k'^2 + p^2z - p^2z^2\\ &= k'^2 + p^2(1-z)z\end{aligned}$$

El término cruzado $2kpz$ ha sido cancelado!

(c)
$$\Sigma(p) = -i\mu^{4-d}g^2\int_0^1 dz\int\frac{d^dk'}{(2\pi)^d}\,\frac{\gamma_\mu(\not p-\not p z-\not k'+m)\gamma^\mu}{[k'^2-m^2z+p^2z(1-z)]^2}$$

De la formula (A.6) vemos que el término en k' integra a cero. De la fórmula (A.5):

$$\int\frac{d^dp}{(p^2+2pq-m^2)^\alpha} = (-1)^{d/2}\, i\pi^{d/2}\,\frac{\Gamma(\alpha-d/2)}{\Gamma(\alpha)}\,\frac{1}{[-q^2-m^2]^{\alpha-d/2}} \qquad (A.5)$$

y definiendo $k'^2-m'^2 = k'^2-m^2z+p^2z(1-z)$

$$m'^2 = m^2z-p^2z(1-z)\,, \quad q=0$$

tenemos:

$$\int\frac{d^dK'}{[K'^2-m^2z+p^2z(1-z)]^2} = i\pi^{d/2}\,\frac{\Gamma(2-d/2)\,\overset{1}{(-1)^{d/2}}}{\underset{1}{\Gamma(2)}[-m^2z+p^2z(1-z)]^{2-d/2}}$$

$\Gamma(n) = (n-1)!$

$\Gamma(2) = (2-1)! =$

$$(2\pi)^{-d}\pi^{d/2} = (4\pi)^{-d/2} = (4\pi)^{2-d/2}(4\pi)^{-2}$$

$$\varepsilon = 4-d \;\Rightarrow\; \varepsilon/2 = 2-d/2$$

$$d = 4-\varepsilon$$

$$\Sigma(p) = \frac{\mu^{4-d}g^2\,\Gamma(\varepsilon/2)}{16\pi^2}\int_0^1 dz\,\gamma_\mu(\not p(1-z)+m)\gamma^\mu/\{[-m^2z+p^2z(1-z)]/4\pi\}^{+\varepsilon/2}$$

$$= \frac{g^2\,\Gamma(\varepsilon/2)}{16\pi^2}\int_0^1 dz\,\gamma_\mu[\not p(1-z)+m]\gamma^\mu \times\left[\frac{-m^2z+p^2z(1-z)}{4\pi\mu^2}\right]^{-\varepsilon/2}$$

Agebra de Dirac en d-dimensiones:

$$\{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu}$$

Donde $g_{\mu\nu}$ es el tensor métrico en un espacio de Minkowski d-dimensional, de modo que $\delta^\mu_\mu = d$, de aquí se sigue que:

$$\gamma^\mu\gamma_\mu = d \qquad \gamma_\mu\gamma_\nu\gamma^\mu = (2-d)\gamma_\nu \qquad (1)$$

En adición

$$\mathrm{Tr}\,(\text{número impar de matrices } \gamma) = 0 \qquad (3)$$

$$\mathrm{Tr}\,I = f(d) \qquad \mathrm{Tr}[\gamma_\mu\gamma_\nu] = f(d)\,g_{\mu\nu} \qquad (4)$$

$$\mathrm{Tr}[\gamma_\mu\gamma_\kappa\gamma_\nu\gamma_\lambda] = f(d)(g_{\mu\kappa}g_{\nu\lambda}-g_{\mu\nu}g_{\kappa\lambda}+g_{\mu\lambda}g_{\kappa\nu}) \qquad (5)$$

De la Ec. (1) tenemos $(2-(4-\varepsilon)) = -2+\varepsilon$

$$\gamma_\mu\gamma_\nu\gamma^\mu = (2-d)\gamma_\nu = (-2+\varepsilon)\gamma_\nu$$

$\gamma_\mu\gamma_\nu\gamma^\mu p(1-z) + \gamma_\mu\gamma^\mu m$
$= (-2+\varepsilon)\gamma_\nu p(1-z) + dm$

$$\Sigma(p) = \frac{g^2\,\Gamma(\varepsilon/2)}{16\pi^2}\int_0^1\{-2\not p(1-z)+4m+\varepsilon[\not p(1-z)-m]\}\times\left[\frac{-m^2z+p^2z(1-z)}{4\pi\mu^2}\right]^{-\varepsilon/2}$$

d) Del ápendice B, ecuación (B.3), tenemos

$$\Gamma(\varepsilon/2) = \frac{2}{\varepsilon}-\gamma+\mathcal{O}(\varepsilon)$$

$$a^\varepsilon = 1+\varepsilon\ln a+\dots$$

$(-2+\varepsilon)\not p(1-z)+4m-\varepsilon m$
$= -2\not p(1-z)+4m+\varepsilon[\not p(1-z)-m]$

$$\Sigma(p) \simeq -\frac{g^2}{16\pi^2}\left(\frac{2}{\varepsilon}-\gamma+\cdots\right)\int_0^1 dz\{2\not p(1-z)-4m-\varepsilon[\not p(1-z)+m]\}\left\{1-\frac{\varepsilon}{2}\ln\left[\frac{m^2z+p^2z(1-z)}{4\pi\mu^2}\right]\right\}$$

$$=-\frac{g^2}{16\pi^2}\left\{\int_0^1 dz\frac{2}{\varepsilon}[2\not p(1-z)-4m]-2\int_0^1 dz[\not p(1-z)+m]-\gamma\int_0^1 dz[2\not p(1-z)-4m]\right.$$
$$\left.-\varepsilon\gamma[\not p(1-z)+m]\right\}\left\{1-\frac{\varepsilon}{2}\ln\left[\frac{m^2z+p^2z(1-z)}{4\pi\mu^2}\right]\right\}$$

$$\simeq -\frac{g^2}{16\pi^2}\left\{\frac{2}{\varepsilon}\left[\int_0^1 dz(2\not p(1-z)-4m)\right]-2\int_0^1 dz[\not p(1-z)+m]-\gamma\int_0^1 dz[2\not p(1-z)-4m]\right.$$
$$\left.-\int_0^1 dz[2\not p(1-z)-4m]\ln\left[\frac{m^2z+p^2z(1-z)}{4\pi\mu^2}\right]+\underbrace{\varepsilon X+\varepsilon^2 Y}_{\to 0,\ \varepsilon\to 0}\right\}$$

Evaluando las integrales:

$$\int_0^1 dz=1 \qquad \int_0^1(1-z)dz=z-\frac{z^2}{2}\Big|_0^1=\frac{1}{2}$$

tenemos

$$\Sigma(p)\simeq-\frac{g^2}{16\pi^2}\left\{\frac{2}{\varepsilon}(\not p-4m)-\frac{2\not p}{2}+2m-\gamma(\not p-4m)-2\int_0^1 dz[\not p(1-z)-2m]\ln\left[\frac{m^2z+p^2z(1-z)}{4\pi\mu^2}\right]\right.$$

$$\simeq-\frac{g^2}{16\pi^2}\left\{\frac{2}{\varepsilon}(\not p-4m)-\not p(1+\gamma)+2m(1+2\gamma)-2\int_0^1 dz[\not p(1-z)-2m]\ln\left[\frac{m^2z+p^2z(1-z)}{4\pi\mu^2}\right]\right.$$

$$=\frac{g^2}{8\pi\varepsilon}(-\not p+4m)+\underbrace{\frac{g^2}{16\pi^2}\left\{\not p(1+\gamma)-2m(1+2\gamma)+2\int_0^1 dz[\not p(1-z)-2m]\ln\left[\frac{m^2z+p^2z(1-z)}{4\pi\mu^2}\right]\right.}$$

$$=\frac{g^2}{8\pi^2\varepsilon}(-\not p+4m)+\text{finite} \qquad (6)$$

Sustituyendo (6) en la expresion para $\Sigma^{ab}(p)$ dada en la primera pagina hayamos

$$\Sigma^{ab}(p)=(T^cT^c)_{ab}\frac{g^2}{8\pi^2\varepsilon}(-\not p+4m)+\text{finite}$$

