# BayesX

Software for Bayesian Inference in Structured Additive Regression Models

Version 1.50



# Reference Manual

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Leo Held (mentally)
German Science Foundation

### **Acknowledgements**

The development of *BayesX* has been supported by grants from the German National Science Foundation (DFG), Collaborative Research Center 386 "Statistical Analysis of Discrete Structures".

Special thanks go to (in alphabetical order of first names):

Achim Zeileis for advertising HCL colors;

Dieter Gollnow for computing and providing the map of Munich (a really hard job);

Leo Held for advertising the program;

Ludwig Fahrmeir for his patience with finishing the program and for carefully reading and correcting the manual:

Ngianga-Bakwin Kandala for being the first user of the program (a really hard job);

Samson Babatunde Adebayo for carefully reading and correcting the manual;

Ursula Becker for carefully reading and correcting the manual;

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BayesX is available at http://www.stat.uni-muenchen.de/~bayesx

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# Chapter 1

# What is BayesX?

### General scope

BayesX is a software tool for estimating structured additive regression models. Structured additive regression embrasses several well-known regression models such as generalized additive models (GAM), generalized additive mixed models (GAMM), generalized geoadditive mixed models (GGAMM), dynamic models, varying coefficient models, and geographically weighted regression within a unifying framework. Besides exponential family regression, BayesX also supports non-standard regression situations such as regression for categorical responses, hazard regression for continuous survival times, and continuous time multi-state models.

### Inferential procedures

Estimation of regression models can be achieved based on two different inferential concepts: Markov chain Monte Carlo simulation techniques corresponding to full Bayesian inference and mixed model methodology corresponding to penalised likelihood or empirical Bayes inference. Both concepts have been implemented in separate regression objects:

- MCMC simulation techniques (bayesreg objects): A fully Bayesian interpretation of structured additive regression models is obtained by specifying prior distributions for all unknown parameters. Estimation can be facilitated using Markov chain Monte Carlo simulation techniques, a general and versatile concept for Bayesian inference. Bayesreg objects provide numerically efficient implementations of MCMC schemes for structured additive regression models. Suitable proposal densities have been developed to obtain rapidly mixing, well-behaved sampling schemes without the need for manual tuning.
- Mixed model based estimation (remlreg objects): An increasingly popular way to estimate semiparametric regression models is the representation of penalisation approaches as mixed models. Within BayesX this concept has been extended to structured additive regression models and several types of non-standard regression situations. The general idea is to take advantage of the close connection between penalty concepts and corresponding random effects distributions. The smoothing parameters of the penalties then transform to variance components in the random effects (mixed) model. While the selection of smoothing parameters has been a difficult task for a long time, several estimation procedures for variance components in mixed models are already available since the 1970's. The most popular one is restricted maximum likelihood in Gaussian mixed models with marginal likelihood as the non-Gaussian counterpart.;br; Remlreg objects employ mixed model methodology for the estimation of structured additive regression

models. While regression coefficients are estimated based on penalised likelihood, restricted maximum likelihood or marginal likelihood estimation forms the basis for the determination of smoothing parameters. From a Bayesian perspective, this yields empirical Bayes / posterior mode estimates for the structured additive regression models. However, estimates can also merely be interpreted as penalised likelihood estimates from a frequentist perspective.

#### Model classes and model terms

BayesX provides functionality for the following types of responses:

- Univariate exponential family: Supported response distributions are Gaussian, Poisson, Binomial, Gamma, negative binomial, zero-inflated Poisson, and zero-inflated negative binomial.
- Categorical responses with unordered responses: For categorical responses with unordered categories, BayesX supports multinomial logit and multinomial probit models. Both effects of category-specific and globally-defined covariates can be estimated. Category-specific offsets or non-availability indicators can be defined to account for varying choice sets.
- Categorical responses with ordered responses: For ordered categorical responses, ordinal as well as sequential models can be specified. Effects can be requested to be category-specific or to be constant over the categories. Supported response functions include the logit and the probit transformation.
- Continuous time survival models: BayesX supports Cox-type hazard regression models with structured additive predictor for continuous time survival analysis. In contrast to the Cox model, the baseline hazard rate is estimated jointly with the remaining effects based on penalized splines. Furthermore, both time-varying effects and time-varying covariates can be included in the predictor. Arbitrary combinations of right, left and interval censored as well as left truncated obserations can be analysed.
- Continuous time multi-state models: In BayesX version 1.5 multi-state models have been added to the model class. Multi-state models form a general class for the analysis of the evolution of discrete phenomena in continuous time. Transition intensities between the discrete states are specified in analogy to the hazard rate in continuous time survival models.

Structured additive regression models can be build from arbitrary combinations of the following model terms:

- Nonlinear effects: Nonlinear effects can be estimated based on either penalised spline or random walk models.
- Seasonal effects: Specific autoregressive priors allow for the estimation of flexible, timevarying seasonal effects.
- Spatial effects: Spatial effects can be specified based on Markov random fields, stationary Gaussian random fields (kriging) or bivariate penalised splines. Both georeferenced regional data as well as point-referenced data based on coordinates are supported.
- Interaction surfaces: Bivariate extensions of penalised splines allow to estimate flexible
  interactions between continuous covariates. Stationary Gaussian random fields can also
  be considered a radial basis function approach and, hence, form a second possibility for
  the specification of interaction surfaces.

- Varying coefficients: Varying coefficient models with both continuous and spatial effect
  modifiers can be estimated. The latter case is also known as geographically weighted
  regression.
- Cluster-specific random effects: BayesX supports i.i.d. Gaussian random intercepts and random slopes.

Note that parts of the functionality may be available for one of the regression objects only. For example, bayesreg objects do not support interval censored survival times while multinomial probit models can not be estimated with remlreg objects. Details can be found in the reference manual.

### **Further functionality**

- **Handling and manipulation of data sets:** BayesX provides a growing number of functions for handling and manipulating data sets, e.g. for reading ASCII data sets, creating new variables, obtaining summary statistics etc. Compare chapter 4 for details.
- Handling and manipulation of geographical maps: BayesX is able to manipulate and draw geographical maps. The regions of the map may be colored according to some numerical characteristics. In BayesX version 1.5, a new color scheme based on HCL colors has been added to obtain a better representation of colored maps independent of the display device. Details can be found in chapter 5 and chapter 6.
- Visualizing data: BayesX provides functions for drawing scatter plots and geographical
  maps. A number of additional options are provided to customize the graphs according
  to the personal needs of the user. Details can be found in chapter 6.
- Model selection for Gaussian and non-Gaussian dags: This tool estimates Gaussian
  and non-Gaussian directed acyclical graphs (dag) via reversible jump MCMC. Details
  can be found in chapter 10.

#### Recommendations for further reading

If you are interested in using BayesX, it is of course not necessary to read the complete manual. Table 1.1 provides a rough guideline for reading this manual and other sources, depending on the features of BayesX you are interested in and your statistical background. In any case, you should read sections 2.1 - 2.5 to make yourself familiar with the general usage of BayesX.

#### References

Green, P.J. (2001): A Primer in Markov Chain Monte Carlo. In: Barndorff-Nielsen, O.E., Cox, D.R. and Klüppelberg, C. (eds.), *Complex Stochastic Systems*. Chapman and Hall, London, 1-62.

Intended use and background	Guideline
Bayesian semiparametric regression based on MCMC simulation techniques. No experience with MCMC.	Read an introductory text about MCMC first. A brief introduction is given for example in Green (2001). Read the methodology manual to make yourself familiar with STAR models. Proceed with chapter 1 of the tutorial manual.
Bayesian semiparametric regression based on MCMC simulation techniques. At least basic knowledge about MCMC.  Semiparametric regression based on mixed model methodology.	Read the methodology manual to make yourself familiar with STAR models. Proceed with chapter 1 of the tutorial manual.  Read the methodology manual to make yourself familiar with STAR models. Proceed with chapter 2 of the tutorial manual.
Model selection for DAGs. No experience with reversible jump MCMC.	Read an introductory text about reversible jump MCMC first. A brief introduction is given for example in Green (2001). Proceed with chapter 10.
Draw and color geographical maps	Read chapter 5 and section 6.1 of the reference manual.

Table 1.1: Recommendations for further reading.

# Chapter 2

# **Getting started**

This chapter provides some useful information for first-time users of *BayesX*: Which versions of BayesX are currently available (section 2.1), how is BayesX installed (section 2.2), what types of manuals exist (section 2.3), and how is the graphical user interface organized (section 2.4). Section 2.5 describes the general usage of *BayesX* and the structure of *BayesX* syntax. The final section contains a description of three data sets that will be used for demonstrating purposes in the later chapters.

# 2.1 Available versions of BayesX

In its current form, BayesX runs only under the various versions of the Windows operating system (e.g. Windows 95, 98, 2000, NT, XP). The graphical user interface and the visualisation tools are implemented in Java, while the computerintensive parts of the program have been implemented in C++. The current version of BayesX can be downloaded from <a href="http://www.stat.uni-muenchen.de/~bayesx">http://www.stat.uni-muenchen.de/~bayesx</a>.

Up to version 1.3, BayesX has been distributed in two versions, the Java based version described above and a non-Java based version, which was written completely in C++. Since the Java based version has additional features, the non-Java based version is no longer supported.

# 2.2 Installing BayesX

After you have downloaded the file installBayesX.exe from the BayesX homepage, proceed by executing this file. The installation process is quite simple and comparable to most standard installations. The installation routine will request all necessary information.

When Bayes X has been installed successfully, it can be started using the Windows Start button or the icon created on the desktop (depending on your specifications during the installation process). The installation directory contains the five subdirectories doc, examples, output, sfunctions and temp. The doc directory contains the program documentation, i.e. the three Bayes X manuals (see the following subsection). The examples directory contains the three data sets, credit.raw, rents.raw and zambia.raw, which will be used for demonstrating purposes throughout the manual. A detailed description of these data sets is given in section 2.6. The examples directory also contains some tutorial programs that illustrate the usage of Bayes X, see the tutorials manual. The output directory is the default directory for program output stored in files. Of course, the output window can be redefined by the user, compare section 7.4 and section 8.2. The sfunctions directory contains some R and S-plus functions for visualizing estimation results obtained with

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bayesreg objects or remlreg objects, see section 9.2 for a detailed description of these functions. Note, however, that the current BayesX version has its own graphics capabilities, see chapter 6 and chapter 9 for details. Finally, temporary files created when estimating regression models will be stored in the temp directory. Usually you will never use this directory.

The subdirectories and their content are briefly summarized in Table 2.1.

Directory	Content
doc	the $BayesX$ manuals
examples	data set examples and tutorial programs
output	default directory for estimation output
sfunctions	R and S-plus functions for visualizing output
temp	temporary files

Table 2.1: Subdirectories of the installation directory and their content.

### 2.3 Manuals

BayesX is shipped with three different manuals. The reference manual (i.e. the manual you are just reading) gives detailed information on the general usage of BayesX, the syntax of BayesX commands and the different objects used by BayesX. The methodology manual provides background information on the statistical methodology that is implemented in BayesX. In this manual you will also find more references on the methodological background. The tutorial manual is intended to make new users familiar with the usage of BayesX by demonstrating examples. It contains two self-contained tutorials, describing how to perform semiparametric regression analyses using BayesX. The manuals are also available from the help menu and can be found in the doc directory (a subdirectory of the installation directory).

# 2.4 Windows and buttons in BayesX

After starting BayesX you will see a main window with a menu bar and four additional subwindows. The four windows are the command window, the output window, the review window and the object browser. The purpose of these windows is described in the following four subsections. Below the menu bar there is a menu bar containing the buttons BREAK, PAUSE and SUPPRESS OUTPUT, and the priority menu. Their functionality is described in subsection 2.4.5 and 2.4.6.

#### 2.4.1 The command window

Allmost all *BayesX* commands are entered and executed in the *command window*. By default, a command will be executed if you press the return key. You can change this default delimiter using the delimiter command, see section 3.4.

### 2.4.2 The output window

In the *output window*, all commands entered in the *command window* or executed through a batch file (see section 3.5) are printed together with the program output.

The content of the *output window* can be saved and processed with your favorite text editor. For saving the output, enter the *file menu* and click on *Save output* or *Save output as*. The file save

dialog will allow you to choose between two different file formats. The default is the rich-text format but it is also possible to store the *output window* in plain ASCII format. This, however, has the disadvantage that all text highlights (for example bold letters) will disappear in the saved file. The *file menu* also allows to clear the *output window* (i.e. delete the content of the window) or to open an already existing file.

Depending on the screen resolution of your computer, letters appearing in the *output window* may be very small or too large. The font size can be varied in the *preferences menu*.

#### 2.4.3 The review window

In many cases, subsequent commands change only slightly. The *review window* gives you convenient access to the last 100 past commands entered during a session. Double click on one of these past commands and it is automatically copied to the *command window*, where it can be modified and / or executed again.

### 2.4.4 The object browser

BayesX is object oriented, i.e. different types of objects are used to store data, estimate regression models, etc. The object browser provides an overview of the objects currently defined and about their contents. The object browser window is split into two parts. The left part displays the different object types currently supported by BayesX (dataset objects, bayesreg objects, remlreg objects, map objects, dag objects and graph objects)s. By clicking on one of the object types, the names of all objects of this type will appear in the right panel of the object browser. Double clicking on one of the names gives a visualization of the object and / or a short summary in the output window, depending on the object type. Double clicking on dataset objects, for example, will open a spreadsheet where the variables and the observations of the data set can be inspected. Clicking on map objects opens a window that contains a graphical representation of the map.

#### 2.4.5 BREAK, PAUSE and SUPPRESS OUTPUT button

The BayesX button panel contains the BREAK button, the PAUSE button and the SUPPRESS OUTPUT button. The purpose of the BREAK button is to interrupt the process that is currently executed (this may take some time). Clicking on the PAUSE button interrupts the current process temporarily until the button is pressed again. If a process is paused, the button caption PAUSE is replaced by CONTINUE, indicating that a second click on the button will continue the current process. Pausing a current process will increase the execution speed of other programs currently running on your computer. Clicking the SUPPRESS OUTPUT button suppresses printing of output in the output window. The button caption changes to SHOW OUTPUT to indicate that an additional click on the button will cause the program to print the output again. Suppressing the output increases the execution speed of BayesX and saves memory. Note, that you can store your output in a log-file even if printing of the output is suppressed (see section 3.2).

#### 2.4.6 Priority menu

When running extensive computations, it may be desirable to reduce the priority of BayesX since otherwise all further programs may be executed very slowly. The priority menu allows you to change the priority of your computations from within BayesX. Usually there should be no need to increase the priority (although it is possible). To pause the current computations use the PAUSE button.

# 2.5 General usage of BayesX

### 2.5.1 Creating objects

BayesX is implemented in an object oriented way, although the object oriented concept does not go too far, i.e. inheritance or other concepts of object oriented programming languages such as S or C++ are not supported. As a consequence, the first thing to do during a session, is to create some objects. Currently, six different object types are available: dataset objects, bayesreq objects, remlreg objects, map objects, dag objects and graph objects. Dataset objects are used to store, handle, and manipulate data sets, see chapter 4 for details. Map objects are used to handle geographical information and are covered in more detail in chapter 5. The main purpose of map objects is to serve as auxiliary objects for bayesreg objects or remlreg objects when estimating spatial effects. Graph objects are used to visualize data (e.g. to create scatterplots or to color geographical maps according to some numerical characteristics), see chapter 6 for details. The most important object types are bayesreg objects and remlreg objects. These objects are used to estimate Bayesian semiparametric regression models based on either Markov Chain Monte Carlo simulation techniques (bayesreg objects) or a mixed model representation of the regression model (remlreg objects). See chapter 7 for a detailed description of bayesrey objects and chapter 8 for a detailed description of remlreg objects. Dag objects are used to estimate Gaussian or non-Gaussian DAGs (direct acyclic graphs) based on reversible jump MCMC simulation techniques (see chapter 10 for details).

The syntax for creating a new object is:

> objecttype objectname

To create for example a dataset object with name mydata, simply type:

### > dataset mydata

Note that some restrictions are imposed on the names of objects, i.e. not all object names are allowed. For example, object names have to begin with an uppercase or lowercase letter rather than a number. Section 4.13 discusses valid variable names but the same rules apply also to object names.

### 2.5.2 Applying methods to previously defined objects

When an object has been created successfully, you can apply methods to that particular object. For instance, dataset objects may be used to read data stored in an ASCII file using method infile, to create new variables using method generate, to modify existing variables using method replace and so on. The syntax for applying methods to the objects is similar for all methods and independent of the particular object type. The general syntax is:

> objectname.methodname [model] [weight varname] [if boolean expression] [, options] [using usingtext]

Table 2.2 explains the syntax parts in more detail.

Note that [...] indicates that this part of the syntax is optional and may be omitted. Moreover for most methods only some of the syntax parts above will be meaningful. The specification of invalid syntax parts is not allowed and will cause an error message.

We illustrate the concept with some simple methods of *dataset objects*. Suppose that a *dataset object* with name mydata has already been created and that some variables should be created. First of all, we have to tell *BayesX* how many observations we want to create. This can be done with the set obs command, see also section 4.10. For example

> mydata.set obs = 1000

Syntax part	Description
objectname	the name of the object to apply the method to
methodname	the name of the method
model	a model specification (for example a regression model)
weight $varname$	specifies varname as weight variable
if boolean expression	indicates that the method should be applied only if a
	certain condition holds
$,\ options$	define (or modify) options for the method
using usingtext	indicates that another object or file is required to
	apply the particular method

Table 2.2: Parts of the general BayesX syntax.

indicates that the data set mydata should have 1000 observations. In this case, the *methodname* is set and the *model* is obs = 1000. Since no other syntax parts (for example if statements) are meaningful for this method, they are not allowed. For instance, specifying an additional weight variable x by typing

> mydata.set obs = 1000 weight x

will cause the error message:

ERROR: weight statement not allowed

In a second step we can now create a new variable X, say, that contains Gaussian (pseudo) random numbers with mean 2 and standard deviation 0.5:

> mydata.generate X = 2+0.5\*normal()

Here, generate is the *methodname* and X = 2+0.5\*normal() is the *model*. In this case the *model* consists of the specification of the new variable name, followed by the equal sign '=' and a mathematical expression for the new variable. Similar as for the set obs command other syntax parts are not meaningful and therefore not allowed. If the negative values of X should be replaced with the constant 0, this can be achieved using the replace command:

> mydata.replace X = 0 if X < 0

Obviously, the if statement is meaningful and is therefore allowed, but not required.

# 2.6 Description of data set examples

This section describes three data sets used to illustrate many of the features of BayesX in the following chapters as well as in the tutorial manual. All data sets are stored columnwise in plain ASCII-format. The first row of each data set contains the variable names separated by blanks. Subsequent rows contain the observations, one observation per row.

#### 2.6.1 Rents for flats

According to the German rental law, owners of apartments or flats can base an increase in the amount that they charge for rent on 'average rents' for flats comparable in type, size, equipment, quality and location in a community. To provide information about these 'average rents', most of the larger cities publish 'rental guides', which can be based on regression analyses with rent as the dependent variable. The file rent94.raw (stored in the examples directory) is a subsample of data collected in 1994 for the Munich rental guide. The variable of primary interest is the monthly rent per square meter in German Marks. Covariates characterizing the flat were constructed from almost

Variable	Description
R	monthly rent per square meter in German marks
F	floor space in square meters
A	year of construction
L	location of the building in subquarters

Table 2.3: Variables of the rent data set.

200 variables out of a questionnaire answered by tenants of flats. The present data set contains a small subset of these variables that are sufficient for demonstration purposes (see Table 2.3).

In addition to the data set, the examples directory contains a map of Munich in the file munich.bnd. This map will be useful for visualizing effects of the location L. See chapter 5 for a description on how to incorporate geographical maps into *BayesX*.

#### References

Lang, S. and Brezger, A. (2004): Bayesian P-splines. Journal of Computational and Graphical Statistics, 13, 183-212.

#### 2.6.2 Credit scoring

The aim of credit scoring is to model and / or predict the probability that a client with certain covariates ('risk factors') will not pay back his credit. The data set contained in the file credit.raw consists of 1000 consumer credits from a bank in southern Germany. The response variable is 'creditability' in dichotomous form (y=0) for creditworthy, y=1 for not creditworthy). In addition, 20 covariates that are assumed to influence creditability were collected. The present data set (stored in the examples directory) contains a subset of these covariates that proved to be the main influential variables on the response variable, see Fahrmeir and Tutz (2001, ch. 2.1). Table 2.4 gives a description of the variables of the data set. Usually a binary logit model is applied to estimate the effect of the covariates on the probability of being not creditworthy. As in the case of the rents for flats example, this data set is used to demonstrate the usage of certain features of BayesX, see primarily subsection 7.6.1 for a Bayesian regression analysis of the data set.

Variable	Description	
y	creditability, dichotomous with $y = 0$ for creditworthy, $y = 1$ for	
	not creditworthy	
account	running account, trichotomous with categories "no running account"	
	(= 1), "good running account" (= 2), "medium running account"	
	("less than 200 DM") $(= 3)$	
duration	duration of credit in months, continuous	
amount	amount of credit in 1000 DM, continuous	
payment	payment of previous credits, dichotomous with categories "good" $(=1)$ ,	
	"bad" $(=2)$	
intuse	intended use, dichotomous with categories "private" $(=1)$ or	
	"professional" $(=2)$	
marstat	marital status, with categories "married" $(=1)$ and "living alone" $(=2)$ .	

Table 2.4: Variables of the credit scoring data set.

#### References

Fahrmeir, L., Tutz, G. (2001): Multivariate Statistical Modelling based on Generalized Linear Models. New York: Springer-Verlag.

#### 2.6.3 Childhood undernutrition in Zambia

Acute and chronic undernutrition is considered to be one of the worst health problems in developing countries. Undernutrition among children is usually determined by assessing the anthropometric status of the child relative to a reference standard. In our example undernutrition is measured through stunting (insufficient height for age), indicating chronic undernutrition. Stunting for child i is determined using the Z-score

$$Z_i = \frac{AI_i - MAI}{\sigma}$$

where AI refers to the child's anthropometric indicator (height at a certain age in our example), MAI refers to the median of the reference population and  $\sigma$  refers to the standard deviation of the reference population.

The data set zambia.raw contains the (standardized) Z-score for 4847 children together with several covariates that are supposed to influence undernutrition (e.g. the body mass index of the mother, the age of the child, and the district the mother lives in). Table 2.5 gives more information on the covariates in the data set.

This data set is used in chapter 1 and 2 of the tutorial manual.

Variable	Description
hazstd	standardized Z-score for stunting
bmi	body mass index of the mother
agc	age of the child
district	district where the mother lives
rcw	mother's employment status with categories "working" (= 1) and "not working"
	(=-1)
edu1	mother's educational status with categories "complete primary but incomplete
edu2	secondary" $(edu1 = 1)$ , "complete secondary or higher" $(edu2 = 1)$ and
	"no education or incomplete primary" $(edu1 = edu2 = -1)$
tpr	locality of the domicile with categories "urban" $(=1)$ and "rural" $(=-1)$
sex	gender of the child with categories "male" $(=1)$ and "female" $(=-1)$

Table 2.5: Variables in the undernutrition data set.

#### References

KANDALA, N. B., LANG, S., KLASEN, S. AND FAHRMEIR, L. (2001): Semiparametric Analysis of the Socio-Demographic and Spatial Determinants of Undernutrition in Two African Countries. *Research in Official Statistics*, 1, 81-100.

# Chapter 3

# **Special Commands**

This chapter describes some commands that are not associated with a particular object type. Among others, there are commands for exiting BayesX, opening and closing log files, saving program output, deleting objects etc.

# 3.1 Exiting BayesX

You can exit BayesX by typing either

> exit

or

> quit

in the *command window*. Of course, *BayesX* can also be closed using the 'exit' entry from the file menu or by clicking on the cross in the upper right corner.

# 3.2 Opening and closing log files

Program output and commands entered by the user can automatically be stored in a log file to make them available for editing in your favorite text editor. Another important application of log files is the documentation of your work. A log file is opened by the command:

> logopen [, option] using filename

Afterwards all commands entered and all program output will be saved in the file *filename*. If the log file specified in *filename* is already existing, new output is appended at the end of the file. To overwrite an existing log file, option replace has to be specified in addition. Note that it is not allowed to open more than one log file simultaneously.

An open log file can be closed by typing:

> logclose

Exiting BayesX automatically closes the current log file.

# 3.3 Saving the contents of the output window

You can save the contents of the *output window* not only with the  $file \rightarrow save$  output or  $file \rightarrow save$  output as menu, but also using the saveoutput command. This is particularly useful for automatic saving in batch files, see section 3.5. The syntax for saving the *output window* is

> saveoutput [, options] using filename

where *filename* is the file (including path) in which the contents of the output will be stored.

# **Options**

#### • replace

By default an error will be raised if you try to store the contents of the *output window* in a file that is already existing. This preserves you from overwriting a file unintentionally. An already existing file can be overwritten by explicitly specifying the replace option.

• type = rtf | txt

The *output window* can be saved in two different file types, namely rich text format (the default) and plain ASCII (requested by specifying type=txt as an option.

# 3.4 Changing the delimiter

By default, commands entered using the *command window* will be executed after pressing the return key. This can be inconvenient, in particular if your statements are long or in batch files. In this case it may be favorable to split a statement into several lines, and execute the command using a different delimiter.

You can change the delimiter using the delimiter command. The syntax is

> delimiter = newdel

where *newdel* is the new delimiter. Only two different delimiters are currently allowed, namely the return key and the ';' (semicolon) key. To specify the semicolon as the delimiter, type

> delimiter = ;

and press return. To return to the return key as the delimiter, type

> delimiter = return;

Note that this statement has to end with a semicolon, since this was previously set to be the current delimiter.

# 3.5 Using batch files

You can execute commands stored in a file just as if they were entered from the keyboard. This may be useful if you want to re-run a certain analysis more than once (possibly with some minor changes) or if you want to run time consuming statistical methods.

Execution of a batch file is started by typing

> usefile filename

This executes the commands stored in *filename* successively. *BayesX* will not stop the execution if an error occurs in one or more commands. Note that it is allowed to invoke another batch file within a currently running batch file.

#### Comments

Comments in batch files are indicated by a % sign, i.e. every line starting with % is ignored by the program.

### Changing the delimiter

In particular in batch files, the readability of your program code may be improved if some (long) commands are split up into several lines. By default this will cause errors, because BayesX interprets each line in your program as one statement. To overcome this problem one has to change the delimiter using the delimiter command, see section 3.4.

# 3.6 Dropping objects

You can delete objects by typing

> drop objectlist

This drops the objects specified in *objectlist*. The names of the objects in *objectlist* must be separated by blanks.

# Chapter 4

# dataset objects

Dataset objects are used to store, manage, and manipulate data. A new dataset object is created by typing

### > dataset objectname

where *objectname* is the name to be assigned to the data set. After the creation of a *dataset object* you can apply the methods discussed below.

Note that in the current version of BayesX only numerical variables are allowed. String valued variables, for example, are not yet supported by BayesX and the attempt to read such variables will raise an error message.

# 4.1 Method descriptive

# Description

Method descriptive calculates and displays univariate summary statistics. To be more specific, the method computes the number of observations, the mean, median, standard deviation, minimum and maximum of the specified variables.

### **Syntax**

> objectname.descriptive varlist [if expression]

Method descriptive computes summary statistics for the variables in *varlist*. An optional if statement may be added to analyze only a part of the data.

# **Options**

not allowed

## **Example**

The statement

> d.descriptive x y

computes summary statistics for the variables x and y. The statement

> d.descriptive x y if x>0

restricts the analysis to observations with x>0.

# 4.2 Method drop

# Description

Method drop deletes variables or observations from the data set.

# **Syntax**

- > objectname.drop varlist
- > objectname.drop if expression

The first command is used to delete the variables specified in *varlist* from the data set. The second statement deletes certain observations, i.e. all observations for which *expression* is true will be deleted.

## **Options**

not allowed

#### **Example**

The statement

> credit.drop account duration

deletes the variables account and duration from the credit scoring data set. With the statement

> credit.drop if marstat = 2

all observations with marstat = 2, i.e. all persons living alone, will be deleted. The statement

> credit.drop account duration if marstat = 2

will raise the error

 ${\tt ERROR}\colon {\tt dropping}$  variables and observations in one step not allowed

because is not allowed to drop variables and observations in one single command.

# 4.3 Functions and Expressions

The primary use of expressions is to generate new variables or change existing variables, see section 4.4 and section 4.9, respectively. Expressions may also be used in if statements to force BayesX to apply a method only to observations where the boolean expression in the if statement is true. The following statements are all examples of expressions:

```
2+2
log(amount)
1*(age <= 30)+2*(age > 30 & age <= 40)+3*(age > 40)
age=30
age+3.4*age^2+2*age^3
amount/1000
```

### 4.3.1 Operators

Three different types of operators can be included in expressions: arithmetic, relational and logical operators.

#### 4.3.1.1 Arithmetic operators

The arithmetic operators are + (addition), - (subtraction), \* (multiplication), / (division), ^ (raise to a power) and the prefix - (negation). Any arithmetic operation on a missing value or an undefined arithmetic operation (such as division by zero) yields a missing value.

# Example

The expression  $(x+y^{(3-x)})/(x*y)$  denotes the formula  $\frac{x+y^{(3-x)}}{x\cdot y}$ 

and evaluates to missing if x or y is missing or zero.

#### 4.3.1.2 Relational operators

The relational operators are > (greater than), < (less than), >= (greater than or equal), <= (less than or equal), = (equal) and != (not equal). Relational expressions are either 1 (if the expression is true) or 0 (if the expression is false).

### Example

Relational operators can for example be used to create indicator variables. The following statement generates a new variable amountcat, with value 1 if amount<=10 and value 2 if amount>10.

```
> credit.generate amountcat = 1*(amount<=10)+2*(amount>10)
```

Another useful application of relational operators is in if statements. For example, changing an existing variable only when a certain condition holds can be achieved by the following command:

```
> credit.replace amount = NA if amount <= 0
```

This sets all observations missing where amount<=0.

#### 4.3.1.3 Logical operators

The logical operators are & (and) and  $\mid$  (or).

#### Example

Suppose you want to generate a variable amountind whose value is 1 for married people with amount greater than 10 and 0 otherwise. This can be achieved by typing

```
> credit.generate amountind = 1*(marstat=1 & amount>10)
```

#### 4.3.1.4 Order of evaluation of the operators

The order of evaluation (from first to last) of operators is

```
/, *
-, +
!=, >, <, <=, >=, =
```

Brackets can be used to change the order of evaluation.

#### 4.3.2 Functions

Functions are a further component of expressions. Any function is indicated by the function name followed by an opening and a closing parenthesis. Inside the parentheses one or more arguments may be specified. The argument(s) of a function may again be expressions, including calls to further functions. Multiple arguments of a function are separated by commas. All functions return missing values when given missing values as arguments or when the result is undefined.

All mathematical function currently available in *BayesX* are referenced in Table 4.1. Statistical functions can be found in Table 4.2.

Function	Description
abs(x)	absolute value
cos(x)	cosine of radians
exp(x)	exponential
floor(x)	returns the integer obtained by truncating $x$ .
	Thus floor(5.2) evaluates to 5 as does floor(5.8).
lag(x)	lag operator
log(x)	natural logarithm
log10(x)	log base 10 of $x$
sin(x)	sine of radians
sqrt(x)	square root

Table 4.1: List of mathematical functions.

#### 4.3.3 Constants

Table 4.3 lists all constants that can be used in expressions.

Function	Description
bernoulli(p)	returns Bernoulli distributed random numbers with probability of
	success $p$ . If $p$ is not within the interval $[0;1]$ , a missing value will
	be returned.
binomial(n,p)	returns $B(n;p)$ distributed random numbers. Both, the number
	of trials $n$ and the probability of success $p$ may be expressions. If
	n < 1, a missing value will be returned. If n is not integer valued,
	the number of trials will be $[n]$ . If $p$ is not within the interval $[0;1]$ ,
	a missing value will be returned.
cumul(x)	empirical cumulative distribution function
cumulnorm(x)	cumulative distribution function $\Phi$ of the standard normal distri-
	bution.
$exponential(\lambda)$	returns exponentially distributed random numbers with parameter
	$\lambda$ . If $\lambda \leq 0$ , a missing value will be returned.
$\operatorname{\mathtt{gamma}}(\mu, \nu)$	returns gamma distributed random numbers with mean $\mu$ and
	variance $\mu^2/\nu$ . If $\nu$ is less than zero, a missing value will be re-
	turned.
normal()	returns standard normally distributed random numbers; $N(\mu, \sigma^2)$
	distributed random numbers may be generated with $\mu$ +
	$\sigma^*$ normal().
$poisson(\lambda)$	returns poisson distributed random numbers with parameter $\lambda$ . If
	$\lambda \leq 0$ , a missing value will be returned.
uniform()	uniform pseudo random number function; returns uniformly dis-
	tributed pseudo-random numbers on the interval $(0,1)$
weibull( $\alpha$ , $\lambda$ )	returns weibull distributed random numbers with density $f(x) =$
	$\alpha \lambda^{\alpha} x^{\alpha-1} \exp(-\lambda x^{\alpha})$ . If $\alpha \leq 0$ and/or $\lambda \leq 0$ , a missing value will
	be returned.

