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CSC 578 Neural Networks and Deep Learning – Section 910
Homework 4

1. Add comments for the following code in the function backprop() in "NN578_network.py".

```
# backward pass
delta = self.cost_derivative(activations[-1], y) * \
        sigmoid_prime(zs[-1])
nabla_b[-1] = delta
nabla_w[-1] = np.dot(delta, activations[-2].transpose())
```

- Add comments for each line.
- Explain in detail what the line is doing, **meaning-wise**. Explain what it is trying to accomplish and why, and how (by utilizing various variables in the network object). Do NOT just what is happening syntactically (for example "a is multiplied by b").
- Include in your explanation on these points (REQUIRED):
  - Why the first line is using * but the third line is using np.dot().
  - Why '.transpose()' in the last line is needed.

```
The backward propagation provides a way of computing the gradient cost
function and using [-1] as the index for the last layer to count backwards.
```

- `delta = self.cost_derivative(activations[-1], y)*sigmoid_prime(zs[-1])` #Output layer Error where $\delta^L = \nabla_a C \odot \sigma'(z^L)$ is computed here the gradient of the activation is multiplied with derivative of the sigmoid output.
  - The output layer is where the derivate of the error function and derivate of the sigmoid function is happening here. The sigmoid prime is applied to the last layers.

- `nabla_b[-1] = delta` #Change in biases is taken by the delta backwards on the last layer and its on the biases.
  - Biases does not have any variables so it goes into the change directly

- `nabla_w[-1] = np.dot(delta, activations[-2].transpose())` #Change in the weights happen where you are computing the previous layers error where you are propagating the error $\delta^l = ((w^{l+1})^T * \delta^{l+1}) \odot \sigma'(z^l)$

- multiplication is used because the values are scalar and the np.dot() is used with transpose() for multiplication because vector with matrix is being multiplied here

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2. An **Exercise** on Cross-entropy cost function in NNDL Chapter 3. Specifically, the 2nd question only:

> In the single-neuron discussion at the start of this section, I argued that the cross-entropy is small if σ(z) ≈ y for all training inputs. The argument relied on y being equal to either 0 or 1. This is usually true in classification problems, but for other problems (e.g., regression problems) y can sometimes take values intermediate between 0 and 1. Show that the cross-entropy is still minimized when **σ(z) = y** for all training inputs. When this is the case the cross-entropy has the value:
>
> $$C = -\frac{1}{n}\sum_x [y \ln y + (1-y)\ln(1-y)]. \qquad (64)$$

- For this problem, you can assume y as a single neuron (in the output layer) or a vector (i.e., the whole output layer).
- If you assume the former, you can do a rigorous proof by using calculus and minimizing the derivative of the function. But if you are not comfortable with calculus, **you can pick at least three values for y (between 0 and 1), and for each value of y, you compute the Cross-entropy value using that y and several varying a (e.g. 0.1, 0.2, 0.3,.. 0.9)** .
- If you assume the latter, you can do a formal proof by calculus as well (although the derivative function will have several variables), but I recommend using Information Theory to prove the formula will minimize when y and a are equal.

Lets look at several values to prove that it is such a case when y=a then it is the minimal value of the cross-entropy. This shown below in the table when we use this expression to solve for Cross-entropy we get these values.

CE= -y*np.log(a)-(1-y)*np.log(1-a))

y	a	CE
0.2	0.1	0.236606
	0.2	0.217322
	0.3	0.228497
	0.4	0.257067
	0.5	0.30103
	0.6	0.362722
	0.7	0.449283
	0.8	0.578558
	0.9	0.809151
0.5	0.1	0.522879
	0.2	0.39794
	0.3	0.33889
	0.4	0.309894
	0.5	0.30103
	0.6	0.309894
	0.7	0.33889
	0.8	0.39794
	0.9	0.522879
0.9	0.1	0.904576
	0.2	0.638764
	0.3	0.486081
	0.4	0.380331
	0.5	0.30103
	0.6	0.239458
	0.7	0.1917
	0.8	0.157116
	0.9	0.141182

Hence, proving that when a **= y** for all training inputs cross-entropy is minimized