Para SU(N)

$$(T^cT^c)_{ab}=\frac{N^2-1}{2N}\delta_{ab}\equiv C_2(F)\delta_{ab}$$

donde

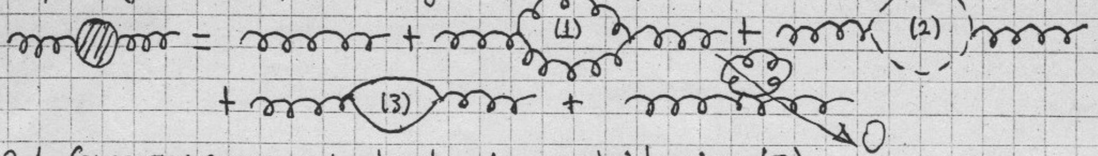
$$C_2(F)\equiv\frac{N^2-1}{2N} \quad \text{para } SU(N).$$

$$\boxed{\Sigma^{ab}(p)=\frac{g^2}{8\pi^2\varepsilon}C_2(F)(-\not p+4m)} \qquad (7)$$

1.2 Polarización del Vacío

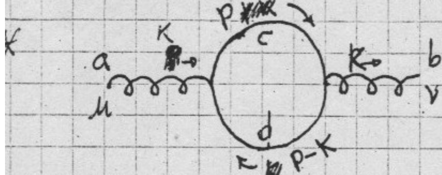
$-ig\gamma_\mu (T^a)_{bc}$

El propagador completo del gluon a un loop es



1.2.1 Comenzaremos calculando la contribucion (3).

$(T^a T^b)_{dd}$ γ_μ



$$= i\Pi^{ab}_{\mu\nu}(3) = -g^2\mu^{4-d}\int\frac{d^dp}{(2\pi)^d}(T^a)_{dc}(T^b)_{cd}\,\times Tr\left[\gamma_\mu\frac{1}{\not p - m}\gamma_\nu\frac{1}{\not p-\not K-m}\right]$$

$$\Pi^{ab}_{\mu\nu}(3) = Tr(T^aT^b)\,\Pi_{\mu\nu}(K) \qquad (8)$$

donde $\Pi_{\mu\nu}(K)$, la expresión correspondiente a QED es:

$$\Pi_{\mu\nu}(K) = i\mu^{4-d}g^2\int\frac{d^dp}{(2\pi)^d}Tr\left[\gamma_\mu\frac{1}{\not p-m}\gamma_\nu\frac{1}{\not p-\not K-m}\right]$$

$$= ig^2\mu^{4-d}\int\frac{d^dp}{(2\pi)^d}\frac{Tr[\gamma_\mu(\not p+m)\gamma_\nu(\not p-\not K+m)]}{[p^2-m^2][(p-K)^2-m^2]}$$

(a) $$\frac{1}{ab} = \int_0^1\frac{dz}{[az+b(1-z)]^2}$$

$a = (p-K)^2-m^2 \qquad b = p^2-m^2$

$$az+b(1-z) = [(p-K)^2-m^2]z + (p^2-m^2)(1-z) = \cancel{p^2z} - 2pKz + K^2z - \cancel{m^2z} + \cancel{p^2z} - m^2 + \cancel{m^2z} + p^2$$

$$az+b(1-z) = -2pKz + p^2 + K^2z - m^2$$

(b). $p' = p - Kz$ ó $p = p' + Kz$

$$az+b(1-z) = \cancel{-2p'Kz} + (p'^2 + \cancel{2p'Kz} + K^2z^2) \underbrace{-2K^2z}_{-K^2z} + K^2z - m^2$$

$$= p'^2 + K^2z^2 - K^2z - m^2$$

$$= p'^2 - m^2 + K^2z(1-z).$$

$$\Pi_{\mu\nu}(K) = ig^2\mu^{4-d}\int_0^1dz\int\frac{d^dp'}{(2\pi)^4}Tr\frac{\{\gamma_\mu(\not p' + \not Kz + m)\gamma_\nu[\not p' - \not K(1-z)+m]\}}{[p'^2-m^2+K^2z(1-z)]^2}$$

La evaluación del numerador es:

$$N = \gamma_\mu(\not p' + \not Kz + m)\gamma_\nu(\not p' - \not K(1-z) + m) + \text{(Productos de 3 matrices } \gamma)$$

$$= \gamma_\mu(\not p'\gamma_\nu\not p') - \gamma_\mu\not Kz\gamma_\nu\not K(1-z) + m^2\gamma_\mu\gamma_\nu + \text{(productos de 3 matrices gamma)}$$

+ productos lineales en p' y K

$$TrN = [p'^\kappa p'^\lambda - K^\kappa K^\lambda z(1-z)]\,Tr[\gamma_\mu\gamma_\kappa\gamma_\nu\gamma_\lambda] + m^2Tr(\gamma_\mu\gamma_\nu)$$

$$= [p'^\kappa p'^\lambda - K^\kappa K^\lambda z(1-z)]f(d)(g_{\mu\kappa}g_{\nu\lambda} - g_{\mu\nu}g_{\kappa\lambda} + g_{\mu\lambda}g_{\nu\kappa}) + m^2f(d)g_{\mu\nu}$$

$Tr(I) = f(d)$, $Tr(\gamma_\mu\gamma_\nu) = f(d)g_{\mu\nu}$

$Tr(\gamma_\mu\gamma_\kappa\gamma_\nu\gamma_\lambda) = f(d)(g_{\mu\kappa}g_{\nu\lambda} - g_{\mu\nu}g_{\kappa\lambda} + g_{\mu\lambda}g_{\nu\kappa})$ || $f(d)\to 4$ cuando $d\to 4$

$$\mathrm{Tr}N = f(d)\{P'_\mu P'_\nu - K_\mu K_\nu z(1-z) - g_{\mu\nu}P'^2 + g_{\mu\nu}K^2 z(1-z) + m^2 f(d) g_{\mu\nu}\} + P'_\nu P'_\mu - K_\nu K_\mu z(1-z)$$

$$= f(d)\{2P'_\mu P'_\nu - 2z(1-z)K_\mu K_\nu + g_{\mu\nu}m^2 - g_{\mu\nu}P'^2 + g_{\mu\nu}K^2 z(1-z)\}$$

$$\boxed{= 2g_{\mu\nu}K^2 z(1-z) - g_{\mu\nu}K^2 z(1-z)}$$

$$= f(d)\{2P'_\mu P'_\nu - 2z(1-z)(K_\mu K_\nu - K^2 g_{\mu\nu}) - g_{\mu\nu}[P'^2 - m^2 + K^2 z(1-z)]\}$$

$P' \to P$

$$\Pi_{\mu\nu}(K) = ig^2\mu^{4-d} f(d)\int_0^1 dz \int \frac{d^dp}{(2\pi)^d}\left\{\frac{2P_\mu P_\nu}{[P^2 - m^2 + K^2 z(1-z)]^2} - \frac{2z(1-z)[K_\mu K_\nu - g_{\mu\nu}K^2]}{[P^2 - m^2 + K^2 z(1-z)]^2} - \frac{g_{\mu\nu}[P^2 - m^2 + K^2 z(1-z)]}{[P^2 - m^2 + K^2 z(1-z)]^2}\right\}$$

$$= ig^2\mu^{4-d} f(d)\int_0^1 dz \int \frac{d^dp}{(2\pi)^d}\left\{\frac{2P_\mu P_\nu}{[P^2 - m^2 + K^2 z(1-z)]^2} - \frac{2z(1-z)[K_\mu K_\nu - g_{\mu\nu}K^2]}{[P^2 - m^2 + K^2 z(1-z)]^2} - \frac{g_{\mu\nu}}{[P^2 - m^2 + K^2 z(1-z)]}\right\}$$

(c) Usando (A.5) y (A.7) obtenemos $\boxed{m'^2 = m^2 - K^2 z(1-z)}$

$$\text{A.5}\quad \int \frac{d^dp}{(p^2+2pq-m^2)^\alpha} = i\pi^{d/2}\frac{\Gamma(\alpha - d/2)}{\Gamma(\alpha)}\frac{1}{[q^2 - m^2]^{\alpha-d/2}}$$

$$\text{A.7}\quad \int \frac{d^dp\, P_\mu P_\nu}{(p^2+2pq-m^2)^\alpha} \overset{q=0}{=} \frac{i\pi^{d/2}}{\Gamma(\alpha)(-m^2)^{\alpha-d/2}}\left[\tfrac{1}{2}g_{\mu\nu}(-m^2)\times\Gamma(\alpha-1-\tfrac{d}{2})\right]$$

$$\int d^dp\left\{\frac{2P_\mu P_\nu}{[P^2 - m^2 + K^2 z(1-z)]^2} - \frac{g_{\mu\nu}}{[P^2 - m^2 + K^2 z(1-z)]}\right\} =$$

$$= \cancel{2}\,\frac{\Gamma(1-d/2)}{\underbrace{\Gamma(2)}_{=1}}\,\frac{1}{\cancel{2}}\,\frac{g_{\mu\nu}}{[m^2 + K^2 z(1-z)]^{1-d/2}} - \frac{\Gamma(1-d/2)}{\underbrace{\Gamma(1)}_{=1}}\,\frac{g_{\mu\nu}}{[m^2 + K^2 z(1-z)]^{1-d/2}} = 0$$