Table 4.2: List of statistical functions

#### **Example**

The following statement generates a variable obsnr whose value is 1 for the first observation, 2 for the second and so on.

> credit.generate obsnr = \_n

The command

> credit.generate nrobs = \_N

generates a new variable nrobs whose values are all equal to the total number of observations, say 1000, for all observations.

### 4.3.4 Explicit subscribing

Individual observations on variables can be referenced by subscribing the variables. Explicit subscripts are specified by the variable name with square brackets that contain an expression. The result of the subscript expression is truncated to an integer, and the value of the variable for the indicated observation is returned. If the value of the subscript expression is less than 1 or greater than the number of observations in the data set, a missing value is returned.

Constant	Description
_n	contains the number of the current observation.
_N	contains the total number of observations in the data set.
_pi	contains the value of $\pi$ .
NA	indicates a missing value
	indicates a missing value

Table 4.3: List of constants

# Example

Explicit subscribing combined with the variable  $_n$  (see Table 4.3) can be used to create lagged values of a variable. For example, the lagged value of a variable x in a data set data can be created by

> data.generate xlag = x[\_n-1]

Note that xlag can also be generated using the lag function

> data.generate xlag = lag(x)

# 4.4 Method generate

# Description

Method generate is used to create a new variable in an existing dataset object.

### **Syntax**

> objectname.generate newvar = expression

Method generate creates a new variable with name newvar (compare section 4.13 for valid variable names) The values of the new variable are specified by expression.

### **Options**

not allowed

### Example

The following command generates a new variable called amount 2 which contains the squared amount in the credit scoring data set.

```
> credit.generate amount2 = amount^2
```

If you try to change the variable currently generated, for example by typing

> credit.generate amount2 = amount^0.5

the error message

ERROR: variable amount2 is already existing

will occur. This prevents you to change an existing variable unintentionally. An existing variable can be changed with method replace, see section 4.9.

If you want to generate an indicator variable largeamount for amounts exceeding a certain value, say 3.5, you have to type

```
> credit.generate largeamount = 1*(amount>3.5)
```

The variable largeamount takes the value 1 if amount is larger than 3.5 and 0 otherwise.

### 4.5 Method infile

### Description

Reads data saved in an ASCII file.

# **Syntax**

> objectname.infile [varlist] [, options] using filename

Reads the data stored in *filename*. The variable names can either be specified in *varlist* or be extracted from the first line of the file. To be more specific, if *varlist* is empty it is assumed that the first row of the file contains the variable names separated by blanks or tabs. The observations do not have to be stored in a special format, except that successive observations should be separated by one or more blanks or tabs. The first value read from the file will be the value of the first variable of the the first observation, the second value will be the value of the second variable of the first observation, and so on. An error message will occur if no values can be read for the last observation for some variables.

The period '.' or 'NA' should be used to indicate missing values.

Note that in the current version of BayesX only numerical variables are allowed. Thus, the attempt to read string valued variables, for example, will cause an error.

### **Options**

 $\bullet$  missing = missing signs

By default the period '.' or 'NA' indicate missing values. If the missing values are indicated by a different sign, you can specify them in the missing option. For example missing = MIS defines 'MIS' as an indicator for a missing value. Note that '.' and 'NA' remain valid indicators for missing values, even if the missing option is specified.

 $\bullet$  maxobs = integer

If you work with large data sets, you may observe the problem that reading in a data set using the infile command is very time consuming. The reason for this problem is that BayesX does not know the number of observations in advance and therefore is unable to allocate enough memory. Therefore new memory has to be allocated whenever a certain amount of memory is exceeded. Specifying option maxobs allows to circumvent this problem and to considerably reduce the computing time. When maxobs is specified, BayesX can allocate enough memory to store at least integer observations before new memory must be reallocated. Suppose for example that your data set consists of approximately 100,000 observations. Then specifying maxobs=105000 allocates enough memory to read in the data set quickly. Note that maxobs=105000 does not require the data set to have exactly 105,000 observations. It only means that new memory will have to be allocated when the number of observations exceeds the 105,000 observations limit.

### Example

Suppose we want to read a data set stored in c:\data\testdata.raw containing the two variables var1 and var2. The first few rows of the datafile could look like this:

var1 var2

2 2.3

 $3 \ 4.5$ 

4.5. Method infile

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...

To read the data set, we have to create a new *dataset object* first, say testdata. We proceed by reading the data using the infile command:

- > dataset testdata
- > testdata.infile using c:\data\testdata.raw

If the first row of the data set file contains no variable names, the second command has to be altered to:

> testdata.infile var1 var2 using c:\data\testdata.raw

If furthermore the data set is large with about 100,000 observations. In this case, the maxobs option is very useful to reduce reading time. Typing for example

> testdata.infile var1 var2 , maxobs=101000 using c:\data\testdata.raw will be much faster than the command without option maxobs.

# 4.6 Method outfile

# Description

Method outfile writes data to a file in ASCII format.

### **Syntax**

> objectname.outfile [varlist] [if expression] [, options] using filename

outfile writes the variables specified in *varlist* to the file with name *filename*. If *varlist* is omitted in the outfile statement, *all* variables in the data set will be included. Each row in the data file will correspond to one observation. Different variables will be separated by blanks. Optionally, an if statement may be used to store only those observations where a certain boolean expression is true.

# **Options**

#### • header

Specifying the header option causes BayesX to write the variable names to the first row of the created data file.

#### • replace

The replace option allows BayesX to overwrite an already existing data file. If replace is omitted and the file specified in *filename* is already existing, an error will be raised. This prevents you from unintentionally overwriting an existing file.

# Example

The statement

> credit.outfile using c:\data\cr.dat

writes the complete credit scoring data set to c:\data\cr.dat. To generate two different ASCII data sets for married people and people living alone, you could type

```
> credit.outfile if marstat = 1 using c:\data\crmarried.dat
```

```
> credit.outfile if marstat = 2 using c:\data\cralone.dat
```

If you want to store only the two variables y and amount, you could type

> credit.outfile y amount using c:\data\cr.dat

This will raise the error message

```
ERROR: file c:\data\cr.dat is already existing
```

because c:\data\cr.dat has already been created. You can overwrite the file using the replace option

> credit.outfile y amount, replace using c:\data\cr.dat

4.7. Method pctile 31

# 4.7 Method pctile

# Description

Method pctile computes and displays the 1%,5%,25%,50%,75%,95% and 99% percentiles of a variable.

# **Syntax**

```
> objectname.pctile varlist [if expression]
```

Method pctile computes and displays the percentiles of the variables specified in *varlist*. An optional if statement may be added to compute the percentiles only for a part of the data.

# **Options**

not allowed

# Example

```
The statement
```

```
> d.pctile x y computes percentiles for the variables x and y. The statement > d.pctile x y if x>0
```

restricts the analysis to observations with x>0.

# 4.8 Method rename

# Description

rename is used to change variable names.

# **Syntax**

> objectname.rename varname newname

rename changes the name of *varname* to *newname*. *newname* must be a valid variable name, see section 4.13 for valid variable names.

# **Options**

not allowed

# 4.9 Method replace

# Description

replace changes values of an existing variable.

# **Syntax**

> objectname.replace varname = expression [if expression]

replace changes some or all values of the variable *varname* to the values specified in *expression*. If *varname* is not existing, an error will be raised. An optional if statement may be used to change the values of the variable only if the boolean expression *expression* is true.

# **Options**

not allowed

### **Example**

The statement

> credit.replace amount = NA if amount<0</pre>

changes the values of the variable amount in the credit scoring data set to missing if amount<0.

# 4.10 Method set obs

# Description

set obs changes the number of observations in a data set.

# **Syntax**

> objectname.set obs = intvalue

set obs raises the number of observations in the data set to *intvalue*. The new number of observations has to be greater or equal than the current number to prevent you from unintentionally deleting parts of the data currently in memory. Observations may be eliminated using the drop statement, see section 4.2. The values of newly created observations will be set to the missing value.

4.11. Method sort

# 4.11 Method sort

# Description

Sorts the data set.

# **Syntax**

> objectname.sort varlist [, options]

Sorts the data set with respect to the variables specified in *varlist*. Missing values are interpreted to be larger than any other number and are thus placed last.

# **Options**

### • descending

If this option is specified, the data set will be sorted in descending order. The default is ascending order.

# 4.12 Method tabulate

# Description

Method tabulate calculates and displays a frequency tables.

# **Syntax**

> objectname.tabulate varlist [if expression]

Method tabulate computes and displays frequency tables of the variables specified in *varlist*. An optional if statement may be added to restrict the analysis to a part of the data.

# **Options**

not allowed

### **Example**

```
The statement
```

```
> d.tabulate x y
```

displays frequency tables for the variables  ${\tt x}$  and  ${\tt y}$ . The statement

```
> d.tabulate x y if x>0
```

restricts the analysis to observations with x>0.

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## 4.13 Variable names

A valid variable name is a sequence of letters (A-Z and a-z), digits (0-9), and underscores (\_). The first character of a variable name must be either a letter or an underscore. BayesX is case-sensitive, i.e. myvar, Myvar and MYVAR are three distinct variable names.

## 4.14 Examples: Working with datasets

This section contains two examples on how to work with *dataset objects*. The first example illustrates some of the methods described in this chapter using the credit scoring data set (see subsection 2.6.2 for a description). The second example demonstrates how to simulate complex statistical models.

## 4.14.1 The credit scoring data set

In this section we illustrate how to code categorical variables according to either dummy or effect coding. This will be useful in regression models, where all categorical covariates must be coded in dummy or effect coding before they can be added to the model.

We start by creating the *dataset object* credit and proceed by reading the data using the infile command.

```
> dataset credit
```

```
> credit.infile using c:\bayesx\examples\credit.raw
```

We can now generate new variables to obtain dummy coded versions of the categorical covariates account, payment, intuse and marstat:

```
> credit.generate account1 = 1*(account=1)
> credit.generate account2 = 1*(account=2)
> credit.generate payment1 = 1*(payment=1)
> credit.generate intuse1 = 1*(intuse=1)
> credit.generate marstat1 = 1*(marstat=1)
```

The reference categories are chosen to be 3 for account and 2 for the other variables. Alternatively, we could code the variables according to effect coding. This is achieved with the following program code:

```
> credit.generate account_eff1 = 1*(account=1)-1*(account=3)
> credit.generate account_eff2 = 1*(account=2)-1*(account=3)
> credit.generate payment_eff1 = 1*(payment=1)-1*(payment=2)
> credit.generate intuse_eff1 = 1*(intuse=1)-1*(intuse=2)
> credit.generate marstat_eff1 = 1*(marstat=1)-1*(marstat=2)
```

## 4.14.2 Simulating complex statistical models

In this section we demonstrate how to simulate complex regression models. Suppose first that we want to simulate data according to the following Gaussian regression model:

$$y_i = 2 + 0.5x_{i1} + \sin(x_{i2}) + \epsilon_i, \quad i = 1, \dots, 1000$$
 (4.1)

$$x_{i1} \sim U(-3,3) \quad i.i.d.$$
 (4.2)

$$x_{i2} \sim U(-3,3) \quad i.i.d.$$
 (4.3)

$$\epsilon_i \sim N(0, 0.5^2) \quad i.i.d. \tag{4.4}$$

First of all, we create a new data set gsim, say, and specify the desired number of observations:

- > dataset gsim
- > gsim.set obs = 1000

In a second step, the covariates x1 and x2 have to be created. We assume that the covariates are uniformly distributed between -3 and 3 and therefore enter the commands

- > gsim.generate x1 = -3+6\*uniform()
- > gsim.generate x2 = -3+6\*uniform()

Finally, we create the response variable by typing

> gsim.generate y = 2 + 0.5\*x1+sin(x2)+0.5\*normal()

Now we could estimate the regression model with the generated data set using one of the regression tools of *BayesX*, see chapter 7 or chapter 8.

Of course, more refined models can also be simulated. We may for example drop the assumption of a constant variance. Suppose the variance is heteroscedastic and growing with order  $\log(i)$  where i is the observation index. Such a heteroscedastic model can be simulated by:

 $> gsim.replace y = 2 + 0.5*x1+sin(x2)+0.1*log(_n+1)*normal()$ 

In this model the standard deviation is given by

$$\sigma_i = 0.1 * \log(i+1), \quad i = 1, \dots, 1000.$$

Suppose now that we want to simulate data from a logistic regression model, where the response variable  $y_i$ , i = 1, ..., n, is binomially distributed with parameters  $n_i$  and  $\pi_i$  ( $n_i$  is the number of replications and  $\pi_i$  is the probability of success.) The probability of success is related to a linear predictor  $\eta_i$  via the logistic distribution function, i.e.

$$\pi_i = \frac{\exp(\eta_i)}{1 + \exp(\eta_i)}.$$

To simulate this model we have to specify the linear predictor  $\eta_i$  and the number of replications  $n_i$ . We make use of a similar linear predictor as in the example above for Gaussian response, namely

$$\eta_i = -1 + 0.5x_{i1} - \sin(x_{i2}).$$

For simplicity, we set  $n_i = 1$  for the number of replications. The following commands generate a data set bin according to the specified model:

- > dataset bin
- > bin.set obs = 1000
- > bin.generate x1 = -3+6\*uniform()
- > bin.generate x2 = -3+6\*uniform()
- > bin.generate eta = -1+0.5\*x1-sin(x2)
- > bin.generate pi = exp(eta)/(1+exp(eta))
- > bin.generate y = binomial(1,pi)

Note that the last three statements can be combined into a single command:

> bin.generate y = binomial(1,exp(-1+0.5\*x1-sin(x2))/(1+exp(-1+0.5\*x1-sin(x2)))

The first version, however, is much easier to read and might therefore be preferred.

## Chapter 5

# map objects

The purpose of *map objects* is to provide functionality for handling and storing geographical maps. In addition, *map objects* serve as auxiliary objects for regression models with spatial effects, where the effect of a location variable is to be included via a spatially correlated prior. In this context, *map objects* store the neighborhood structure of a map and provide functionality to compute weights associated with this neighborhood structure.

The typical approach will be as follows: A map object is created, the boundary information of a geographical map is read from an external file and stored in the map object. This can be achieved using the infile command, see section 5.1 below. Based on the boundary information, the map object automatically computes the neighborhood structure of the map and the weights associated with the neighborhood structure. Since there are several proposals in the spatial statistics literature for defining weights, the user is given the choice between a couple of alternatives. Afterwards the map object can be passed to the regress function of a regression object to estimate regression models with spatial covariates, see chapter 7 and chapter 8, and in particular the subsections about spatial covariates therein.

## 5.1 Method infile

## 5.1.1 Description

Method infile is used to read the boundary information of a geographical map stored in an external file. Currently, two file formats are supported: boundary files and graph files. A boundary file contains information about the boundaries of the different regions of a map in terms of closed polygons, i.e. the boundary of each region is represented by a set of connected straight lines. A detailed description of the structure of boundary files is given below.

A graph file stores the nodes and edges of a graph representing the neighborhood structure of a map. In addition, weights associated with the edges of the graph can be specified. In terms of geographical maps, the nodes of the graph correspond to the regions of the map while the edges specify the neighborhood structure. As a consequence, the neighborhood structure of a geographical map is immediately available in a graph file while it has to be computed from the polygons in case of a boundary file (a task which may be time-consuming). Therefore an advisable strategy is to read a boundary files only once to compute the neighborhood structure and to store it as a graph file afterwards (using the outfile command, see section 5.2). This is particularly the case for geographical maps with a lot of regions. However, using graph files also has a disadvantage: Visualization of geographical information is only possible with boundary files since only these contain the information on the boundaries required for visual representation.

## 5.1.2 Syntax

> objectname.infile [, options] using filename

Method infile reads the map information stored in the boundary or graph file filename. If option graph is specified, BayesX expects a graph file, otherwise a boundary file is expected.

## Structure of a boundary file

A boundary file provides the boundary information of a geographical map in terms of closed polygons. For each region of the map, the boundary files contains a block of lines defining the name of the region, the number of lines the polygon consists of, and the polygons themselves. The first line of such a block contains the region code surrounded by quotation marks and the number of lines the polygon of the region consists of. The region code and the number of lines must be separated by a comma. The subsequent lines contain the coordinates of the straight lines that form the boundary of the region. The straight lines are represented by the coordinates of their end points. Coordinates must be separated by a comma.

To give an example we print a (small) part of the boundary file of Germany:

:	$2726.61646,\!4310.54248$	2359.06665,3951.18677
	2716.08154,4256.69775	2285.32275,3969.91553
"6634",31	2710.22900,4227.43408	2258.40015,4061.21753
2319.26831,4344.48828	2680.96533,4234.45752	2197.53223,4049.51221
2375.45435,4399.50391	2583.81055,4165.39551	2162.41602,4086.96948
2390.67139,4446.32520	2568.59351,4096.33398	2204.55542,4091.65161
2470.26807, 4405.35645	2520.60132,4042.48901	2192.85010,4125.59717
2576.78735,4379.60449	2535.81836.3941.82251	2284.15210.4220.41113
2607.22144,4337.46533	2490.16724,3920.75269	2339.16748,4292.98438
2627.12061,4356.19385	2451.53955,3903.19458	2319.26831,4344.48828
2662.23682,4355.02344	2437.49292.3924.26440	2010.20001,4044.40020
2691.50024,4311.71338	2369.60156,3933.62866	:

5.1. Method infile

The map corresponding to this part of the *boundary file* can be found in Figure 5.1. Note that the first and the last point of a polygon must be identical (see the example above) to obtain a closed polygon.

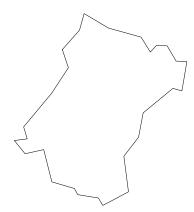


Figure 5.1: Corresponding graph of the section of the boundary file

In some cases a region may be divided into a number of subregions that are not connected. As an example, Figure 5.2 shows a region within Germany that is separated into 8 subregions. In such a case each subregion has to be included separately in the *boundary file* but all subregions should share the same region code. Note that the subregions can be placed anywhere in the *boundary file* and, in particular, do not have to follow each other.

Another special case that requires additional attention is illustrated in Figure 5.3: A region is completely surrounded by another region. In this case an additional line has to be added to the boundary description of the *surrounded* region. This additional line is placed between the first line and the polygons and has to contain the region code of the *surrounding* region. The full syntax is: is.in," region code"

The following lines show a section of the *boundary file* of Germany, where region "9361" is totally surrounded by region "9371":

```
: "9361",7
is.in,"9371"
4155.84668,2409.58496
4161.69922,2449.38330
4201.49756,2461.08862
4224.90820,2478.64673
4250.66016,2418.94922
4193.30371,2387.34448
4155.84668,2409.58496
:
```

Finally, we want to draw attention to an important limitation of the current version of BayesX. In most cases map objects serve as auxiliary objects to estimate spatial effects with bayesreg objects or remlreg objects. In this case the names of the regions of the map and the values of the corresponding spatial covariate have to match. Since only numerical variables allowed in dataset objects, the names of the regions in the map object have to be numbers, although, in principle, there is no limitation for the names of regions in map objects.

## Structure of a graph file

A graph file stores the nodes and the edges of a graph G = (N, E), see for example George and Liu (1981, Ch. 3) for a brief introduction to graph theory. A graph is a convenient way to represent the neighborhood structure of a geographical map, where the nodes of the graph correspond to the region codes while the neighborhood structure is represented by the edges of the graph. In some situations it can be useful to define weights associated with the edges of a graph which can be stored in the graph file as well.

In BayesX, the first line of a graph file has to contain the total number of nodes of the graph. Afterwards, each region (or node) is represented by a block of three lines. The first of these lines contains the name of the node (typically the name of the geographical region). The second line states the number of edges for that particular node. The third line contains the corresponding edges of the node, represented by the index of a neighboring node. Note that the index starts with zero, i.e. the first node is represented by the index 0, the second node by the index 1, and so on.

We illustrate the structure of a *graph file* with an example. The following few lines are the beginning of the *graph file* corresponding to the map of (former) West Germany:

```
327
9162
3
1 2 3
9174
6
0 4 2 3 5 6
9179
6
0 1 7 3 8 6
```

The first line specifies the total number of nodes, 327 in the present example. The subsequent three lines correspond to the node with name '9162', which is the first region in the map of West Germany. Region '9162' has 3 neighbors, namely the second, third and fourth node appearing in the graph file. Once again, note that the index starts with zero, i.e. 0 corresponds to the first node, 1 corresponds to the second node and so on. Lines 5 to 7 in the example correspond to node '9174' and its neighbors and lines 8 to 10 correspond to node '9179'.

Graph files also allow to specify weights associated with the edges of the nodes. Since no weights

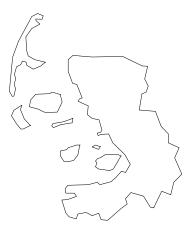


Figure 5.2: Example for a region that is divided into subregions.

5.1. Method infile 43

have been explicitly specified in the preceding example, all weights are automatically defined to be equal to one. Nonequal weights can specified in the *graph file* by adding them following the edges of a particular node. An example of the beginning of a *graph file* with weights is given below:

```
327

9162

3

1 2 3 0.4 1.2 0.7

9174

6

0 4 2 3 5 6 0.4 0.3 0.8 0.8 1.4 1.6

9179

6

0 1 7 3 8 6 1.2 0.8 0.2 1.8 1.7 1.3

:
```

In this case, the edges of the first node '9162' have weights 0.4, 1.2 and 0.7.

## **Options**

• graph

If graph is specified as an additional option, BayesX expects a graph file rather than a boundary file.

• weightdef=adjacency | centroid | combnd

Option weightdef allows to request the computation of weights associated with each pair of neighbors. Currently there are three weight specifications available: weightdef=adjacency, weightdef=centroid and weightdef=combnd. For weightdef=adjacency all weights are set equal to one. This is the most common choice in spatial statistics and also the default weight method.

Specifying weightdef=centroid results in weights which are inverse proportional to the distance of the centroids of neighboring regions. More specifically, the weight  $w_{us}$  of two neighboring regions u and s is set to  $w_{us} = c \cdot \exp(-d(u, s))$ , where d is the Euclidian distance between the centroids of the two sites and c is a normalizing constant. In analogy to adjacency weights, the constant c is chosen in such a way that the total sum of weights is equal to the total number of neighbors.

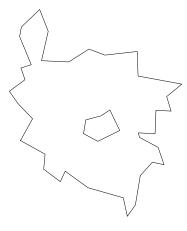


Figure 5.3: Example for a region that is totally surrounded by another region.

The third choice weightdef=combnd results in weights proportional to the length of the common boundary of two regions. Similarly to weightdef=centroid, the weights are normalized, i.e. the total sum of weights is equal to the number of neighbors.

Note that the specification of the weightdef option is only meaningful if a boundary file is read. For graph files the option has no effect since the boundary information of regions is missing and the computation of weights is therefore not possible.

5.2. Method outfile 45

## 5.2 Method outfile

## Description

Method outfile performs the reverse of the infile command, i.e. the current map information is written to an external file. The map information can be written either in *boundary* file or in graph file format.

## **Syntax**

> objectname.outfile [, options] using filename

outfile writes the map information to the external file specified in *filename*. If graph is specified as an additional option, the file format will be a *graph file* and a *boundary file* otherwise.

## **Options**

#### • graph

Forces the program to store the map information in *graph file* format rather than in *boundary file* format.

#### • includeweights

Option includeweights is meaningful only if the storing format is a *graph file*, i.e. if option graph is specified. In this case, the weights associated with the edges (neighbors) of the nodes (regions) are stored in addition to the graph structure.

#### • replace

The replace option allows BayesX to overwrite an already existing file. If replace is omitted in the optionlist an error will be raised if you attempt to overwrite an existing file. This prevents you from overwriting an existing file unintentionally.

## 5.3 Method reorder

## Description

Method **reorder** reorders the regions of a map to obtain an adjacency matrix with minimal envelope. A new map should always be reordered before using it with *bayesreg objects* since MCMC updates for spatial covariates will be much faster with an optimised envelope. *BayesX* uses the reverse Cuthill Mc-Kee algorithm to reorder maps, see George and Liu (1981) p. 58 ff.

## **Syntax**

#### > objectname.reorder

reorder reorders the regions of a map in order to obtain the smallest envelope of the corresponding adjacency matrix.

## **Options**

not allowed.

## Reference

George, A., Liu, J. W. (1981). Computer Solution of Large Sparse Positive Definite Systems. Series in computational mathematics, Prentice-Hall.

# Chapter 6

# graph objects

Graph objects are used to visualize data or estimation results obtained with the regression objects in BayesX. Currently, graph objects can be used to draw scatterplots between variables (section 6.2, method plot), or to draw and color geographical maps stored in map objects (section 6.1, method drawmap). The resulting plots are either printed on the screen or stored as postscript files for further use in other documents (e.g. IATEX documents). A graph object is created by typing

> graph objectname

in the command window.

## 6.1 Method drawmap

## Description

Method drawmap is used to draw geographical maps and color the regions according to some numerical characteristics.

## **Syntax**

- > objectname.drawmap [plotvar regionvar] [if expression], map=mapname [options]
- > [using dataset]

Method drawmap draws the map stored in the map object mapname and prints the graph either on the screen or stores it as a postscript file (if option outfile is specified). The regions with regioncode regionvar are colored according to the values of the variable plotvar. The variables plotvar and regionvar are supposed to be stored in the dataset object dataset. An if statement may be specified to use only a part of the data in dataset. Several options are available, e.g. for changing from grey scale to color scale or to store the map as a postscript file. See the options list below for more details.

## **Options**

The most important option, which therefore is obligatory, is the map option. This option specified the name of the *map object* containing the boundary information to be drawn. Additional options for method drawmap (in alphabetical order) are:

#### • color

The color option allows to choose between a grey scale and a colored scale. If the keyword color is specified, a colored scale is used instead of a grey scale.

#### • drawnames

In some situations it may be useful to print the names of the regions into the graph (although the result will probably be confusing in most cases). This can be achieved by specifying the additional option drawnames. By default the names of the regions are omitted in the map.

## ullet fontsize = integer

Specifies the font size (in pixels) for labelling the legend and writing the names of the regions (if specified). Note, that the title is scaled accordingly (see option titlesize). The default is fontsize=12.

#### • hcl

Requests that a color palette from the HCL color space should be used instead of an RGB palette. The HCL colors will be selected diverging from a neutral center (grey) to two different extreme colors (red and green) in contrast to the RGB colors diverging from yellow to red and green. HCL colors are particularly useful for electronic presentations since they are device-independent. The option hcl is only meaningful in combination with the option color.

#### • lowerlimit = realvalue

Lower limit of the range to be drawn. If lowerlimit is omitted, the minimum numerical value in *plotvar* will be used as the lower limit instead.

#### $\bullet$ map = mapname

mapname specifies the name of the map object containing the boundary information to be drawn. This option is obligatory.

#### • nolegend

By default a legend is drawn into the graph. Specifying the option nolegend will exclude the legend from the graph.

#### • nrcolors = integer

To color the regions according to their numerical characteristics, the data are divided into a (typically large) number of ordered categories. Afterwards a color is associated with each category. The number option can be used to specify the number of categories (and therefore the number of different colors). The maximum number of colors is 256, which is also the default value.

#### • outfile = filename

If option outfile is specified the graph will be stored as a postscript file rather than being printed on the screen. The full path and the filename have to be specified in *filename*. By default, an error will be raised if the specified file is already existing or if the path is invalid. To overwrite an already existing file, option replace has to be specified in addition. This prevents you from overwriting your files unintentionally.

#### • pcat

If you want to visualize posterior probabilities, it is convenient to specify pcat. In this case, method drawmap expects a variable containing only the values -1, 0 and 1. Of course you can achieve the same result by setting nrcolors=3, lowerlimit=-1 and upperlimit=1.

#### • replace

The replace option is only meanigful in combination with option outfile. Specifying replace as an additional option allows the program to overwrite an already existing file (specified in outfile), otherwise an error will be raised.

#### swapcolors

In some situations it may be favorable to swap the order of the colors, i.e. black (red) shades corresponding to large values and white (green) shades corresponding to small values. This is achieved by specifying swapcolors. By default, small values are colored in black shades (red shades) and large values in white shades (green shades).

#### • title = characterstring

Adds a title to the graph. If the title contains more than one word, *characterstring* must be enclosed by quotation marks (e.g. title="my first map").

#### • titlesize = realvalue

Specifies the factor by which the size of the title is scaled relative to the size of the labels of the legend (compare option fontsize). The default is titlesize=1.5.

#### • upperlimit = realvalue

Upper limit of the range to be drawn. If upper limit is omitted, the maximum numerical value in *plotvar* will be used as the upper limit instead.

## **Example**

This example shows how to draw the map of Munich and how to color the subquarters in Munich according to some numerical characteristics. The boundary file of Munich (munich.bnd) as well as the data set rent94means.raw containing the distribution of the average rents across subquarters are included in the subfolder examples of the BayesX installation directory. In the following we assume that BayesX is installed in the folder c:\bayesx. We start by creating a  $dataset\ object\ d$  and a  $map\ object\ m$  and proceed with reading the rent data set and the map of Munich:

- > dataset d
- > d.infile using c:\bayesx\examples\rent94means.raw
- > map m
- > m.infile using c:\bayesx\examples\munich.bnd

Afterwards we create a graph object g and draw the map of Munich:

- > graph g
- > g.drawmap , map=m

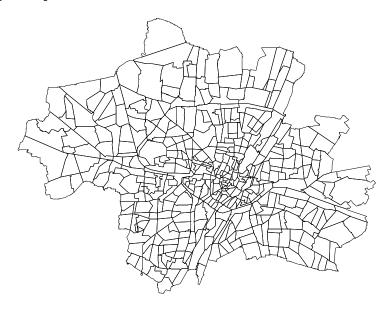


Figure 6.1: Map of Munich

The map of Munich appears on the screen in a separate window, compare Figure 6.1. Before closing the window you are asked whether you want to save the map or not. If you agree the map will be stored as a postscript file in the folder you specify. Of course, the map can be directly stored in postscript format using the outfile option. In this case the map is not shown on the screen. Typing

> g.drawmap , map=m outfile=c:\temp\munich.ps

stores the map of Munich in the file c:\temp\munich.ps and the graph is not printed on the screen.

Usually maps are drawn to visualize numerical characteristics of their regions. For instance, typing

> g.drawmap R L , map=m color using d

displays the distribution of the average rents R across subquarters L, see Figure 6.2. The areas in the figure shaded with diagonal lines mark subquarters for which no data are available. The specification of the second variable L is required to match the names of the subquarters stored in the *map object* m with the data set d. Option color is specified to obtain a colored graph. Specifying option hcl in addition yields the same information visualized in HCL colors (see Figure 6.3).

## > g.drawmap R L , map=m color hcl using d

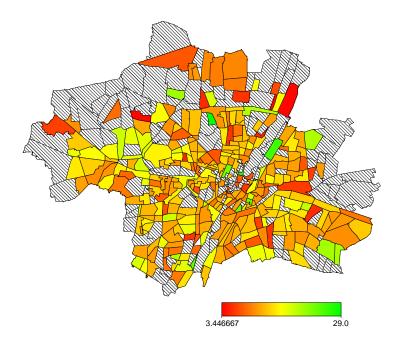


Figure 6.2: Distribution of the average rents per square meter in Munich visualized in RGB colors.

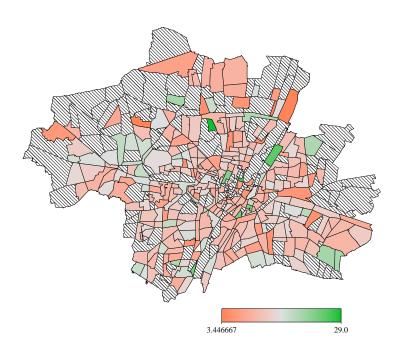


Figure 6.3: Distribution of the average rents per square meter in Munich visualized in HCL colors.

6.2. Method plot 53

## 6.2 Method plot

## Description

Method plot is used to draw scatterplots between two or more variables. Several options for labelling axes, connecting points, saving the graph etc. are available.

## **Syntax**

> objectname.plot xvar yvar1 [yvar2 yvar3 ...] [if expression] [, options] using dataset

Method plot draws scatterplots of yvar1, yvar2, yvar3 ... against xvar into a single graph using the data set specified in dataset. An if statement may be used to apply the method only to a part of the data. In addition, several options may be specified for labelling axes, connecting points, saving the graph in postscript format etc., see the options list below.

## **Options**

The following options are available for method plot (listed in alphabetical order):

• connect = 1|2|3|4|5[specifications for further variables]

Option connect specifies how points in the scatterplot are connected. There are currently 5 different specifications:

- draw straight lines between the points (default)
- 2, 3, 4 draw dashed lines (numbers 2 4 indicate different variants)
- 5 do not connect, i.e. plot points only

If you draw more than one scatterplot in the same graph (i.e. more than one *yvar* is specified) the points for each *yvar* can be connected differently by specifying the corresponding number (1,2,3,4,5) separately for every *yvar*. Typing for example

#### connect=15

connects the points corresponding to yvar1 and xvar by straight lines, but does not connect the points corresponding to yvar2 (if specified) and xvar. Points corresponding to additional variables yvar3, etc. are connected by straight lines (the default).

An equivalent way of specifying the different variants is available via the symbols 'l', 'd', '\_', '-' and 'p', which correspond to the numbers 1-5, i.e.

connect=12345 is equivalent to connect=ld\_-p

 $\bullet$  fontsize = integer

Specifies the font size (in pixels) for labelling axes etc. Note that the title is scaled accordingly. The default is fontsize=12.

