Reference

Thursday, July 4, 2024 7:39 PM

1. https://www.rareskills.io/zk-book
2. https://vitalik.eth.limo/general/2022/03/14/trustedsetup.html

Trusted Setup (powers of tau / KZG)

Recall:
-> Problem statement
-> R1CS (L, R, O matrices)
-> Lagrange Interpolation (U, V, W array of polys)
-> Evaluate QAP at a random point (guaranteed by Schwartz-Zippel)

↑

Need trusted setup
But why ?

Suppose random point tau is publicly known, QAP formula becomes:

$$((U \cdot a)(\tau))\,((V \cdot a)(\tau)) = ((W \cdot a)(\tau)) + h(\tau)\,t(\tau)$$

Prover can come up with fake numbers that still satisfy the equation, therefore forge fake proof and trick the verifier.

Q: How to generate a fake proof if tau is leaked

A: If prover knows tau, he can construct polys without following the protocol. He only needs to make sure that the QAP equation is balanced when the fake polys are evaluated at tau. That will result in different polys but the forged proof will still go through verifier step successfully.

Trusted setup computes and publishes "encryptions" of tau, called powers of tau:

$$[\tau^0 G_1, \tau^1 G_1, \tau^2 G_1, \tau^3 G_1, \cdots, \tau^b G_1]$$
$$[\tau^0 G_2, \tau^1 G_2, \tau^2 G_2, \tau^3 G_2, \cdots, \tau^b G_2]$$

tau: toxic waste

$[\,]_1$ means G_1 point (either G_1 or G_2)
↑ $[\,]_1$ ↑ $[\,]_2$

Prover computes evaluation of polys in QAP according to powers of tau. For example, U * a can be computed as:

$$[A]_1 = \left[(U \cdot a)(\tau)\right]_1 = \left[(C_n x^n + C_{n-1} x^{n-1} + \cdots + C_1 x + C_0)(\tau)\right]_1$$

inner product, still poly

$$= \left[C_n \tau^n + C_{n-1} \tau^{n-1} + \cdots + C_1 \tau + C_0\right]_1$$

EC points addition $= [C_n \tau^n]_1 + [C_{n-1} \tau^{n-1}]_1 + \cdots + [C_1 \tau]_1 + [C_0]_1$

(a+b) * G = aG + bG

EC points scalar multiplication $= C_n(\tau^n G_1) + C_{n-1}(\tau^{n-1} G_1) + \cdots + C_1(\tau G_1) + C_0(G_1)$

↑ ↑ ↑ come from trusted setup

$= (\cdots, \cdots)$

Compute $[B]_2 = [(V \cdot a)(\tau)]_2 = ((\cdots, \cdots), (\cdots, \cdots))$

$[C']_1 = [(W \cdot a)(\tau)]_1 = (\cdots, \cdots)$

$[HT]_1 = [(h(x)t(x))(\tau)]_1 = (\cdots, \cdots)$

not that easy, will discuss later

$[C]_1 = [C']_1 + [HT]_1 = (\cdots, \cdots)$

Verifier verifies π:

pretty bad ↙ at this moment

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pretty bad at this moment

$$e([A]_1, [B]_2) == e([C]_1, G_2)$$

G_1 point, G_2 point, G_1 point

Issue: evaluating h(x)t(x)

When we evaluate h(tau)t(tau), we are "multiplying" two G1 points, which will introduce a pairing -> unwanted

(Why not compute h(x) * t(x) first and evaluate ht(tau)? h(x) * t(x) computation is unwanted too because it is expensive. 1. We don't want to evaluate both h and t too early since it introduces unwanted pairing 2. We don't want to evaluate h and t too late so we have to multiply two long polys. So we choose the tradeoff in the middle.)

Solution: embed t(tau) in another powers of tau

$$(\tau^0 t(\tau) G_1, \tau^1 t(\tau) G_1, \tau^2 t(\tau) G_1, \cdots, \tau^5 t(\tau) G_1) \rightarrow$$ 3rd powers of tau generated by trusted setup

Then the computation of h(tau)t(tau) by prover becomes:

$$[h(\tau)t(\tau)]_1 = [(ht)(\tau)]_1$$

e.g.: say $t(x) = x - 1$, $h(x) = 2x + 1$, $\tau = 5$

① $h(\tau)t(\tau) = 11 \cdot 4 = 44$

② $(h(x) \cdot t(\tau))(\tau) = (2x+1) \cdot 4 = 8x + 4 = 40 + 4 = 44$

$$= [(h(x) \cdot t(\tau))(\tau)]_1$$

$$= [((h_0 + h_1 x + h_2 x^2 + \cdots + h_n x^n) \cdot t(\tau))(\tau)]_1$$

$$= [h_0 t(\tau) + h_1 t(\tau) x + h_2 t(\tau) x^2 + \cdots + h_n t(\tau) x^n)(\tau)]_1$$

$$= [h_0 t(\tau) + h_1 \tau t(\tau) + h_2 \tau^2 t(\tau) + \cdots + h_n \tau^n t(\tau)]_1$$