Así, solo el término de la mitad contribuye a la integral de $\Pi_{\mu\nu}(K)$. Este se evalua usando de nuevo (A.5)

$$\Pi_{\mu\nu}(K) = ig^2\mu^{4-d}\frac{f(d)}{(2\pi)^d}\left[-\int_0^1 \frac{dz\,[2z(1-z)(K_\mu K_\nu - g_{\mu\nu}K^2)]}{[m^2 + K^2 z(1-z)]^{2-d/2}}\, i\pi^{d/2}\frac{\Gamma(2-d/2)}{\Gamma(2)}\right]$$

(d) Extrayendo el polo. $\frac{\varepsilon}{2} = 2 - \frac{d}{2}$, $(2\pi)^d\pi^{d/2} = (4\pi)^{\varepsilon/2}(4\pi)^{-2}$

$$\Pi_{\mu\nu}(K) = g^2\cancel{\mu^{4-d}}\frac{f(d)}{(4\pi)^2}\frac{\Gamma(\varepsilon/2)}{(4\pi\mu^2)^{-\varepsilon/2}}\int_0^1 \frac{dz[2z(1-z)(K_\mu K_\nu - g_{\mu\nu}K^2)}{[m^2 + K^2 z(1-z)]^{\varepsilon/2}}$$

$$\int_0^1 dz[2z(1-z)(K_\mu K_\nu - g_{\mu\nu}K^2)] \times \left[\frac{-m^2 + K^2 z(1-z)}{4\pi\mu^2}\right]^{-\varepsilon/2}$$

$$= 2g^2\frac{f(d)}{16\pi^2}(K_\mu K_\nu - g_{\mu\nu}K^2)\left[\frac{2}{\varepsilon} - \gamma + \ldots\right]\int_0^1 dz\, z(1-z)\left\{1 - \frac{\varepsilon}{2}\ln\left[\frac{-m^2 + K^2 z(1-z)}{4\pi\mu^2}\right]\right\}$$

$$= 2g^2\frac{f(d)}{16\pi^2}(K_\mu K_\nu - g_{\mu\nu}K^2)\left\{\left(\frac{2}{\varepsilon} - \gamma\right)\underbrace{\int_0^1 dz\, z(1-z)}_{=\frac{1}{6}} - \int_0^1 dz\, z(1-z)\ln\left[\frac{-m^2 + K^2 z(1-z)}{4\pi\mu^2}\right]\right\}$$

$f(d) \to 4$, $\varepsilon \to 0$

$$= \cancel{2}g^2\frac{4}{16\pi^2}(K_\mu K_\nu - g_{\mu\nu}K^2)\left\{\frac{1}{3\varepsilon} - \frac{\gamma}{6} - \int_0^1 dz\, z(1-z)\ln\left[\frac{m^2 + K^2 z(1-z)}{4\pi\mu^2}\right]\right\}$$

$$\Pi_{\mu\nu}(K) = \frac{e^2}{6\pi^2\varepsilon}(K_\mu K_\nu - g_{\mu\nu}K^2) + \text{finite.} \qquad (9)$$

Sustituyen la anterior expresion en la Ecuación para $\Pi^{ab}_{\mu\nu}(3)$ tenemos finalmente

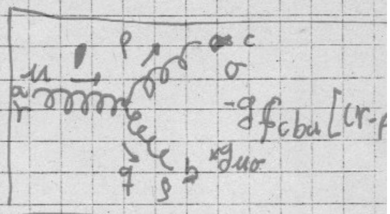
$$\Pi^{ab}_{\mu\nu}(3) = \mathrm{Tr}(T^a T^b)\frac{g^2}{6\pi^2\varepsilon}(P_\mu P_\nu - g_{\mu\nu}P^2)$$

Para el grupo $SU(N)$ se usa la normalización

$$\mathrm{Tr}[T^a T^b] = \frac{1}{2}\delta^{ab}$$

Pero si hay n_f fermiones contribuyendo a la polarización del vacío, tendremos que adicionar al factor n_F

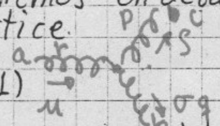
Para la QCD $n_f \equiv n_F$ = numero de quarks



Entonces:

$$\boxed{\Pi^{ab}_{\mu\nu}(3) = \frac{g^2}{6\pi^2\varepsilon}\frac{n_f}{2}\delta_{ab}(P_\mu P_\nu - g_{\mu\nu}P^2)} \qquad (10)$$

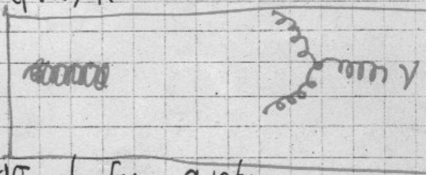
Como los calculos para los otros procesos que contribuyen a la polarización del vacío son muy similares no los haremos en detalle. Retornaremos a los cálculos detallados en el cálculo del vertice.



$-gC_{cba}[(r-p)_\sigma g_{\mu\rho} + (r-q)_\rho g_{\mu\sigma} + (q-p)_\mu g_{\rho\sigma}]$

1.2.2 Cálculo de la contribución (1)

$$= i\Pi^{ab}_{\mu\nu}(1) = -\frac{1}{2}g^2\mu^{4-d}f^{acd}f^{bde}\int\frac{d^dK}{(2\pi)^d}\frac{E_{\mu\nu}}{(P+K)^2K^2} \qquad (11)$$



donde

$$E_{\mu\nu} = [(-2P-K)_\sigma g_{\mu\rho} + (P+2K)_\mu g_{\sigma\rho} + (P-K)_\rho g_{\mu\sigma}]$$
$$\times [(P-K)^\rho g^\sigma_\nu + (P+2K)_\nu g^{\sigma\rho} + (-K-2P)^\sigma g^\rho_\nu]$$

El factor $\frac{1}{2}$ en (11) es el factor de simetría. Usando $g^\sigma_\sigma = d$, $E_{\mu\nu}$ queda

$$E_{\mu\nu} = P_\mu P_\nu(d-6) + (P_\mu K_\nu + K_\mu P_\nu)(2d-3) + K_\mu K_\nu(4d-6) + g_{\mu\nu}[(2P+K)^2 + (P-K)^2]$$

El procedimiento es ahora estandar. (a) Introducir los parámetros de Feynman, (b) poner $K' = K + Pz$, (c) usar las formulas (A4–A.6) (Realmente, los terminos lineales en K' integran a zero), (d) Finalmente los terminos con polos son extraidos y el resultado final es

$$\Pi^{ab}_{\mu\nu}(1) = \frac{-g^2}{16\pi^2\varepsilon}f^{acd}f^{bcd}\left[\frac{11}{3}P_\mu P_\nu - \frac{19}{6}g_{\mu\nu}P^2\right] \qquad (12)$$
$$= -\frac{g^2}{16\pi^2\varepsilon}\delta^{ab}C_2(G)\left[\frac{11}{3}P_\mu P_\nu - \frac{19}{6}g_{\mu\nu}P^2\right]$$

1.2.3 La contribución del fantasma es

$$= g^2 f^{cad}f^{dbc}\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{(P+K)_\mu K_\nu}{(P+K)^2K^2}$$

y por una aplicación directa de las técnicas usadas antes, obtenemos

$$\Pi^{ab}_{\mu\nu}(2) = \frac{g^2}{16\pi^2\varepsilon}f^{acd}f^{bcd}\left[\frac{1}{3}P_\mu P_\nu + \frac{1}{6}g_{\mu\nu}P^2\right] \qquad (13)$$

Por inducción se muestra que.