 $\bullet$  height = integer

Specifies the height (in pixels) of the graph. The default is height=210.

• linecolor = B|b|c|G|g|o|m|r|y [specifications for further variables]

Option linecolor specifies the color to be used for drawing lines (or points, see option connect) in the scatterplot. Currently the following specifications are available:

- B black (default)
- b blue
- c cyan
- G gray
- g green
- o orange
- m magenta
- r red
- y vellow

If you draw more than one scatterplot in the same graph (i.e. more than one *yvar* is specified) you can use different colors for each *yvar* by simply specifying the corresponding symbol (B,b,c,G,g,o,m,r,y) for each *yvar*. Typing for example

## linecolor = Bgr

colors the lines (points) corresponding to yvar1 and xvar in black, whereas the points corresponding to yvar2 and yvar3 (if specified) and xvar are colored in green and red, respectively.

#### • linewidth = integer

Specifies how thick lines should be drawn. The default is linewidth=5.

#### • outfile = filename

If option outfile is specified, the graph will be stored as a postscript file rather than being printed on the screen. The full path and the filename have to be specified in *filename*. By default, an error will be raised if the specified file is already existing or if the specified folder is not valid. To overwrite an already existing file, option replace must be specified in addition. This prevents you from overwriting your files unintentionally.

#### $\bullet$ pointsize = integer

Specifies the size of the points (in pixels) if drawing points rather than lines. The default is pointsize=20.

## • replace

The replace option is useful only in combination with option outfile. Specifying replace as an additional option allows the program to overwrite an already existing file (specified in outfile), otherwise an error will be raised.

#### • title = characterstring

Adds a title to the graph. If the title contains more than one word, *characterstring* must be enclosed by quotation marks (e.g. title="my first title").

#### • titlesize = realvalue

Specifies the factor by which the size of the title is scaled relative to the size of the labels of the axes (compare option fontsize). The default is titlesize=1.5.

#### • width = integer

Specifies the width (in pixels) of the graph. The default is width=356.

#### • xlab = characterstring

Labels the x-axis. If the label contains more than one word, *characterstring* must be enclosed by quotation marks (e.g. xlab="x axis").

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#### $\bullet$ xlimbottom = realvalue

Specifies the minimum value at the x-axis to be drawn. The default is the minimum value in the data set. If xlimbottom is above the minimum value in the data set, only a part of the graph will be visible.

#### • xlimtop = realvalue

Specifies the maximum value at the x-axis to be drawn. The default is the maximum value in the data set. If xlimtop is below the maximum value in the data set, only a part of the graph will be visible.

#### • xstart = realvalue

Specifies the value where the first tick on the x-axis should be drawn. The default is the minimum value on the x-axis.

#### $\bullet$ xstep = realvalue

If xstep is specified, ticks are drawn at the x-axis with stepwidth *realvalue* starting at the minimum value on the x-axis (or at the value specified in option xstart). By default, five equally spaced ticks are drawn at the x-axis.

#### • ylab = characterstring

Labels the y-axis. If the label contains more than one word, *characterstring* must be enclosed by quotation marks (e.g. ylab="y axis").

#### • ylimbottom = realvalue

Specifies the minimum value at the y-axis to be drawn. The default is the minimum value in the data set. If ylimbottom is above the minimum value in the data set, only a part of the graph will be visible.

#### • ylimtop = realvalue

Specifies the maximum value at the y-axis to be drawn. The default is the maximum value in the data set. If ylimtop is below the maximum value in the data set, only a part of the graph will be visible.

#### • ystart = realvalue

Specifies the value where the first tick on the y-axis should be drawn. The default is the minimum value on the y-axis.

#### • ystep = realvalue

If ystep is specified, ticks are drawn at the y-axis with stepwidth *realvalue* starting at the minimum value on the y-axis (or at the value specified in option ystart). By default, five equally spaced ticks are drawn at the y-axis.

#### • Further options for representing dates

In the following we describe options that may be useful if the variable on the x-axis represents dates. An example is a variable with values ranging from 1 to 19, representing the time period from January 1983 to July 1984. In this case, we might prefer that the x-axis is labelled in terms of dates rather than in the original coding (from 1 to 19). To achieve this, BayesX provides the options month, year and xstep. Options year and month are used to specify the year and the month (1 for January, 2 for February, ...) corresponding to the minimum covariate value. In the example mentioned above year=1983 and month=1 will produce the

correct result. In addition, option xstep may be specified to define the periodicity in which your data are collected. For example xstep=12 (the default) corresponds to monthly data, while xstep = 4, xstep = 2 and xstep = 1 correspond to quarterly, half yearly and yearly data, respectively.

#### Example

We use the Munich rent data set rent94.raw to demonstrate the usage of method plot. The data set is included in the subfolder examples of the *BayesX* installation directory. In the following we assume that *BayesX* has been installed to the folder c:\bayesx. We start by reading the data by typing:

```
> dataset d
> d.infile using c:\bayesx\examples\rent94.raw
```

Then we generate a *graph object* g and draw a scatterplot between floor space (variable F) and rent per square meter (variable R):

```
> graph g
> g.plot R F using d
```

The strange picture shown in Figure 6.4 appears on the screen. The problem is that the points are connected by straight lines although the values of F are not sorted. Hence, to obtain an improved scatterplot, we could either sort the data set with respect to F or simply avoid connecting the points. Typing

```
> d.sort F
> g.plot R F using d
yields the first option. Typing
> g.plot R F, connect=p using d
```

yields the second option mentioned above. The corresponding graphs are shown in Figure 6.5 and Figure 6.6, respectively. To further improve the appearance of the scatterplot we add a title and label the x- and y-axes by typing

```
> g.plot R F, title="scatterplot between F and R" ylab="rent"
   xlab="floor space in square meters" connect=p using d
```

The result is shown in Figure 6.7. Finally, we add the outfile option to save the graph in postscript format:

```
> g.plot R F, title="scatterplot between F and R" ylab="rent"
   xlab="floor space in square meters" connect=p
   outfile=c:\temp\plotrf.ps using d
```

6.2. Method plot

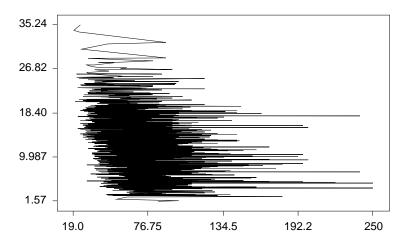


Figure 6.4: Scatterplot between floor space and rent per square meters (first try).

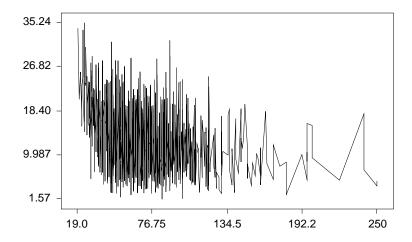


Figure 6.5: Scatterplot between floor space and rent per square meters (second try).

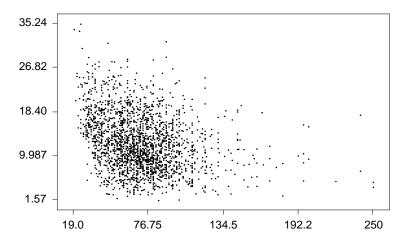


Figure 6.6: Scatterplot between floor space and rent per square meters (third try).

## scatterplot between $\boldsymbol{F}$ and $\boldsymbol{R}$

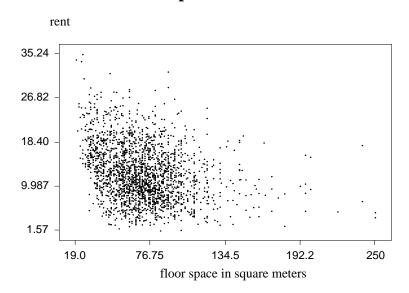


Figure 6.7: Scatterplot between floor space and rent per square meters (final try).

## 6.3 Method plotautocor

## Description

Method plotautocor visualizes the autocorrelation functions obtained with method autocor of bayesreg objects, see also section 7.2.

## Syntax

#### > objectname.plotautocor [,options] using dataset

Plots the autocorrelation functions stored in *dataset*. The data set must have the special structure described in section 7.2, i.e. method plotautocor is meaningful only if Bayesian regression models have been estimated in advance using *bayesreg objects* and autocorrelation functions of sampled parameters have been computed using method autocor of *bayesreg objects*.

## **Options**

#### • mean

If option mean is specified, only minimum, mean and maximum autocorrelations are plotted for each lag number and model term. This typically leads to a considerable reduction in computing time and storing size.

#### • outfile = filename

If option outfile is specified, the graph will be stored as a postscript file instead of being printed on the screen. The path and the filename have to be specified in *filename*. An error will be raised if the specified file is already existing and the replace option has not been specified.

#### • replace

The replace option is only meaningful in combination with option outfile. Specifying replace as an additional option allows the program to overwrite an already existing file (specified in outfile), otherwise an error will be raised.

## 6.4 Method plotsample

## Description

Method plotsample visualizes the sampling paths of sampled parameters obtained with method getsample of bayesreg objects, see also section 7.3. The application of method plotsample is meaningful only if Bayesian regression models have been estimated in advance using bayesreg objects and sampled parameters have been computed and stored using method getsample.

## **Syntax**

> objectname.plotsample [,options] using dataset

Plots sampled parameters stored in *dataset*. The data set must have the special structure described in section 7.3.

## **Options**

• outfile = filename

If option outfile is specified, the graph will be stored as a postscript file instead of being printed on the screen. The full path and the filename have to be specified in *filename*. An error will be raised if the specified file is already existing and the replace option has not been specified.

#### • replace

The replace option is only useful in combination with option outfile. Specifying replace as an additional option allows the program to overwrite an already existing file (specified in outfile), otherwise an error will be raised.

## Chapter 7

# bayesreg objects

bayesreg objects are used to fit (multivariate) exponential family, hazard rate or multi-state models with structured additive predictor subsumed in the class of structured additive regression (STAR) models, see Fahrmeir, Kneib, and Lang (2004). Inference is based on fully Bayesian approach implemented via Markov Chain Monte Carlo (MCMC) simulation techniques. The methodology manual provides brief introduction to structured additive regression and MCMC-based inference. More details can be found in Fahrmeir and Lang (2001a), Fahrmeir and Lang (2001b), Lang and Brezger (2004), Brezger and Lang (2006), Fahrmeir and Osuna (2003), Hennerfeind, Brezger and Fahrmeir (2006), and Fahrmeir and Hennerfeind (2003). Good introductions to generalized linear models are the monographs of Fahrmeir and Tutz (2001) and Mc Cullagh and Nelder (1989). Introductions to semi- and nonparametric models are given in Green and Silverman (1994), Hastie and Tibshirani (1990), Hastie and Tibshirani (1993) and Hastie, Tibshirani and Friedman (2001). The paper of Chib and Greenberg (1995), the monograph Markov Chain Monte Carlo in Practice edited by Gilks, Richardson and Spiegelhalter (1996) and the article by Green (2001) give a thorough overview over MCMC simulation techniques.

First steps with *bayesreg objects* can be done with the tutorial like example in chapter 1 of the tutorials manual which provides a self-contained demonstrating example.

## 7.1 Method regress

## 7.1.1 Description

Method regress estimates a structured additive regression model.

## **7.1.2** Syntax

> objectname.regress model [weight weightvar] [if expression] [, options] using dataset

Method regress estimates the regression model specified in *model* using the data specified in *dataset*. *dataset* has to be the name of a *dataset object* created before. The details of correct models are covered in subsubsection 7.1.2.2. The distribution of the response variable can be either Gaussian, gamma, binomial, multinomial, Poisson, negative binomial, zero inflated Poisson or zero inflated negative binomial. In addition, *BayesX* supports continuous time survival and multi-state models, see also Table 7.5 for a more detailled overview. The response distribution is specified using option family, see subsubsection 7.1.2.4 below and the options list in subsection 7.1.3. The default value is family=binomial with a logit link. An if statement may be specified to analyze only parts of the data set, i.e. the observations where *expression* is true.

#### 7.1.2.1 Optional weight variable

An optional weight variable weightvar may be specified to estimate weighted regression models. For Gaussian responses BayesX assumes that  $y_r|\eta_r,\sigma^2\sim N(\eta,\sigma^2/weightvar_r)$ . Thus, for grouped Gaussian responses the weights must be the number of observations in the groups if the  $y_r$ 's are the average of individual responses. If the  $y_r$ 's are the sum of responses in every group, the weights have to be the reciprocal of the number of observations in the groups. Of course, estimation of usual weighted regression models with heteroscedastic errors is also possible. In this case the weights should be proportional to the reciprocal of the heteroscedastic variances. If the response distribution is binomial, it is assumed that the values of the weight variable correspond to the number of replications and that the values of the response variable itself correspond to the number of successes. If weight is omitted, BayesX assumes that the number of replications is one, i.e. the values of the response must be either zero or one. For grouped Poisson data the weights have to be the number of observations in a group and the  $y_i$ 's are assumed to be the average of individual responses. In the case of gamma distributed responses, BayesX assumes  $y_r \sim G(\exp(\eta_r), \nu/weightvar_r)$  where  $\mu_r = \exp(\eta_r)$  is the mean and  $s_r = \nu/weightvar_r$  is the scale parameter.

If estimation is based on latent utility representations, the specification of weights is not allowed. Similarly, for negative binomial, zero inflated Poisson and zero inflated negative binomial models as well as hazard regression and multi-state models, weighted regression is not implemented yet.

#### 7.1.2.2 Syntax of possible model terms

The general syntax of models is:

```
depvar = term_1 + term_2 + \cdots + term_r
```

where depvar specifies the dependent variable in the model and  $term_1, \ldots, term_r$  define the specific form of covariate effects on the dependent variable. The different terms have to be separated by '+' signs. A constant intercept is automatically included in the models and does not have to be requested by the user. This section reviews all possible model terms that are currently supported by  $bayesreg\ objects$  and provides some specific examples. Note that all described terms may be

combined in arbitrary order. An overview about the capabilities of *bayesreg objects* is given in Table 7.1. Table 7.2 shows how interactions between covariates are specified. Full details about all available options are given in subsubsection 7.1.2.3.

Throughout this section Y will denote the dependent variable.

#### Offset

```
Description: Adds an offset term to the predictor.
```

Predictor:  $\eta = \cdots + offs + \cdots$ 

Syntax: offs(offset)

Example:

The following model statement can be used to estimate a Poisson model with offs as offset term and W1 and W2 as fixed effects (if family=poisson is specified in addition):

$$Y = offs(offset) + W1 + W2$$

#### Fixed effects

Description: Incorporates covariate W1 as a fixed effect into the model.

Predictor:  $\eta = \cdots + \gamma_1 W 1 + \cdots$ 

Syntax: W1

Example:

The following model statement specified a model with q fixed (linear) effects:

$$Y = W1 + W2 + \cdots + Wq$$

#### Nonlinear effects of continuous covariates and time scales

#### First or second order random walk

Description: Defines a first or second order random walk prior for the effect of X1.

Predictor:  $\eta = \cdots + f_1(X1) + \cdots$ 

Syntax:

X1(rw1[, options])

X1(rw2[, options])

Example:

Suppose that X1 is a continuous covariate with possibly nonlinear effect. The following model statement defines a second order random walk prior for  $f_1$ :

```
Y = X1(rw2,a=0.001,b=0.001)
```

The term X1(rw2,a=0.001,b=0.001) indicates, that the effect of X1 should be included nonparametrically using a second order random walk prior. A first order random walk is obtained by modifying the first argument in X1(rw2,a=0.001,b=0.001) from rw2 to rw1 yielding X1(rw1,a=0.001,b=0.001). The second and third argument in the expression above are

used to specify the hyperparameters of the inverse gamma prior for the variance of the random walk. Besides the options a and b, some more options are available, see subsubsection 7.1.2.3 for details.

## P-spline with first or second order random walk penalty

Description: Defines a P-spline with a first or second order random walk penalty for the parameters of the spline.

```
Predictor: \eta = \cdots + f_1(X1) + \cdots
```

Syntax:

X1(psplinerw1[, options])

X1(psplinerw2[, options])

Example:

A P-spline with second order random walk penalty is obtained by:

```
Y = X1(psplinerw2)
```

By default, the degree of the spline is 3 and the number of inner knots is 20. The following model term defines a quadratic P-spline with 30 knots:

```
Y = X1(psplinerw2,degree=2,nrknots=30)
```

Full details about all possible options for P-splines are given in subsubsection 7.1.2.3.

#### Seasonal effect of a time scale

Description: Defines a seasonal effect of time.

```
Predictor: \eta = \cdots + f_{season}(time) + \cdots
```

Syntax:

time(season[, options])

Example:

A seasonal component for a time scale time is specified by

```
Y = time(season, period=12).
```

where the second argument specifies the period of the seasonal effect. In the example above the period is 12, corresponding to monthly data.

#### **Spatial Covariates**

#### Markov random field

Description:

Defines a Markov random field prior for the spatial covariate region. BayesX allows to incorporate spatial covariates with geographical information stored in the map object specified in option map.

```
Predictor: \eta = \cdots + f_{spat}(region) + \cdots
```

Syntax:

region(spatial,map=characterstring[, options])

#### Example:

For the specification of a Markov random field prior, map is an obligatory argument that represents the name of a map object (see chapter 5) containing all necessary spatial information about the geographical map, i.e. the neighbors of each region and the weights associated with the neighbors. For example the statement

#### Y = region(spatial, map=germany)

defines a Markov random field prior for region where the geographical information is stored in the *map object* germany. An error will be raised if germany is not existing. It is advisable to reorder the regions of a map prior to estimation of a spatial effect to obtain a band matrix like precision matrix. This can be achieved using method reorder of *map objects*, see section 5.3 for details.

#### Two-dimensional P-spline with first order random walk penalty

#### Description:

Defines a two-dimensional P-spline for the spatial covariate region with a two-dimensional first order random walk penalty for the parameters of the spline. Estimation is based on the coordinates of the centroids of the regions. The centroids are computed using the geographical information stored in the *map object* specified in the option map.

```
Predictor: \eta = \cdots + f(centroids) + \cdots

Syntax:

region(geospline,map=characterstring[, options])
```

#### Example:

For the specification of a two-dimensional P-spline (geospline) map is an obligatory argument indicating the name of a map object (see chapter 5) that contains all necessary spatial information about the geographical map, i.e. the neighbors of each region and the weights associated with the neighbors. The model term

```
Y = region(geospline, map=germany)
```

specifies a two-dimensional cubic P-spline with first order random walk penalty where the geographical information is stored in the *map object* germany.

#### Unordered group indicators

#### Unit- or cluster-specific unstructured effect

Description: Defines an unstructured (uncorrelated) random effect with respect to grouping variable grvar.

```
Predictor: \eta = \cdots + f(grvar) + \cdots
Syntax:
grvar(random[, options])
```

#### Example:

Gaussian i.i.d. random effects allow to cope with unobserved heterogeneity among units or clusters of observations. Suppose the analyzed data set contains a group indicator grvar that

gives information about the individual or cluster a particular observation belongs to. Then an individual-specific uncorrelated random effect is defined by

```
Y = grvar(random)
```

The inclusion of more than one random effect term in the model is possible, leading to the estimation of multilevel models. However, we have only limited experience with multilevel models so that it is not clear how well these models can be estimated in BayesX.

#### Nonlinear baseline effect in hazard regression or multi-state models

#### P-spline with second order random walk penalty

Description: Defines a P-spline with second order random walk penalty for the parameters of the spline for the log-baseline effect  $\log(\lambda_0(\texttt{time}))$ .

```
Predictor: \eta = \log(\lambda_0(time)) + \cdots
Syntax:
time(baseline[, options])
```

Example:

Suppose continuous-time survival data (time, delta) together with additional covariates (W1,X1) are given where time denotes the vector of observed duration times, delta is the vector of corresponding indicators of non-censoring, W1 is a discrete covariate, and X1 is a continuous covariate. The following continuous time survival model with hazard rate  $\lambda$  and log-baseline effect  $\log(\lambda_0(\texttt{time}))$ 

```
\lambda(time) = \lambda_0(time) \exp(\gamma_0 + \gamma_1 W 1 + f(X1)) = \exp(\log(\lambda_0(time)) + \gamma_0 + \gamma_1 W 1 + f(X1))
```

is estimated by the model statement

```
delta = time(baseline) + W1 + X1(psplinerw2)
```

Similarly, baseline effects on the transition intensities can be specified in multi-state models.

#### Varying coefficients with continuous covariates as effect modifier

#### First or second order random walk

Description:

Defines a varying coefficient term, where the effect of X1 varies smoothly over the range of X2. Therefore covariate X2 is called the effect modifier. The smoothness prior for f(X2) is a first or second order random walk.

```
Predictor: \eta = \cdots + f(X2)X1 + \cdots

Syntax:

X1*X2(rw1[, options])

X1*X2(rw2[, options])
```

Example:

A varying coefficient term with a second order random walk smoothness prior is defined as follows:

```
Y = X1*X2(rw2)
```

#### P-spline with first or second order random walk penalty

Description:

Defines a varying coefficient term, where the effect of X1 varies smoothly over the range of X2. The smoothness prior for f(X2) is a P-spline with first or second order random walk penalty.

Predictor:  $\eta = \cdots + f(X2)X1 + \cdots$ 

Syntax:

X1\*X2(psplinerw1[, options])

X1\*X2(psplinerw2[, options])

Example:

A varying coefficient term with a second order random walk smoothness prior is defined as follows:

Y = X1\*X2(psplinerw2)

#### Seasonal prior

Description:

Defines a varying coefficients term where the effect of X1 varies over the range of the effect modifier time. A seasonal prior is assumed for the effect of time.

Predictor:  $\eta = \cdots + f_{season}(time)X1 + \cdots$ 

Syntax:

X1\*time(season[, options])

Example:

The inclusion of a varying coefficients term with a seasonal prior may be meaningful if we expect a different seasonal effect with respect to a binary variable X1. In this case we can include an additional seasonal effects for observations with X1=1 by

Y = X1\*time(season)

#### Time-varying effects in hazard regression or multi-state models

#### P-spline with second order random walk penalty

Description: Defines a varying coefficients term where the effect of X1 varies over the range of the effect modifier time, i.e. variable X1 is assumed to have a time-varying effect. The smoothness prior for f(time) is a P-spline with second order random walk penalty.

Predictor:  $\eta = \log(\lambda_0(time)) + f(time)X1 \cdots$ 

Syntax:

X1\*time(baseline[, options])

Example:

Suppose continuous-time survival data (time, delta) together with an additional covariate X1 are given, where time denotes the vector of observed duration times and delta is the

vector of corresponding indicators of non-censoring. The following Cox model with hazard rate

$$\lambda(time) = \lambda_0(time) \exp(\gamma_0 + f(time)X1)$$
  
= 
$$\exp(\log(\lambda_0(time)) + \gamma_0 + f(time)X1)$$

is estimated by the model statement

```
delta = time(baseline) + X1*time(baseline)
```

Similarly, time-varying effects on the transition intensities can be specified in multi-state models.

#### Varying coefficients with spatial covariates as effect modifiers

#### Markov random field

Description:

Defines a varying coefficients term where the effect of X1 varies smoothly over the range of the spatial covariate region. A Markov random field is estimated for  $f_{spat}(region)$ . The geographical information is assumed to be stored in the  $map\ object$  specified in the option map.

```
Predictor: \eta = \cdots + f_{spat}(region)X1 + \cdots
```

Syntax:

X1\*region(spatial,map=characterstring[,options])

Example:

The statement

```
Y = X1*region(spatial, map=germany)
```

defines a varying coefficient term with the spatial covariate **region** as the effect modifier and a Markov random field as spatial smoothness prior. Weighted Markov random fields can be estimated by including an appropriate weight definition when creating the *map object* **germany** (see section 5.1).

#### Varying coefficients with unordered group indicators as effect modifiers (random slopes)

#### Unit- or cluster-specific unstructured effect

Description:

Defines a varying coefficient term where the effect of X1 varies over the range of the group indicator grvar. Models of this type are usually referred to as models with random slopes. A Gaussian i.i.d. random effect with respect to grouping variable grvar is assumed for f(grvar). A main effect  $\gamma X1$  is additionally estimated using a diffuse prior for  $\gamma$ . Therefore the random slope effect f(grvar)X1 can be seen as the deviation from the main effect. Estimation is carried out using hierarchical centering, see Gelfand, Sahu and Carlin (1995). Note that nonsensical results are obtained if an additional fixed effect of X1 is added in the model statement because the fixed effect is automatically included.

```
Predictor: \eta = \cdots + \gamma X1 + f(grvar)X1 + \cdots
```

7.1. Method regress

Syntax:

X1\*grvar(random[, options])

Example:

A random slope with additional incorporation of X1 as fixed effect is specified as follows:

Y = X1\*grvar(random)

If the linear effect of X1 should be omitted, the option nofixed has to be specified:

Y = X1\*grvar(random, nofixed)

#### **Surface estimators**

#### Two-dimensional P-spline with first order random walk penalty

Description:

Defines a two-dimensional P-spline with a two-dimensional first order random walk penalty for the parameters of the spline.

```
Predictor: \eta = \cdots + f(X1, X2) + \cdots
```

Syntax:

X1\*X2(pspline2dimrw1[, options])

Example:

The model term

```
Y = X1*X2(pspline2dimrw1)
```

specifies a two-dimensional cubic P-spline with first order random walk penalty.

In many applications it is favorable to additionally incorporate the one-dimensional main effects of X1 and X2 into the model. In this case the two-dimensional surface can be seen as the deviation from the main effects. Note, that the number of inner knots has to be the same for the main effects and the interaction effect. For example, splines with 10 inner knots are estimated by

```
Y = X1(psplinerw2,nrknots=10) + X2(psplinerw2,nrknots=10)
+ X1*X2(pspline2dimrw1,nrknots=10)
```

#### 7.1.2.3 Description of additional options for terms of bayesreg objects

All arguments described in this section are optional and can therefore be omitted. Generally, all options are specified by adding the option name to the specification of the model term type in the parentheses, separated by commas. Boolean options are specified by simply adding the option name. For example, a random intercept term with a=b=0.001 as parameters for the inverse gamma prior of the variance parameter, with updating according to IWLS and without incorporation of X1 as fixed effect is specified as follows:

```
X1*grvar(random, a=0.001, b=0.001, proposal=iwls, nofixed)
```

Note that all options may be specified in arbitrary order. Table 7.3 provides explanations and the default values of all possible options. All reasonable combinations of model terms and options can be found in Table 7.4.

Type	Syntax example	Description
Offset	offs(offset)	Variable offs is an offset term.
Linear effect	W1	Linear effect of W1.
First or second or-	X1(rw1)	Nonlinear effect of X1.
der random walk	X1(rw2)	
P-spline	X1(psplinerw1)	Nonlinear effect of X1.
	<pre>X1(psplinerw2)</pre>	
Seasonal prior	time(season,period=12)	Time-varying seasonal effect of time with period
		12.
Markov random	region(spatial,map=m)	Spatial effect of region where region indicates the
field		region an observation pertains to. The boundary
		information and the neighborhood structure are
		stored in the map object m.
Two dimensional	region(geospline,map=m)	Spatial effect of region. Estimates a two dimen-
P-spline		sional P-spline based on the centroids of the re-
		gions. The centroids are obtained from the map
		object m.
Random intercept	grvar(random)	I.i.d. Gaussian (random) effect of the group indi-
		cator grvar, e.g. grvar may be an individual indi-
		cator when analyzing longitudinal data.
Baseline in Cox	time(baseline)	Nonlinear shape of the baseline effect $\lambda_0(time)$ of
or multi-state		a Cox model. $\log(\lambda_0(time))$ is modelled by a P-
models		spline with second order penalty.

Table 7.1: Overview over different model terms for bayesreg objects.

Type of interaction	Syntax example	Description
Varying coefficient term	X1*X2(rw1)	Effect of X1 varies smoothly over the range
	X1*X2(rw2)	of the continuous covariate X2 or time.
	X1*X2(psplinerw1)	
	X1*X2(psplinerw2)	
	X1*time(season)	
Random slope	X1*grvar(random)	The regression coefficient of X1 varies with
		respect to the unit- or cluster-index vari-
		able grvar.
Geographically weighted	<pre>X1*region(spatial,map=m)</pre>	Effect of X1 varies geographically. Covari-
regression		ate region indicates the region an obser-
		vation pertains to.
Two dimensional	X1*X2(pspline2dimrw1)	Two dimensional surface for the continu-
surface		ous covariates X1 and X2.
Time-varying effect in		
Cox or multi-state	_	
models	X1*time(baseline)	Nonlinear, time-varying effect of X1.

Table 7.2: Possible interaction terms for bayesreg objects.

	Description	Default		
a,b	The options a and b specify the hyperparameters of the inverse Gamma	a=0.001, b=0.001		
	prior for the variance $\tau^2$ .			
min,max	The options min and max define the minimum and maximum block	automaticdetermination		
	sizes of block move updates. In every iteration, BayesX randomly			
	chooses the block size within this range. If omitted, the minimum			
	and maximum block sizes are automatically determined during the			
	burnin period such that the average acceptance rate is between $30\%$			
	and 70%. The specification of minimum and maximum block sizes			
	is only meaningful in combination with conditional prior proposals			
	and therefore has no effect for Gaussian responses, categorical probit			
	models, or if proposal=iwls or proposal=iwlsmode is specified.			
lambda	Provides a starting value for the smoothing parameter $\lambda$ .	lambda=0.1		
proposal	Specifies the type of proposal. proposal=cp means conditional prior	proposal=iwls		
	proposal, proposal=iwls stands for iteratively weighted least squares			
	(IWLS) proposal and proposal=iwlsmode indicates IWLS based on			
1	posterior mode estimation.			
updateW	The option updateW may be used to specify how often the IWLS weight	updateW=1		
	matrix should be updated. updateW=0 means never, updateW=1 means			
	in every iteration (which is the default), updateW=2 means in every second iteration and so on.			
dograa	Specifies the degree of B-spline basis functions.	dograp-3		
degree	Specifies the degree of B-spline basis functions.  Specifies the number of inner knots for a P-spline term.	degree=3 nrknots=20		
nrknots	The option gridsize can be used to restrict the number of points	gridsize=-1		
gridsize	(on the x-axis) for which estimates are computed. By default, esti-	gridsizei		
	mates are computed at every distinct covariate value in the data set			
	(indicated by gridsize=-1). This may be relatively time consuming			
	in situations where the number of distinct covariate values is large. If			
	gridsize=nrpoints is specified, estimates are computed on an equidis-			
	tant grid with nrpoints knots.			
derivative	If specified, first order derivatives of the function estimate are com-	-		
	puted (for P-splines only).			
period	Period of the seasonal effect. The default is period=12 which corre-	period=12		
1	sponds to monthly data.	r		
nofixed	The option nofixed suppresses the estimation of the main effect $\gamma X1$	-		
	for random slopes.			

Table 7.3: Optional arguments for bayesreg object terms.

	rw1/rw2	season	psplinerw1/psplinerw2	spatial	random	geospline	pspline2dimrw1	baseline
a	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue
b	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue
min	*	*	*	×	×	*	*	integer
max	*	*	*	×	×	*	*	integer
lambda	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue
proposal	•	•	•	•	0	•	•	×
updateW	integer	integer	integer	integer	×	integer	integer	×
degree	×	×	integer	×	×	integer	integer	integer
nrknots	×	×	integer	×	×	integer	integer	integer
gridsize	×	×	integer	×	×	×	integer	integer
derivative	×	×	$\triangle$	×	×	×	×	×
period	×	integer	×	×	×	×	×	×
nofixed	×	×	×	×	Δ	×	×	×
map	×	X	×	map object	×	map object	×	X
×	not available							
*	available only if proposal = cp							
0	admissible values are iwls,iwlsmode							
•	admissible values are cp,iwls,iwlsmode							
Δ	available as boolean option (specified without supplying a value)							

Table 7.4: Terms and options for bayesreg objects.

7.1. Method regress

### 7.1.2.4 Specifying the response distribution

Supported univariate distributions are Gaussian, binomial (with logit or probit link), Poisson, negative binomial, gamma, zero inflated Poisson and zero inflated negative binomial. Supported multivariate models are multinomial logit or probit models for categorical responses with unordered categories and the cumulative threshold model with probit link for categorical responses with ordered categories. Continuous survival times as well as multi-state models can be analysed based on semiparametric models with Cox-type hazard rates, see subsections 7.1.2.5 and 7.1.2.6. An overview over the supported models is given in Table 7.5. The distribution of the response is specified by adding the additional option family to the (global) options list of the regression call. For instance, family=gaussian defines the response to be Gaussian distributed. In some cases, one or more additional options associated with the specified response distribution can be specified. An example is the reference option for multinomial responses, which defines the reference category. In the following we give detailed instructions on how to specify the various models.

### Gaussian responses

For Gaussian responses BayesX assumes  $y_i|\eta_i, \sigma^2 \sim N(\eta_i, \sigma^2/weightvar_i)$  or, equivalently, in matrix notation  $y|\eta, \sigma^2 \sim N(\eta, \sigma^2C^{-1})$ , where  $C = diag(weightvar_1, \dots, weightvar_n)$  is a known weight matrix. Gaussian regression models are obtained by adding

# family=gaussian

to the options list.