$\forall \{ y = a \}$, CE is minimized

3. A **Problem** in NNDL Chapter 3,

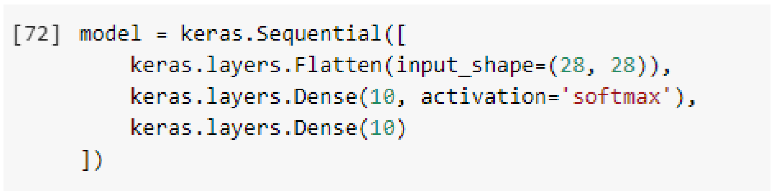
> We've discussed at length the learning slowdown that can occur when output neurons saturate, in networks using the quadratic cost to train. Another factor that may inhibit learning is the presence of the x_j term in Equation (61). Because of this term, when an input x_j is near to zero, the corresponding weight w_j will learn slowly. Explain why it is not possible to eliminate the x_j term through a clever choice of cost function.

Whenever a partial derivate is taken using any of the activation function for the neural network. The cost function with respect to weight and the derivate of the weighted input always has an x_j because of the chain rule. It is not possible to remove this x_j due to the constraint that cost must be a function of the output activations. x_j is the activation of the j node in the hidden layer.

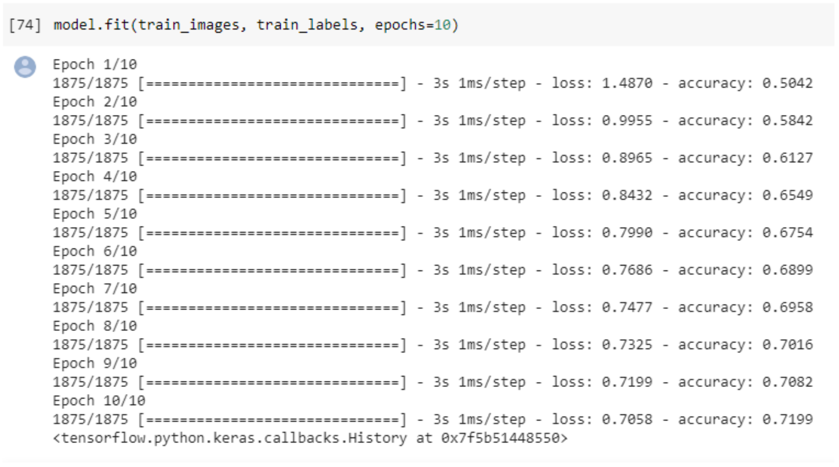
4.

Experimenting with hidden layers. I expect as I reduce the number of the hidden layers the accuracy drops but it calculates faster.

While choosing 10 for hidden layer with the activation at softmax as mentioned in the preliminary



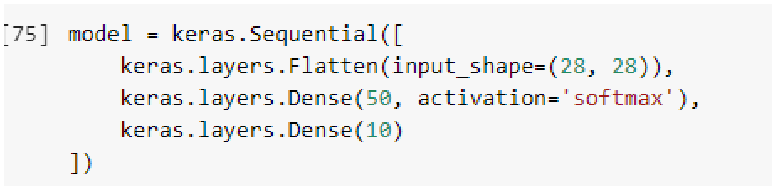
```
[72] model = keras.Sequential([
         keras.layers.Flatten(input_shape=(28, 28)),
         keras.layers.Dense(10, activation='softmax'),
         keras.layers.Dense(10)
     ])
```