$$f^{acd} f^{bcd} = \delta^{ab} C_2(G)$$

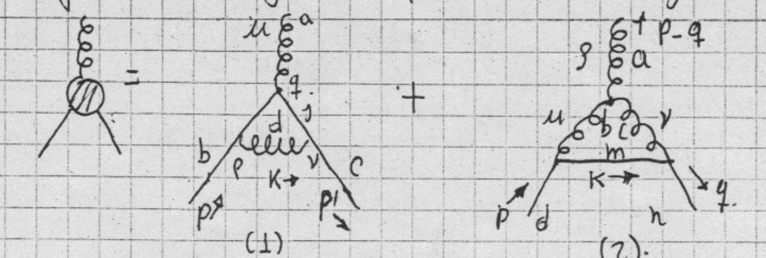
$$C_2(g) = N \text{ para } SU(N) \quad , \quad C_2(g) = 0 \text{ para } U(1)$$

colocando todos los resultados juntos, el tensor de polarización del vacio llegua a ser

$$\boxed{\Pi^{ab}_{\mu\nu}(1+2+3) = \frac{g^2}{8\pi^2 \varepsilon}(g_{\mu\nu}p^2 - p_\mu p_\nu)\left(\frac{5}{3}C_2(G) - \frac{2n_f}{3}\right)\delta^{ab}} \qquad (15)$$

1.3 EL VERTICE.

Dos diagramas de Feynman diferentes contribuyen al vertice:



1.3.1. La contribución de la Fig. (1) a la función del vértice es.

$$-igμ^{2-d/2}(\Lambda^a_μ)_{cd}(p,q,p')(1)$$

$$=(-igμ^{2-d/2})^3\int\frac{d^dk}{(2\pi)^d}\gamma_\nu(T^d)_{cj}\frac{i}{\not p'-\not k-m}\gamma_μ(T^a)_{ji}\frac{i}{\not p-\not k-m}(T^d)_{ib}\gamma_\rho\frac{-ig^{\nu\rho}}{k^2}$$

$$\Lambda^a_μ(p,q,p')(1)=(T^dT^aT^d)\Lambda_μ(p,q,p') \qquad (15)$$

donde $\Lambda_μ(p,q,p')$ es el vertice idéntico para la QED

$$-igμ^{2-d/2}\Lambda_μ(p,q,p')=(-ieμ^{2-d/2})^3\int\frac{d^dk}{(2\pi)^d}\gamma_\nu\frac{i}{\not p-\not k-m}\gamma_μ\frac{i}{\not p'-\not k-m}\gamma_\rho\frac{-ig^{\nu\rho}}{k^2}$$

$$=-(eμ^{2-d/2})^3\int\frac{d^dk}{(2\pi)^4}\frac{\gamma_\nu(\not p-\not k+m)\gamma_μ(\not p-\not k+m)\gamma^\nu}{k^2[(p-k)^2-m^2][(p'-k)^2-m^2]}$$

(a) $$\frac{1}{abc}=2\int_0^1dx\int_0^{1-x}dy\frac{1}{[a(1-x-y)+bx+cy]^3}$$

$a=k^2 \quad b=(p-k)^2-m^2 \quad c=(p'-k)^2-m^2$

$$a(1-x-y)+bx+cy=k^2(1-x-y)+[(p-k)^2-m^2]x+[(p'-k)^2-m^2]y$$

$$=k^2(1-x-y)+(p^2-2pk+k^2-m^2)x+[(p'^2-2p'k+k^2)-m^2]y$$

$$=k^2(1-\cancel{x}-\cancel{y})+p^2x-2pkx+\cancel{k^2x}-m^2x+p'^2y-2p'ky+\cancel{k^2y}-m^2y$$

$$=k^2-m^2(x+y)-2k(px+p'y)+p^2x+p^2y$$

$$\Lambda_μ(p,q,p')=\frac{-2ig^2μ^{4-d}}{(2\pi)^d}\int_0^1dx\int_0^{1-x}dy\int d^dk\frac{\gamma_\nu(\not p-\not k+m)\gamma_μ(\not p-\not k+m)\gamma^\nu}{(k^2-m^2(x+y)-2k(px+p'y)+p^2x+p'^2y)^3}$$

(b) $k'=k-px-p'y \Rightarrow k=k'+px+p'y$

$$k^2-2k(px+p'y)=2k'px+2k'p'y-2k'(px+p'y)+2pp'xy+k'^2+p^2x^2+p'^2y^2$$
$$-2px(px+p'y)-2p'y(px+p'y)$$

$$k^2 - m^2(x+y) - 2k(px + p'y) + p^2x + p'^2y = k'^2 - m^2(x+y) + p^2x^2 - 2p^2x^2 + p'^2y^2$$
$$-2p'^2y^2 + p^2x + p'^2y - 2pp'xy$$
$$= k'^2 - m^2(x+y) + p^2x(1-x) + p'^2y(1-y) - 2pp'xy$$

$$\Lambda_\mu(p,q,p') = \frac{-2ig^2\mu^{4-d}}{(2\pi)^d}\int_0^1 dx\int_0^{1-x} dy \int \frac{d^dk\, \gamma_\nu[\not{p}'(1-y) - \not{p}x - \not{k} + m]\gamma_\mu[\not{p}(1-x) - \not{p}'y - \not{k}}{[k^2 - m^2(x+y) + p^2x(1-x) + p'^2y(1-y) - 2p\cdot p'xy]^3}$$
$$\times \gamma^\nu$$

La parte divergente es:

$$\Lambda_\mu(p,q,p') = \frac{-2ig^2\mu^{4-d}}{(2\pi)^d}\int_0^1 dx\int_0^{1-x} dy \int \frac{d^dk\, \gamma_\nu \not{k}\gamma_\mu \not{k}\gamma^\nu}{[k^2 - m^2(x+y) + p^2x(1-x) + p'^2y(1-y) - 2p\cdot p'xy]^3}$$

(c) $-m'^2 \equiv -m^2(x+y) + p^2x(1-x) + p'^2y(1-y) - 2p\cdot p'xy$ y usando (A.7)

$$\Lambda_\mu(p,q,p') = -\frac{\cancel{2}ig^2\mu^{4-d}}{(2\pi)^d}\int_0^1 dx\int_0^{1-x} dy\, \frac{i\pi^{d/2}\Gamma(2-d/2)[-m'^2]}{\Gamma(3)[m'^2]^{3-d/2}}\gamma_\nu\gamma_\rho\gamma_\mu\gamma_\sigma\gamma^\nu \frac{1}{\cancel{2}}g_{\rho\sigma}$$

$$= \frac{g^2}{2}\left(\frac{1}{4\pi}\right)^{d/2}\mu^{4-d}\,\Gamma(2-d/2)\int_0^1 dx\int_0^{1-x} \frac{\gamma_\nu\gamma_\rho\gamma_\mu\gamma^\rho\gamma^\nu}{[-m'^2]^{2-d/2}}$$

Como

$$\gamma_\nu\gamma_\rho\gamma_\mu\gamma_\sigma\gamma^\nu = (2-d)\gamma_\rho\gamma_\mu\gamma_\sigma + 2(\gamma_\mu\gamma_\sigma\gamma_\rho - \gamma_\rho\gamma_\sigma\gamma_\mu)$$

entonces

$$\gamma_\nu\gamma_\rho\gamma_\mu\gamma^\rho\gamma^\nu = (2-d)\gamma_\rho\gamma_\mu\gamma^\rho + 2(\cancel{\gamma_\mu\gamma^\rho\gamma_\rho - \gamma_\rho\gamma^\rho\gamma_\mu})$$
$$= (2-d)^2\gamma_\mu.$$

(d)

$$\Lambda_\mu(p,q,p') = \frac{g^2}{2}\frac{1}{16\pi^2}\left(\frac{2}{\varepsilon} - \gamma + \cdots\right)\int_0^1 dx\int_0^{1-x} dy\,(2-d)^2\gamma_\mu\left[\frac{-m'^2}{4\pi\mu^2}\right]^{-\varepsilon/2}$$
$$\left\{1 - \frac{\varepsilon}{2}\ln\left[\frac{-m'^2}{4\pi\mu^2}\right]\right\}$$

$$\Lambda_\mu(p,q,p') = \frac{g^2}{2}\frac{1}{16\pi^2}\left[\frac{2}{\varepsilon}\int_0^1 dx\int_0^{1-x} dy\,(2-d)^2\gamma_\mu\right] + \text{finite}.$$

como $\int_0^1 (1-x)dx = \frac{1}{2}$ entonces,

$$\boxed{\Lambda_\mu(p,q,p') = \frac{e^2}{8\pi^2\varepsilon}\gamma_\mu + \text{finite}.} \qquad (16)$$