An optional weight variable weightvar can be specified to estimate weighted regression models, see subsubsection 7.1.2.1 for details. For grouped Gaussian responses, the weights represent the number of observations in the groups if the  $y_i$ 's are the average of individual responses. If the  $y_i$ s are the sum of responses in every group, the weights are given by the reciprocal of the number of observations in the groups. Of course, estimation of usual weighted regression models with heteroscedastic errors is also possible. In this case, the weights should be proportional to the reciprocal of the heteroscedastic variances. If no weight variable is specified, BayesX assumes  $weightvar_i = 1, i = 1, ..., n$ .

For Gaussian responses, the additional parameter  $\sigma^2$  for the error variancehas to be estimated. An additional inverse gamma prior with hyperparameters aresp and bresp is defined for  $\sigma^2$ . The default for the hyperparameters is aresp=0.001 and bresp=0.001. The default values may be changed using the aresp and bresp option. For instance, adding

aresp=0.01 bresp=0.01

to the options list set both a and b equal to 0.01.

### Gamma distributed responses

In the literature, the density function of the gamma distribution is parameterized in various ways. In the context of regression analysis, the density is usually parameterized in terms of the mean  $\mu$  and the scale parameter s. Then, the density of a gamma distributed random variable y is given by

$$p(y) \propto y^{s-1} \exp\left(-\frac{s}{\mu}y\right)$$
 (7.1)

for y > 0. For the mean and the variance we obtain  $E(y) = \mu$  and  $Var(y) = \mu^2/s$ . We write  $y \sim G(\mu, s)$ .

A second parameterization is typically employed for hyperparameters **a** and **b** of priors for variance parameters in the context of Bayesian hierarchical models. In this case, the density is given by

$$p(y) \propto y^{a-1} \exp(-by) \tag{7.2}$$

for y > 0. In this parameterization we obtain E(y) = a/b and  $Var(y) = a/b^2$  for the mean and the variance, respectively. We write  $y \sim G(a, b)$ 

In BayesX a gamma distributed response variable is parameterised in the first form (7.1). For the rth observation BayesX assumes  $y_r|\eta_r, \nu \sim G(\exp(\eta_r), \nu/weightvar_r)$  where  $\mu_r = \exp(\eta_r)$  is the mean and  $s = \nu/weightvar_r$  is the scale parameter. A gamma distributed response is specified by adding

### family=gamma

to the options list. An optional weight variable weightvar can be specified to estimate weighted regression models, see subsubsection 7.1.2.1 for details.

In analogy to the variance parameter in Gaussian response models, we assume a Gamma prior (second parameterization (7.2)) with hyperparameters  $a_{\nu}$  and  $b_{\nu}$  for the scale parameter  $\nu$ , i.e.  $\nu \sim Gamma(a_{\nu}, b_{\nu})$ . The default for the hyperparameters is  $a_{\nu} = 1$  and  $b_{\nu} = 0.005$ . The default values may be changed using the aresp and bresp option.

Updating of the scale parameter  $\nu$  is implemented via MH-steps based on a gamma proposal distribution with mean  $E(\nu^{prop}) = \nu^c$  equal to the current state of the chain  $\nu^c$  and a fixed variance  $Var(\nu^{prop})$ . The variance  $Var(\nu^{prop})$  can be considered a tuning parameter. It is specified by the additional option gammavar. For example, when adding

### gammavar=0.0001

 $Var(\nu^{prop}) = 0.0001$  is used in the proposal distribution. The default is gammavar=0.001.

It is also possible to assume a fixed deterministic scale parameter. The scale parameter is defined to be fixed by adding

scalegamma = fixed

to the options list. The (fixed) value of the scale parameter is specified by adding:

scale = realvalue

Typing e.g.

scale = 1

defines the scale parameter to be fixed at the value  $\nu = 1$ .

# Binomial logit and probit models

A binomial logit model is requested by the option

family=binomial

while a probit model is obtained by adding

family=binomialprobit

to the option list.

For logit models a weight variable may be specified in addition, see subsubsection 7.1.2.1 for details. BayesX assumes that the weight variable corresponds to the number of replications and the response variable to the number of successes. If a weight variable is omitted, BayesX assumes that the number of replications is one, i.e. the values of the response must be either zero or one. For probit models the specification of a weight variable is not allowed.

### Multinomial logit and probit models

A multinomial logit model is specified by adding the option

family=multinomial

to the options list, while a multinomial probit model is requested by

family=multinomialprobit

A further option (reference) can be added to the options list to define the reference category. If the response variable has three categories 1, 2 and 3, the reference category can be set to 2, by adding

#### reference=2

to the options list. If the option is omitted, the *smallest* number will be used as the reference category.

### Cumulative threshold models

So far, bayesreg objects support only cumulative probit models. Such a model is specified by adding family=cumprobit

to the options list. The reference category will always be the largest value of the response and can not be changed by the user.

An important problem with Bayesian cumulative threshold models is the mixing and convergence of MCMC samples for the threshold parameters. Usually the mixing is relatively poor implying the necessity for quite large MCMC samples in order to obtain reliable estimation results. An exception are cumulative models with three categories of the response. In this case, BayesX uses a reparameterized model for which the mixing of the threshold parameters is typically quite satisfactory. A description of this reparameterization can be found in Fahrmeir and Lang (2001b) or in Chen and Dey (2000). However, parameter estimates are given in the original parameterization. To estimate three categorical response models without reparameterization the additional option notransform can be added to the options list (not recommended).

### Poisson regression

A Poisson regression model is specified by adding

family=poisson

to the options list.

A weight variable may be specified in addition, see <u>subsubsection 7.1.2.1</u> for details. For grouped Poisson data, the weights must be the number of observations in a group and the responses are assumed to be the average of individual responses.

### Negative binomial regression

A negative binomial regression is specified by adding

family=nbinomial

to the options list. Weights are not allowed in combination with the negative binomial distribution. For negative binomial responses, BayesX assumes  $y_i|\eta_i, \delta \sim NB(\eta_i, \delta)$  for a direct negative binomial formulation or equivalently  $y_i|\eta_i \sim Po(\nu_i\eta_i)$  with  $\nu_i|\delta \sim G(\delta, \delta)$  for a Poisson-Gamma formulation. These alternatives are specified by setting the option distopt=nb (default) or distopt=poga re-

spectively. The first formulation works with a negative binomial likelihood and provides estimates for the parameters in the predictor and for  $\delta$ . The second formulation works with a Poisson likelihood but has an extra vector of multiplicative random effects with prior  $\nu_i | \delta \sim G(\delta, \delta)$ . It provides estimates for the parameters in the predictor, for  $\delta$  and for the  $\nu_i$ .

The prior for the scale parameter  $\delta$  is G(a,b) in both formulations, where a=1 is fixed and b is estimated. Its prior is again a gamma distribution with fixed parameters 1 and 0.005.

### Zero inflated count data models

A zero inflated regression model for count data is specified by adding family=zip

to the options list. Weights are not allowed in combination with zero-inflated count data.

Zero inflated count data distributions are used when the number of zero counts in the data exceeds the number of zero counts expected by the standard distribution. Zero inflation is introduced based on two processes: The first process is an underlying count data process that can not be observed directly, but only after a transformation through the so called selection process. This process is defined in terms of a 0/1 variable. If we observe a 0 in the selection process, the observed count will be zero, independently of the value generated by the underlying count data process. Otherwise, if we observe a 1, the result will be directly generated by the underlying count data process. The implemented zero inflated distributions in BayesX do not work with both processes directly. They are marginalized versions of a given count data distribution with respect to a  $0/1 \sim Bern(1-\theta)$  selection process. For the count data process we have two possibilities, that can be controlled through the option zipdistopt. If we choose a Poisson distribution, we obtain a zero inflated Poisson distribution (zipdistopt = zip) denoted by  $y_i|\eta_i, \theta \sim ZIP(\eta_i, \theta)$ . We can also combine both zero inflation and overdispersion based on a negative binomial distribution yielding the zero inflated negative binomial (zipdistopt = zinb) distribution denoted by  $y_i|\eta_i, \delta, \theta \sim ZINB(\eta_i, \delta, \theta)$ .

### 7.1.2.5 Continuous time survival analysis

Bayes X offers two alternatives of estimating continuous time survivals models with semiparametric predictor  $\eta$ , both of which are described in subsection 5.2 of the methodology manual. The first alternative is to assume that all time-dependent values are piecewise constant, leading to the so called *piecewise exponential model* (p.e.m.). The second alternative is to estimate the log-baseline effect  $\log(\lambda_0(t)) = f_0(t)$  based on a P-spline with second order random walk penalty.

# Piecewise exponential model (p.e.m.)

In subsection 5.2 of the methodology manual we demonstrated how continuous time survival data has to be manipulated to transform it to a Poisson for model estimation. Suppose that the following modified data set is available

У	indnr	a	δ	$\Delta$	x1	x2
0	1	0.1	1	$\log(0.1)$	0	3
0	1	0.2	1	$\log(0.1)$	0	3
1	1	0.3	1	$\log(0.05)$	0	3
0	2	0.1	0	$\log(0.1)$	1	5
0	2	0.2	0	$\log(0.02)$	1	5
:	:	:	:	:	:	:

with indicator y, interval limit a, indicator of non-censoring  $\delta$  and offset  $\Delta$  defined as in subsection 5.2 of the methodology manual. Let x1 be a covariate with linear effect and x2 a continuous covariate with nonlinear effect. Then the correct syntax for estimating a p.e.m. with a bayesreg object named b would be as follows:

```
> b.regress y = a(rw1) + Delta(offset) + x1 + x2(psplinerw2), family=poisson... or
```

Note that a time-varying effect of an additional covariate X may be estimated by simply adding the term

X\*a(rw1) or X\*a(rw2)

to the model statement.

# Specifying a P-spline prior for the log-baseline

For a continuous time survival model with a P-spline prior with second order random walk penalty for the baseline effect,

### family=cox

has to be specified in the options list. The number of knots and degree of the P-spline prior for  $f_0(t)$  can be specified as additional options for the baseline term. Note that it is obligatory that there is a baseline term specified for the vector of observed duration times. The indicator of non-censoring  $\delta_i$  has to be specified as the dependent variable in the model statement. Data augmentation and the specification of an offset term are not required here. To handle left truncation and time-varying covariates, the additional variable beginvar, that records when the observation became at risk, may be specified by adding begin=beginvar to the options list.

In the example above with survival data

t	δ	x1	x2
0.25	1	0	3
0.12	0	1	5
		•	•

a continuous time survival model with a quadratic P-spline prior with 15 knots for the log-baseline would be estimated as follows:

```
> b.regress delta = t(baseline,degree=2,nrknots=15)+ x1 + x2(psplinerw2),
family=cox
```

Again a time-varying effect of a covariate X can be estimated by simply adding the term

X\*time(baseline)

to the model statement.

#### 7.1.2.6 Continuous time multi-state models

Multi-state models describe the temporal development of discrete phenomena in continuous time based on transition intensities for each of the observable transition types. Consider for example a multi-state model for human sleep as depicted in Figure 8.1 and that the transition intensities for the four possible transitions are specified as

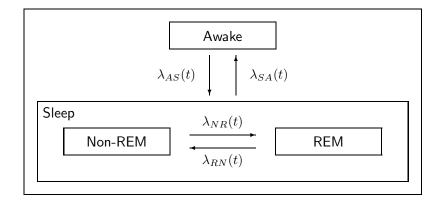


Figure 7.1: Schematic representation of sleep stages and transitions of interest.

$$\lambda_{AS,i}(t) = \exp \left[ g_0^{(AS)}(t) + b_i^{(AS)} \right], 
\lambda_{SA,i}(t) = \exp \left[ g_0^{(SA)}(t) + b_i^{(SA)} \right], 
\lambda_{NR,i}(t) = \exp \left[ g_0^{(NR)}(t) + c_i(t) g_1^{(NR)}(t) + b_i^{(NR)} \right] 
\lambda_{RN,i}(t) = \exp \left[ g_0^{(RN)}(t) + c_i(t) g_1^{(RN)}(t) + b_i^{(RN)} \right]$$

Each of the transitions is parameterised in terms of a baseline effect  $g_0^{(h)}(t)$  and a transition specific frailty term (random effect)  $b_i^{(h)}$ . In addition, time-varying effects  $g_1^{(h)}(t)$  of binary indicators  $c_i(t)$  for a high blood level of cortisol are introduced for the transitions between REM and Non-REM. The corresponding data set should be arranged as follows:

id	st	beg	end	tas	tsa	$\operatorname{trn}$	tnr	cort	corthigh
1	2	0	1	0	1	0	0	52.6	0
1	1	1	5	1	0	0	0	52.6	0
1	2	5	8	0	1	0	0	52.6	0
1	1	8	10	1	0	0	0	52.6	0
1	2	10	36	0	0	0	0	52.6	0
1	2	36	76	0	0	0	0	46.9	0
1	2	76	108	0	0	0	1	47.5	0
1	3	108	109	0	0	1	0	47.5	0
1	2	109	110	0	0	0	1	47.5	0
1	3	110	111	0	0	1	0	47.5	0
1	2	111	115	0	0	0	1	47.5	0
1	3	115	116	0	0	0	0	47.5	0
1	3	116	126	0	0	1	0	37.4	0
		•	•	•	•	•			
•	•		•	•	•	•			•
•		•	•	•	•	•	•		•
2	2	0	12	0	1	0	0	22.5	0
2	1	12	15	1	0	0	0	22.5	0
2	1	15	28	0	1	0	0	88.6	1
			•						•
•			•		•				
		•							

Each path observed for the multi-state model is transformed into several lines in the data set, where id identifies the original paths. In the above example, parts of the first two observations are displayed. Each line of the data set represents a time interval identified by the variables beg and end. Variable st indicates the current state of the process. Note that the states have to be numbered consecutively from 1 to H. Since we are considering continuous time scales, an observation should start at t=0 (unless the observation is left truncated) and the variables beg and end should be generated so that within each observation process beg equals the value of end in the previous row (unless observations are fragmentary only).

The variables tas, tsa, trn and tnr are binary indicators for the four transitions sleep  $\rightarrow$  awake (tsa), awake  $\rightarrow$  sleep (tas), Non-REM  $\rightarrow$  REM (tnr) and REM  $\rightarrow$  Non-REM (trn). Such an indicator equals one if the corresponding transition is observed at the end of the interval and zero otherwise. Note that there are lines in the data set, where none of the transitions is observed. These correspond to intervals where the value of the time-varying covariate cort (cortisol-level) changes. The variable corthigh is a dichotomized version of cort which indicates a high level of cortisol (cort>60).

The model specified above is estimated by entering the following command

Note that a separate model equation has to specified for each transition with the binary transition indicator as response. Instead of method regress, method mregress has to be called since multiple model equations are combined. The right and the left boundary of the time intervals have to specified as covariate for the the baseline effect and as global option begin, respectively. Similarly, the state variable has to specified via the global option state.

More details about Bayesian semiparametric multi-state models including a detailed description of the human sleep data application can be found in Kneib and Hennerfeind (2006).

# **7.1.3** Options

# Options for controlling MCMC simulations

Options for controlling MCMC simulations are listed in alphabetical order.

• burnin = integer

Changes the number of burn-in iterations to *integer*, where *integer* must be a positive integer number or zero (i.e. no burn-in period). The number of burn-in iterations must be smaller than the number of iterations (see option iterations).

DEFAULT: burnin=2000

• iterations = integer

Changes the number of MCMC iterations to *integer*, where *integer* must be a positive integer number. The number of iterations must be larger than the number of burn-in iterations.

DEFAULT: iterations=52000

### $\bullet$ maxint = integer

If first or second order random walk priors are specified, in some cases the data will be slightly grouped: The range between the minimal and maximal observed covariate values will be divided into (small) intervals, and for each interval one parameter will be estimated. The grouping has almost no effect on estimation results as long as the number of intervals is large enough. With the maximt option the amount of grouping can be determined by the user. integer is the maximum number of intervals allowed. For equidistant data, maximt = 150 for example, means that no grouping will be done as long as the number of different observations is equal to or below 150. For non equidistant data some grouping may be done even if the number of different observations is below 150.

DEFAULT: maxint=150

# $\bullet$ step = integer

Defines the thinning parameter for MCMC simulation. For example, step = 50 means, that only every 50th sampled parameter will be stored and used to compute characteristics of the posterior distribution as means, standard deviations or quantiles. The aim of thinning is to reach a considerable reduction of disk storing and autocorrelations between sampled parameters.

DEFAULT: step=50

# Options for specifying the response distribution

Options for specifying the response distribution are listed in alphabetical order below.

### • aresp = realvalue

Defines the value of the hyperparameter **a** for the inverse gamma prior of the overall variance parameter  $\sigma^2$ , if the response distribution is Gaussian. *realvalue* must be a positive real valued number.

DEFAULT: aresp=1

# • bresp = realvalue

Defines the value of the hyperparameter b for the inverse gamma prior of the overall variance parameter  $\sigma^2$ , if the response distribution is Gaussian. *realvalue* must be a positive real valued number.

DEFAULT: bresp=0.005

### • distopt = characterstring

Defines the implemented formulation for the negative binomial model if the response distribution is negative binomial. The two possibilities are to work with a negative binomial likelihood (distopt=nb) or to work with the Poisson likelihood and the multiplicative random effects (distopt=poga)

DEFAULT: distopt=nb

### • family = characterstring

Defines the distribution of the response variable in the model. Models supported are Gaussian regression models with the identity link, binomial logit or probit models, multinomial logit or probit models for unordered categories of the response, cumulative threshold models with probit link for ordered categories of the response, and Poisson, negative binomial or their zero inflated versions with the log-link. For some distributions (e.g. multinomial) additional options may be specified to control MCMC inference. A detailed description on how to specify the distribution of the response is given in subsubsection 7.1.2.4. Table 7.5 lists all possible

specifications for the distribution of the response currently supported by BayesX. In addition, a list of options associated with the particular response distribution is given.

DEFAULT: family=binomial

# • reference = realvalue

Option reference is meaningful only if either family=multinomial or family=multinomialprobit is specified as the response distribution. In this case reference defines the reference category to be chosen. Suppose, for instance, that the response is three categorical with categories 1, 2 and 3. Then reference=2 defines the value 2 to be the reference category.

### • zipdistopt = characterstring

Defines the zero inflated distribution for the regression analysis. The two possibilities are to work with a zero inflated Poisson distribution (zipdistopt=zip) or to work with the zero inflated negative binomial likelihood (zipdistopt=zinb).

value of family	response distribution	link	additional options
family=gaussian	Gaussian	identity	aresp, bresp
family=binomialprobit	binomial	probit	
family=binomial	binomial	logit	
family=multinomialprobit	unordered multinomial	probit	reference
family=multinomial	unordered multinomial	logit	reference
family=cumprobit	cumulative threshold	probit	
family=poisson	Poisson	log-link	
family=nbinomial	Negative Binomial	log-link	distopt
family=zip	Zero inflation	log-link	zipdistopt
family=cox	continuous-time survival data		begin
family=multistate	continuous-time multi-state data		begin, state

Table 7.5: Summary of supported response distributions.

### **Further options**

Options are listed in alphabetical order:

# $\bullet$ begin = variable name

Option begin is meaningful only if family=cox is specified as the response distribution. In this case begin specifies the variable that records when the observation became at risk. This option can be used to handle left truncation and time-varying covariates. If begin is not specified, all observations are assumed to have become at risk at time 0.

### • level1 = integer

Besides the posterior means and medians, BayesX provides pointwise posterior credible intervals for every effect in the model. In a Bayesian approach based on MCMC simulation techniques credible intervals are estimated by computing the respective quantiles of the sampled effects. By default, BayesX computes (pointwise) credible intervals for nominal levels of 80% and 95%. The option level1 allows to redefine one of the nominal levels (95%). Adding, for instance,

level1=99

to the options list computes credible intervals for a nominal level of 99% rather than 95%.

### • level2 = integer

Besides the posterior means and medians, BayesX provides pointwise posterior credible intervals for every effect in the model. In a Bayesian approach based on MCMC simulation techniques credible intervals are estimated by computing the respective quantiles of the sampled effects. By default, BayesX computes (pointwise) credible intervals for nominal levels of 80% and 95 %. The option level2 allows to redefine one of the nominal levels (80%). Adding, for instance,

#### leve12=70

to the options list computes credible intervals for a nominal level of 70% rather than 80%.

### • predict

Option predict may be specified to compute samples of the deviance D, the effective number of parameters  $p_D$  and the deviance information criterion DIC of the model, see Spiegelhalter et al. (2002). The computation of these quantities is based on the unstandardized deviance which is defined as  $D(\theta) = -2\log(p(y|\theta))$  where  $\theta = (\mu, \sigma^2)$  for Gaussian responses,  $\theta = (\mu, \delta)$  for negative binomial responses and  $\theta = \mu$  for the rest of non-Gaussian responses. The effective number of parameters is defined by  $p_D = \overline{D}(\theta) - D(\overline{\theta})$  where  $D(\theta)$ is the posterior mean deviance and  $D(\bar{\theta})$  is the deviance of the posterior mean of  $\theta$ . The deviance information criterion is defined as  $DIC = \overline{D(\theta)} + p_D$ . Bayes prints sample properties of the deviance, the effective number of parameters  $p_D$  and the DIC in the output window or in an open log file. The complete sample of the deviance is stored in a file with ending deviance.raw. The complete filename including the storage folder is given in the output window or the log file. The last two entries of that file contain again the effective number of parameters  $p_D$  and the DIC. Additionally, a file with ending predictmean.raw is created that contains for every observation the posterior mean of the predictor  $\eta_i$  and the expectation  $E(y_i|\eta_i) = \mu_i$  as well as the saturated deviance  $D_i^{sat}$  and leverage statistics  $p_{D_i}$ . The saturated deviance is defined as  $D(\mu, \sigma^2) = -2\log(p(y|\mu, \sigma^2)) + 2\log(p(y|\mu = y, \sigma^2))$ . For non-Gaussian responses the variance  $\sigma^2$  disappears and for negative binomial responses we have  $\delta$  instead. The individual saturated deviance  $D_i^{sat}$  can be used to compute deviance residuals. The deviance residuals are given by  $r_i = sign(y_i - \mu_i)\sqrt{D_i^{sat}}$ . The leverage statistics  $p_{D_i}$  is defined as the contribution of the ith observation to  $p_D$ . More details about the quantities discussed above can be found in Spiegelhalter et al. (2002). To clarify the computation of D,  $p_D$ , DICetc. Table 7.6 provides formulas of the p.d.f. and the (unstandardized) deviance D for the different response distributions provided in *BayesX*.

# 7.1.4 Estimation output

The way the estimation output is presented depends on the estimated model. Estimation results of fixed effects are displayed in a tabular form in the *output window* and/or in a log file (if created before). Shown will be the posterior mean, the standard deviation, the 2.5% and the 97.5% quantiles. Other quantiles may be obtained by specifying the level1 and/or level2 option, see subsection 7.1.3 for details. Additionally a file is created where estimation results for fixed effects are replicated. The name of the file is given in the *output window* and/or in a log file. Estimation effects of nonlinear effects of continuous and spatial covariates as well as unstructured random effects are presented in a different way. Results are stored in an external ASCII-file whose contents can be read into any general purpose statistics program (e.g. STATA, R, S-plus) to further analyze and/or visualize the results. The structure of the files is as follows: There will be one file

distribution	density	D = -2 Loglikelihood
Gaussian	$p(y \mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2/c}}$	$\log(\frac{2\pi\sigma^2}{c}) + \frac{c}{\sigma^2}(y-\mu)^2$
	$\exp(-\frac{c}{2\sigma^2}(y-\mu)^2)$	
Binomial	$p(y \mu) \propto \mu^y (1-\mu)^{c-y}$	$-2y\log(\mu) - 2(c-y)\log(1-\mu)$
Poisson	$p(y \mu) \propto \exp((y\log(\mu) - \mu)c)$	$-2c(y\log(\mu) - \mu)$
Negative Binomial	$p(y \mu,\delta) \propto \frac{\Gamma(y+\delta)}{\Gamma(\delta)} \left(\frac{\delta}{\delta+\mu}\right)^{\delta} \left(\frac{\mu}{\delta+\mu}\right)^{y}$	$-2\{\log(\Gamma(y+\delta)) - \log(\Gamma(\delta)) + \delta\log(\delta)$
		$+y\log(\mu) - (\delta+y)\log(\delta+\mu)$
Zero inflated	$p(0 \mu,\theta) = \theta + (1-\theta)\exp(-\mu)$	$-2\log(\theta + (1-\theta)\exp(-\mu))$
Poisson	$p(y \mu,\theta) \propto (1-\theta) \exp(-\mu)\mu^y$	$-2\log(1-\theta) - 2(y\log(\mu) - \mu)$
Zero inflated	$p(0 \mu, \delta, \theta) = \theta + (1 - \theta) \left(\frac{\delta}{\delta + \mu}\right)^{\delta}$	$-2\log\left(\theta + (1-\theta)\left(\frac{\delta}{\delta+\mu}\right)^{\delta}\right)$
Negative Binomial	$p(y \mu, \delta, \theta) \propto (1 - \theta) \frac{\Gamma(y+\delta)}{\Gamma(\delta)}$	$-2\{\log(1-\theta)$
	$\left(\frac{\delta}{\delta+\mu}\right)^{\delta}\left(\frac{\mu}{\delta+\mu}\right)^{y}$	$+\log(\Gamma(y+\delta)) - \log(\Gamma(\delta)) + \delta\log(\delta)$
		$+y\log(\mu) - (\delta+y)\log(\delta+\mu)$
Multinomial logit	$p(y \mu) \propto \prod \mu_j^{y_j}$	$-2(\sum y_j \log(\mu_j))$
Multinomial probit		not available
Cumulative probit	$p(y \mu) \propto \prod \mu_j^{y_j}$	$-2(\sum y_j \log(\mu_j))$

Table 7.6: Formulas of the probability densities, the unstandardized deviance and the saturated deviance for the various response distributions. The quantity c in the formulas corresponds to the weights specified in a weight statement, see weightspecification for details on how to specify weights. In the case of multinomial logit and cumulative probit models the variables  $y_j$  are indicator variables where  $y_j = 1$  denotes that the j-th category of the response y has been observed.

for every nonparametric effect in the model. The name of the files and the storing directory are displayed in the *output window* and/or a log file. The files contain ten or eleven columns depending on whether the corresponding model term is an interaction effect. The first column contains a parameter index (starting with one), the second column (and the third column if the estimated effect is a two-dimensional P-spline) contain the values of the covariate(s) whose effect is estimated. In the following columns the estimation results are given in form of the posterior means and the 2.5%, 10%, 50%, 90% and 97.5% quantiles. The last two columns contain posterior probabilities based on nominal levels of 95% and 80%. A value of 1 corresponds to a strictly positive 95% or 80% credible interval and a value of -1 to a strictly negative credible interval. A value of 0 indicates that the corresponding credible interval contains zero. Other quantiles may be obtained by specifying the level1 and/or level2 option, see subsection 7.1.3 for details. As an example compare the following few lines, that are the beginning of a file containing the results for a particular covariate, x say:

```
pqu90
intnr
       x
           pmean
                     pqu2p5
                                pqu10
                                         pmed
                                                           pqu97p5
                                                                      pcat95
                                                                                pcat80
    -2.778436
                -0.0730973
                              -0.349922
                                          -0.259827
                                                       -0.0765316
                                                                    0.109233
                                                                                0.211572
   -2.723671
                -0.167492
                             -0.39718
                                        -0.322043
                                                     -0.168924
                                                                 -0.0167056
                                                                               0.075335
   -2.633617
3
                -0.320366
                             -0.497797
                                         -0.433861
                                                      -0.321034
                                                                  -0.198619
                                                                               -0.129246
                             -0.623495
4
   -2.547761
                -0.455913
                                         -0.560266
                                                      -0.458746
                                                                  -0.347443
                                                                               -0.296006
                                                                                           -1
                                                                                                -1
5
   -2.455208
                -0.591498
                             -0.744878
                                         -0.694039
                                                      -0.592381
                                                                  -0.484629
                                                                               -0.440857
                                                                                           -1
                                                                                                -1
6
   -2.385378
                -0.687709
                             -0.858153
                                         -0.802932
                                                      -0.687944
                                                                  -0.577029
                                                                               -0.522391
                                                                                           -1
7
    -2.34493
               -0.736406
                            -0.914646
                                        -0.851548
                                                     -0.73536
                                                                -0.623369
                                                                            -0.561035
8
    -2.291905
                -0.785899
                             -0.962212
                                         -0.895262
                                                      -0.783646
                                                                  -0.674511
                                                                               -0.609532
                                                                                           -1
                                                   -0.877516
    -2.178096
                -0.876173
                            -1.0428
                                       -0.982029
                                                               -0.768126
                                                                            -0.708452
```

Note that the first row always contains the names of the variables in the ten columns.

The estimated nonlinear effects can be visualized by using either the graphics capabilities of *BayesX* or a couple of R and S-plus functions, see section 9.1 and section 9.2, respectively. Of course, any other (statistics) software package with plotting facilities may be used as well.

# 7.1.5 Examples

Here we give only a few examples about the usage of method regress. More detailed examples can be found in chapter 1 of the tutorial manual.

Suppose that we have a data set test with a binary response variable y, and covariates x1, x2, x3 and t, where t is assumed to be a time scale measured in months. Suppose further that we have already created a *bayesreg object* b.

#### Fixed effects

We first specify a model with y as the response variable and fixed effects for the covariates x1, x2 and x3. Hence the predictor is

$$\eta = \gamma_0 + \gamma_1 x 1 + \gamma_2 x 2 + \gamma_3 x 3$$

This model is estimated by typing:

> b.regress y = x1 + x2 + x3, iterations=12000 burnin=2000
step=10 family=binomial using test

Here, step=10 defines the thinning parameter, i.e. only every 10th sampled parameter will be stored and used for estimation. test is the data set that is used for estimation. By specifying option family=binomial, a binomial logit model is estimated. A probit model can be estimated by specifying family=binomialprobit.

#### Additive models

Suppose now that we want to allow for possibly nonlinear effects of x2 and x3. Defining cubic P-splines with second order random walk penalty as smoothness priors, we obtain

> b.regress y = x1 + x2(psplinerw2) + x3(psplinerw2), iterations=12000 burnin=2000 step=10 family=binomial using test

which corresponds to the predictor

$$\eta = \gamma_0 + \gamma_1 x 1 + f_1(x2) + f_2(x3).$$

Suppose now for a moment that the response is not binary but multicategorical with unordered categories 1, 2 and 3. In that case we can estimate either a multinomial logit or a probit model. A logit model is estimated by typing:

> b.regress y = x1 + x2(psplinerw2) + x3(psplinerw2), iterations=12000
burnin=2000 step=10 family=multinomial reference=2 using test

That is, family=binomial was altered to family=multinomial, and the option reference=2 was added in order to define the value 2 as the reference category. Accordingly, a multinomial probit model is estimated by typing

> b.regress y = x1 + x2(psplinerw2) + x3(psplinerw2), iterations=12000
burnin=2000 step=10 family=multinomialprobit reference=2 using test

7.2. Method autocor

# Time scales

In our next step we extend the model by incorporating an additional trend and a flexible seasonal component for the time scale t:

```
> b.regress y = x1 + x2(psplinerw2) + x3(psplinerw2) + t(psplinerw2) +
t(season,period=12), iterations=12000 burnin=2000 step=10
family=binomial using test
```

Note that we passed the period of the seasonal component as a second argument.

# **Spatial covariates**

Suppose now that we have an additional spatial covariate region, which indicates the geographical region an observation belongs to. To incorporate a structured spatial effect, we first have to create a *map object* and read in the boundary information of the different regions (polygons that form the regions, neighbors etc.). If you are unfamiliar with *map objects* please read chapter 5 first.

```
> map m
> m.infile using c:\maps\map.bnd
```

In a second step we reorder the regions of the map using the **reorder** command to obtain minimal bandwidths of the corresponding adjacency matrix of the map. This usually speeds up MCMC simulation for spatial effects.

```
> m.reorder
```

Since we normally need the map again in further sessions, we store the reordered map in *graph file* format, because reading *graph files* is much faster than reading *boundary files*.

```
> m.outfile , graph using c:\maps\mapgraph.gra
```

We can now extend our predictor with a spatial effect:

```
> b.regress y = x1 + x2(psplinerw2) + x3(psplinerw2) + t(psplinerw2) +
t(season,period=12) + region(spatial,map=m), iterations=12000 burnin=2000
step=10 family=binomial using test
```

In some situations it may be reasonable to incorporate an additional unstructured random effect into the model in order to split the total spatial effect into a structured and an unstructured component. This is done by typing

```
> b.regress y = x1 + x2(psplinerw2) + x3(psplinerw2) + t(psplinerw2) +
t(season,period=12) + region(spatial,map=m) + region(random), iterations=12000
burnin=2000 step=10 family=binomial using test
```

# 7.2 Method autocor

# Description

This method is a post estimation command, i.e. its usage is meaningful only if method regress has been applied before. Method autocor computes the autocorrelation functions of all sampled (and stored) parameters. The computed functions will be written to an external file whose name and storing path is printed in the *output window* and/or an additional log file. The computed autocorrelations can be visualized by using either method plotautocor of *graph objects* or the R / S-plus function plotautocor.