EC addition $= [h_0 t(\tau)]_1 + [h_1 \tau t(\tau)]_1 + [h_2 \tau^2 t(\tau)]_1 + \cdots + [h_n \tau^n t(\tau)]_1$

EC scalar multiplication $= h_0(t(\tau) G_1) + h_1(\tau t(\tau) G_1) + h_2(\tau^2 t(\tau) G_1) + \cdots + h_n(\tau^n t(\tau) G_1)$

Come from trusted setup

Q: What if I evaluate h(tau)t(tau) directly without doing the extra powers of tau

A: Prover does not know tau, so he can't compute h(tau)t(tau) on his own without the help from trusted setup. h(tau) needs an encrypted evaluation, and t(tau) needs another one. That will result in a "multiplication" of two G1 points, which is undesired.

Implementation of powers of tau

```
# polynomial degree is 6

# Powers of tau for A
def generate_powers_of_tau_G1(tau):
    return [multiply(G1, int(tau ** i)) for i in range(t.degree)] # up to tau**6
```

```
# Powers of tau for B
def generate_powers_of_tau_G2(tau):
    return [multiply(G2, int(tau ** i)) for i in range(t.degree)] # up to tau**6
```

```
# Powers of tau for h(tau)t(tau)
def generate_powers_of_tau_HT(tau):
    before_delta_inverse = [multiply(G1, int(tau ** i * t_evaluated_at_tau)) for i in range(t.degree - 1)] # up to tau**5
    return [multiply(entry, int(delta_inverse)) for entry in before_delta_inverse]
```

Python list comprehension
https://realpython.com/list-comprehension-python/

Syntax:
new_list = [expression for member in iterable if conditional]

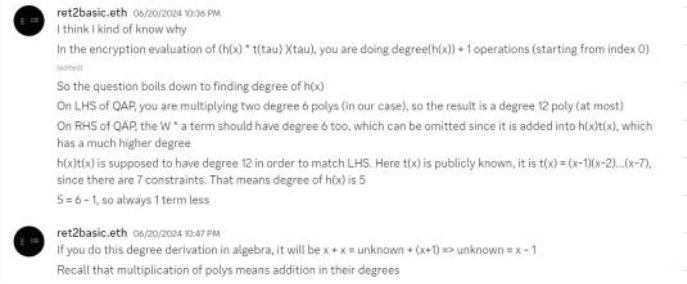
Optional

Example:

```
ret2basic@PwnieIsland: ~ 80x24
ret2basic@PwnieIsland:~$ python3
Python 3.10.12 (main, Nov 20 2023, 15:14:05) [GCC 11.4.0] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> new_list = [10*x for x in range(5) if x % 2 == 1]
>>> print(new_list)
[10, 30]
>>> new_list2 = [10*x for x in range(5)]
>>> print(new_list2)
[0, 10, 20, 30, 40]
>>>
```

Q: Why the 3rd powers of tau has 1 less term?

A: Because the length equals to deg(h(x)). Do the math: x + x = unknown + (x+1) => unknown = x - 1.
My derivation:



Implementation for encrypted poly evaluation:

```python
def inner_product(ec_points, coeffs):
    return reduce(add, (multiply(point, int(coeff)) for point, coeff in zip(ec_points, coeffs)), Z1)

def encrypted_evaluation_G1(poly):
    powers_of_tau = generate_powers_of_tau_G1(tau)
    evaluate_on_ec = inner_product(powers_of_tau, poly.coeffs[::-1])

    return evaluate_on_ec

def encrypted_evaluation_G2(poly):
    powers_of_tau = generate_powers_of_tau_G2(tau)
    evaluate_on_ec = inner_product(powers_of_tau, poly.coeffs[::-1])

    return evaluate_on_ec

def encrypted_evaluation_HT(poly):
    powers_of_tau = generate_powers_of_tau_HT(tau)
    evaluate_on_ec = inner_product(powers_of_tau, poly.coeffs[::-1])

    return evaluate_on_ec
```

py_ecc.bn128.Z1 -> point at infinity over FQ

G
G+G
G+G+G
...
curve_order * G = O -> point at infinity

Z/6Z -> 5+1 = 0

Python generator comprehension

https://www.pythonlikeyoumeanit.com/Module2_EssentialsOfPython/Generators_and_Comprehensions.html

Introducing Generators

Now we introduce an important type of object called a **generator**, which allows us to generate arbitrarily-many items in a series, without having to store them all in memory at once.

Definition:

A **generator** is a special kind of iterator, which stores the instructions for how to *generate* each of its members, in order, along with its current state of iterations. It generates each member, one at a time, only as it is requested via iteration.