Retornando a la ecuación (15) queda por evaluar el factor de Teoria de grupos

$$T^d T^a T^d = T^d[T^a,T^d] + T^d T^d T^a = i f^{acd} T^d T^c + C_2(F) T^a.$$

Usando la relación de conmutación de nuevo.

$$T^d T^a T^d = -\frac{1}{2} f^{adc} f^{dcb} T^b + C_2(F) T^a.$$

$$f^{adc} f^{dcb} = f^{adc} f^{bdc} = \delta^{ab} C_2(G)$$

donde $C_2(G) = N$ para $SU(N)$.

entonces

$$T^d T^a T^d = \left[-\frac{1}{2} C_2(G) + C_2(F)\right] T^a \qquad (17)$$

Sustituyendo (17) y (16) en (15), obtenemos

$$\boxed{\Lambda^a_\mu(1) = \frac{g^2}{8\pi^2\epsilon}\left[-\frac{C_2(G)}{2} + C_2(F)\right]\gamma_\mu T^a.} \qquad (18)$$

1.3.2 La contribución de la Fig (2) a la función del vértice es.

$$-ig\mu^{2-d/2}\Lambda^a_\rho(2) = (-ig)^2(-g)(\mu^{4-d})^{3/2}\int\frac{d^dk}{(2\pi)^d}\gamma_\mu(T^b)_{dm}\frac{-i}{(k-p)^2} f_{abc}[(p-q-k+p)_\nu g_{\mu\rho} + (k-p-q+k)_\rho g_{\mu\nu} + (q-k-p+q)_\mu g_{\nu\rho}]\frac{-i}{(q-k)^2}\frac{i}{(\not k-m)}\gamma_\nu (T^c)_{mn}.$$

ó:

$$\Lambda^a_\rho(2) = \frac{g^2\mu^{4-d}}{(2\pi)^d} f^{abc} T^b T^c I_\rho$$

donde

$$I_\rho = \int d^dk \frac{\gamma_\mu(\not k+m)\gamma_\nu[(2p-q-k)_\nu g_{\mu\rho} + (2k-p-q)_\rho g_{\mu\nu} + (2q-p-k)_\mu g_{\nu\rho}]}{(k^2-m^2)(k-p)^2(q-k)^2}.$$

(a) $a = k^2 - m^2$ $b = (k-p)^2$ $c = (q-k)^2$.

$$a(1-x-y) + bx + cy = k^2(1-\cancel{x}-y) - m^2(1-x-y) + \cancel{k^2x} + p^2x - 2kpx + q^2y + k^2y - 2qky$$

$$= k^2 - m^2(1-x-y) + p^2x + q^2y - 2kpx - 2qky$$

(b) $k = k' + px + py \Rightarrow$

$$a(1-x-y) + bx + cy = (k'+px+py)^2 - m^2(1-x-y) + p^2x + q^2y - 2p(k'+px+qy)x - 2q(k'+px+py)y$$

$$= k'^2 - m^2(1-x-y) + p^2x + q^2y + \cancel{2k'px} + \cancel{2k'qy} + p^2x^2 + q^2y^2 - \cancel{2k'px} - 2p^2x^2 - 2pqxy$$
$$+ 2pxpy - \cancel{2k'qy} - 2q^2y^2 - 2pqxy$$

$$= k'^2 - m^2(1-x-y) + p^2x + q^2y - p^2x^2 - 2pxpy - q^2y^2.$$

$$= k'^2 - m^2(1-x-y) + p^2x + q^2y - (px+qy)^2.$$

$$I_\rho = 2\int_0^1 dx \int_0^{1-x} dy \int d^dk' \, \gamma^\mu(\not k' + \not p x + \not q y + m)\gamma^\nu \{[(2-x)p - (1+y)q - k']_\nu g_{\mu\rho}$$
$$+ [2k' + (2x-1)p + (2y-1)q]_\rho g_{\mu\nu} + [(2-y)q - (1+x)p - k']_\mu g_{\nu\rho}\} \times$$

$\times\left[K'^2 - m^2(1-x-y) + p^2 x + q^2 y - (px+qy)^2\right]^{-3}$ (11)

La parte divergente viene de los términos del numerador cuadráticos en K' que son:

2. Renormalización a 1 loop de una Teoría no Abeliana

2.1 AUTO ENERGIA, renormalizacion de ψ y m.

Resumimos a continuación las cantidades divergentes encontradas antes.

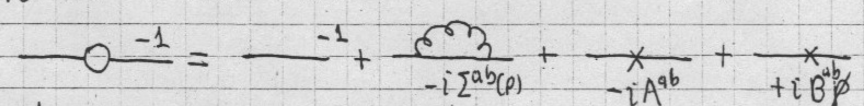
$$\Sigma^{ab}(p) = \frac{g^2}{8\pi^2\varepsilon}(-\not{p}+4m)\delta^{ab}C_2(F) \qquad (7)$$

$$\Pi^{ab}_{\mu\nu}(1+2+3) = \frac{g^2}{8\pi^2\varepsilon}(g_{\mu\nu}p^2 - p_\mu p_\nu)\left(\frac{5}{3}C_2(G) - \frac{2n_f}{3}\right)\delta^{ab} \qquad (10)$$

$$\Lambda^a_\mu = \frac{g^2}{8\pi^2\varepsilon}\left[C_2(G)+C_2(F)\right]\gamma_\mu T^a \qquad (20)$$

Con $C_2(F) = \frac{N^2-1}{2N}$ y $C_2(G) = N$ para $SU(N)$

Si queremos que el propagador inverso del fermión permanezca finito debemos adicionar contraterminos



$$S'_F(p)^{-1} \equiv \Gamma^2(p) = S_F(p)^{-1} - \Sigma^{ab}(p) - A^{ab} + B^{ab}\not{p}$$

$$= \not{p} - m + \frac{g^2}{8\pi^2\varepsilon}\not{p}\,\delta^{ab}C_2(F) - \frac{g^2}{8\pi^2\varepsilon}C_2(F)(4m)\delta^{ab} - A^{ab} + B^{ab}\not{p} = \text{finito}$$

de aqui que, ignorando terminos finitos

$$A^{ab} = -\frac{g^2 m}{2\pi^2\varepsilon}C_2(F)\delta^{ab}$$

$$B = -\frac{g^2}{8\pi^2\varepsilon}C_2(F)\delta^{ab}$$

Asi para

$$\mathcal{L}_1 = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi$$

adicionamos

$$(\mathcal{L}_1)_{CT} = iB\bar{\psi}\not{\partial}\psi - A\bar{\psi}\psi$$

dando el lagrangiano desnudo total

$$(\mathcal{L}_1)_B = \mathcal{L}_1 + (\mathcal{L}_1)_{CT} = i(1+B)\bar{\psi}\not{\partial}\psi - (m+A)\bar{\psi}\psi$$

Definiendo la función de onda desnuda por

$$\psi_B = \sqrt{1+B}\,\psi \equiv \sqrt{Z_2}\,\psi \qquad (21\text{-}a)$$

con

$$\boxed{Z_2 \equiv 1 + B = 1 - \frac{g^2}{8\pi^2\varepsilon}C_2(F)} \qquad (21\text{-}b)$$

podemos escribir el lagrangiano desnudo como

$$(\mathcal{L}_1)_B = i\bar{\psi}_B \partial\!\!\!/ \psi_B - m_B \bar{\psi}_B \psi_B \qquad (22)$$

donde la masa desnuda m_B estadada por

$$m_B = Z_2^{-1}(m+A)$$

$$= m\left(1 - \frac{g^2}{2\pi^2 \varepsilon} C_2(F) \right)\left(1 + \frac{g^2}{8\pi^2 \varepsilon} C_2(F) \right)$$

$$\simeq m\left(1 + \left(\frac{1}{8} - \frac{1}{2}\right)\frac{g^2}{\pi^2 \varepsilon} C_2(F) \right)$$

$$= m\left(1 - \frac{3\,g^2}{8\pi^2 \varepsilon} C_2(F) \right) = m + \delta m.$$

El hecho de que el lagrangiano dado en la ecuación (22) se haya podido mantener de la misma forma que el lagrangiano original significa que, a este orden, la teoría es renormalizable.