# Syntax

# > objectname.autocor [, options]

The execution of this command computes autocorrelation functions for all sampled and stored parameters. An error will be raised if regression results are not yet available. The computed functions will be stored in an external file. The storing directory will be the current output directory of the bayesreg object. By default, this directory is <INSTALLDIRECTORY>\output, but the current output directory may be changed by redefining the global option outfile, see section 7.4. The filename will be the current output name extended by the ending \_autocor.raw. By default, the output name is the name of the particular bayesreg object. Thus, if for example your bayesreg object name is bayes, the complete filename will be bayes\_autocor.raw. Once again, the default output name may be changed using the global option outfile (section 7.4). Note that the autocorrelation file will be overwritten whenever method autocor is applied. As a remedy the current output directory and/or output name should be changed before estimating a new model, using the global option outfile, see section 7.4.

The structure of the file with the stored autocorrelation functions is the following: The computed functions are stored in a matrix like fashion. For every parameter the autocorrelation function will be stored columnwise, with autocorrelation for lag 1 in row 1, for lag 2 in row 2 and so on. The first column of the file contains the lag number. In addition, for each term in the estimated model, minimum, mean and maximum autocorrelations will be computed and stored. Note finally that the very first row of the file contains the column names.

The computed autocorrelations can be visualized by using either method plotautocor of graph objects or the R / S-plus function plotautocor. However, since the structure of the file is very simple, the visualization of the functions can be done with every software package which has graphics capabilities.

# **Options**

• maxlag = integer

With the maxlag option, the maximum lag number for computing autocorrelations may be specified. *integer* must be a positive integer valued number.

DEFAULT: maxlag=250

# **Examples**

Suppose we have already defined a *bayesreg object* b and estimated a (simple) regression model with Gaussian responses using the following regress statement

# > b.regress Y = X, family=gaussian using d

where d is the analyzed data set. The model contains only a fixed effect for covariate X and an intercept. We may now want to check the mixing of the sampled parameters (one for the overall variance parameter, one for the intercept and one for the fixed effect of X) by computing autocorrelation functions. The following statement computes autocorrelations up to lag 100 and stores the result in the default output directory with filename b\_autocor.raw:

# > b.autocor, maxlag=100

The default output directory is <INSTALLDIRECTORY>\output. So if, for instance, BayesX is installed in c:\bayes, the autocorrelation functions will the stored in c:\bayes\output\b\_autocor.raw. If you wish to store the file in another directory, c:\data say, and under another name, for example estimate1, you must use the global option outfile before estimation (see also section 7.4). The following commands produce the desired result (program output between the different statements omitted):

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```
> b.outfile = c:\data\estimate1
> b.regress Y = X, family=gaussian using d
> b.autocor , maxlag=100
```

Now the autocorrelation functions will be stored under c:\data\estimate1\_autocor.raw.

The computed autocorrelation functions may now be visualized using the R / S-plus function plotautocor. The function plots the autocorrelation functions for all estimated parameters (in our example only three) against the lag number. If the option mean.autocor=T is specified, only minimum, mean and maximum autocorrelations for each term in the model are plotted against the lag number. Obviously, this is much faster than the first alternative, where the autocorrelation functions of all parameters are plotted.

The following statement in plots and stores the autocorrelations in the postscript file c:\data\estimate1\_autocor.ps:

```
> plotautocor("c:\\data\\estimate1_autocor.raw",
    "c:\\data\\estimate1_autocor.ps")
```

Note that double backslashes are required in R / S-plus to specify the directory of a file correctly. For more details about the function plotautocor compare subsection 9.2.5.

The autocorrelation functions can be plotted directly in BayesX using method plotautocor of  $graph\ objects$ . For that purpose, the computed autocorrelation functions must be first read into BayesX as a new data set, a say, and then visualized using method plotautocor of  $graph\ objects$ . The following commands produce the desired results:

```
> b.outfile = c:\data\estimate1
> b.regress Y = X, family=gaussian using d
> b.autocor, maxlag=100
> dataset a
> a.infile using c:\data\estimate1_autocor.raw
> graph g
> g.plotautocor using a
```

A third way for plotting autocorrelation functions is given by applying method plotautocor of bayesreg objects. Here autocorrelations are computed and plotted in one step.

# 7.3 Method getsample

# Description

This method is a post estimation command, that is only meaningful if method regress has been applied before. With method getsample all sampled parameters will be stored in (one or more) ASCII file(s). Afterwards, sampling paths can be plotted and stored in a postscript file either by using method plotsample of graph objects or by using the R / S-plus function plotsample. Of course, any other program with graphics capacities could be used as well.

# **Syntax**

### > objectname.getsample

This command stores all sampled parameters in ASCII file(s). An error will be raised, if regression results are not yet available. The storing directory will be the current output directory of the bayesreg object. By default, this directory is <INSTALLDIRECTORY>\output, but you can change the current output directory by redefining the global option outfile. The filenames will be the current output name extended by an ending depending on the type of the estimated effect. For example for fixed effects, the complete filename will be b\_FixedEffects1\_sample.raw, if b is the name of the bayesreg object. The total number of created files and their filenames are printed in the output window and/or an open log file. By default, the output name is the name of the bayesreg object. Once again, the default name may be changed using the global option outfile. Note that it can happen that some or all files will be overwritten, if method getsample is applied more than once with the same bayesreg object. To avoid such problems change the current output directory and/or output name before estimating a new model using the global option outfile.

The structure of the created files is as follows: The very first row contains the parameter names. In the following lines, the parameters are stored in a matrix like fashion. In the first row (to be precise the second row, since the first contains the names) the first sampled value of each parameter separated by blanks is stored. In the second row the second value is stored and so on. In the first column of each row the sampling number is printed.

# **Options**

not allowed

### **Examples**

Suppose we have already defined a *bayesreg object* b and estimated a (simple) regression model with Gaussian errors using the following regress statement:

# > b.regress Y = X, family=gaussian using d

The model contains only a fixed effect for covariate X and an intercept. We may now want to check the mixing of the sampled parameters (one for the overall variance  $\sigma^2$ , one for the intercept and one for X) by storing sampled parameters in an ASCII-file and visualizing sampling paths. The statement

# > b.getsample

forces BayesX to store the sampled parameters in files named:

```
b_FixedEffects1_sample.raw
b_intercept_sample.raw
b_scale_sample.raw
The storing directory is the surrent output of
```

The storing directory is the current output directory, which is by default <INSTALLDIRECTORY\output. The current output directory can be changed using the global option outfile. For example the two commands

```
> b.outfile = c:\data\estimate1
> b.getsample
```

force the program to store the sampled parameters in the files:

```
c:\data\estimate1_FixedEffects1_sample.raw.
```

- c:\data\estimate1\_intercept\_sample.raw
- c:\data\estimate1\_scale\_sample.raw

The first few lines of the first file look like this:

```
intnr b_1
1 -2.00499
2 -1.97745
3 -1.98498
4 -1.97108
5 -2.00004
:
```

We can now visualize the sampling paths directly using method plotsample of graph objects. We first read in sampled parameters by typing:

```
> dataset s1
```

> s1.infile using c:\data\estimate1\_FixedEffects1\_sample.raw

We proceed by creating a *graph object* g and applying method plotsample:

```
> graph g
> g.plotsample using s1
```

Alternatively we could use R / S-plus and the function plotsample for visualizing sampling paths. We type, for instance

```
> plotsample("c:\\data\\estimate1_FixedEffects1_sample.raw")
```

# 7.4 Global options

The purpose of global options is to affect the global behavior of a *bayesreg object*. The main characteristic of global options is, that they are not associated with a certain method.

The syntax for specifying global options is

> objectname.optionname = newvalue

where *newvalue* is the new value of the option. The type of the value depends on the respective option.

The following global options are currently available for *bayesreg objects*:

• outfile = filename

By default, the estimation output produced by the **regress** procedure will be written to the default output directory, which is

# <INSTALLDIRECTORY>\output.

The default filename is composed of the name of the *bayesreg object* and the type of the file. For example, if you estimated a nonparametric effect for a covariate X, say, then the estimation output will be written to

# <INSTALLDIRECTORY>\output\b\_nonpX.res

where b is the name of the bayesreg object. In most cases, however, it may be necessary to save estimation results into a different directory and/or under a different filename than the default. This can be done using the outfile option. With the outfile option you have to specify the directory where the output should be stored to and in addition a base filename. The base filename should not be a complete filename. For example specifying

# > b.outfile = c:\data\res

would force BayesX to store the estimation result for the nonparametric effect of X in file

c:\data\res\_nonpX.res

### • iterationsprint = integer

By default, the current iteration number is printed in the *output window* (or in an additional log file) after every 100th iteration. This can lead to rather big and complex output files. The iterationsprint option allows to redefine after how many iterations the current iteration number is printed. For example iterationsprint=1000 forces *BayesX* to print the current iterations number only after every 1000th iteration rather than after every 100th iteration.

# 7.5 Visualizing estimation results

Visualization of estimation results is described in chapter 9.

# 7.6 Examples

In this Section we present a couple of complex examples about the usage of bayesreg objects. The first example contains a reanalysis of the 'credit scoring' data set that is described in subsection 2.6.2, which contains also a (incomplete) list of some publications where the 'credit scoring' data set has already been analyzed. The second example is a Bayesian analysis of determinants of childhood undernutrition in Zambia. The data set is described in subsection 2.6.3. This section contains also a list of publications where the data set has been analyzed. Both data sets are shipped together with BayesX and are stored in the directory examples, which is a subdirectory of the installation directory. Since the main focus here is on illustrating the usage of bayesreg objects, we omit any interpretation of estimated effects.

# 7.6.1 Binary data: credit scoring

All BayesX statements of this section can be found in the examples directory in the file credit.prg. In principle, the commands in credit.prg can be executed using the usefile command for running batch files, see section 3.5. Note, however, that the specified directories therein may not exist on your computer. Thus, to avoid errors, the file must be modified first to execute correctly.

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# 7.6.1.1 Reading the data into BayesX

In order to analyze the 'credit scoring' data set, we first have to load the data set into BayesX. For the rest of this section we assume that BayesX is installed in the directory c:\bayes. In this case, the 'credit scoring' data set can be found in c:\bayes\examples under the name credit.raw. With the following two commands (entered in the command window) we first create a dataset object credit and afterwards load the data set into BayesX using the infile command:

- > dataset credit
- > credit.infile using c:\bayes\examples\credit.res

Since the first row of the file already contains the variable names, it is not necessary to specify variable names in the infile statement. If the first row of the data set does not contain the variable names, they must be additionally specified in the infile command, e.g. for the 'credit scoring' data set we get

> credit.infile y account duration amount payment intuse marstat
 using c:\bayes\examples\credit.res

We now compute effect coded versions of the categorical covariates account, payment, intuse and marstat:

```
> credit.generate account1 = 1*(account=1)-1*(account=3)
> credit.generate account2 = 1*(account=2)-1*(account=3)
> credit.generate payment1 = 1*(payment=1)-1*(payment=2)
> credit.generate intuse1 = 1*(intuse=1)-1*(intuse=2)
> credit.generate marstat1 = 1*(marstat=1)-1*(marstat=2)
```

The reference categories for the covariates are chosen to be 3 for account and 2 for the others.

# 7.6.1.2 Creating a bayesreg object

Before we are able to estimate Bayesian regression models, we first have to create a bayesreg object:

- > bayesreg b
- > b.outfile = c:\results\credit

The second command changes the default output directory and name (which is c:\bayes\output\b) to c:\results\credit. This means that subsequent regression output is stored in the directory c:\results and that all filenames start with credit.

### 7.6.1.3 Probit models

We can now start estimating models. We first describe how probit models are estimated. The estimation of probit models is slightly faster than logit models because the full conditionals of the effects are Gaussian.

We first estimate a model with fixed effects only:

```
> b.regress y = account1 + account2 + duration + amount + payment1 + intuse1
+ marstat1, predict iterations=6000 burnin=1000 step=5 family=binomialprobit
using credit
```

Here we specified 6000 iterations, a burnin period of 1000 iterations and a thinning parameter of 5, i.e. every 5th sampled parameter will be stored and used for estimation. The additional option **predict** is used to compute samples of the deviance, the effective number of parameters, the deviance information criterion (DIC), predicted means etc.

Executing the command yields the following output (simulation output omitted):

### SIMULATION TERMINATED

SIMULATION RUN TIME: 13 seconds

#### ESTIMATION RESULTS:

Predicted values:

Estimated mean of predictors, expectation of response and individual deviances are stored in file c:\results\credit\_predictmean.raw

Estimation results for the Deviance:

Unstandardized Deviance (-2\*Loglikelihood(y|mu))

Mean: 1027.05 Std. Dev: 4.22501 2.5% Quantile: 1021.01 10% Quantile: 1022.48 50% Quantile: 1026.29 90% Quantile: 1032.73 97.5% Quantile: 1037.28

Saturated Deviance (-2\*Loglikelihood(y|mu) + 2\*Loglikelihood(y|mu=y))

Mean: 1027.05 Std. Dev: 4.22501 2.5% Quantile: 1021.01 10% Quantile: 1022.48 50% Quantile: 1026.29 90% Quantile: 1032.73 97.5% Quantile: 1037.28

Samples of the deviance are stored in file
c:\results\credit\_deviance\_sample.raw

Estimation results for the DIC:

DIC based on the unstandardized Deviance

Deviance(bar\_mu): 1018.85 pD: 8.20049 DIC: 1035.26

DIC based on the saturated Deviance

Deviance(bar\_mu): 1018.85 pD: 8.20049 DIC: 1035.26

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#### FixedEffects1

Acceptance rate: 100 %

Variable	mean	Std. Dev.	2.5% quant.	median	97.5% quant.
const	-0.715437	0.117673	-0.9464	-0.710836	-0.49298
account1	-0.629553	0.0684571	-0.764526	-0.6244	-0.50343
account2	0.50699	0.0625032	0.389608	0.506698	0.63888
duration	0.0204795	0.00471734	0.0111948	0.0205776	0.0298505
amount	0.0172111	0.0189096	-0.0173778	0.0167277	0.0542619
payment1	-0.2919	0.0762402	-0.438703	-0.288309	-0.152512
intuse1	-0.137287	0.0477423	-0.228492	-0.137755	-0.041511
marstat1	-0.158195	0.0476991	-0.254198	-0.157534	-0.0663364

Results for fixed effects are also stored in file
c:\results\credit\_FixedEffects1.res

Somewhat surprisingly, we observe that the amount of credit seems to have no ('significant') influence on the response. To check this phenomenon more carefully, we run a second estimation, now allowing for possibly nonlinear effects of the continuous covariates amount and duration. We choose cubic P-splines with second order random walk penalty as smoothness priors and modify the regress statement above according to the new model:

> b.regress y = account1 + account2 + duration(psplinerw2) + amount(psplinerw2)
+ payment1 + intuse1 + marstat1, predict iterations=6000 burnin=1000 step=5
family=binomialprobit using credit

We get the following output for the nonlinear functions (output for the rest omitted):

### f\_duration\_pspline

Acceptance rate: 100 %

Results are stored in file c:\results\credit\_f\_duration\_pspline.res

Postscript file is stored in file c:\results\credit\_f\_duration\_pspline.ps

Results may be visualized using method 'plotnonp' Type for example: objectname.plotnonp 1

f\_duration\_pspline\_variance

Acceptance rate: 100 %

Estimation results for the variance component:

Mean: 0.00787338
Std. dev.: 0.0100301
2.5% Quantile: 0.00128376
10% Quantile: 0.00190016
50% Quantile: 0.00487717

90% Quantile: 0.0157856 97.5% Quantile: 0.0315107

Results for the variance component are also stored in file c:\results\credit\_f\_duration\_pspline\_var.res

f\_amount\_pspline

Acceptance rate: 100 %

Results are stored in file c:\results\credit\_f\_amount\_pspline.res

Postscript file is stored in file c:\results\credit\_f\_amount\_pspline.ps

Results may be visualized using method 'plotnonp' Type for example: objectname.plotnonp 3

f\_amount\_pspline\_variance

Acceptance rate: 100 %

Estimation results for the variance component:

Mean: 0.00870842
Std. dev.: 0.0115895
2.5% Quantile: 0.0013844
10% Quantile: 0.00220351
50% Quantile: 0.00570725
90% Quantile: 0.0164858
97.5% Quantile: 0.0340326

Results for the variance component are also stored in file c:\results\credit\_f\_amount\_pspline\_var.res

We visualize estimated effects for amount and duration using method plotnonp (as advised by the program):

```
> b.plotnonp 1, outfile="c:\results\credit_duration.ps"
```

> b.plotnonp 3, outfile="c:\results\credit\_amount.ps"

This produces the graphs (stored in postscript files) shown in Figure 7.2.

We add a title, x-axis and y-axis labels by typing

- > b.plotnonp 1, outfile="c:\results\credit\_duration.ps" replace
   xlab="duration" ylab="f(duration)" title="effect of duration"
- > b.plotnonp 3, outfile="c:\results\credit\_amount.ps" replace
  xlab="amount" ylab="f(amount)" title="effect of amount"

and obtain the improved graphs shown in Figure 7.3. The option replace is specified to allow BayesX to overwrite the previously generated postscript files. If the outfile option is omitted, the graphs are printed on the screen rather than being stored as postscript files.

We now want to check the mixing of the generated Markov chains, although the mixing for probit models is usually excellent. For that reason we compute and plot the autocorrelation functions by

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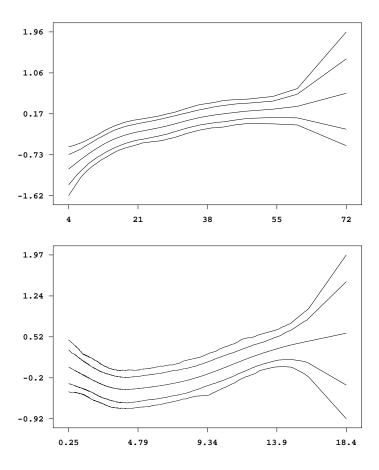
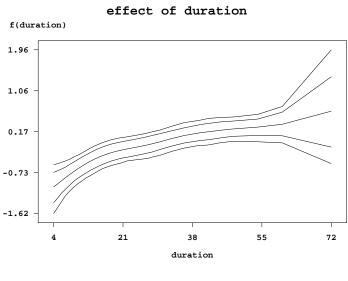


Figure 7.2: Estimated effects of duration and amount of credit. Shown is the posterior mean within 80% and 95% credible regions.

# typing:

> b.plotautocor, outfile="c:\results\credit\_autocor.ps"

We obtain the file c:\results\credit\_autocor.ps containing 9 pages of autocorrelation functions for all parameters in the model. The first page of this file is shown in Figure 7.4. We see that autocorrelations die off very quickly.



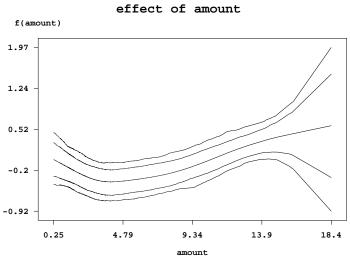


Figure 7.3: Improved plots of the effect of duration and amount.

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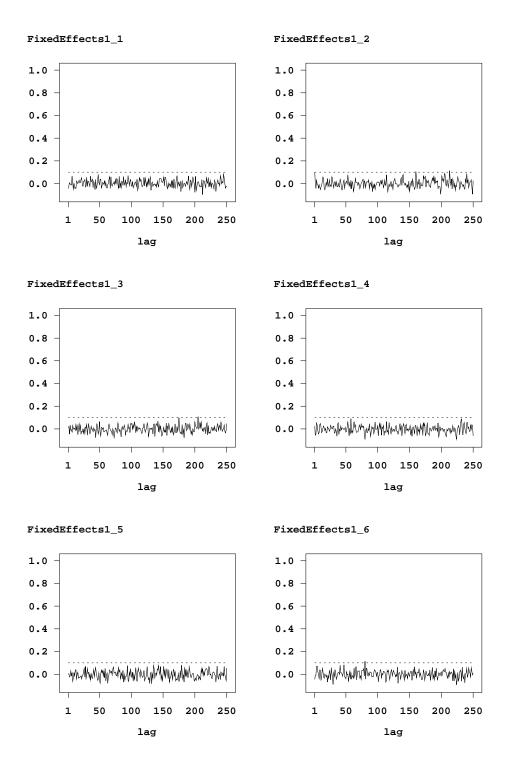


Figure 7.4: First page of the autocorrelation file.

### 7.6.1.4 Logit models

A logit model rather than a probit model is estimated by replacing family=binomialprobit with family=binomial:

```
> b.regress y = account1 + account2 + duration(psplinerw2) + amount(psplinerw2)
+ payment1 + intuse1 + marstat1, predict iterations=6000 burnin=1000 step=5
family=binomial using credit
```

In contrast to binary probit models, the full conditionals for the regression coefficients are no longer Gaussian. BayesX offers 3 different types of proposal densities. These are iteratively weighted least squares (IWLS) proposals based either on the current state of the parameters or on the posterior modes as described in subsubsection 4.1.3 or Brezger and Lang (2006), and conditional prior proposals as described in Fahrmeir and Lang (2001b). We recommend the usage of IWLS proposals, since no tuning is required and mixing properties are superior to those of conditional prior proposals. The default are IWLS proposals based on the current state of the parameters. The following statement causes BayesX to use IWLS proposals based on posterior modes, which usually yield even higher acceptance probabilities compared to ordinary IWLS proposals:

```
> b.regress y = account1 + account2 + duration(psplinerw2,proposal=iwlsmode)
+ amount(psplinerw2,proposal=iwlsmode) + payment1 + intuse1 + marstat1,
predict iterations=6000 burnin=1000 step=5
family=binomial using credit
```

As for the probit model, we visualize the estimated nonlinear effects of duration and amount using method plotnonp:

- > b.plotnonp 1 , outfile="c:\results\credit\_logit\_duration.ps" replace
   xlab="duration" ylab="f(duration)" title="effect of duration"
- > b.plotnonp 3 , outfile="c:\results\credit\_logit\_amount.ps" replace
   xlab="amount" ylab="f(amount)" title="effect of amount"

The resulting graphs are shown in Figure 7.5. As could have been expected only the scale of the estimated effects differs (because of the logit link).

Once again, to check the mixing of the sampled parameters we compute and plot the autocorrelation functions using method plotautocor:

> b.plotautocor, outfile="c:\results\credit\_logit\_autocor.ps"

The first page of the resulting postscript file is shown in Figure 7.6. As can be seen, the autocorrelations for the logit model with IWLS proposals are almost as low as for the probit model.

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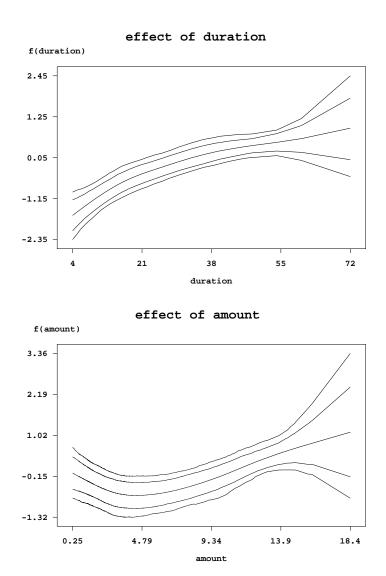


Figure 7.5: Effect of duration and amount, if a logit model is estimated rather than a probit model.

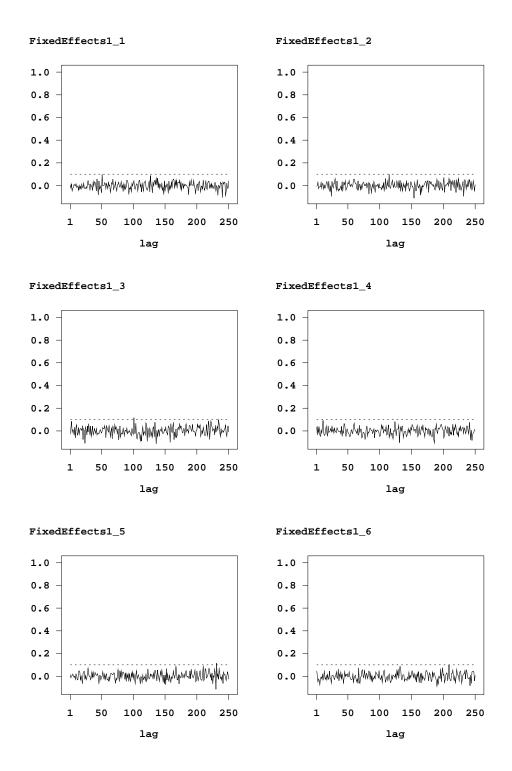


Figure 7.6: First page of the autocorrelation file, if a logit model is estimated.

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# 7.6.1.5 Varying the hyperparameters

In the preceding examples we used the default hyperparameters a=0.001 and b=0.001 for the inverse gamma prior of the variances. In some situations, however, the estimated nonlinear functions may considerably depend on the particular choice of hyperparameters a and b. This may be the case for very low signal to noise ratios or/and small sample sizes. It is therefore highly recommended to estimate all models under consideration using a (small) number of different choices for a and b (e.g. a=1,b=0.005; a=0.001,b=0.001; a=0.0001,b=0.0001) to assess the dependence of results on minor changes in the model assumptions. In that sense, the variation of hyperparameters can be used as a tool for model diagnostics.

We estimate our probit model from subsubsection 7.6.1.3 again, but now with hyperparameters a=1.0, b=0.005 and a=0.0001, b=0.0001, respectively.

- > b.regress y = account1 + account2 + duration(psplinerw2,a=1.0,b=0.005) +
  amount(psplinerw2,a=1.0,b=0.005) + payment1 + intuse1 + marstat1,
  predict iterations=6000 burnin=1000 step=5 family=binomialprobit using credit
- > b.regress y = account1 + account2 + duration(psplinerw2,a=0.0001,b=0.0001) +
  amount(psplinerw2,a=0.0001,b=0.0001) + payment1 + intuse1 + marstat1,
  predict iterations=6000 burnin=1000 step=5 family=binomialprobit using credit

Figure 7.7 shows the estimated nonlinear effects of variables duration and amount with the different choices for a and b. We see that in this example estimation results differ only slightly for the different choices of a and b.

# 7.7 References

- BESAG, J., GREEN, P., HIGDON, D. AND MENGERSEN, K. (1995): Bayesian computation and stochastic systems (with discussion). *Statistical Science*, 10, 3-66.
- Besag, J. and Kooperberg, C. (1995): On conditional and intrinsic autoregressions. *Biometrika*, 82, 733-746.
- Besag, J., York, J. and Mollie, A. (1991): Bayesian image restoration with two applications in spatial statistics (with discussion). *Annals of the Institute of Statistical Mathematics*, 43, 1-59.
- BILLER, C. (2000): Bayesianische Ansätze zur nonparametrischen Regression. Skaker Verlag, Aachen.
- Brezger, A. (2000): Bayesianische P-splines. Master thesis, University of Munich.
- Brezger, A. and Lang, S. (2006): Generalized additive regression based on Bayesian P-splines. *Computational Statistics and Data Analysis* 50, 967-991.
- CLAYTON, D. (1996): Generalized linear mixed models. In: Gilks, W., Richardson S. and Spiegelhalter D. (eds), *Markov Chain Monte Carlo in Practice*. London: Chapman and Hall, 275-301.
- Chen, M.H. and Dey, D.K. (2000): Bayesian Analysis for Correlated Ordinal Data Models. Generalized linear models: A Bayesian perspective (ed. Dey, D.K., Ghosh, S.K. and Mallick, B.K.), 8,133-159, Marcel Dekker, New York.
- Chib, S. and Greenberg, E. (1995): Understanding the Metropolis-Hastings Algorithm. *The American Statistician*, 49, 327-335.

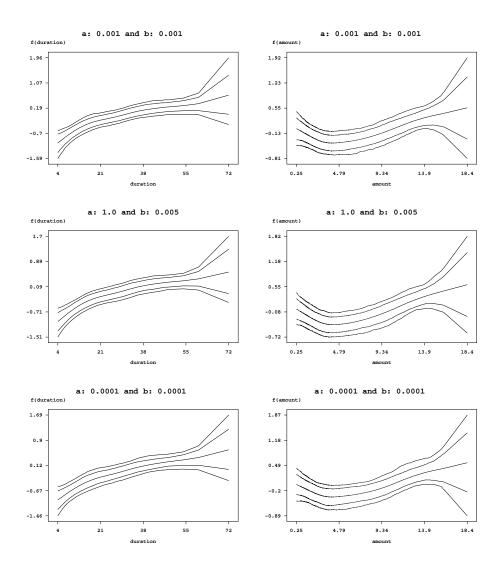


Figure 7.7: Results for the effect of duration and amount for different values of the hyperparameters for the variances.

EILERS, P.H.C. AND MARX, B.D. (1996): Flexible smoothing using B-splines and penalized likelihood (with comments and rejoinder). *Statistical Science*, 11 (2), 89-121.

Fahrmeir, L. and Lang, S. (2001a): Bayesian Inference for Generalized Additive Mixed Models Based on Markov Random Field Priors. *Journal of the Royal Statistical Society C*, 50, 201-220.

Fahrmeir, L. and Lang, S. (2001b): Bayesian Semiparametric Regression Analysis of Multicategorical Time-Space Data. *Annals of the Institute of Statistical Mathematics*, 53, 10-30.

Fahrmeir, L. and Tutz, G. (2001): Multivariate Statistical Modelling based on Generalized Linear Models. New York: Springer-Verlag.

Gamerman, D. (1997): Efficient Sampling from the posterior distribution in generalized linear models. *Statistics and Computing*, 7, 57-68.

Gelfand, A.E., Sahu, S.K. and Carlin, B.P. (1996): Efficient Parametrizations for Genera-

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- lized Linear Mixed Models. In: Bernardo, J.M., Berger, J.O., Dawid, A.P. and Smith, A.F.M. (eds.), *Bayesian Statistics*, 5. Oxford University Press, 165-180.
- GEORGE, A. AND LIU, J.W. (1981). Computer Solution of Large Sparse Positive Definite Systems. Series in computational mathematics, Prentice-Hall.
- Green, P.J. (2001): A Primer in Markov Chain Monte Carlo. In: Barndorff-Nielsen, O.E., Cox, D.R. and Klüppelberg, C. (eds.), *Complex Stochastic Systems*. Chapmann and Hall, London, 1-62.
- Green, P.J. and Silverman, B. (1994): Nonparametric Regression and Generalized Linear Models. Chapman and Hall, London.
- Hastie, T. and Tibshirani, R. (1990): Generalized additive models. Chapman and Hall, London.
- Hastie, T. and Tibshirani, R. (1993): Varying-coefficient Models. *Journal of the Royal Statistical Society B*, 55, 757-796.
- Hastie, T. and Tibshirani, R. (2000): Bayesian Backfitting. Statistical Science, 15, 193-223.
- Hastie, T., Tisbshirani, R. and Friedman, J. (2001): The Elements of Statistical Learning: Data Mining, Inference and Prediction. New York: Sprigner-Verlag.
- Kneib, T. and Hennerfeind, A. (2006) Bayesian Semiparametric Multi-State Models. Under revision for *Statistical Modelling*. A preliminary version is available as SFB 386 Discussion Paper 502.
- Knorr-Held, L. (1999): Conditional Prior Proposals in Dynamic Models. Scandinavian Journal of Statistics, 26, 129-144.
- KRAGLER, P. (2000): Statistische Analyse von Schadensfällen privater Krankenversicherungen. Master thesis, University of Munich.
- Lang, S. (1996): Bayesianische Inferenz in Modellen mit variierenden Koeffizienten. Master thesis, University of Munich.
- LANG, S. AND BREZGER, A. (2004): Bayesian P-splines. *Journal of Computational and Graphi*cal Statistics, 13, 183-212.
- McCullagh, P. and Nelder, J.A. (1989): Generalized Linear Models. Chapman and Hall, London.
- OSUNA, L. (2004): Semiparametric Bayesian Count Data Models. Dr. Hut Verlag, München.
- Rue, H. (2001): Fast Sampling of Gaussian Markov Random Fields with Applications. *Journal of the Royal Statistical Society B*, 63, 325-338.
- SPIEGELHALTER, D.J., BEST, N.G., CARLIN, B.P. AND VAN DER LINDE, A. (2002): Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society B*, 65, 583-639.

# Chapter 8

# remlreg objects

Remlreg objects are used to fit (multivariate) exponential family, hazard rate or multi-state models with structured additive predictor subsumed in the class of structured additive regression (STAR) models, see Fahrmeir, Kneib, and Lang (2004). Inference is based on a mixed model representation of the regression model and yields either penalised likelihood estimates (from a frequentist perspective) or empirical Bayes / posterior mode estimates (from a Bayesian perspective). The methodological background is provided in considerable detail in the methodology manual. More details on models for univariate responses can be found in Fahrmeir, Kneib and Lang (2004). Kneib and Fahrmeir (2006) describes models for categorical responses. Models for continuous time survival analysis based on structured hazard regression can be found in Kneib and Fahrmeir (2007). Interval censoring and some further extensions are discussed in Kneib (2006).

First steps with *remlreg objects* can be done with the example in chapter 2 of the tutorial manual which provides a self-contained demonstrating example.

# 8.1 Method regress

# **8.1.1** Syntax

> objectname.regress model [weight weightvar] [if expression] [, options] using dataset

Method regress estimates the regression model specified in *model* using the data specified in *dataset*. *dataset* has to be the name of a *dataset object* created before. The details of correct model specification are covered in subsubsection 8.1.1.2. The distribution of the response variable can be either Gaussian, binomial, multinomial, Poisson or gamma. In addition, *BayesX* supports continuous time survival and multi-state models, see also Table 7.5 for a more detailled overview.. The response distribution is specified using option family, see subsubsection 8.1.1.4 below. The default is family=binomial with a logit link. An if statement can be specified to analyze only parts of the data set, i.e. the observations where *expression* is true.

# 8.1.1.1 Optional weight variable

An optional weight variable weightvar can be specified to estimate weighted regression models. For Gaussian responses, BayesX assumes that  $y_i|\eta_i, \sigma^2 \sim N(\eta_i, \sigma^2/weightvar_i)$ . Thus, for grouped Gaussian responses the weights represent the number of observations in the groups if the  $y_i$ 's are the average of individual responses. If the  $y_i$ s are the sum of responses in every group, the weights have to be the reciprocal of the number of observations in the groups. Of course, estimation of usual weighted regression models with heteroscedastic errors is also possible. In this case, the

weights should be proportional to the reciprocal of the heteroscedastic variances. If the response distribution is binomial, the weight variable should correspond to the number of replications while the values of the response variable should represent the number of successes. If weight is omitted, BayesX assumes that the number of replications is one, i.e. the values of the response must be either zero or one. For grouped Poisson data, the weights have to specify the number of observations in a group while the  $y_i$ s are assumed to be the average of individual responses. Weights are not allowed for models with categorical response, continuous survival time models and multi-state models.

# 8.1.1.2 Syntax of possible model terms

The general syntax of models for remlreg objects is:

```
depvar = term_1 + term_2 + \cdots + term_r
```

depvar specifies the dependent variable whereas  $term_1, \ldots, term_r$  define the form of covariate influences. The different terms must be separated by '+' signs. A constant intercept is automatically included in the model and therefore has not to be specified by the user.

This section reviews all possible model terms supported in the current version of *remlreg objects* and provides some specific examples. Note that all described terms may be combined in arbitrary order. An overview about the capabilities of *remlreg objects* is given in Table 8.1. Table 8.2 shows how interactions between covariates are specified. Full details about all available options are given in subsubsection 8.1.1.3.

Throughout this section Y denotes the dependent variable.

Type	Syntax example	Description
Offset	offs(offset)	Variable offs is an offset term.
Linear effect	W1	Linear effect of W1.
Category-specific	W1(catspecific)	Category-specific linear effect of W1 (in cumulative
linear effect		or sequential models only).
First or second or-	X1(rw1)	Nonlinear effect of X1.
der random walk	X1(rw2)	
P-spline	X1(psplinerw1)	Nonlinear effect of X1.
	X1(psplinerw2)	
Seasonal prior	time(season,period=12)	Varying seasonal effect of time with period 12.
Markov random	region(spatial,map=m)	Spatial effect of region where region indicates the
field		region an observation pertains to. The bound-
		ary information and the neighborhood structure
		is stored in the map object m.
Two dimensional	region(geospline,map=m)	Spatial effect of region. Estimates a two dimen-
P-spline		sional P-spline based on the centroids of the re-
		gions. The centroids are stored in the map object
		m.
Stationary	region(geokriging)	Spatial effect of region. Estimates a stationary
Gaussian random		Gaussian random field based on the centroids of
field		the regions. The centroids are stored in the $map$
		object m.
Random intercept	grvar(random)	I.i.d. Gaussian (random) effect of the group indi-
		cator grvar, e.g. grvar may be an individual indi-
		cator when analyzing longitudinal data.
Baseline in Cox or	time(baseline)	Nonlinear shape of the baseline effect $\lambda_0(time)$ of
multi-state models		a Cox model. $\log(\lambda_0(time))$ is modelled by a P-
		spline with second order random walk penalty.

Table 8.1: Overview over different model terms for remlreg objects.

Type of interaction	Syntax example	Description
Varying coefficient term	X1*X2(rw1)	Effect of X1 varies smoothly over the range
	X1*X2(rw2)	of the continuous covariate X2 or time.
	X1*X2(psplinerw1)	
	X1*X2(psplinerw2)	
	X1*time(season)	
Random slope	X1*grvar(random)	The regression coefficient of X1 varies with
		respect to the unit- or cluster index vari-
		able grvar.
Geographically weighted	<pre>X1*region(spatial,map=m)</pre>	Effect of X1 varies geographically. Covari-
regression		ate region indicates the region an obser-
		vation pertains to.
Two dimensional	X1*X2(pspline2dimrw1)	Two dimensional surface for the continu-
surface		ous covariates X1 and X2.
Stationary Gaussian	X1*X2(kriging)	Stationary Gaussian random field for co-
random field		ordinates X1 and X2.
Time-varying effect in	X1*time(baseline)	Nonlinear, time-varying effect of X1.
Cox or multi-state mod-		
els		

Table 8.2: Possible interaction terms for remlreg objects.

### Offset

Description: Adds an offset term to the predictor.

Predictor:  $\eta = \cdots + offs + \cdots$ 

Syntax:

offs(offset)

Example:

For example, the following model statement can be used to estimate a poisson model with offs as offset term and W1 and W2 as fixed effects (if family=poisson is specified in addition):

$$Y = offs(offset) + W1 + W2$$

### **Fixed effects**

Description: Incorporates covariate W1 as a fixed effect into the model.

Predictor:  $\eta = \cdots + \gamma_1 W 1 + \cdots$ 

Syntax:

W1

Example:

The following model statement causes regress to estimate a model with q fixed (linear) effects:

$$Y = W1 + W2 + \cdots + Wq$$

# Category-specific fixed effects

Description: In cumulative and sequential models for ordered categorical responses, fixed effects may either be defined globally or category-specific. To request the estimation of category-specific fixed effects, the keyword catspecific has to be specified. Category-specific effects can only be estimated for the response families cumlogit, cumprobit, seqlogit, and seqprobit.

```
Predictor: \eta^{(j)} = \cdots + \gamma_1^{(j)}W1 + \cdots

Syntax: W1(catspecific)
```

Example:

The following model statement causes regress to estimate a model with category-specific effect of covariate W1 and a global effect of covariate W2:

$$Y = W1(catspecific) + W2$$

#### Nonlinear effects of continuous covariates and time scales

### First or second order random walk

Description: Defines a first or second order random walk prior for the effect of X1.

Predictor:  $\eta = \cdots + f_1(X1) + \cdots$ Syntax:

X1(rw1[, options])
X1(rw2[, options])

Example:

Suppose that X1 is a continuous covariate with possibly nonlinear effect. The following model statement defines a second order random walk prior for  $f_1$ :

$$Y = X1(rw2)$$

The term X1(rw2,a=0.001,b=0.001) indicates, that the effect of X1 should be incorporated nonparametrically using a second order random walk prior. A first order random walk can be requested by specifying X1(rw1) instead.

### P-spline with first or second order random walk penalty

Description: Defines a P-spline with first or second order random walk penalty for the parameters of the spline.

```
Predictor: \eta = \cdots + f_1(X1) + \cdots

Syntax:

X1(psplinerw1[, options])

X1(psplinerw2[, options])
```

Example:

For example, a P-spline with second order random walk penalty is obtained using the following model statement:

```
Y = X1(psplinerw2)
```

By default, the degree of the spline is 3 and the number of inner knots is 20. The following model term defines a quadratic P-spline with 30 knots:

```
Y = X1(psplinerw2,degree=2,nrknots=30)
```

# Seasonal component for time scales

Description: Defines a time-varying seasonal effect of time.

```
Predictor: \eta = \cdots + f_{season}(time) + \cdots
```

Syntax:

time(season[, options])

Example:

A seasonal component for a time scale time is specified by

```
Y = time(season,period=12)
```

where the second argument indicates the period of the seasonal effect. In the example above, the period is 12 corresponding to monthly data.

# Nonlinear baseline effect in continuous time survival or multi-state models

### P-spline with second order random walk penalty

Description: Defines a P-spline with second order random walk penalty for the parameters of the spline for the log-baseline effect  $\log(\lambda_0(\texttt{time}))$ .

```
Predictor: \eta = \log(\lambda_0(time)) + \cdots
```

Syntax:

time(baseline[, options])

Example:

Suppose continuous-time survival data (time, delta) with additional covariates (W1, X1) are given, where time denotes the vector of observed duration times, delta is the vector of corresponding indicators of non-censoring, W1 is a discrete covariate, and X1 a continuous covariate. The following Cox-type model with hazard rate  $\lambda$  and log-baseline effect  $\log(\lambda_0(\text{time}))$ 

$$\lambda(time) = \lambda_0(time) \exp(\gamma_0 + \gamma_1 W 1 + f(X1))$$
  
= 
$$\exp(\log(\lambda_0(time)) + \gamma_0 + \gamma_1 W 1 + f(X1))$$

can be estimated by the model statement

```
delta = time(baseline) + W1 + X1(psplinerw2)
```

Similarly, baseline effects for the transition intensities in multi-state models can be specified.

### **Spatial Covariates**

### Markov random field

Description:

Defines a Markov random field prior for the spatial covariate region. BayesX allows to incorporate spatial covariates with geographical information stored in the map object specified in option map.

```
Predictor: \eta = \cdots + f_{spat}(region) + \cdots
```

Syntax:

region(spatial,map=characterstring[, options])

Example:

For the specification of a Markov random field prior, map is an obligatory argument that represents the name of a *map object* (see chapter 5) containing all necessary spatial information about the geographical map, i.e. the neighbors of each region and the weights associated with the neighbors. For example the statement

```
Y = region(spatial, map=germany)
```

defines a Markov random field prior for region where the geographical information is stored in the map object germany. An error will be raised if germany is not existing.

## Two-dimensional P-spline with first order random walk penalty

Description:

Defines a two-dimensional P-spline for the spatial covariate region with a two-dimensional first order random walk penalty for the parameters of the spline. Estimation is based on the coordinates of the centroids of the regions. The centroids are computed using the geographical information stored in the *map object* specified in the option map.

```
Predictor: \eta = \cdots + f(centroids) + \cdots
```

Syntax

region(geospline,map=characterstring[, options])

Example:

For the specification of a two-dimensional P-spline (geospline) map is an obligatory argument indicating the name of a map object (see chapter 5) that contains all necessary spatial information about the geographical map, i.e. the neighbors of each region and the weights associated with the neighbors. The model term

```
Y = region(geospline, map=germany)
```

specifies a two-dimensional cubic P-spline with first order random walk penalty where the geographical information is stored in the map object germany.

### Stationary Gaussian random field

Description:

Defines a stationary Gaussian random field for the spatial covariate region. Estimation is based on the coordinates of the centroids of the regions an observation pertains to. The centroids are computed using the geographical information stored in the *map object* specified in option map.

```
Predictor: \eta = \cdots + f(centroids) + \cdots

Syntax:

region(geokriging, map=characterstring[, options])
```

Example:

For the specification of a stationary Gaussian random field (geokriging), map is an obligatory argument indicating the name of a map object (see chapter 5). The model term

```
Y = region(geokriging, map=germany)
```

specifies a stationary Gaussian random field where the geographical information is stored in the *map object* germany.

## Unordered group indicators

## Unit- or cluster specific unstructured effect

Description: Defines an unstructured (uncorrelated) random effect with respect to grouping variable grvar.

```
Predictor: \eta = \cdots + f(grvar) + \cdots
Syntax:
grvar(random[, options])
```

Example:

Gaussian i.i.d. random effects allow to cope with unobserved heterogeneity among units or clusters of observations. Suppose the analyzed data set contains a group indicator grvar that gives information about the individual or cluster a particular observation belongs to. Then an individual-specific uncorrelated random effect is defined by

```
Y = grvar(random)
```

The inclusion of more than one random effect term in the model is possible, allowing the estimation of multilevel models. However, we have only limited experience with multilevel models so that it is not clear how well these models can be estimated using *remlreg objects*.

### Varying coefficients with continuous covariates as effect modifier

### First or second order random walk

Description:

Defines a varying coefficient term, where the effect of X1 varies smoothly over the range of X2. Therefore covariate X2 is called the effect modifier. The smoothness prior for f(X2) is a first or second order random walk.

```
Predictor: \eta = \cdots + f(X2)X1 + \cdots

Syntax:

X1*X2(rw1[, options])

X1*X2(rw2[, options])
```

### Example:

For example, a varying coefficient term with a second order random walk smoothness prior is defined as follows:

$$Y = X1*X2(rw2)$$

## P-spline with first or second order random walk penalty

### Description:

Defines a varying coefficient term, where the effect of X1 varies smoothly over the range of X2. The smoothness prior for f is a P-spline with first or second order random walk penalty.

```
Predictor: \eta = \cdots + f(X2)X1 + \cdots
```

Syntax:

X1\*X2(psplinerw1[, options])

X1\*X2(psplinerw2[, options])

### Example:

For example, a varying coefficient term with a second order random walk smoothness prior is defined as follows:

If the effect of a covariate should vary according to different types of effect modifiers, this leads to similar identification problems as in usual additive models. To avoid such problems, option center can be specified to request the estimation of centered effects. For example, if both X2 and Z2 are assumed to modify the effect of X1, the specification of

```
Y = X1*X2(psplinerw2) + X1*Z2(psplinerw2)
```

yields a non-identifiable model. In contrast

```
Y = X1 + X1*X2(psplinerw2, center) + X1*Z2(psplinerw2, center)
```

is well-identified. Note that the main effect of X1 has to be included separately. Equivalently, we could absorb the main effect into the first term, yielding

```
Y = X1*X2(psplinerw2) + X1*Z2(psplinerw2, center)
```

However, the former specification has the advantage that the model terms are clearly separated.

Models of the type just discussed arise for example if X1 is a binary dummy-variable indicating two different groups of data. In this case the model

Y = X1 + X2(psplinerw2) + X1\*X2(psplinerw2, center) + Z2(psplinerw2) + X1\*Z2(psplinerw2, center) assumes different effects of both X2 and Z2 in the groups.

### Seasonal prior

### Description:

Defines a varying coefficients term where the effect of X1 varies over the range of the effect modifier time. A seasonal prior is assumed for the effect of time.

Predictor: 
$$\eta = \cdots + f_{season}(time)X1 + \cdots$$

Syntax:

X1\*time(season[, options])

Example:

The inclusion of a varying coefficients term with a seasonal prior may be meaningful if we expect a different seasonal effect with respect to a binary variable X1. In this case we can include an additional seasonal effects for observations with X1=1 by

```
Y = X1*time(season)
```

### Time-varying effects in continuous-time or multi-state models

### P-spline with second order random walk penalty

Description: Defines a varying coefficients term where the effect of X1 varies over the range of the effect modifier time, i.e. variable X1 is assumed to have a time-varying effect. The smoothness prior for f(time) is a P-spline with second order random walk penalty.

```
Predictor: \eta = \log(\lambda_0(time)) + f(time)X1\cdots
```

Syntax:

X1\*time(baseline[, options])

Example:

Suppose continuous-time survival data (time, delta) together with an additional covariate X1 are given, where time denotes the vector of observed duration times and delta is the vector of corresponding indicators of non-censoring. The following Cox model with hazard rate

$$\lambda(time) = \lambda_0(time) \exp(\gamma_0 + f(time)X1)$$
  
= 
$$\exp(\log(\lambda_0(time)) + \gamma_0 + f(time)X1)$$

is estimated by the model statement

```
delta = time(baseline) + X1*time(baseline)
```

Similarly, time-varying effects on the transition intensities in multi-state models can be specified.

### Varying coefficients with spatial covariates as effect modifiers

### Markov random field

Description:

Defines a varying coefficient term where the effect of X1 varies smoothly over the range of the spatial covariate region. A Markov random field is estimated for  $f_{spat}$ . The geographical information is stored in the map object specified through the option map.

```
Predictor: \eta = \cdots + f_{spat}(region)X1 + \cdots
```

Syntax:

X1\*region(spatial,map=characterstring[, options])

Example:

For example the statement

Y = X1\*region(spatial, map=germany)

defines a varying coefficient term with the spatial covariate **region** as the effect modifier and a Markov random field as spatial smoothness prior. Weighted Markov random fields can be estimated by including an appropriate weight definition when creating the *map object* **germany** (see section 5.1).

Similarly as for varying coefficient terms with continuous effect modifiers, varying coefficients with spatial effect modifier can be centered to avoid identifiability problems:

```
Y = X1*region(spatial, map=germany, center)
```

## Varying coefficients with unordered group indicators as effect modifiers (random slopes)

### Unit- or cluster specific unstructured effect

Description:

Defines a varying coefficient term where the effect of X1 varies over the range of the group indicator grvar. Models of this type are usually referred to as models with random slopes. A Gaussian i.i.d. random effect with respect to grouping variable grvar is assumed for f.

```
Predictor: \eta = \cdots + f(grvar)X1 + \cdots
```

Syntax:

X1\*grvar(random[, options])

Example:

For example, a random slope is specified as follows:

```
Y = X1*grvar(random)
```

Note, that in contrast to *bayesreg objects*, the main effects are *not* included automatically. If main effects should be included in the model, they have to be specified as additional fixed effects. The syntax for obtaining the predictor

```
\eta = \cdots + \gamma X 1 + f(grvar)X 1 + \cdots
would be
X1 + X1*grvar(random[, options])
```

### Surface estimators

### Two-dimensional P-spline with first order random walk penalty

Description:

Defines a two-dimensional P-spline based on the tensor product of one-dimensional P-splines with a two-dimensional first order random walk penalty for the parameters of the spline.

```
Predictor: \eta = \cdots + f(X1, X2) + \cdots
Syntax:

X1*X2(pspline2dimrw1[, options])

Example:
The model term
```

Y = X1\*X2(pspline2dimrw1)

specifies a tensor product cubic P-spline with first order random walk penalty.

In many applications it is favorable to additionally incorporate the one-dimensional main effects of X1 and X2 into the models. In this case the two-dimensional surface can be seen as the deviation from the main effects. Note, that in contrast to *bayesreg objects* the number of inner knots and the degree of the spline may be different for the main effects and for the interaction. For example, a model with 20 inner knots for the main effects and 10 inner knots for the two-dimensional P-spline is estimated by

```
Y = X1(psplinerw2,nrknots=20) + X2(psplinerw2,nrknots=20)
+ X1*X2(pspline2dimrw1,nrknots=10)
```

### Stationary Gaussian random field

Description:

Defines that the parameters of the locations follow a stationary Gaussian random field. Depending on the options chosen, locations are given either by the distinct pairs of X1 and X2 or by a subset of these pairs, which we will also refer to as knots. Note that in principle stationary Gaussian random fields can be used to estimate surfaces depending on arbitrary variables X1 and X2, but they are defined based on *isotropic* correlation functions. This means that correlations between sites that have the same distance also have the same correlation, regardless of direction and the sites location. Therefore, if Gaussian random fields shall be used to estimate interactions between variables that do not represent longitude and latitude, these variables have to be standardized appropriately.

```
Predictor: \eta = \cdots + f(X1, X2) + \cdots
Syntax:
X1*X2(\text{kriging}[, options])
```

Example:

The model term

```
Y = X1*X2(kriging,nrknots=100)
```

specifies a stationary Gaussian random field for the effect of X1 and X2 with 100 knots, which are computed based on the space filling algorithm mentioned in section 3.2 of the methodology manual. If all distinct pairs of X1 and X2 shall be used as knots, we have to specify

```
Y = X1*X2(kriging,full)
```

Note, that the knots computed by the space filling algorithm will be stored in a file in the outfile directory of the *remlreg object* (the file name will be printed in the output window with the estimated effects). These knots can be read into a *dataset object* which may be passed to the kriging term if we want to use the same knots as in previous calls:

```
dataset kn
kn.infile using knotfile
Y = X1*X2(kriging,knotdata=kn)
```

To determine the actual number of knots, the options are interpreted in a specific sequence. If option full is specified, both nrknots and knotdata are ignored. Similarly, nrknots is ignored if knotdata is specified.

## 8.1.1.3 Description of additional options for terms of remlreg objects

All arguments described in this section are optional and may be omitted. Generally, options are specified by adding the option name to the specification of the model term type in the parentheses, separated by commas. All options may be specified in arbitrary order. Table 8.3 provides explanations and the default values of all possible options. All reasonable combinations of model terms and options can be found in Table 8.4.

optionname	description	default
lambdastart	Starting value for the smoothing parameter $\lambda$ .	lambdastart=10
degree	Degree of B-spline basis functions.	degree=3
nrknots	Number of inner knots for a P-spline term or number of knots for a	nrknots=20 (P-splines)
	kriging term.	nrknots=100 (kriging)
knotdata	Dataset object containing the knots to be used with the kriging term	no default.
full	Specifies that all distinct locations should be used as knots in the	-
	kriging term.	
nu	The smoothness parameter $\nu$ of the Matèrn correlation function for	nu=1.5
	kriging terms.	
maxdist	Specifies the value $c$ that is used to determine the scale parameter $\rho$	default depends on nu
	of the Matèrn correlation function for kriging terms. Compare section	
	3.2 of the methodology manual.	
р	Parameter $p$ of the coverage criterion for the space filling algorithm	p=-20
	that determines the knots of a kriging term.	00
q	Parameter $q$ of the coverage criterion for the space filling algorithm	q=20
	that determines the knots of a kriging term.	
maxsteps	Maximum number of steps to be performed by the space filling algorithm.	maxsteps=300
mmi dahai aa	How to choose grid points for numerical integration in Cox and multi-	gridchoice=quantiles
gridchoice	state models. May be either 'quantiles', 'equidistant' or 'all'.	gridchoice-quantiles
tgrid	Number of equidistant time points to be used for numerical in-	tgrid=100
Ugila	tegration in Cox and multi-state models. Only meaningful if	ugiiu-100
	gridchoice=equidistant.	
nrquantiles	Number of quantiles that are used to define the grid points for nu-	nrquantiles=50
quairozzo	merical integration in Cox and multi-state models. First a grid of	quantities of
	nrquantiles quantiles is computed, then the grid for integration is	
	defined by nrbetween equidistant points between each quantile. Only	
	meaningful if gridchoice=quantiles.	
nrbetween	Number of points between quantiles that are used to define the grid	nrbetween=5
	points for numerical integration in Cox and multi-state models. First	
	a grid of nrquantiles quantiles is computed, then the grid for integra-	
	tion is defined by nrbetween equidistant points between each quantile.	
	Only meaningful if gridchoice=quantiles.	
map	Map object for spatial effects.	no default
period	Period of a seasonal effect. The default (period=12) corresponds to	period=12
	monthly data.	
catspecific	Requests that the corresponding effect should be modelled category-	-
	specific. Can only be used in cumulative and sequential models for	
	categorical responses, i.e. with response families cumlogit, cumprobit,	
	seqlogit and seqprobit.	
center	For varying coefficient terms this option requests that the effect should	-
	be centered to avoid identifiability problems	

Table 8.3: Optional arguments for remlreg object terms.

## 8.1.1.4 Specifying the response distribution

Supported univariate distributions are Gaussian, binomial (with logit, probit or cumulative log-log link), Poisson and gamma. For multivariate responses, *BayesX* supports multinomial logit models

	rw1/rw2	season	psplinerw1/psplinerw2	spatial	random	geospline	pspline2dimrw1	kriging	geokriging	baseline	
${\tt lambdastart}^*$	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	realvalue	
degree	×	×	integer	×	×	integer	integer	×	×	integer	
nrknots	×	×	integer	×	×	integer	integer	integer	×	integer	
knotdata	×	×	×	×	×	×	×	dataset object	dataset object	×	
full	×	×	×	×	×	×	×	Δ	Δ	×	
nu	×	×	×	×	×	×	×	•	•	×	
$\mathtt{maxdist}^*$	×	×	×	×	×	×	×	realvalue	realvalue	×	
p**	×	×	×	×	×	×	×	realvalue	realvalue	×	
q*	×	×	×	×	×	×	×	realvalue	realvalue	×	
maxsteps	×	×	×	×	×	×	×	integer	integer	×	
gridchoice	×	×	×	×	×	×	×	×	×	0	
tgrid	×	×	×	×	×	×	×	×	×	integer	
nrquantiles	×	×	×	×	×	×	×	×	×	integer	
nrbetween	×	×	X	×	×	×	×	×	×	integer	
period	×	integer	X	×	×	×	×	×	×	×	
map	×	×	X	map object	×	$map\ object$	×	×	$map\ object$	×	
catspecific	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ	×	
center	Δ	$\triangle$	$\triangle$	Δ	×	Δ	×	×	Δ	×	
*	positive va	positive values only									
**	negative values only										
×	not availal	not available									
•		admissible values are 0.5,1.5,2.5,3.5									
Δ	available a	s boolean o	ption (specified without s	upplying a va	lue)						
0	admissible	values are	quantiles, equidistant a	and all	•						

 ${\it Table~8.4:~Terms~and~options~for~remlreg~objects.}$ 

for categorical responses with unordered categories and cumulative as well as sequential logit and probit models for categorical responses with ordered categories. Continuous survival times as well as multi-state models can be analysed based on semiparametric models with Cox-type hazard rates. An overview over the supported models is given in Table 8.5. The distribution of the response is specified by adding the additional option family to the (global) options list of the regression call. For instance, family=gaussian defines the response to be Gaussian distributed. In some cases, one or more additional options associated with the specified response distribution can be specified. An example is the reference option for multinomial responses, which defines the reference category. In the following we give detailed instructions on how to specify the various models.

value of family	response distribution	link	options
family=gaussian	Gaussian	identity	
family=binomial	binomial	logit	
family=binomialprobit	binomial	probit	
family=binomialcomploglog	binomial	complementary	
		log-log	
family=multinomial	unordered multinomial	logit	reference
family=multinomialcatsp	unordered multinomial (with	logit	reference
	category-specific covariates)		
family=cumprobit	cumulative multinomial	probit	
family=cumlogit	cumulative multinomial	logit	
family=seqprobit	sequential multinomial	probit	
family=seqlogit	sequential multinomial	logit	
family=poisson	Poisson	log	
family=gamma	gamma	log	
family=cox	continuous-time survival data		leftint,
			lefttrunc
family=multistate	continuous-time multi-state		state,
	data		lefttrunc

Table 8.5: Summary of supported response distributions.

### Gaussian responses

For Gaussian responses BayesX assumes  $y_i|\eta_i, \sigma^2 \sim N(\eta_i, \sigma^2/weightvar_i)$  or, equivalently, in matrix notation  $y|\eta, \sigma^2 \sim N(\eta, \sigma^2C^{-1})$ , where  $C = diag(weightvar_1, \dots, weightvar_n)$  is a known weight matrix. Gaussian regression models are obtained by adding

### family=gaussian

to the options list.

An optional weight variable weightvar can be specified to estimate weighted regression models, see subsubsection 7.1.2.1 for details. For grouped Gaussian responses, the weights represent the number of observations in the groups if the  $y_i$ 's are the average of individual responses. If the  $y_i$ s are the sum of responses in every group, the weights are given by the reciprocal of the number of observations in the groups. Of course, estimation of usual weighted regression models with heteroscedastic errors is also possible. In this case, the weights should be proportional to the reciprocal of the heteroscedastic variances. If no weight variable is specified, BayesX assumes  $weightvar_i = 1, i = 1, ..., n$ .

### Binomial logit, probit and complementary log-log models

A binomial logit model is requested by the option

family=binomial

while a probit model is obtained with

family=binomialprobit

and a complementary log-log model with

### family=binomialcomploglog

A additional weight variable may be specified, see subsection 8.1.1 for the syntax. BayesX assumes that the weight variable corresponds to the number of replications and the response variable to the number of successes. If the weight variable is omitted, BayesX assumes that the number of replications is one, i.e. the values of the response must be either zero or one.

### Multinomial logit models

So far, remlreg objects support only multinomial logit models and no probit models. A multinomial logit model without category-specific covariates is specified by adding the option

family=multinomial

to the options list.

If there are category-specific covariates, the option has to be altered to

## family=multinomialcatsp

Category-specific covariates are included as follows: Suppose that covariate **x** has been observed for a response variable with the three categories 1, 2 and 3. Then you have to include the three variable **x1**, **x2** and **x3** into your dataset. Within the regression syntax, you have to specify

## x\_catspecific

to request a parametric effect of x

x\_catspecific(psplinerw2)

for a nonparametric effect of x and

### x\_catspecific\*id(psplinerw2)

to obtain a random effect of x with respect to the grouping variable id. Currently BayesX only supports these three term types for effects of category-specific effects.

For both family=multinomial and family=multinomialcatsp a second option (reference) can be added to the options list to define the reference category. If the response variable has three categories 1, 2 and 3, the reference category can be set to 2, by adding

### reference=2

to the options list. If the option is omitted, the *smallest* number will be used as the reference category.

If some categories are not available for some observations, BayesX can account for this by including either category-specific offsets or non-availability indicators. Suppose again that the response has the three categories 1, 2 and 3. Then offset terms o1, o2 and o3 can be used to account for varying choice sets. If a category is available, the offset is simply set to zero, while a large negative value (i.e. -1000) has to be assigned to the offset term of categories which are not available. To account for the offset term,

### o\_catspecific(offset)

has to be added to the model specification. Of course, you can also assign different values to the

offsets, e.g. to account for a priori differences in the availability of some categories.

The usage of offset terms to account for non-availability may in some cases be numerically unstable (e.g. if several categories are not available or if the reference category is not available). Therefore an alternative possibility is to include non-availability indicators na1, na2 and na3. Each of the indicators is assigned the value one if the corresponding category is not available and zero otherwise. Within the regression syntax, the non-availability indicator has to be specified as a global option, i.e.

..., family=multinomialcatsp naindicator=na\_catspecific

### Cumulative logit and probit models

A cumulative logit model is specified by adding

family=cumlogit

to the options list, a cumulative probit is obtained by

family=cumprobit

In both cases, the reference category will always be the largest value of the response.

Note, that in contrast to *bayesreg objects remlreg objects* can deal with an arbitrary number of ordered categories. However, for more than about 5 categories estimation may become rather computer intensive and time demanding (depending on the size of your data set).

By default, all effects in cumulative logit and probit models are considered to be defined globally. To obtain category-specific effects, the additional keyword catspecific has to be specified. For example, the specification of the predictor

```
Y = W1(catspecific) + W2 + X1(psplinerw2, catspecific) + X2(psplinerw2)
```

requests category-specific effects for the covariates W1 and X1, and global effects for the covariates W2 and X2. Note that complicated ordering restrictions have to be fulfilled for the covariate-dependent thresholds defined implicitly by category-specific effects. Therefore numerical problems are likely to be observed in models with sparse data or a lot of category-specific effects.

## Sequential logit and probit models

A sequential logit model is specified by adding

family=cumlogit

to the options list, while a sequential probit can be requested by

family=cumprobit

The reference category will always be the largest value of the response.

Similar as in cumulative models, all effects in sequential logit and probit models are considered to be defined globally by default. To obtain category-specific effects, the additional keyword catspecific has to be specified. For example, the specification of the predictor

```
Y = W1(catspecific) + W2 + X1(psplinerw2, catspecific) + X2(psplinerw2)
```

requests category-specific effects for the covariates W1 and X1, and global effects for the covariates W2 and X2. In contrast to cumulative models no ordering restrictions are imposed in sequential models.

### Poisson regression

A Poisson regression model is specified by adding

## family=poisson

to the options list.

A weight variable may be specified in addition, see subsection 8.1.1 for the syntax. For grouped Poisson data, the weights must be the number of observations in a group and the responses are assumed to be the average of individual responses.

### Gamma distributed responses

In the literature, the density function of the gamma distribution is parameterized in various ways. In the context of regression analysis, the density is usually parameterized in terms of the mean  $\mu$  and the scale parameter s. Then, the density of a gamma distributed random variable y is given by

$$p(y) \propto y^{s-1} \exp(-\frac{s}{\mu}y) \tag{8.1}$$

for y > 0. For the mean and the variance we obtain  $E(y) = \mu$  and  $Var(y) = \mu^2/s$ . We write  $y \sim G(\mu, s)$ .

A second parameterization is typically employed for hyperparameters a and b of priors for variance parameters in the context of Bayesian hierarchical models. In this case, the density is given by

$$p(y) \propto y^{a-1} \exp(-by) \tag{8.2}$$

for y > 0. In this parameterization we obtain E(y) = a/b and  $Var(y) = a/b^2$  for the mean and the variance, respectively. We write  $y \sim G(a, b)$ 

In BayesX a gamma distributed response variable is parameterised in the first form (8.1). For the rth observation BayesX assumes  $y_r|\eta_r, \nu \sim G(\exp(\eta_r), \nu/weightvar_r)$  where  $\mu_r = \exp(\eta_r)$  is the mean and  $s = \nu/weightvar_r$  is the scale parameter. A gamma distributed response is specified by adding

### family=gamma

to the options list. An optional weight variable *weightvar* can be specified to estimate weighted regression models, see subsection 8.1.1 for the syntax.

### Continuous time survival analysis

Bayes X offers two alternatives of estimating continuous time survivals models with semiparametric predictor  $\eta$ , both of which are described in subsection 5.2 of the methodology manual. The first alternative is to assume that all time-dependent values are piecewise constant, leading to the so called *piecewise exponential model* (p.e.m.). The second alternative is to estimate the log-baseline effect  $\log(\lambda_0(t)) = f_0(t)$  based on a P-spline with second order random walk penalty.

## Piecewise exponential model (p.e.m.)

In subsection 5.2 of the methodology manual we demonstrated how continuous time survival data has to be manipulated to transform it to a Poisson for model estimation. Suppose that the following modified data set is available

У	indnr	a	δ	$\Delta$	x1	<b>x</b> 2
0	1	0.1	1	$\log(0.1)$	0	3
0	1	0.2	1	$\log(0.1)$	0	3
1	1	0.3	1	$\log(0.05)$	0	3
0	2	0.1	0	$\log(0.1)$	1	5
0	2	0.2	0	$\log(0.02)$	1	5
:	:	:		:	:	:

with indicator y, interval limit a, indicator of non-censoring  $\delta$  and offset  $\Delta$  defined as in subsection 5.2 of the methodology manual. Let x1 be a covariate with linear effect and x2 a continuous covariate with nonlinear effect. Then the correct syntax for estimating a p.e.m. with a remlreg object named r is e.g. as follows:

- > r.regress y = a(rw1) + Delta(offset) + x1 + x2(psplinerw2), family=poisson...
  or
- > r.regress y = a(rw2) + Delta(offset) + x1 + x2(psplinerw2), family=poisson...

Note that a time-varying effect of an additional covariate X may be estimated by simply adding the term

X\*a(rw1) or X\*a(rw2)

to the model statement.

### Specifying a P-spline prior for the log-baseline

For a continuous time survival model with a P-spline prior with second order random walk penalty for the baseline effect,

### family=cox

has to be specified in the options list. The number of knots and degree of the P-spline prior for  $f_0(t)$  can be specified as additional options for the baseline term. Note that it is obligatory that there is a baseline term specified for the vector of observed duration times. The indicator of non-censoring  $\delta_i$  has to be specified as the dependent variable in the model statement. Data augmentation and the specification of an offset term are not required here. In the example above with survival data

t	δ	x1	x2
0.25	1	0	3
0.12	0	1	5
:	:	:	:

a continuous time survival model with a quadratic P-spline prior with 15 knots for the log-baseline would be estimated as follows:

```
> r.regress delta = t(baseline,degree=2,nrknots=15)+ x1 + x2(psplinerw2),
family=cox...
```

Again a time-varying effect of a covariate X can be estimated by simply adding the term

## X\*time(baseline)

to the model statement.

## Interval censoring and left truncation

Interval censoring and left truncation can be incorporated using the additional options leftint and lefttrunc of remlreg objects. These two variables represent the lower interval boundary  $T_{lo}$  and the left truncation time  $T_{tr}$  as discussed in section 5.3 of the methodology manual. The time variable specified in the baseline statement corresponds to  $T_{up}$ , the upper boundary of the interval. In general an observation can now be described completely by the quadruple  $(T_{tr}, T_{lo}, T_{up}, \delta)$ , with

```
T_{lo} = T_{up}, \ \delta = 1 if the observation is uncensored,

T_{lo} = T_{up}, \ \delta = 0 if the observation is right censored,

T_{lo} < T_{up}, \ \delta = 0 if the observation is interval censored.
```

For left truncated observations we have  $T_{tr} > 0$  while  $T_{tr} = 0$  for observations which are not truncated.

An example for a statement that estimates a model with left truncation and interval censoring is given by

```
> r.regress delta = tup(baseline)+ x1 + x2(psplinerw2), family=cox
lefttrunc=ttr leftint=tlo...
```

### Continuous time multi-state models

Multi-state models describe the temporal development of discrete phenomena in continuous time based on transition intensities for each of the observable transition types. Consider for example a multi-state model for human sleep as depicted in Figure 8.1 and that the transition intensities for the four possible transitions are specified as

$$\begin{split} \lambda_{AS,i}(t) &= \exp\left[g_0^{(AS)}(t) + b_i^{(AS)}\right], \\ \lambda_{SA,i}(t) &= \exp\left[g_0^{(SA)}(t) + b_i^{(SA)}\right], \\ \lambda_{NR,i}(t) &= \exp\left[g_0^{(NR)}(t) + c_i(t)g_1^{(NR)}(t) + b_i^{(NR)}\right] \\ \lambda_{RN,i}(t) &= \exp\left[g_0^{(RN)}(t) + c_i(t)g_1^{(RN)}(t) + b_i^{(RN)}\right] \end{split}$$

Each of the transitions is parameterised in terms of a baseline effect  $g_0^{(h)}(t)$  and a transition specific frailty term (random effect)  $b_i^{(h)}$ . In addition, time-varying effects  $g_1^{(h)}(t)$  of binary indicators  $c_i(t)$  for a high blood level of cortisol are introduced for the transitions between REM and Non-REM. The corresponding data set should be arranged as follows:

id	st	beg	end	tas	tsa	$\operatorname{trn}$	tnr	cort	corthigh
1	2	0	1	0	1	0	0	52.6	0
1	1	1	5	1	0	0	0	52.6	0
1	2	5	8	0	1	0	0	52.6	0
1	1	8	10	1	0	0	0	52.6	0
1	2	10	36	0	0	0	0	52.6	0
1	2	36	76	0	0	0	0	46.9	0
1	2	76	108	0	0	0	1	47.5	0
1	3	108	109	0	0	1	0	47.5	0
1	2	109	110	0	0	0	1	47.5	0
1	3	110	111	0	0	1	0	47.5	0

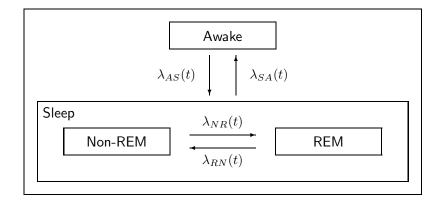


Figure 8.1: Schematic representation of sleep stages and transitions of interest.

1	2	111	115	0	0	0	1	47.5	0
1	3	115	116	0	0	0	0	47.5	0
1	3	116	126	0	0	1	0	37.4	0
•	•				•	•	•	•	
•	•	•	•	•	•	•	•	•	•
		•						•	
2	2	0	12	0	1	0	0	22.5	0
2	1	12	15	1	0	0	0	22.5	0
2	1	15	28	0	1	0	0	88.6	1
•	•	•	•		•	•	•	•	
•	•	•	•	•	•	•	•	•	•

Each path observed for the multi-state model is transformed into several lines in the data set, where id identifies the original paths. In the above example, parts of the first two observations are displayed. Each line of the data set represents a time interval identified by the variables beg and end. Variable st indicates the current state of the process. Note that the states have to be numbered consecutively from 1 to H. Since we are considering continuous time scales, an observation should start at t=0 (unless the observation is left truncated) and the variables beg and end should be generated so that within each observation process beg equals the value of end in the previous row (unless observations are fragmentary only).

The variables tas, tsa, trn and tnr are binary indicators for the four transitions sleep  $\rightarrow$  awake (tsa), awake  $\rightarrow$  sleep (tas), Non-REM  $\rightarrow$  REM (tnr) and REM  $\rightarrow$  Non-REM (trn). Such an indicator equals one if the corresponding transition is observed at the end of the interval and zero otherwise. Note that there are lines in the data set, where none of the transitions is observed. These correspond to intervals where the value of the time-varying covariate cort (cortisol-level) changes. The variable corthigh is a dichotomized version of cort which indicates a high level of cortisol (cort>60).

The model specified above is estimated by entering the following command

### family=multistate lefttrunc=beg state=st using sleep

Note that a separate model equation has to specified for each transition with the binary transition indicator as response. Instead of method regress, method mregress has to be called since multiple model equations are combined. The right and the left boundary of the time intervals have to specified as covariate for the baseline effect and as global option lefttrunc, respectively. Similarly, the state variable has to specified via the global option state.

## 8.1.2 Options

## Options for controlling the estimation process

## $\bullet$ eps = realvalue

Defines the termination criterion of the estimation process. If both the relative changes in the regression coefficients and the variance parameters are less than eps, the estimation process is assumed to have converged.

DEFAULT: eps = 0.00001

### • lowerlim = realvalue

Since small variances are close to the boundary of their parameter space, the usual Fisher-scoring algorithm for their determination has to be modified. If the fraction of the penalized part of an effect relative to the total effect is less than lowerlim, the estimation of the corresponding variance is stopped and the estimator is defined to be the current value of the variance (see section 4.2 of the methodology manual for details).

DEFAULT: lowerlim = 0.001

### • maxit = integer

Defines the maximum number of iterations to be used in estimation. Since the estimation process will not necessarily converge, it may be useful to define an upper bound for the number of iterations. Note, that BayesX returns results based on the current values of all parameters even if no convergence could be achieved within maxit iterations, but a warning message will be printed in the  $output\ window$ .

DEFAULT: maxit=400

### • maxchange = realvalue

Defines the maximum value that is allowed for relative changes in parameters in one iteration to prevent the program from crashing because of numerical problems. Note, that BayesX produces results based on the current values of all parameters even if the estimation procedure is stopped due to numerical problems, but an error message will be printed in the output window.

DEFAULT: maxchange=1000000

### Options for the analysis of survival times and multi-state models

### • leftint = variable name

Gives the name of the variable that contains the lower (left) boundary  $T_{lo}$  of the interval  $[T_{lo}, T_{up}]$  for an interval censored observation. For right censored or uncensored observations we have to specify  $T_{lo} = T_{up}$ . If leftint is missing, all observations are assumed to be right censored or uncensored, depending on the corresponding value of the censoring indicator.

### • lefttrunc = variablename

Option lefttrunc specifies the name of the variable containing the left truncation time  $T_{tr}$ .

For observations that are not truncated, we have to specify  $T_{tr} = 0$ . If lefttrunc is missing, all observations are assumed to be not truncated. For multi-state models variable lefttrunc specifies the left endpoint of the corresponding time interval (compare page 123).

• state = variablename

For multi-state models, state specifies the current state of the process (compare page 123).

### **Further options**

### $\bullet$ level1 = integer

Besides the posterior mode, regress provides (approximate) pointwise posterior credible intervals for every effect in the model. By default, BayesX computes credible intervals for nominal levels of 80% and 95%. The option level1 allows to redefine one of the nominal levels (95%). Adding, for instance,

#### level1=99

to the options list leads to the computation of credible intervals for a nominal level of 99% rather than 95%.

### • level2 = integer

Besides the posterior mode, regress provides (approximate) pointwise posterior credible intervals for every effect in the model. By default, BayesX computes credible intervals for nominal levels of 80% and 95%. The option level2 allows to redefine one of the nominal levels (80%). Adding, for instance,

### leve12=70

to the options list leads to the computation of credible intervals for a nominal level of 70% rather than 80%.

## 8.1.3 Estimation output

The way the estimation output is presented depends on the estimated model. Estimation results for fixed effects are displayed in a tabular form in the *output window* and/or in a log file (if created before). This table will contain the posterior mode, the standard deviation, p-values and an approximate 95% credible interval. Other credible intervals may be obtained by specifying the level1 option, see subsection 8.1.2 for details. Additionally, a file replicating results for the fixed effects is created. The name of this file is supplied in the *output window* and/or in a log file.

Estimated nonparametric effects are presented in a different way. Here, results are stored in external ASCII-files that can be read into any general purpose statistics program (e.g. STATA, R, S-plus) to further analyze and/or visualize the results. The structure of these files is as follows: There will be one file for every nonparametric effect in the model. The names of the files and the storing directory are displayed in the *output window* and/or a log file. The files contain ten columns (for main effects) or eleven columns (for interaction effects). The first column contains a parameter index (starting with one), the second column (and the third column if the estimated effect is an interaction) contain the values of the covariate(s) whose effect has been estimated. In the following columns the estimation results are given in form of the posterior mode, the lower boundaries of the (approximate) 95% and 80% credible intervals, the standard deviation and the upper boundaries of the 80% and 95% credible intervals. The last two columns contain approximations to the posterior probabilities based on nominal levels of 95% and 80%. A value of 1 corresponds to a strictly positive 95% or 80% credible interval while a value of -1 to a strictly negative credible interval. A value of 0 indicates that the corresponding credible interval contains zero. Other credible intervals

and posterior probabilities may be obtained by specifying the level1 and/or level2 option, see subsection 8.1.2 for details. As an example, compare the following lines, which are the beginning of a file containing the results for a nonparametric effect of a particular covariate, x say:

```
intnr x pmode ci95lower ci80lower std ci80upper ci95upper pcat95
  -2.87694 -0.307921 -0.886815 -0.686408 0.295295
                                                        0.070567
                                                                  0.270973 \quad 0
  -2.86203 -0.320479
                        -0.885375
                                  -0.689815
                                             0.288154
                                                        0.0488558
   -2.8515 -0.329367 -0.88473 -0.69247 0.283292 0.0337362 0.225997
                        -0.884692 -0.692689 0.282913
  -2.85066
             -0.330072
                                                        0.0325457
  -2.82295
           -0.3535 -0.884544 -0.700703 0.270887 -0.00629671 0.177545
            -0.37418 -0.886192 -0.708939 0.261178 -0.0394208
  -2.79856
                                                                  0.137832
  -2.79492 \quad -0.377272
                        -0.886579 -0.710263 0.259798 -0.0442813 0.132035
  -2.79195
            -0.379788
                        -0.886921 -0.711358 0.258689 -0.0482183 0.127345
9 \quad -2.78837 \quad -0.382834 \quad -0.887367 \quad -0.712704 \quad 0.257363 \quad -0.0529641 \quad 0.1217 \quad 0 \quad -1
```

Note that the first row of the files always contains the names of the columns.

The estimated nonlinear effects can be visualized using either the graphics capabilities of *BayesX* or a couple of R and S-plus functions, see section 9.1 and section 9.2, respectively. Of course, any other (statistics) software package with plotting facilities can be used as well.

Estimation results for the variances and the smoothing parameters of nonparametric effects are printed in the *output window* and/or a log file. Additionally, a file is created containing the same information. For example, the file corresponding to the nonparametric effect presented above contains:

```
 \begin{array}{cccc} \text{variance} & \text{smoothpar} & \text{stopped} \\ 0.0492324 & 20.3118 & 0 \end{array}
```

The value in the last row indicates whether the estimation of the variance has been stopped before convergence. A value of 1 corresponds to a 'stopped' variance.

## 8.1.4 Examples

Here we give only a few examples about the usage of method regress. A more detailed, tutorial like example can be found in chapter 2 of the tutorial manual.

Suppose that we have a data set test with a binary response variable y, and covariates x1, x2, x3, t and region, where t is assumed to be a time scale measured in months and region indicates the geographical region an observation belongs to. Suppose further that we have already created a remlreg object r.

### Fixed effects

We first specify a model with y as the response variable and fixed effects for the covariates x1, x2 and x3. Hence the predictor is

$$\eta = \gamma_0 + \gamma_1 x 1 + \gamma_2 x 2 + \gamma_3 x 3$$

This model is estimated by typing:

```
> r.regress y = x1 + x2 + x3, family=binomial using test
```

By specifying option family=binomial, a binomial logit model is estimated. A probit model can be obtained by specifying family=binomialprobit.

### Additive models

Suppose now that we want to allow for possibly nonlinear effects of x2 and x3. Defining cubic P-splines with second order random walk penalty as smoothness priors, we obtain

> r.regress y = x1 + x2(psplinerw2) + x3(psplinerw2), family=binomial using test which corresponds to the predictor

$$\eta = \gamma_0 + \gamma_1 x 1 + f_1(x2) + f_2(x3).$$

If the response is not binary but categorical with unordered categories 1, 2 and 3, we can estimate a multinomial logit model by typing:

```
> r.regress y = x1 + x2(psplinerw2) + x3(psplinerw2), family=multinomial
  reference=2 using test
```

In this case, family=binomial has to be altered to family=multinomial, and the option reference=2 was added to define the value 2 as the reference category.

### Time scales

In the next step we extend the model by incorporating an additional trend and a flexible seasonal component for the time scale t:

```
> r.regress y = x1 + x2(psplinerw2) + x3(psplinerw2) +
t(psplinerw2) + t(season,period=12), family=binomial using test
```

Note that we passed the period of the seasonal component as a second argument.

## **Spatial covariates**

To incorporate a structured spatial effect, we have to create a *map object* first. Afterwards we read the boundary information of the different regions (polygons that form the regions, neighbors etc.). If you are unfamiliar with *map objects* please read chapter 5 first.

```
> map m
```

```
> m.infile using c:\maps\map.bnd
```

Since we usually need the map again in further sessions, we store it in *graph file* format, because reading *graph files* is much faster than reading *boundary files*.

```
> m.outfile , graph using c:\maps\mapgraph.gra
```

We can now augment our predictor with a spatial effect:

```
> r.regress y = x1 + x2(psplinerw2) + x3(psplinerw2) + t(psplinerw2)
+ t(season,period=12) + region(spatial,map=m), family=binomial using test
```

In some situations it may be reasonable to incorporate an additional unstructured random effect into the model in order to split the total spatial effect into a structured and an unstructured component. This is achieved by

```
> r.regress y = x1 + x2(psplinerw2) + x3(psplinerw2) + t(psplinerw2)
+ t(season,period=12) + region(spatial,map=m) + region(random),
family=binomial using test
```

## 8.2 Global options

The purpose of global options is to affect the global behavior of a *remlreg object*. The main characteristic of global options is, that they are not associated with a certain method.

The syntax for specifying global options is

objectname.optionname = newvalue

where *newvalue* is the new value of the option. The type of the value depends on the respective option.

Currently only one global option is available for remlreg objects:

### • outfile = filename

By default, the estimation output produced by the **regress** procedure will be written to the default output directory, which is

```
<\!INSTALLDIRECTORY\!>\!\!\setminusoutput
```

The default file name is composed of the name of the *remlreg object* and the type of the file. For example, if you estimated a nonparametric effect for a covariate X, say, using a P-spline, then the estimation output will be written to

```
<INSTALLDIRECTORY>\output\r_f_X_pspline.res
```

where r is the name of the *remlreg object*. In most cases, however, it may be necessary to save estimation results into a different directory and/or under a different file name than the default. This can be achieved using the outfile option. Here, you have to specify the directory where the output should be stored and a base file name. This base file name should not be a complete file name. For example specifying

```
outfile = c:\data\res1
```

would cause BayesX to store the estimation result for the nonparametric effect of X in file c:\data\res1\_f\_X\_pspline.res

## 8.3 Visualizing estimation results

Visualization of estimation results is described in chapter 9

## 8.4 References

- Fahrmeir, L., Kneib, T. and Lang, S. (2004): Penalized structured additive regression for space-time data: A Bayesian perspective. *Statistica Sinica*, 14, 715-745.
- Fahrmeir, L. and Tutz, G. (2001): Multivariate Statistical Modelling based on Generalized Linear Models. New York: Springer-Verlag.
- Green, P.J. and Silverman, B. (1994): Nonparametric Regression and Generalized Linear Models. Chapman and Hall, London.
- HASTIE, T. AND TIBSHIRANI, R. (1990): Generalized additive models. Chapman and Hall, London.
- Hastie, T. and Tibshirani, R. (1993): Varying-coefficient Models. *Journal of the Royal Statistical Society B*, 55, 757-796.

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Hastie, T., Tisbshirani, R. and Friedman, J. (2001): The Elements of Statistical Learning: Data Mining, Inference and Prediction. New York: Springer-Verlag.

- Kneib, T. (2006): Geoadditive hazard regression for interval censored survival times. Computational Statistics and Data Analysis, 51, 777-792
- Kneib, T. and Hennerfeind, A. (2006) Bayesian Semiparametric Multi-State Models. Under revision for *Statistical Modelling*. A preliminary version is available as SFB 386 Discussion Paper 502.
- Kneib, T. and Fahrmeir, L., (2006): Structured additive regression for categorical space-time data: A mixed model approach. *Biometrics*, 62, 109-118.
- Kneib, T. and Fahrmeir, L., (2007): A mixed model approach to structured hazard regression. Scandinavian Journal of Statistics, 34, 207-228..
- Brezger, A. (2000): Bayesianische P-splines. Master thesis, University of Munich.
- LIN, X. AND ZHANG, D. (1999): Inference in generalized additive mixed models by using smoothing splines. *Journal of the Royal Statistical Society B*, 61, 381–400.
- McCullagh, P. and Nelder, J.A. (1989): Generalized Linear Models. Chapman and Hall, London.

# Chapter 9

# Visualizing estimation results

In this chapter we show, how estimation results produced with one of the regression tools described in the two previous chapters can be visualized. In general, there are two possibilities to visualize results: Within BayesX, special functions can be applied to regression objects. Since both regression tools provide almost the same possibilities to visualize results, we describe them simultaneously in the next section. Tools for the visualization of autocorrelations for MCMC samples are described in subsection 9.1.3. An alternative way to visualize results is to use the R / S-Plus functions shipped together with BayesX. These functions are described in section 9.2.

## 9.1 BayesX functions

BayesX allows to visualize estimation results immediately after estimation. The output window and/or the log file describe how to do this for a particular model term. Nonlinear effects of continuous covariates and time scales are plotted with method plotnonp. Spatial effects are visualized with method drawmap. When using bayesreg objects, autocorrelation functions of sampled parameters can be visualized with method plotautocor.

## 9.1.1 Method plotnonp

## Description

Method plotnonp is a post estimation command, i.e. it is meaningful only if method regress has been applied before. The method allows to plot estimated effects of nonlinear covariate effects immediately after estimation. T

## **Syntax**

> objectname.plotnonp termnumber [, options]

Plots the estimated effect with term number termnumber. The term number will be printed in the output window and/or an open log file. Several options are available for labelling axis, adding a title, etc., see the options list below. Note that method plotnonp can be applied only if random walks, P-splines or seasonal components are used as priors.

## **Options**

The following options are available for method plotnonp (listed in alphabetical order):

• connect=1|2|3|4|5[specifications for further variables]

Option connect specifies how points in the scatterplot are connected. There are currently 5 different specifications:

- draw straight lines between the points (default)
- 2, 3, 4 draw dashed lines (numbers 2-4 indicate different variants)
- 5 do not connect, i.e. plot points only

If you draw more than one scatterplot in the same graph (i.e. more than one *yvar* is specified) you can connect points for every *yvar* differently by simply specifying the corresponding number (1,2,3,4,5) for every *yvar*. Typing for example

```
connect=15
```

connects the points corresponding to yvar1 and xvar by straight lines, but does not connect the points corresponding to yvar2 (if specified) and xvar. Points corresponding to additionally specified variables yvar3, etc. are connected by straight lines.

An equivalent way of specifying the different variants is available via the symbols 1, d,  $\setminus$ \_, - and p, which correspond to the numbers 1-5, i.e.

connect=12345 is equivalent to connect=ld\_-p

• fontsize = integer

Specifies the font size (in pixels) for labelling axes etc. Note that the title is scaled accordingly. The default is fontsize=12.

 $\bullet$  height = integer

Specifies the height (in pixels) of the graph. The default is height=210.

• levels = all|1|2|none

By default, plotnonp plots the estimated nonlinear covariate effect together with the pointwise credible intervals based on nominal levels of 80% and 95% (the nominal levels may be changed using the options level1 and/or level2). Option levels allows to omit completely pointwise credible intervals in the graphs (levels=none), print only the 95% credible intervals (levels=1) or to print only the 80% credible intervals (levels=2).

## • linecolor = B|b|c|G|g|o|m|r|y [specifications for further variables]

Option linecolor specifies the color to be used for drawing lines (or points, see option connect) in the scatterplot. Currently the following specifications are available:

- B black (default)
- b blue
- c cyan
- G gray
- g green
- o orange
- m magenta
- r red
- y yellow

If you draw more than one scatterplot in the same graph (i.e. more than one *yvar* is specified) you can use different colors for each *yvar* by simply specifying the corresponding symbol (B,b,c,G,g,o,m,r,y) for each *yvar*. Typing for example

### linecolor = Bgr

colors the lines (points) corresponding to yvar1 and xvar in black, whereas the points corresponding to yvar2 and yvar3 (if specified) and xvar are colored in green and red, respectively.

## • linewidth = integer

Specifies how thick lines should be drawn. The default is linewidth=5.

## • outfile = characterstring

If option outfile is specified the graph will be stored as a postscript file rather than being printed on the screen. The path and the filename must be specified in *characterstring*. By default, an error will be raised if the specified file is already existing or the specified folder is not existing. To overwrite an already existing file, option replace must be additionally specified. This prevents you from unintentionally overwriting your files.

### • pointsize = integer

Specifies the size of the points (in pixels) if drawing points rather than lines is specified. The default is pointsize=20.

### • replace

The replace option is useful only in combination with option outfile. Specifying replace as an additional option allows the program to overwrite an already existing file (specified in outfile), otherwise an error will be raised.

### • title = characterstring

Adds a title to the graph. If the title contains more than one word, *characterstring* must be enclosed by quotation marks (e.g. title="my first title").

### • titlesize = realvalue

Specifies the factor by which the size of the title is scaled relative to the size of the labels of the axes (compare option fontsize). The default is titlesize=1.5.

### • width = integer

Specifies the width (in pixels) of the graph. The default is width=356.

### • xlab = characterstring

Labels the x-axis. If the label contains more than one word, *characterstring* must be enclosed by quotation marks (e.g. xlab="x axis").

### $\bullet$ xlimbottom = realvalue

Specifies the minimum value at the x-axis to be plotted. The default is the minimum value in the data set. If xlimbottom is above the minimum value in the data set, only a part of the graph will be visible.

### • xlimtop = realvalue

Specifies the maximum value at the x-axis to be plotted. The default is the maximum value in the data set. If xlimtop is below the maximum value in the data set, only a part of the graph will be visible.

### • xstart = realvalue

Specifies the value where the first 'tick' on the x-axis should be drawn. The default is the minimum value on the x-axis.

## • xstep = realvalue

If xstep is specified, ticks are drawn at the x-axis with stepwidth *realvalue* starting at the minimum value on the x-axis (or at the value specified in option xstart). By default, five equally spaced ticks are drawn at the x-axis.

### • ylab = characterstring

Labels the y-axis. If the label contains more than one word, *characterstring* must be enclosed by quotation marks (e.g. ylab="y axis").

## $\bullet$ ylimbottom = realvalue

Specifies the minimum value at the y-axis to be plotted. The default is the minimum value in the data set. If ylimbottom is above the minimum value in the data set, only a part of the graph will be visible.

### • ylimtop = realvalue

Specifies the maximum value at the y-axis to be plotted. The default is the maximum value in the data set. If ylimtop is below the maximum value in the data set, only a part of the graph will be visible.

### • ystart = realvalue

Specifies the value where the first 'tick' on the y-axis should be drawn. The default is the minimum value on the y-axis.

### • ystep = realvalue

If ystep is specified, ticks are drawn at the y-axis with stepwidth *realvalue* starting at the minimum value on the y-axis (or at the value specified in option ystart). By default, five equally spaced ticks are drawn at the y-axis.

## **Examples**

Suppose we have already created a regression object reg and have estimated a regression model with Gaussian errors using a command like

```
> reg.regress Y = X(psplinerw2), family=gaussian using d
```

where Y is the response variable and X the only explanatory variable. The effect of X is modelled nonparametrically using Bayesian P-splines. In the  $output\ window$  we obtain the following estimation output for the effect of X:

f\_x

Results are stored in file c:\results\reg\_f\_x\_pspline.res Results may be visualized using method plotnonp Type for example: objectname.plotnonp 0

The term number of the effect of X is 0, i.e. by typing

> reg.plotnonp 0

we obtain the plot shown in Figure 9.1.

Of course, a title, axis labels etc. can be added. For example by typing

> reg.plotnonp 0 , title="my title" xlab="x axis" we obtain the plot shown in Figure 9.2.

By default, the plots appear in an additional window on the screen. They can be directly stored in postscript format by adding option outfile. For example by typing

> reg.plotnonp 0 , title="my title" xlab="x axis" outfile="c:\results\result1.ps" the graph is stored in postscript format in the file c:\results\result1.ps.

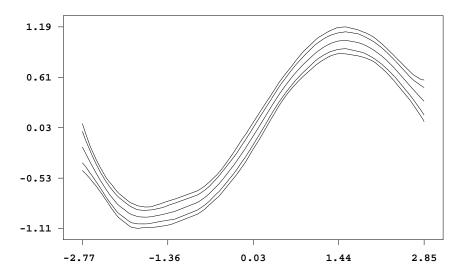


Figure 9.1: Illustration for the usage of method plotnonp

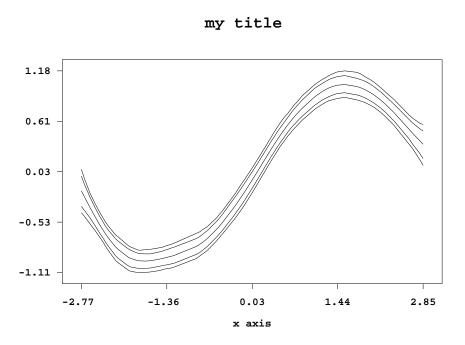


Figure 9.2: Second illustration for the usage of method plotnonp

## 9.1.2 Method drawmap

## Description

Method drawmap is a post estimation command, i.e. it is meaningful only if method regress has been applied before. The method allows to visualize estimated effects of spatial covariates immediately after estimation.

## **Syntax**

## > objectname.drawmap termnumber [, options]

Visualizes the effect of a spatial covariate by coloring the regions of the corresponding geographical map according to the estimated effect (or other characteristics of the posterior). The term number termnumber identifies the model term and can be found in the output window and/or an open log file. Several options are available for adding a title or changing the color scale etc., see the options list below. Note that method drawmap can be applied only if Markov random fields, geosplines or geokriging are used as priors.

## **Options**

The following options are available for method drawmap (in alphabetical order):

### • color

The color option allows to choose between a grey scale for the colors and a colored scale. If color is specified a colored scale is used instead of a grey scale.

#### drawnames

In some situations it may be useful to print the names of the regions into the graph (although the result may be confusing in most cases). This can be done by specifying the additional option drawnames. By default the names of the regions are omitted in the map.

### • fontsize = integer

Specifies the font size (in pixels) for labelling the legend and writing the names of the regions (if specified). Note, that the title is scaled accordingly (see option titlesize). The default is fontsize=12.

### • hcl

Requests that a color palette from the HCL color space should be used instead of an RGB palette. The HCL colors will be selected diverging from a neutral center (grey) to two different extreme colors (red and green) in contrast to the RGB colors diverging from yellow to red and green. HCL colors are particularly useful for electronic presentations since they are device-independent. The option hcl is only meaningful in combination with the option color.

### • lowerlimit = realvalue

Lower limit of the range to be drawn. If lowerlimit is omitted, the minimum numerical value in plotvar will be used instead as the lower limit.

## • nolegend

By default a legend is drawn into the graph. By specifying the option **nolegend** the legend will be omitted.

### • nrcolors = integer

To color the regions according to their numerical characteristics, the data are divided into a (typically large) number of ordered categories. Afterwards a color is associated with each category. The nrcolors option can be used to specify the number of categories (and with it the number of different colors). The maximum number of colors is 256, which is also the default value.

### • outfile = characterstring

If option outfile is specified the graph will be stored as a postscript file rather than being printed on the screen. The path and the filename must be specified in *characterstring*. By default, an error will be raised if the specified file is already existing or the specified folder is not existing. To overwrite an already existing file, option replace must be additionally specified. This prevents you from unintentionally overwriting your files.

### • pcat

If you want to visualize the values of the columns pcat80 or pcat95 it is convenient to specify pcat. This forces drawmap to expect a column that consists only of the values -1, 0 and 1. Of course you can achieve the same result by setting nrcolors=3, lowerlimit=-1 and upperlimit=1.

### • plotvar = variable name

By default, the regions of the map are colored according to the estimated spatial effect. Option plotvar allows to color the map according to other characteristics of the posterior by explicitly specifying the name of the variable to be plotted. Compare the header of the file containing the estimation results to see all variables available for plotting.

### • replace

The replace option is only useful in combination with option outfile. Specifying replace as an additional option allows the program to overwrite an already existing file (specified in outfile), otherwise an error will be raised.

#### • swapcolors

In some situations it may be favorable to swap the order of the colors, i.e. black (red) shades corresponding to large values and white (green) shades corresponding to small values. This is achieved by specifying swapcolors. By default, small values are colored in black shades (red shades) and large values in white shades (green shades).

### • title = characterstring

Adds a title to the graph. If the title contains more than one word, *characterstring* must be enclosed by quotation marks (e.g. title="my first map").

### • titlesize = realvalue

Specifies the factor by which the size of the title is scaled relative to the size of the labels of the legend (compare option fontsize). The default is titlesize=1.5.

### • upperlimit = realvalue

Upper limit of the range to be plotted. If upperlimit is omitted, the maximum numerical value in plotvar will be used instead as the upper limit.

## **Examples**

Suppose we have already created a regression object reg and have estimated a regression model with Gaussian errors using something like

```
> map m
> m.infile using c:\maps\map1.bnd
```

> reg.regress Y = region(spatial,map=m), family=gaussian using d

where Y is the response variable and region the only explanatory variable. The effect of the spatial covariate region is modelled nonparametrically using a Markov random field. In the *output window* we obtain the following estimation output for the effect of region:

```
f_spat_region
```

```
Results are stored in file

c:\results\reg_f_region_spatial.res

Results may be visualized using method 'drawmap'

Type for example: objectname.drawmap 0
```

The term number of the effect of region is 0, i.e. by typing

```
> reg.drawmap 0
```

we obtain the map shown in Figure 9.3 where the regions are colored according to the estimated spatial effect.

By default the regions are colored in grey scale. A color scale is obtained by adding option color. A title can be added as well. For example by typing