Recall that a list readily stores all of its members; you can access any of its contents via indexing. A generator, on the other hand, *does not store any items*. Instead, it stores the instructions for generating each of its members, and stores its iteration state; this means that the generator will know if it has generated its second member, and will thus generate its third member the next time it is iterated on.

The whole point of this is that you can use a generator to produce a long sequence of items, without having to store them all in memory.

Example:

```
ret2basic@PwnIslands: ~/Desktop/Groth16 80x24
>>> even_gen = (i for i in range(100) if i%2 == 0)
>>> print(even_gen)
<generator object <genexpr> at 0x7c16a5fddc40>
>>> next(even_gen)
0
>>> next(even_gen)
2
>>> next(even_gen)
4
>>> next(even_gen)
6
>>>
```

Python zip() -> return the direct product of two iterators

Direct product: G, G' -> (G, G')
(a , b)

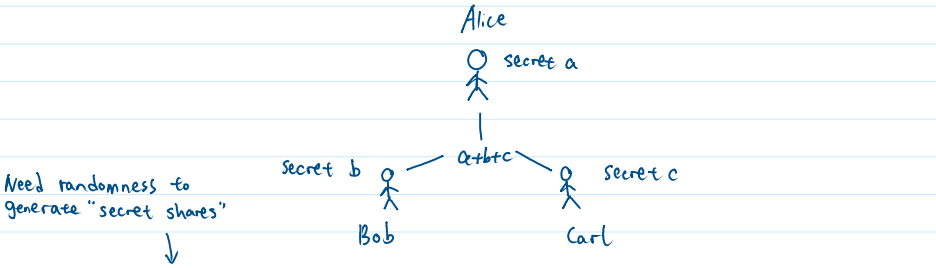
Example:

```
ret2basic@PwnIslands: ~/Desktop/Groth16 80x24
>>> list1 = ["a", "b", "c"]
>>> list2 = [123, 456, 789]
>>> zip(list1, list2)
<zip object at 0x7c16a5530400>
>>> print(list(_))
[('a', 123), ('b', 456), ('c', 789)]
>>>
```

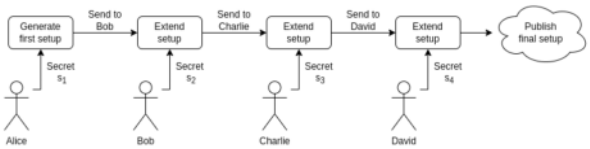
Blend MPC into trusted setup https://vitalik.eth.limo/general/2022/03/14/trustedsetup.html

To minimize centralization risk -> MPC, **1-of-N** trust model (N is typically in the hundreds)

Digress: the very basic idea of MPC



1. How to generate new powers of tau in MPC setting



Start with Alice : $[G_1, s_1G_1, s_1^2G_1, \cdots, s_1^{n-1}G_1]$

then Bob : $[G_1, s_1s_2G_1, s_1^2s_2^2G_1, \cdots, s_1^{n-1}s_2^{n-1}G_1]$

then Charlie , and David

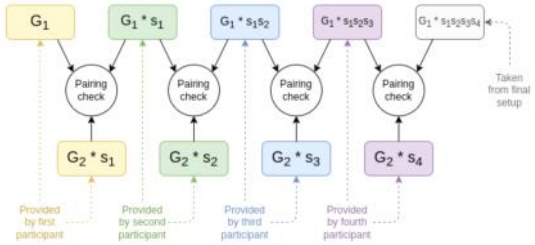
Final result : $[G_1, s_1s_2s_3s_4G_1, s_1^2s_2^2s_3^2s_4^2G_1, \cdots, s_1^{n-1}s_2^{n-1}s_3^{n-1}s_4^{n-1}G_1]$

then Charlie , and David

Final result: $[G_1, s_1 s_2 s_3 s_4 G_1, s_1^2 s_2^2 s_3^2 s_4^2 G_1, \ldots\ldots, s_1^{n-1} s_2^{n-1} s_3^{n-1} s_4^{n-1} G_1]$

As long as **ONE of** the participants is honest (not leaking random number s_i to other parties), the trusted setup is just as secure as one single trusted entity handles it.

2. How to verify that each participant actually participated in the computation



yellow: $e(G_1, s_1 \cdot G_2) == e(s_1 \cdot G_1, G_2)$

green: $e(s_1 \cdot G_1, s_2 \cdot G_2) == e(s_1 s_2 \cdot G_1, G_2)$

blue: $e(s_1 \cdot s_2 \cdot G_1, s_3 \cdot G_2) == e(s_1 \cdot s_2 \cdot s_3 \cdot G_1, G_2)$

purple: $e(s_1 \cdot s_2 \cdot s_3 \cdot G_1, s_4 \cdot G_2) == e(s_1 \cdot s_2 \cdot s_3 \cdot s_4 \cdot G_1, G_2)$