2.2. Para el tensor de Polarización del Vacío (RENORMALIZACION DE A_μ).

Para mantener el propagador del gluon finito veamos primero las modificaciones al propagador (los detalles de la teoría no Abeliana no son importantes aquí)

$$D'_{\mu\nu}(K) = D_{\mu\nu}(K) - D_{\mu\alpha}(K)\Pi^{\alpha\beta}(K) D_{\nu\alpha} + \cdots$$

$$= -\frac{g_{\mu\nu}}{K^2} - \frac{g_{\mu\alpha}}{K^2}\frac{g^2}{8\pi^2\varepsilon}\left(+g_{\alpha\beta}K^2 - K_\alpha K_\beta\right)\underbrace{\left(\frac{5}{3}C_2(G) - \frac{2n_f}{3}\right)}\Big]\frac{g_{\beta\nu}}{K^2} + \cdots$$

$$= -\frac{g_{\mu\nu}}{K^2}\left(1 + \frac{g^2}{8\pi^2\varepsilon}\left(\frac{5}{3}C_2(G) - \frac{2n_f}{3}\right)\right) + \frac{g^2}{8\pi^2\varepsilon}\left(\frac{5}{3}C_2(G) - \frac{2n_f}{3}\right)\frac{1}{K^2}\frac{K_\mu K_\nu}{K^2} + \cdots \quad (23)$$

$\therefore$ los términos infinitos en $D'_{\mu\nu}$ deben ser removidos adicionando contra-términos al lagrangiano original. El lagrangiano que da lugar al propagador gauge es

$$\mathcal{L}_2 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_\mu A^\mu)^2.$$

El contratermino requerido es.

$$(\mathcal{L}_2)_{CT} = -\frac{C}{4}F_{\mu\nu}F^{\mu\nu} - \frac{E}{2}(\partial_\mu A^\mu)^2.$$

Entonces el lagrangiano desnudo es

$$(\mathcal{L}_2)_B = \mathcal{L}_2 + (\mathcal{L}_2)_{CT} = -\left(\frac{1+C}{4}\right)F_{\mu\nu}F^{\mu\nu} + \text{términos gauge}.$$

Definiendo el campo desnudo

$$A^\mu_B \equiv Z_3^{1/2}A^\mu = \sqrt{1+C}\,A^\mu \qquad (24)$$

y comparando con (23) obtenemos

$$\boxed{Z_3 = 1 + \frac{g^2}{8\pi^2\varepsilon}\left(\frac{5}{3}C_2(G) - \frac{2n_f}{3}\right)} \qquad (25)$$

2.3 FUNCION DEL VERTICE, renormalización de la carga de color.

Es claro que la parte divergente de Λ^a_μ puede ser eliminada adicionando un contratermino al lagrangiano

$$\mathcal{L}_3 = -g\mu^{2-d/2} A^\mu \bar{\psi}\gamma_\mu \psi = -g\mu^{2-d/2}\bar{\psi}\not{A}\psi$$

$$(\mathcal{L}_3)_{CT} = -Dg\mu^{2-d/2}\bar{\psi}\not{A}\psi$$

De la ecuación (20)

$$\Lambda^a_\mu = \frac{g^2}{8\pi^2\varepsilon}\left[C_2(G)+C_2(F)\right] \times \gamma_\mu T^a$$

$$D = -\frac{g^2}{8\pi^2\varepsilon}\left[C_2(G)+C_2(F)\right]$$

de modo que

$$(\mathcal{L}_3)_B = -(1+D)g\mu^{\varepsilon/2}A^\mu\bar{\psi}\gamma_\mu\psi$$

$$\equiv -Z_1 g\mu^{\varepsilon/2}A^\mu\bar{\psi}\gamma_\mu\psi \qquad (26)$$

con

$$\boxed{Z_1 = 1 - \frac{g^2}{8\pi^2\varepsilon}\left[C_2(G)+C_2(F)\right]} \qquad (27)$$

Expresando $(\mathcal{L}_3)_B$ en la Ec (26) en términos de los campos desnudos definidos en las ecs. (21.a) y (24) obtenemos

$$(\mathcal{L}_3)_B = -\frac{Z_1 g\mu^{\varepsilon/2}}{Z_2 Z_3^{1/2}} A^\mu_B\bar{\psi}_B\gamma_\mu\psi_B \equiv -g_B A^\mu_B\bar{\psi}_B\gamma_\mu\psi_B$$

de modo que podemos finalmente definir la carga desnuda como

$$\boxed{g_B \equiv g\mu^{\varepsilon/2} Z_1 Z_2^{-1} Z_3^{-1/2}} \qquad (28)$$

Usando las ecuaciones (21.b), (25) y (27) obtenemos

$$g_B \cong g\mu^{\varepsilon/2}\left[1-\frac{g^2}{8\pi^2\varepsilon}\left[C_2(G)+C_2(F)\right]\right]\left[1+\frac{g^2}{8\pi^2\varepsilon}C_2(F)\right]\left\{1-\frac{g^2}{16\pi^2\varepsilon}\left(\frac{5}{3}C_2(G)-\frac{2n_f}{3}\right)\right\}$$

$$= g\mu^{\varepsilon/2}\left[1+\frac{g^2}{8\pi^2\varepsilon}\left[-C_2(G)-\cancel{C_2(F)}+\cancel{C_2(F)}\right]\right]\left[1-\frac{g^2}{8\pi^2\varepsilon}\left(\frac{5}{6}C_2(G)-\frac{2n_f}{6}\right)\right]$$

$$= g\mu^{\varepsilon/2}\left\{1+\frac{g^2}{8\pi^2\varepsilon}\left[-C_2(G)-\frac{5}{6}C_2(G)+\frac{n_f}{3}\right]\right.$$

$$= g\mu^{\varepsilon/2}\left\{1+\frac{g^2}{8\pi^2\varepsilon}\left[-\frac{11}{6}C_2(G)+\frac{n_f}{3}\right]\right\}$$

$$g_B = g\mu^{\varepsilon/2}\left\{1+\frac{g^2}{16\pi^2\varepsilon}\left[-\frac{11}{3}C_2(G)+\frac{2n_f}{3}\right]\right\} \qquad (29)$$

3) ECUACIONES DEL GRUPO DE LA RENORMALIZACION PARA LAS CONSTANTES GAUGE DEL MODELO ESTANDAR A UN LOOP

Las ecuación grupo de la renormalización para la constante gauge es

$$\mu \frac{dg}{\partial \mu} = \beta(g) \qquad (30)$$

$$\boxed{Y^2_{new} = \frac{3}{5} Y_{old} \qquad \tilde{Y}^2 = a^2 Y_a^2}$$

$\alpha = \frac{g^2}{4\pi}$

$\frac{d}{d} g^{-2} \times 4\pi$

$= 2g^{-3} \frac{dg}{dt} \times 4\pi$

$= \frac{ba}{2\pi}$

$\Rightarrow \frac{dg}{dt} = \frac{g^3}{16\pi^2}$

donde $\beta(g) = \lim_{\varepsilon \to 0} \mu \frac{\partial g_B}{\partial \mu}$

Usando la Ec. (29) tenemos

$$\mu \frac{\partial g_B}{\partial \mu} = g \frac{\varepsilon}{2} \mu^{\varepsilon/2 - 1} \mu + \frac{\varepsilon}{2} \frac{g^3}{16\pi^2 \varepsilon} \mu^{\varepsilon/2 - 1} \mu \left(-\frac{11}{3} C_2(G) + \frac{2n_f}{3} \right)$$

$$\beta(g) = \lim_{\varepsilon \to 0} \frac{\partial g_B}{\partial \mu} = \frac{g^3}{16\pi^2} \left(-\frac{11}{3} C_2(G) + \frac{2n_f}{3} \right)$$

haciendo el cambio de variable en (30) $t = \ln \mu \Rightarrow dt = \frac{1}{\mu} d\mu$, entonce

$\alpha = g^2/4\pi \quad \frac{d\alpha^{-1}}{dt}$

$\frac{d}{dt} g^{-2}(4\pi) =$

$-2g^{-3} \frac{dg}{dt}(4\pi)$

$\frac{d\alpha^{-1}}{dt} = -2g^{-3} \left(\frac{bg^3}{16\pi^2} \right)$

$\boxed{\frac{d\alpha^{-1}}{dt} \times 4\pi = \frac{b}{2\pi}}$

$$\frac{\partial g}{\partial t} = \beta(g) \quad ó$$

$$\frac{\partial g}{\partial t} = \frac{g^3}{16\pi^2} \left(-\frac{11}{3} C_2(G) + \frac{2n_f}{3} \right)$$

$$\frac{\partial g}{\partial t} = b \frac{g^3}{16\pi^2} \Rightarrow \frac{d}{dt} \alpha^{-1} = -\frac{ba}{2\pi}$$