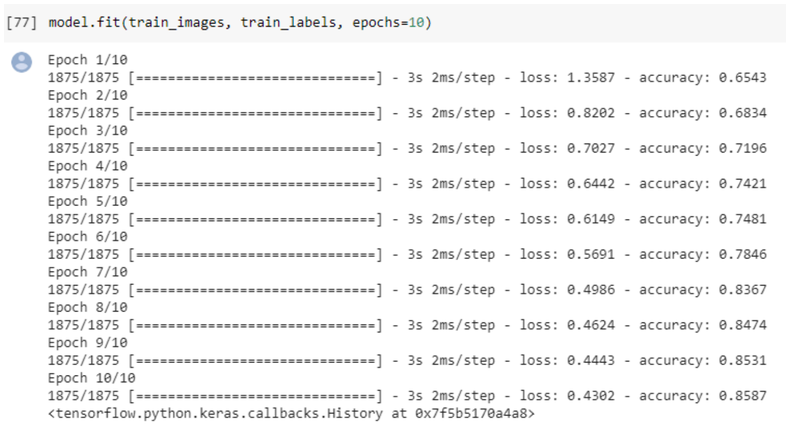
```
[74] model.fit(train_images, train_labels, epochs=10)

Epoch 1/10
1875/1875 [==============================] - 3s 1ms/step - loss: 1.4870 - accuracy: 0.5042
Epoch 2/10
1875/1875 [==============================] - 3s 1ms/step - loss: 0.9955 - accuracy: 0.5842
Epoch 3/10
1875/1875 [==============================] - 3s 1ms/step - loss: 0.8965 - accuracy: 0.6127
Epoch 4/10
1875/1875 [==============================] - 3s 1ms/step - loss: 0.8432 - accuracy: 0.6549
Epoch 5/10
1875/1875 [==============================] - 3s 1ms/step - loss: 0.7990 - accuracy: 0.6754
Epoch 6/10
1875/1875 [==============================] - 3s 1ms/step - loss: 0.7686 - accuracy: 0.6899
Epoch 7/10
1875/1875 [==============================] - 3s 1ms/step - loss: 0.7477 - accuracy: 0.6958
Epoch 8/10
1875/1875 [==============================] - 3s 1ms/step - loss: 0.7325 - accuracy: 0.7016
Epoch 9/10
1875/1875 [==============================] - 3s 1ms/step - loss: 0.7199 - accuracy: 0.7082
Epoch 10/10
1875/1875 [==============================] - 3s 1ms/step - loss: 0.7058 - accuracy: 0.7199
<tensorflow.python.keras.callbacks.History at 0x7f5b51448550>
```

The model reaches about 72% accuracy.

Modifying this number to 50 hidden layer should increase the accuracy and decrease the time as well.



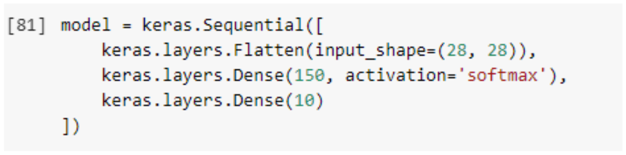
```
[75] model = keras.Sequential([
         keras.layers.Flatten(input_shape=(28, 28)),
         keras.layers.Dense(50, activation='softmax'),
         keras.layers.Dense(10)
     ])
```



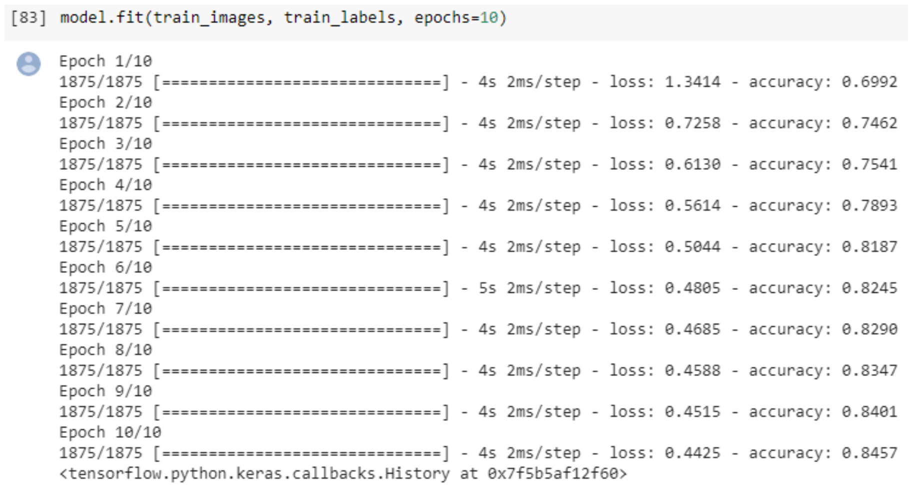
```
[77] model.fit(train_images, train_labels, epochs=10)

     Epoch 1/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 1.3587 - accuracy: 0.6543
     Epoch 2/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 0.8202 - accuracy: 0.6834
     Epoch 3/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 0.7027 - accuracy: 0.7196
     Epoch 4/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 0.6442 - accuracy: 0.7421
     Epoch 5/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 0.6149 - accuracy: 0.7481
     Epoch 6/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 0.5691 - accuracy: 0.7846
     Epoch 7/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 0.4986 - accuracy: 0.8367
     Epoch 8/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 0.4624 - accuracy: 0.8474
     Epoch 9/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 0.4443 - accuracy: 0.8531
     Epoch 10/10
     1875/1875 [==============================] - 3s 2ms/step - loss: 0.4302 - accuracy: 0.8587
     <tensorflow.python.keras.callbacks.History at 0x7f5b5170a4a8>
```

