ECON 251

Problem Set #1

(18 points)

Part I: Expected value, variance and covariance (8 points in total)

This exercise illustrates the properties of expected value, variance and covariance that we are going to use further in this course.

Note: You solved a similar example during your Section #1.

Suppose that for variables X and Y the following holds:

$$\mathbb{E}(X) = 1$$
$$\mathbb{E}(Y) = -1$$
$$var(X) = 4$$
$$var(Y) = 1$$
$$cov(X, Y) = -2.$$

Now suppose you generate two new variables equal to (2X+Y) and (X+4Y).

(i) Show that $\mathbb{E}(2X + Y) = 1$ and $\mathbb{E}(X + 4Y) = -3$.
(ii) Show that $var(2X + Y) = 9$.
(iii) Show that $cov(2X + Y, X + 4Y) = -6$.
(iv) In part *(iii)* you showed that $cov(2X + Y, X + 4Y) < 0$.
Intuitively, what does this imply for the relationship between the two variables (2X+Y) and (X+4Y)?

(2 points each, 8 points in total)

Part II: The summation operator (10 points in total)

This exercise illustrates the properties of the summation operator that we are going to use further in this course.

(i) Prove that given data on a variable X, and its sample mean $\bar{X}$ (defined as $\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N}$) the following holds: the sum of the deviations from the sample mean is always zero, i.e.

$$\sum_{i=1}^{N} (X_i - \bar{X}) = 0$$

Hint: We demonstrated this in class.

(*ii*) Prove that

$$\sum_{i=1}^{N} X_i Y_i \neq \sum_{i=1}^{N} X_i \sum_{i=1}^{N} Y_i$$

Hint: Provide a counterexample (easiest for *N*=2). You solved a similar example during your Section #1.

(iii) Prove that given data on a variable X, and its sample mean $\bar{X}$ (defined as $\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N}$) the following holds:

$$\sum_{i=1}^{N} (X_i - \bar{X})^2 = \sum_{i=1}^{N} X_i^2 - N\bar{X}^2$$

Hint: You solved virtually the same example during sections, except that then you had two variables X and Y:

$$\sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y}) = \sum_{i=1}^{N} X_i Y_i - N\,\bar{X}\bar{Y},$$

while in this example $X = Y$.

(iv) Suppose that you have data on a variable *X*. Denote the sample mean of the data by $\bar{X}$. Suppose that you multiply every observation in the data by a constant *B* and you add another constant *A*, such that you obtain a new observation $X_i^* = A + BX_i$ (this is often called a *linear transformation*). Denote the sample mean of the new data by $\overline{X^*}$.

a) Show that $\overline{X^*} = A + B\bar{X}$.

Hint: The sample mean of the *original data* is defined as: $\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N}$. The sample mean of the *new data* is defined as: $\overline{X^*} = \frac{\sum_{i=1}^{N} X_i^*}{N} = \frac{\sum_{i=1}^{N}(A+BX_i)}{N}$. Now simply show that $\frac{\sum_{i=1}^{N}(A+BX_i)}{N} = A + B\bar{X}$.

b) Show that

$$\sum_{i=1}^{N} (X_i^* - \overline{X^*})^2 = B^2 \sum_{i=1}^{N} (X_i - \bar{X})^2$$

Hint: Use the result from **a)** and substitute X_i^* and $\overline{X^*} = A + B\bar{X}$ into the left hand side.

(2 points each, 10 points in total)