En general

$$\frac{dg}{dt} = \frac{g^3}{16\pi^2} \underbrace{\left(-\frac{11}{3} C_2(G) + \frac{2n_f}{3} + \frac{1}{6} n_H \right)}_{b}$$

$C_2(G) = 2$

SU(2): $n_f = 6$ dobletes

$n_H = 1$ doblete de Higgs

$b_{SU(2)} = -\frac{11}{3} \times 2 + \frac{2}{3} \times 6 + \frac{1}{6} \times 1 = -\frac{19}{6}$

U(1): $C_2(G) = 0$

$n_f = 10 = \frac{10}{3} \times 3$

$n_H = 1 = \frac{1}{2} + \frac{1}{2}$

$b_{U(1)} = 0 + \frac{2}{3} \times \frac{10}{3} \times \frac{1}{2} + \frac{1}{6} = \frac{41}{6} \times \frac{3}{5}$

$\frac{10}{3} = \frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{2}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 1 = \frac{10}{3}$

$1 = \frac{1}{2} + \frac{1}{2}$

Para SU(3) $\quad b|_{SU(3)} \equiv b_3 = -\frac{11}{3} \cdot 3 + \frac{2 \cdot 6}{3} = -7 < 0$

donde $n_F = n_f =$ número de quarks $= 6$.

Similarmente para SU(2) $\quad b|_{SU(2)} = -\frac{19}{6} < 0$

mientras que para U(1) $\quad b|_{U(1)} = 41/10 > 0 \quad$ si $g_1 \equiv \sqrt{\frac{5}{3}}\, g'$.

Así, las Ecuaciones del grupo de la renormalización a 1 loop para los acoplamientos gauge del modelo estandar g_1, g_2, g_3 se pueden escribir en forma compacta como

$\frac{41}{6}$ para g'

$$\frac{d}{dt} g_a = \frac{1}{16\pi^2} b_a g_a^3 \qquad (31)$$

donde $t = \ln \mu$, con μ la escala del grupo de la renormalización, $b_a^{SM} = \left(\frac{41}{10}, -\frac{19}{6}, -7 \right)$; la normalización de g_1 aquí es escogida para que este de acuerdo con la derivada covariante canónica para la Gran unificación del grupo gauge $SU(3)_c \times SU(2)_L \times U(1)$

en SU(5) o SU(10). Así en términos de los acoplamientos gauge electrodebiles convencionales g y g' con $e = g\sin\theta_W = g'\cos\theta_W$ uno tiene $g_2 = g$ y $g_1 = \sqrt{\frac{5}{3}}\,g'$

Dentro del contexto del modelo estandar los acoplamientos, gauge no unifican. Sin embargo el modelo mínimo supersimétrico incluye justo el contenido de partículas correcto para asegurar que los acoplamientos gauge unifiquen. Los coeficientes de la RGE son en este caso mas grande debido a los efectos virtuales de los nuevas partículas en los loops:

$$b_a^{MSSM} = \left(\frac{33}{5}, 1, -3\right)$$

APENDICE A.

Las integrales en el espacio de momentum que nos aparecieron fueron de la forma

$$I_d(q) = \int \frac{d^d p}{(p^2 + 2pq - m^2)^\alpha} \qquad (A.1)$$

Introduciendo coordenadas polares

$$(p_0, r, \phi, \theta_1, \theta_2, \dots, \theta_{d-3})$$

de modo que

$$d^d p = dp_0\, r^{d-2}\, dr\, d\phi \sin\theta_1\, d\theta_1 \sin^2\theta_2\, d\theta_2 \dots \sin^{d-3}\theta_{d-3}\, d\theta_{d-3}$$
$$= dp_0\, r^{d-2}\, dr\, d\phi \prod_{k=1}^{d-3} \sin^k\theta_k\, d\theta_k$$

Entonces

$$I_d(q) = 2\pi \int_{-\infty}^{\infty} dp_0 \int_0^\infty r^{d-2}\, dr \int_0^\pi \frac{\prod_{k=1}^{d-3} \sin^k\theta_k\, d\theta_k}{(p^2 + 2pq - m^2)^\alpha}$$

Ahora usaremos la fórmula

$$\int_0^{\pi/2} (\sin\theta)^{2n-1} (\cos\theta)^{2m-1}\, d\theta = \frac{1}{2}\frac{\Gamma(n)\Gamma(m)}{\Gamma(n+m)}$$

colocando $m = \frac{1}{2}$ y recordando que $\boxed{\Gamma(\frac{1}{2}) = \sqrt{\pi}}$ da. , $k = 2n-1$, $n = \frac{k+1}{2}$

$$\int_0^\pi (\sin\theta)^k\, d\theta = \frac{\Gamma(\frac{k+1}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{k+2}{2})}$$

y de aquí

$$I_d(q) = 2\pi \int_{-\infty}^{\infty} dp_0 \int_0^\infty \frac{r^{d-2}\, dr}{(p^2 + 2pq - m^2)^\alpha} \left(\sqrt{\pi}\frac{\Gamma(1)}{\cancel{\Gamma(3/2)}}\right)\left(\sqrt{\pi}\frac{\cancel{\Gamma(3/2)}}{\cancel{\Gamma(5/2)}}\right) \cdots \left(\frac{\cancel{\Gamma(\frac{d-3+1}{2})}}{\Gamma(\frac{d-3+2}{2})}\sqrt{\pi}\right), \quad \Gamma(1) = 1$$

$$(\sqrt{\pi})^{(d-3)/2}$$

$$I_d(q) = \frac{2\pi^{\frac{1}{2}(d-3)+1}}{\Gamma\left(\frac{d-1}{2}\right)} \int_{-\infty}^{\infty} dp_0 \int_0^\infty \frac{r^{d-2}}{(p_0^2 - r^2 + 2pq - m^2)^\alpha}$$

Esta integral es invariante Lorentz, de modo que la podemos evaluar en el sistema de referencia $q_\mu=(\mu,\vec{0})$, entonces $2pq=2\mu p_0$. Cambiando variables a $p'_\mu=p_\mu+q_\mu$, que implica $p_0'^2-q^2=p_0^2+2\mu p_0$, obtenemos:

$$I_d(q)=\frac{2\pi^{(d-1)/2}}{\Gamma\left(\frac{d-1}{2}\right)}\int_{-\infty}^{\infty}dp'_0\int_0^{\infty}\frac{r^{d-2}dr}{[p_0'^2-r^2-(q^2+m^2)]^\alpha}\qquad \text{(A2)}$$

$\to p_0^2=p_0'^2-q^2-2\mu p_0$

$$\begin{aligned} p_\mu'^2&=p_\mu^2+2p_\mu q_\mu+q_\mu^2\\ &=p_\mu^2+2p_\mu q_\mu+q_\mu^2\\ &=p_\mu^2+2\mu p_0+q_0^2\\ p_0'^2-q^2&=p_0^2+2\mu p_0 \end{aligned}$$
con $q^2=q_0^2$

La función beta de Euler es $[(r^2-p_0'^2+(q^2+m^2))(-1)]^\alpha$

$$B(x,y)=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}=2\int_0^{\infty}dt\,t^{2x-1}(1+t^2)^{-x-y},$$

válida para $\mathrm{Re}\,x>0$, $\mathrm{Re}\,y>0$, así poniendo

$$x=\frac{1+\beta}{2}\qquad y=\alpha-\frac{1+\beta}{2}\qquad t=\frac{s}{M}\Rightarrow\alpha=y+\frac{1+\beta}{2}=y+x$$

tenemos que

$$\int_0^{\infty}ds\frac{s^\beta}{(s^2+M^2)^\alpha}=\frac{\Gamma\left(\frac{1+\beta}{2}\right)\Gamma\left(\alpha-\frac{1+\beta}{2}\right)}{2(M^2)^{\alpha-(1+\beta)/2}\Gamma(\alpha)}\qquad \text{(A.3)}$$