Accuracy goes up to 85% and the step size increased to 2m/s

Moving the hidden layers to 150 should fairy improved accuracy but also increase the stepsize time.



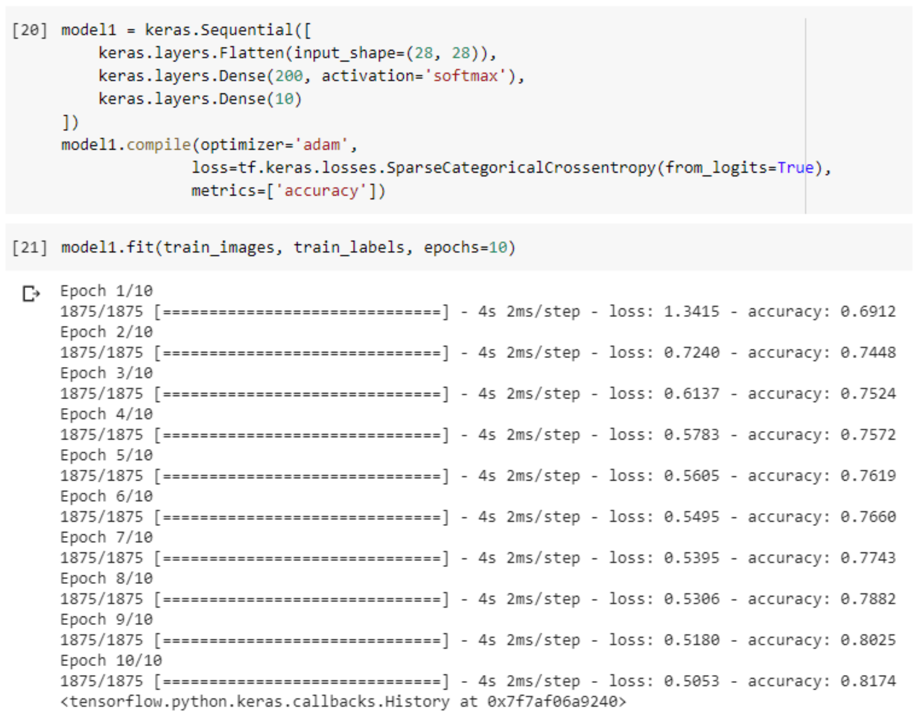
```
[81] model = keras.Sequential([
         keras.layers.Flatten(input_shape=(28, 28)),
         keras.layers.Dense(150, activation='softmax'),
         keras.layers.Dense(10)
     ])
```



```
[83] model.fit(train_images, train_labels, epochs=10)

     Epoch 1/10
     1875/1875 [==============================] - 4s 2ms/step - loss: 1.3414 - accuracy: 0.6992
     Epoch 2/10
     1875/1875 [==============================] - 4s 2ms/step - loss: 0.7258 - accuracy: 0.7462
     Epoch 3/10
     1875/1875 [==============================] - 4s 2ms/step - loss: 0.6130 - accuracy: 0.7541
     Epoch 4/10
     1875/1875 [==============================] - 4s 2ms/step - loss: 0.5614 - accuracy: 0.7893
     Epoch 5/10
     1875/1875 [==============================] - 4s 2ms/step - loss: 0.5044 - accuracy: 0.8187
     Epoch 6/10
     1875/1875 [==============================] - 5s 2ms/step - loss: 0.4805 - accuracy: 0.8245
     Epoch 7/10
     1875/1875 [==============================] - 4s 2ms/step - loss: 0.4685 - accuracy: 0.8290
     Epoch 8/10
     1875/1875 [==============================] - 4s 2ms/step - loss: 0.4588 - accuracy: 0.8347
     Epoch 9/10
     1875/1875 [==============================] - 4s 2ms/step - loss: 0.4515 - accuracy: 0.8401
     Epoch 10/10
     1875/1875 [==============================] - 4s 2ms/step - loss: 0.4425 - accuracy: 0.8457
     <tensorflow.python.keras.callbacks.History at 0x7f5b5af12f60>
```