Sustituyendo en (A2) con $\beta=d-2$, $M^2=-p_0'^2+q^2+m^2$, obtenemos

$$\begin{aligned} I_d(q)&=\frac{(-1)^\alpha 2\pi^{(d-1)/2}\,\Gamma\left(\frac{1+d-2}{2}\right)\Gamma\left(\alpha-\frac{d-1}{2}\right)}{\Gamma\left(\frac{d-1}{2}\right)2\,\Gamma(\alpha)}\int_{-\infty}^{\infty}\frac{dp'_0}{(q^2+m^2-p_0'^2)^{\alpha-(d-1)/2}}\\ &=(-1)^{-\alpha}\pi^{(d-1)/2}\frac{\Gamma\left(\alpha-\frac{d-1}{2}\right)}{\Gamma(\alpha)}\int_{-\infty}^{\infty}\frac{dp'_0}{(q^2+m^2-p_0'^2)^{\alpha-(d-1)/2}}\\ &=\underbrace{(-1)^{2\alpha+(d-1)/2}}_{(-1)^{d/2}(-1)^{-1/2}}\pi^{(d-1)/2}\frac{\Gamma\left(\alpha-\frac{d-1}{2}\right)}{\Gamma(\alpha)}\int_{-\infty}^{\infty}\frac{d'p_0}{[p_0'^2-(q^2+m^2)]^{\alpha-(d-1)/2}} \end{aligned}$$

Usando de nuevo (A-3) $M=-(q^2+m^2)$, $\beta=0$, $\alpha'=\alpha-(d-1)/2$

$$\begin{aligned} I_d(q)&=\underset{(-1)^{-1/2}}{i}\pi^{(d-1)/2}\frac{\cancel{\Gamma\left(\alpha-\frac{d-1}{2}\right)}}{\Gamma(\alpha)}\times\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\alpha-\left(\frac{d-1}{2}\right)-\frac{1}{2}\right)}{\cancel{\Gamma\left(\alpha-\frac{d-1}{2}\right)}[-(q^2+m^2)]^{\alpha-(d-1)/2-1/2}}(-1)^{d/2}\\ &=i\pi^{d/2}\frac{\Gamma(\alpha-d/2)}{\Gamma(\alpha)}\frac{1}{[-(q^2+m^2)]^{\alpha-d/2}}(-1)^{d/2}\\ &=(-1)^\alpha i\pi^{d/2}\frac{\Gamma(\alpha-d/2)}{\Gamma(\alpha)}\frac{1}{(q^2+m^2)^{\alpha-d/2}}\qquad \text{(A.4)} \end{aligned}$$

$\Gamma(\frac{1}{2}) \to \sqrt{\pi}$

no es necesario

Usando (A.1)

$$\int\frac{d^dp}{(p^2+2pq-m^2)^\alpha}=i\pi^{d/2}\frac{\Gamma\left(\alpha-\frac{d}{2}\right)}{\Gamma(\alpha)}\frac{1}{[-q^2-m^2]^{\alpha-d/2}}(-1)^{d/2}\qquad \text{(A.5)}$$

$q=0$ $q=0$

diferenciando esta expresión con respecto a q^u.

$$-\alpha\int d^dp\,\frac{2p_u}{(p^2+2pq-m^2)^{\alpha+1}} = i\pi^{d/2}\,\frac{\Gamma(\alpha-d/2)}{\Gamma(\alpha)}\left(-\alpha+\frac{d}{2}\right)\frac{2q_u}{[-q^2-m^2]^{\alpha-d/2+1}}$$

$-\Gamma(\alpha-d/2)(\alpha-d/2) = -\Gamma(\alpha+1-d/2)$

$\alpha\Gamma(\alpha)=\Gamma(\alpha+1)$

usando $\beta\Gamma(\beta)=\Gamma(\beta+1)$ y poniendo $\alpha+1\to\alpha$, da.

$$\int d^dp\,\frac{p_u}{(p^2+2pq-m^2)^{\alpha}} \overset{+}{=}- i\pi^{d/2}\,\frac{\Gamma(\alpha-d/2)}{\Gamma(\alpha)}\,\frac{q_u}{(-q^2-m^2)^{\alpha-d/2}} \qquad (A.6)$$

diferenciando de nuevo con respecto a q_v

$$\int d^dp\,\frac{p_u p_v}{(p^2+2pq-m^2)^{\alpha}} = \frac{i\pi^{d/2}}{\Gamma(\alpha)}\,\frac{1}{(-q^2-m^2)^{\alpha-d/2}}\left[q_uq_v\Gamma\left(\alpha-\frac{d}{2}\right)+\frac{1}{2}g_{uv}(-q^2-m^2)\Gamma\left(\alpha-1-\frac{d}{2}\right)\right] \qquad (A.7)$$

Contrayendo

$$\int d^dp\,\frac{p^2}{(p^2+2pq-m^2)^{\alpha}} = \frac{i\pi^{d/2}}{\Gamma(\alpha)}\,\frac{1}{(-q^2-m^2)^{\alpha-d/2}}\times\left[q^2\Gamma\left(\alpha-\frac{d}{2}\right)+\frac{d}{2}(-q^2-m^2)\Gamma\left(\alpha-1-\frac{d}{2}\right)\right] \qquad (A.8)$$

De (III) $n\to r$

$$-\ln\Gamma(z) = \ln z+\gamma z+\sum_{r=1}^{\infty}\left[\ln\left(1+\frac{z}{r}\right)-\frac{z}{r}\right]$$

$$-\frac{d}{dz}\ln\Gamma(z) = -\frac{\Gamma'(z)}{\Gamma(z)} = \frac{1}{z}+\gamma+\sum_{r=1}^{\infty}\left[\underbrace{\frac{1}{1+\frac{z}{r}}\cdot\frac{1}{r}}_{\frac{1}{z+r}}-\frac{1}{r}\right]$$

APENDICE B.

Extracción de los polos.

Necesitaremos las formulas

$$z\Gamma(z)=\Gamma(z+1) \qquad (I)$$

$$\psi_1(z)=\frac{d\ln\Gamma(z)}{dz}=\frac{\Gamma'(z)}{\Gamma(z)} \qquad (II)$$

$$\frac{1}{\Gamma(z)} = z e^{\gamma z}\prod_{n=1}^{\infty}\left(1+\frac{z}{n}\right)e^{-z/n} \qquad (III)$$

(III) es la representacion de Weirstrass de $\Gamma(z)$, γ es la constante de Euler

$$\gamma=\lim_{n\to\infty}\left(1+\frac{1}{2}+\dots+\frac{1}{n}-\ln n\right)=0.5772157$$

De (III)

$$\cancel{-\ln\Gamma(z)=\ln z+\gamma z+\sum_{r=1}^{\infty}\left[\frac{1}{1+\frac{z}{r}}\frac{1}{r}-\frac{1}{r}\right]}$$

$$-\frac{d}{dz}\ln\Gamma(z) = -\frac{\Gamma'(z)}{\Gamma(z)} = \frac{1}{z}+\gamma+\sum_{r=1}^{\infty}\left(\frac{1}{z+r}-\frac{1}{r}\right)$$

De (II) $\psi_1(z) = -\frac{1}{z}-\gamma+\sum_{r=1}^{\infty}\left(\frac{1}{r}-\frac{1}{z+r}\right)$

Cuando $z=n=$ entero.

$$\psi_1(z) = -\gamma-\cancel{\frac{1}{n}}+\left\{\frac{1}{1}+\frac{1}{2}+\dots+\frac{1}{n-1}+\cancel{\frac{1}{n}}+\cancel{\frac{1}{n+1}}+\cancel{\frac{1}{n+2}}+\dots-\left[\cancel{\frac{1}{1+n}}+\cancel{\frac{1}{2+n}}+\dots\right]\right\}$$

$$\psi_1(n) = -\gamma + \sum_{r=1}^{n-1} \frac{1}{r}, \qquad \psi_1(1) = -\gamma$$

Haciendo una expansión de Taylor. $(\varepsilon \ll 1)$

$$\begin{aligned} \Gamma(1+\varepsilon) &= \Gamma(1) + \varepsilon\Gamma'(1) + \mathcal{O}(\varepsilon^2) \\ &= 1 + \varepsilon\Gamma(1)\psi_1(1) + \mathcal{O}(\varepsilon^2) \\ &= 1 - \varepsilon\gamma + \mathcal{O}(\varepsilon^2). \end{aligned}$$

De aquí se deduce usando (I): $\Gamma(1+\varepsilon) = \varepsilon\Gamma(\varepsilon)$

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon}\Gamma(1+\varepsilon) = \frac{1}{\varepsilon}\left[1 - \varepsilon\gamma + \mathcal{O}(\varepsilon^2)\right] = \frac{1}{\varepsilon} - \gamma + \mathcal{O}(\varepsilon).$$

$$\boxed{\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma + \mathcal{O}(\varepsilon)} \qquad (B.1).$$

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