Observing from the increase in the hidden layers to 150 didn't allow the model to get the accuracy to higher but decreased from the previous 50 hidden layers. It does seem like it requires more time to learn for many hidden layers.

The next experiment is to decrease the hidden layers to get optimum result. The choice is 200 hidden layers.



```
[20] model1 = keras.Sequential([
         keras.layers.Flatten(input_shape=(28, 28)),
         keras.layers.Dense(200, activation='softmax'),
         keras.layers.Dense(10)
     ])
     model1.compile(optimizer='adam',
                  loss=tf.keras.losses.SparseCategoricalCrossentropy(from_logits=True),
                  metrics=['accuracy'])
```

```
[21] model1.fit(train_images, train_labels, epochs=10)
```

```
Epoch 1/10
1875/1875 [==============================] - 4s 2ms/step - loss: 1.3415 - accuracy: 0.6912
Epoch 2/10
1875/1875 [==============================] - 4s 2ms/step - loss: 0.7240 - accuracy: 0.7448
Epoch 3/10
1875/1875 [==============================] - 4s 2ms/step - loss: 0.6137 - accuracy: 0.7524
Epoch 4/10
1875/1875 [==============================] - 4s 2ms/step - loss: 0.5783 - accuracy: 0.7572
Epoch 5/10
1875/1875 [==============================] - 4s 2ms/step - loss: 0.5605 - accuracy: 0.7619
Epoch 6/10
1875/1875 [==============================] - 4s 2ms/step - loss: 0.5495 - accuracy: 0.7660
Epoch 7/10
1875/1875 [==============================] - 4s 2ms/step - loss: 0.5395 - accuracy: 0.7743
Epoch 8/10
1875/1875 [==============================] - 4s 2ms/step - loss: 0.5306 - accuracy: 0.7882
Epoch 9/10
1875/1875 [==============================] - 4s 2ms/step - loss: 0.5180 - accuracy: 0.8025
Epoch 10/10
1875/1875 [==============================] - 4s 2ms/step - loss: 0.5053 - accuracy: 0.8174
<tensorflow.python.keras.callbacks.History at 0x7f7af06a9240>
```

It seems like with the just 10 epochs it is not just enough to see the accuracy improved. It is hard to establish what is the right amount of hidden layers and it functions very randomly. But Can make an estimated guess.

Experimentation on different activation functions of hidden layers

Activation	Max Accuracy at 10 Epocs on 128 hidden layers
Sigmoid	0.9067
Relu	0.9106
Tanh	0.9112
softmax	0.8410

Tanh performed good at the different experiments.

Performing experimentation on batch size. The expectation is that as the batch size increases the accuracy should increase. The configuration of tanh is used for activation and 50 hidden layers and 10 epocs

Batch_size	Accuracy
20	0.905
25	0.9037
30	0.9026
35	0.9024
40	0.9112

Looking at the results I see that at size of 40 the accuracy is at 91% which is good. But it did decrease from 30 batch size to 35 batch size.

Extermination on Regularizes to see how they effect the model differently. Using the similar configuration from the last experiment with having 40 as batch size and 10 epochs with tanh activation I expect to see shorter timings for

L1	L2	Accuracy
0.01	0.01	0.8375
0.02	0.02	0.8271
0.03	0.03	0.8224
0.04	0.04	0.8228

Looking at these values. It seems as the regularization increases in values the accuracy also decreases and even the time increased for the last data experiment.