```
> reg.drawmap 0 , color title="my title" we obtain the map shown in Figure 9.4.
```

By default, the maps appear in an additional window on the screen. They can be directly stored in postscript format by adding option outfile. For example by typing

```
> reg.drawmap 0 , color title="my title" outfile="c:\results\result1.ps"
the colored map is stored in postscript format in the file c:\results\result1.ps.
```

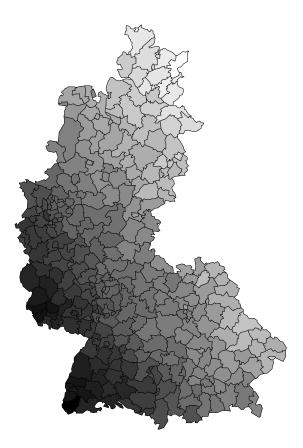


Figure 9.3: Illustration for the usage of method drawmap

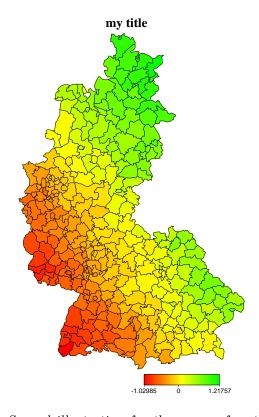


Figure 9.4: Second illustration for the usage of method drawmap

## 9.1.3 Method plotautocor

## Description

Method plotautocor is a post estimation command, i.e. it is meaningful only if method regress has been applied before. Method plotautocor computes and visualizes the autocorrelation functions of the parameters in the model. This method is only applicable to bayesreg objects.

## **Syntax**

> objectname.plotautocor [, options]

Computes and visualizes the autocorrelation functions in the model. Several options are available for specifying the maximum lag for autocorrelations, storing the graphs in postscript format etc., see the options list below.

## **Options**

The following options are available for method plotautocor (in alphabetical order):

 $\bullet$  maxlag = integer

Option maxlag may be used to specify the maximum lag for autocorrelations. The default is maxlag=250.

• mean

If option mean is specified, for each lag number and model term only minimum, mean and maximum autocorrelations are plotted. This can lead to a considerable reduction in computing time and storing size.

• outfile = characterstring

If option outfile is specified the graph will be stored as a postscript file and not printed on the screen. The path and the filename must be specified in *characterstring*. An error will be raised if the specified file is already existing and the replace option is not specified.

• replace

The replace option is only useful in combination with option outfile. Specifying replace as an additional option allows the program to overwrite an already existing file (specified in outfile), otherwise an error will be raised.

### **Examples**

Suppose we have already created a bayesreg object reg and have estimated a regression model with Gaussian errors using

```
> reg.regress Y = X(psplinerw2), family=gaussian using d
```

where Y is the response variable and X the only explanatory variable. The effect of X is modelled nonparametrically using Bayesian P-splines. We can now check the mixing of sampled parameters by computing and drawing the autocorrelation functions up to a maximum lag of 150:

> reg.plotautocor , maxlag=150 outfile="c:\results\autocor.ps"

In this example the autocorrelation functions are not shown on the screen but stored in postscript format in the file c:\results\autocor.ps. If option outfile is omitted, the functions are plotted on the screen. The resulting file contains 5 pages. As an example, the first page of the file is shown in Figure 9.5.

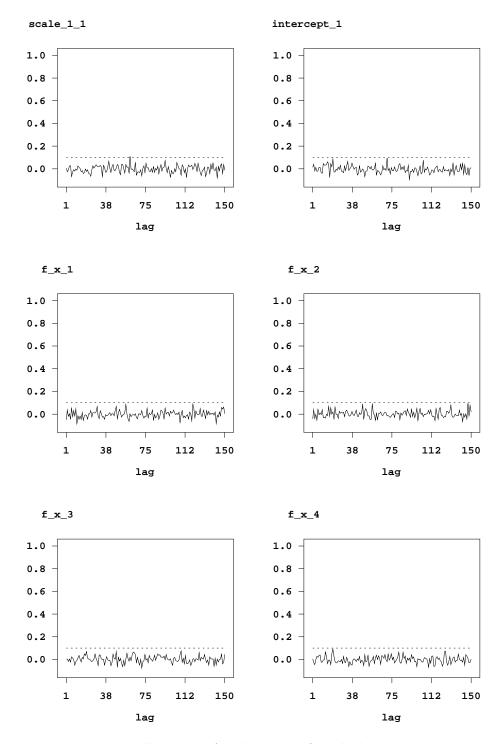


Figure 9.5: Illustration for the usage of method plotautocor

## 9.2 R / S-plus functions

Some S-plus and R functions for plotting estimated functions are shipped together with BayesX. These functions can be found in the subdirectory sfunctions of the installation directory. Table 9.1 gives a first overview over the different functions and their abilities. The usage of the functions is very simple so that also users not familiar with the R / S-plus environment should be able to apply the functions without any difficulties. The following subsections describe how to install the functions in R / S-plus and give a detailed description of the usage of the respective functions.

Functionname	Description
plotnonp	visualizes estimated nonparametric functions
plotautocor	visualizes autocorrelation functions
plotsample	visualizes sampling paths of sampled parameters
readbndfile	reads in boundaries of geographical maps
drawmap	visualizes estimation results for spatial covariates
plotsurf	visualizes estimated two-dimensional surfaces

Table 9.1: Overview over R / S-plus functions

### 9.2.1 Installation of the functions

Installation of the different functions is very easy. The S-plus code for the functions is stored in the directory < INSTALLDIRECTORY>\sfunctions in the ASCII text file plot.s. To install the functions you first have to start S-plus. Afterwards the functions will be installed by entering > source("<INSTALLDIRECTORY>\\sfunctions\\plot.s")

in the Commands Window of S-plus. Note that a double backslash is required in S-plus to specify a directory correctly. For use with R the file plot.r is supplied, which contains slightly modified versions of the S-plus functions.

## 9.2.2 Plotting nonparametric functions

This subsection describes the usage of the function plotnonp for visualizing nonparametric function estimates.

Suppose that a Bayesian regression model has already been estimated with predictor

$$\eta = \ldots + f(X) + \ldots,$$

where the effect of X is modelled nonparametrically using for example a first or second order random walk prior. Unless the directory for estimation output has been changed using the global option outfile (see section 7.4 and section 8.2), estimation results for the nonparametric effect of X are stored in the directory

< INSTALLDIRECTORY > \output

that is in the subdirectory output of the installation directory. The filename is  $objectname\_f_X_rw.res$ 

that is it is composed of the name of the regression object and the covariate name. For the following we assume that c:\bayes is the installation directory and reg is the name of the regression object. In this case results for the effect of X are stored in:

### c:\bayes\output\reg\_f\_X\_rw.res

The structure of such a file has already been described in section 7.1 (bayesreg objects) and section 8.1 (remlreg objects). Although it is possible (and very easy) to visualize the estimated nonparametric function with any software package that has plotting capabilities, a fast and easy way of plotting estimation results without knowing the particular structure of the results-file is desirable. This is the task of function plotnonp.

The function has only one required and many optional arguments. The required argument is the directory and the filename where nonparametric estimation results are stored. For example by entering the command

## > plotnonp("c:\\bayes\\output\\reg\_f\_X\_rw.res")

a graphic window will be opened with the plotted function estimate. The function plots the estimated effect together with the posterior 2.5%, 10%, 90% and 97.5% quantiles. One advantage of the function is that after its application no permanent objects will remain in the R / S-plus environment.

Besides the required argument a lot of optional arguments may be passed to the function. Among others there are options for plotting the graphs in a postscript file rather than the screen, labelling the axes, specifying the minimum/maximum value on the x/y axes and so on. The following optional arguments can be passed to plotnonp:

- psname = "filename (including path)"

  Name of the postscript output file. If psname is specified the graph will be stored in a postscript file and will not appear on the screen.
- level = 0—1—2 Specifies whether to plot only the 95% credible intervals (level=1) or only the 80% credible intervals (level=2). Default value is level=0, i.e. both.
- ylimtop = realvalue Specifies the maximum value on the y-axis (vertical axis).
- ylimbottom = realvalue Specifies the minimum value on the y-axis.
- xlab = "characterstring" xlab is used to label the x-axis (horizontal axis).
- ylab = "characterstring"
  ylab is used to label the y-axis.
- maintitle = "characterstring" Adds a title to the graph.
- subtitle = "characterstring" Adds a subtitle to the graph.
- linecol = *integer*Specifies the color of the credible intervals. Default value is linecol=3.
- linetype = integer

  Specifies the line type for the credible intervals. Default value is linetype=1 (solid).

As an illustration compare the following statement:

> plotnonp("c:\\bayes\\reg\_f\_X\_rw.res", psname="c:\\bayes\\reg\_f\_X\_rw.ps",
 maintitle="Maintitle",ylab="effect of X",xlab="X")

## Maintitle

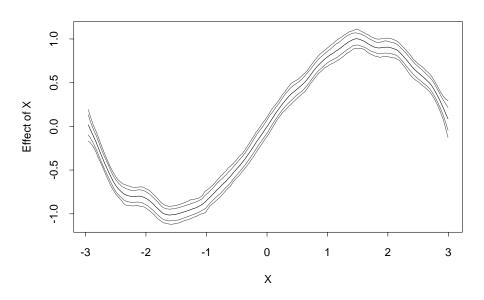


Figure 9.6: Illustration for the usage of plotnonp

This statement draws the estimated effect of X and stores the graph in the postscript file "c:\\bayes\\reg\_f\_X\_rw.ps". A title, a x-axis and y-axis label are added to the graph. For illustration purposes, the resulting graph is shown in Figure 9.6.

In some situations the effect of a covariate representing dates must be plotted. Suppose for example that a covariate has values ranging from 1 to 19 representing the time period from January 1983 to July 1984. In this case, we naturally prefer that the x-axis is labelled in terms of dates rather than in the original coding (from 1 to 19). To achieve this, function plotnonp provides the three additional options year, month and step. Options year and month are used to specify the year and the month (1 for January, 2 for February, ...) corresponding to the minimum covariate value. In the example mentioned above year=1983 and month=1 will produce the correct result. In addition, option step may be specified to define the periodicity in which your data are collected. For example step=12 (the default) corresponds to monthly data, while step=4, step=2 and step=1 correspond to quarterly, half yearly and yearly data. We illustrate the usage of year, month and step with our example. Suppose we estimated the effect of calendar time D, say, on a certain dependent variable, where the range of the data is as described above. Then the following function call will produce the postscript file shown in Figure 9.7:

> plotnonp("c:\\bayes\\reg\_f\_D\_pspline.res",psname="c:\\bayes\\reg\_f\_D\_pspline.ps",
 year=1983,month=1,step=12,xlab="date",ylab= " ")

Note, that ylab=" " forces R / S-plus to omit the y-axis label. If ylab (as well as xlab) is omitted, default labels will be given to the two axis.

Finally, we note that all options that can be passed to the plot function of R / S-plus may also be passed to function plotnonp. Thus, function plotnonp is more or less a specialized version of the plot function.

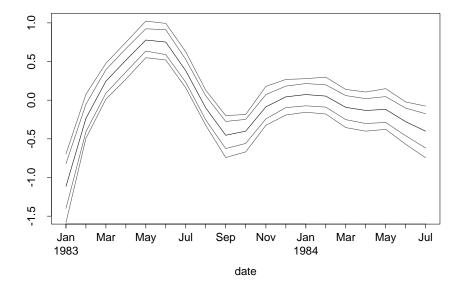


Figure 9.7: Illustration for the usage of plotnonp

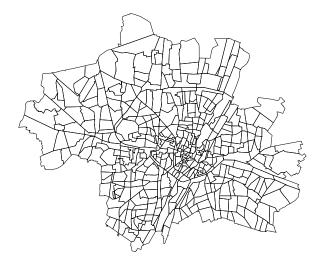


Figure 9.8: Map of Munich

## 9.2.3 Drawing geographical maps

This subsection describes how to visualize estimation results of spatial covariates, where the observations represent the location or site in connected geographical regions. A typical example for a spatial covariate is given in the 'rents for flats' example, see subsection 2.6.1, where the covariate L indicates the location (in subquarters) of the flat in Munich. Figure 9.8 shows a map of Munich separated into subquarters.

Typically, the effect of such a spatial covariate is incorporated into a regression model via an unstructured or structured random effect. In the latter case a spatial smoothness prior for the spatial covariate is specified that penalizes too abrupt changes of the estimated effect in neighboring sites. In some situations the incorporation of both, an unstructured and a structured effect, may also be appropriate. Details on how to incorporate spatial covariates into a semiparametric regression model are given in section 7.1 (bayesreg objects) and section 8.1 (remlreg objects). For the rest of this section we assume that an effect of a spatial covariate has already been estimated and that we now want to visualize the estimation results. This can be easily done with the two functions

readbndfile and drawmap. Function readbndfile is used to read the boundary information of a map that is stored in a so called boundary file and to store this information as a permanent map object. The boundary file contains mainly the polygons which form the different geographical regions of the map. The required structure of such a file is described below. After the successful reading of the boundary information of a map, the second function drawmap may be used to draw and print the map either on the screen or into a postscript file. There are several possible ways to draw the map. In the simplest case the map can be drawn without any estimation effects, i.e. only the boundaries of the different regions or sites are drawn, see Figure 9.8 for an example. In practice, however, one usually wants to color the regions of the map according to some numerical characteristics. As an example compare Figure 9.9 in which the subquarters of Munich are colored according to the frequency of flats in the 'rents for flats' data set located in the respective subquarter. Subquarters colored in red contain less flats compared to subquarters colored in green. In striped areas no observations are available.

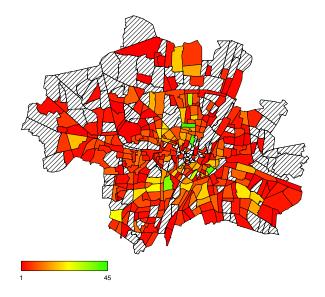


Figure 9.9: relative frequencies of observed flats in the 'rents for flats' data set

In the following we give a detailed description of the usage of the functions readbndfile and drawmap.

#### **Function readbndfile**

Function readbndfile is used to read in boundary information stored in a boundary file. The function has two required arguments. The first argument is the filename of the boundary file to read in. The second argument specifies the name of the map object (recall that the map information is stored as a permanent object). To give an example, suppose that BayesX is installed in the directory c:\bayes and that we want to read in the map of Munich. In this case the boundary file of the map is stored in the subdirectory examples of the installation directory, that is in c:\bayes\examples. The name of the boundary file is simply munich.bnd. The following function call reads in the boundary information of Munich and stores the map permanently in the object munich:

#### > readbndfile("c:\\bayes\\examples\\munich.bnd","munich")

Once again, note that double backslashes are required to specify a directory. The second argument in the statement above is "munich", i.e. the name of the map object is simply munich. To refer

to the map of Munich in subsequent statements and function calls, the quotation marks must be omitted.

#### **Function drawmap**

Function drawmap is used to draw geographical maps and color the regions according to some numerical characteristics. There is only one required argument that must be passed to drawmap, that is the name of the map to be drawn. Provided that the map has already been read into R / S-plus (via function readbndfile), the following statement draws the map of Munich in a graphic-window on the screen:

> drawmap(map=munich)

Storing the map in a postscript file rather than drawing it on the screen can be achieved by specifying the name of the postscript file using the outfile option. For example the command

> drawmap(map=munich,outfile="c:\\bayes\\munich.ps")

produces a postscript file named munich.ps with the map of Munich.

However, in most cases one not only wants to draw the boundaries of a geographical map, but also color the regions according to some numerical characteristics. Suppose for example that we have already estimated a location specific effect on the monthly rents in the 'rents for flats' data set. Suppose further that the estimated effects are stored in c:\bayes\output\reg\_f\_L\_spatial.res. The structure of the file is described in detail in section 7.1 (bayesreg objects) and section 8.1 (remlreg objects).

Suppose now that we want to visualize estimation results for the spatial covariate L by coloring the subquarters of Munich according to the posterior mean estimated using a bayesreg object. Compared to the statement above, (at least) three more arguments must be passed to function drawmap; the argument dfile that specifies the filename of estimated results, the argument plotvar that specifies the variable to be plotted and the argument regionvar that specifies which column of the file, containing estimation results, stores the region names. The following statement produces the desired result:

```
> drawmap(map=munich,outfile="c:\\bayes\\munich.ps",
   dfile="c:\\bayes\\output\\reg_f_L_spatial.res", plotvar="pmean",regionvar="L")
```

Note that the right hand side of options plotvar and regionvar must be enclosed by quotation marks. If we want to color the subquarters of Munich according to the posterior mode estimated using a remlreg object we have to change the statement to

```
> drawmap(map=munich,outfile="c:\\bayes\\munich.ps",
   dfile="c:\\bayes\\output\\reg_f_L_spatial.res", plotvar="pmode",regionvar="L")
```

## Optional arguments of function drawmap

Besides the arguments discussed so far there are some more optional arguments that can be passed to drawmap. They are listed and described below together with a summary of the arguments already described:

• cex.legend = realvalue

This option is only available for the R-version of drawmap. cex.legend specifies the magnification to be used for legend annotation. The default value is cex.legend=0.7.

• color = T/F

The color option allows to choose between a grey scale for the colors and a colored scale. The default is color=F, which means a grey scale.

#### • dfile = "filename (including path)"

Filename (including path) of the file containing numerical characteristics of the regions of the map. The file must contain at least two columns, one column that lists the names of the regions and one column containing the numerical characteristics of the respective regions. It is important that the names of the regions listed match with the region names stored in the map object. The first row of the file must contain the names of the columns.

#### • drawnames = T/F

In some situations it may be favorable to print the names of the regions into the graph (although the result may be confusing in most cases). This can be done by specifying the additional option drawnames=T. By default the names of the regions are omitted in the graph.

## • hcl = T/F

This option is only available for the R-version of drawmap and depends on the package vcd. Specifying hcl=T requests that a color palette from the HCL color space should be used instead of an RGB palette. The HCL colors will be selected diverging from a neutral center (grey) to two different extreme colors (red and green) in contrast to the RGB colors diverging from yellow to red and green. HCL colors are particularly useful for electronic presentations since they are device-independent. Option hcl=T is only meaningful in combination with the option color=T.

## • legend = T/F

By default a legend is drawn into the graph. To omit the legend in the graph, legend=F must be passed as an additional argument.

## • lowerlimit = realvalue

Lower limit of the range to be drawn. If lowerlimit is omitted, the minimum numerical value in the plotvar column will be used instead as the lower limit.

#### • map = "characterstring"

Name of the map object. Use function readbndfile to read in geographical maps.

#### $\bullet$ nrcolors = integer

To color the regions according to their numerical characteristics, the data are divided into a (typically large) number of ordered categories. Afterwards a color is associated with each category. The nrcolors option can be used to specify the number of categories (and with it the number of different colors). Default value is 100.

## • outfile = "filename (including path)"

Filename (including path) of the postscript file where the map should be stored.

## • pcat = T/F

If you want to visualize the values of the columns pcat80 or pcat95 it is convenient to specify pcat=T. This forces drawmap to expect a column that consists only of the values -1, 0 and 1. Of course you can achieve the same result by setting nrcolors=3, lowerlimit=-1 and upperlimit=1. The default is pcat=F.

• plotvar = "characterstring"

Name of the column in the data file containing the numerical characteristics of the regions (see also argument dfile). Note that the right hand side must be enclosed by quotation marks.

• pstitle = "characterstring"

Adds a title to the graph. Note that the right hand side must be enclosed by quotation marks.

• regionvar = "characterstring"

Name of the column in the data file containing the region names (see also argument dfile). Note that the right hand side must be enclosed by quotation marks.

• swapcolors = T/F

In some situations it may be favorable to swap the order of the colors, i.e. red shades corresponding to large values and green shades corresponding to small values. This is achieved by specifying swapcolors=T. By default small values are colored in red shades and large values in green shades.

• upperlimit = realvalue

Upper limit of the range to be drawn. If upperlimit is omitted, the maximum numerical value in the plotvar column will be used instead as the upper limit.

## 9.2.4 Plotting two-dimensional surfaces

This subsection describes the usage of the function plotsurf for visualizing two-dimensional surfaces. The function plotsurf merely invokes different R / S-plus functions for visualizing two-dimensional data. Thus, users familiar with R / S-plus may prefer to use this functions directly to gain more flexibility.

Suppose that a Bayesian regression model has already been estimated with predictor

$$\eta = \ldots + f(X1, X2) + \ldots,$$

where the interaction effect of X1 and X2 is modelled nonparametrically using two-dimensional P-splines and that the estimation results are stored in file:

c:\bayes\output\reg\_f\_X1\_X2\_pspline.res

The function plotsurf requires at least one argument, which is the name (including path) of the file containing the estimation results. For example the command

> plotsurf("c:\\bayes\\output\\reg\_f\_X1\_X2\_pspline.res")

plots the estimated surface against X1 and X2 on the screen. There are several additional options that can be passed, for example for changing the plot type or storing the graph as a postscript file rather than displaying it on the screen. The following list describes all possible arguments that may be passed to the function plotsurf:

• cols = 3 column vector

This option is only meaningful, if the argument data is specified. In this case cols gives the columns of the object or data file passed to the argument data, that should be used as values for the x, y and z axis. The default is cols=c(2,3,4) which corresponds to plotting the estimated surface against X1 and X2.

• data = "filename (including path)"

Name (including path) of the file containing the estimation results. The file must contain at least 3 columns, one for the x-axis, one for the y-axis and one for the z-axis. The file must contain a header.

• mode = 1/2/3/4/5

This option specifies the plot type. In S-Plus there are five different types available, where the default value mode=1 corresponds to what is called a 'Surface Plot' in S-plus. In R three different types are available corresponding to a perspective plot (mode=1, function persp in R), a contour plot (mode=2, function contour in R), and an image plot (mode=2, function image in R) of the data. In R plotsurf depends on the package akima for the interpolation of irregular data.

• outfile = "filename (including path)"

Name (including path) of the postscript file where the graph should be stored. This option is only meaningful for mode=2 and mode=3.

## 9.2.5 Plotting autocorrelation functions

This section describes how to visualize autocorrelation functions of sampled parameters using the function plotautocor.

To compute autocorrelation functions, the post-estimation command autocor must be applied, see section 7.2 for details. For the rest of this section we assume that autocorrelations are already computed and stored in file:

c:\bayes\output\reg\_autocor.raw

The minimum number of arguments required for the function is one, namely the file where the computed autocorrelation functions are stored. In this case a *graphic window* will be opened and the autocorrelation functions are plotted on the screen. To store autocorrelations in a postscript file, an output filename must be specified as a second argument. Thus, the command

- > plotautocor("c:\\bayes\\output\\reg\_autocor.raw")
  prints autocorrelations on the screen, while the statement
- > plotautocor("c:\\bayes\\output\\reg\_autocor.raw","c:\\bayes\\output\\reg\_autocor.ps") forces R / S-plus to store the autocorrelation graphs in the postscript file c:\bayes\output\reg\_autocor.ps.

In particular for regression models with a large number of parameters the execution of function plotautocor can be very time consuming. Moreover, the size of the resulting postscript file can be very large. To avoid such problems plotautocor provides the additional argument mean.autocor. If mean.autocor=T is specified, for each lag number and model term only minimum, mean and maximum autocorrelations are plotted, leading in most cases to a considerable reduction in computing time and storing size.

## 9.2.6 Plotting sampled parameters

This section describes how to plot sampled parameters using the function plotsample. Before applying function plotsample, sampled parameters must be stored in ASCII-format using the postestimation command getsample. See section 7.3 for details, but note that sampled parameters will be stored in several different files, typically one file for each term in the model.

Suppose now that we want to visualize sampling paths for the parameters of the nonlinear effect of a covariate X. Assume further that sampled parameters are stored in the ASCII file

c:\bayes\output\reg\_X\_sample.raw.

As most other functions, plotsample provides two possibilities of drawing sampled parameters. The first possibility is to print the graphs on the screen, and the second is to store them into a postscript file. To print the sampling paths on the screen, only the filename (including path) of the ASCII file where sampled parameters are stored must be passed to the function. For the example mentioned above the corresponding command is:

> plotsample("c:\\bayes\\output\\reg\_X\_sample.raw")

If sampling paths should be drawn into a postscript file rather than on the screen, the filename of the resulting postscript file must be specified as a second argument. Thus, for our example we get:

> plotsample("c:\\bayes\\output\\reg\_X\_sample.raw", "c:\\bayes\\output\\reg\_X\_sample.ps") In addition, all options that are available for the R / S-plus function plot may be passed to function plotsample, see the S-plus documentation for details.

## Chapter 10

## **DAG** Objects

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Dag objects are needed to estimate dag models using reversible jump MCMC. The considered variables may be Gaussian or binary, even the mixed case of a conditional Gaussian distribution is possible. A general introduction into graphical models can be found in Lauritzen (1996). For a description of the more particular Gaussian dags see for instance Geiger and Heckerman (1994). We refer to Brooks (1998) or Gilks (1996) for an introduction into MCMC simulation techniques. For the more general reversible jump MCMC have a look at Green (1995); for reversible jump MCMC in context of graphical models at Giudici and Green (1999). The following explanations to the statistical background of the program can be found in more detail in Fronk and Giudici (2000).

## 10.1 Method estimate

## 10.1.1 Description

The method estimate estimates the dependency structure of the given variable which is represented by a dag. Furthermore, the parameters of this model are estimated. This is done within a Bayesian framework; we assume prior distributions for the unknown parameters and use MCMC techniques for estimation. In the following we first focus on the Gaussian case and describe the statistical model which is assumed for the variables. Some factorizations which result from the properties of dags are also given. To represent the dags we rely on the concept of adjacency matrices which is briefly explained and necessary to understand the output. We finally give some brief information about the used algorithm without going into details. Finally, we address the situation of binary and mixed (i.e. continuous and binary) variables, too, which is reduced to the Gaussian case by introducing latent variables.

#### Model Assumptions

A Gaussian dag d can be represented as a regression model for each variable  $X_i$ , i = 0, ..., p - 1, given the parents of  $X_i$ , denoted by  $\mathbf{X}_{pa(i)}$ ,

$$X_i \mid \boldsymbol{x}_{pa(i)}, \boldsymbol{\beta}_{i|pa(i)}, \sigma^2_{i|pa(i)}, d \sim \mathrm{N}(\beta_{i0} + \sum_{x_l \in pa(x_i)} \beta_{il} x_l, \sigma^2_{i|pa(i)}).$$

The joint distribution of all variables  $\mathbf{X} = (X_0, \dots, X_{p-1})'$  is then given by

$$p(\boldsymbol{x} \mid \boldsymbol{\beta}, \boldsymbol{\sigma}^2) = \prod_{i=0}^{p-1} p(x_i \mid \boldsymbol{x}_{pa(i)}, \boldsymbol{\beta}_{i|pa(i)}, \sigma_{i|pa(i)}^2),$$

where  $\beta_{i|pa(i)}$  is the |pa(i)| + 1-dimensional vector of the intercept  $\beta_{i0}$  and the |pa(i)| regression coefficients of  $X_i$ . Furthermore,  $\sigma^2_{i|pa(i)}$  is the partial variance of  $X_i$  given its parents  $\boldsymbol{x}_{pa(i)}$ . Let  $\boldsymbol{\beta} = (\beta'_{0|pa(1)}, \dots, \beta'_{p-1|pa(p)})'$  denote the vector of the  $\boldsymbol{\beta}_{i|pa(i)}$ 's and accordingly  $\boldsymbol{\sigma}^2 = (\sigma^2_{0|pa(1)}, \dots, \sigma^2_{p-1|pa(p)})'$  the vector of the conditional variances  $\sigma^2_{i|pa(i)}$ .

The vector  $\boldsymbol{\beta}_{i|pa(i)}$  is assumed to be normally distributed with mean  $\boldsymbol{b}_{i|pa(i)}$  and covariance matrix  $\frac{1}{\alpha_i}\sigma_{i|pa(i)}^2\boldsymbol{I}$ , where  $\alpha_i$  is a known scaling factor. For the sake of simplicity, we shall assume  $\alpha_i = \alpha$ . Formally:

$$\boldsymbol{\beta}_{i|pa(i)} \mid \sigma_{i|pa(i)}^2, d \sim N_{|pa(i)|+1} \left( \boldsymbol{b}_{i|pa(i)}, \frac{1}{\alpha} \sigma_{i|pa(i)}^2 \boldsymbol{I} \right).$$

This implies that the coefficients of a regression model are assumed to be mutually independent. For the partial variance  $\sigma_{i|pa(i)}^2$  we use an inverse gamma prior with parameters  $\delta_{i|pa(i)}$  and  $\lambda_{i|pa(i)}$ :

$$\sigma_{i|pa(i)}^2 \mid d \sim \text{IG}\left(\delta_{i|pa(i)}, \lambda_{i|pa(i)}\right).$$

Finally, by supposing that there exist D possible dags, which, in the absence of subject-matter information, have all the same probability, we get a discrete uniform distribution for d: p(d) = 1/D. Taking advantage of the well-known factorization property of the joint distribution

$$p(\boldsymbol{x} \mid \boldsymbol{\beta}, \boldsymbol{\sigma}^2, d) = \prod_{i=0}^{p-1} p(x_i \mid \boldsymbol{x}_{pa(i)}, \boldsymbol{\beta}_{i \mid pa(i)}, \sigma^2_{i \mid pa(i)})$$

and the "global parameter independences"

$$p(\boldsymbol{\beta} \mid \boldsymbol{\sigma}^2, d) = \prod_{i=0}^{p-1} p(\boldsymbol{\beta}_{i|pa(i)} \mid \boldsymbol{\sigma}_{i|pa(i)}^2),$$
 and 
$$p(\boldsymbol{\sigma}^2 \mid d) = \prod_{i=0}^{p-1} p(\sigma_{i|pa(i)}^2)$$

(for a detailed description see Geiger and Heckerman, 1999), we get the joint distribution:

$$\begin{aligned} p(\boldsymbol{x}, \boldsymbol{\beta}, \boldsymbol{\sigma}^2, d) &= & p(\boldsymbol{x} \mid \boldsymbol{\beta}, \boldsymbol{\sigma}^2, d) \, p(\boldsymbol{\beta} \mid \boldsymbol{\sigma}^2, d) \, p(\boldsymbol{\sigma}^2 \mid d) \, p(d) \\ &= & \prod_{i=0}^{p-1} p(\boldsymbol{x}_i \mid \boldsymbol{x}_{pa(i)}, \boldsymbol{\beta}_{i|pa(i)}, \sigma^2_{i|pa(i)}) \prod_{i=0}^{p-1} p(\boldsymbol{\beta}_{i|pa(i)} \mid \sigma^2_{i|pa(i)}) \\ &= & \prod_{i=0}^{p-1} p(\sigma^2_{i|pa(i)}) \, p(d) \end{aligned}$$

#### Representation of DAGs

To represent dags we rely on the concept of adjacency matrices. For a given graph  $\mathcal{G} = (V, E)$  with |V| = p, the adjacency matrix of  $\mathcal{G}$  is defined as the  $(p \times p)$ -matrix A,  $[A]_{ij} = a_{ij}$ , with

$$a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in E \\ 0, & \text{if } (v_i, v_j) \notin E. \end{cases}$$

In general, all three types of graphs (undirected, directed and chain graphs) can be uniquely represented by the corresponding adjacency matrix. Note that regarding dags, as we do, the parents of the vertex *i* are indicated by the *i*-th column, while its children are given in the *i*-th row. We use the representation via adjacency matrices also to check the acyclicity of the graph. For an illustration of this concept consider the graph in Figure 10.1.



Figure 10.1: A directed acyclic graph containing and the corresponding adjacency matrix A.

## Reversible Jump Algorithm for Continuous Variables

We are not only interested in estimating the parameters for a given dag d but also want to learn about the structure of d itself. So we need to construct a Markov chain which has  $\pi(d, \mu, \beta, \sigma^2 \mid x)$  as its invariant distribution. Changing the dag like adding or deleting a directed edge implies also a changing in the dimension of the parameter space. To deal with this situation we use a reversible jump algorithm. Reversible jump MCMC was proposed and described by Green (1995); it can be regarded as a generalization of the usual MCMC and allows to sample simultaneously from parameter spaces of different dimensions.

Our algorithm can be briefly summarized by the following moves, which produce a Markov chain in the state space that is made up by the vector of unknowns  $(d, \beta, \sigma^2)$ :

- 1. Updating the dag d by adding, switching or deleting a directed edge, remaining always in the class of directed acyclic graphs. When adding or deleting an edge this move involves a change in dimensionality of the parameter space.
- 2. Update  $\beta_{i|pa(i)}$ , i = 0, ..., p 1.
- 3. Update  $\sigma_{i|pa(i)}^2$ , i = 0, ..., p 1.

For a detailed explanation of the different steps in the continuous case, see Fronk and Giudici (2000). For a simplification of the oftentimes crucial switch step, see Fronk (2002) and also the explanations of the option *switch* in 10.1.3.

#### Reversible Jump Algorithm for Binary Variables

Now we consider the situation of p binary variables of which the joint distribution is assumed to be multinomial. The influence to a variable  $X_i$  from its known parents  $x_{pa(i)}$  shall be given by a probit model, i.e.

$$p_i = E(X_i \mid \boldsymbol{x}_{pa(i)}) = \Phi(\boldsymbol{x}'_{pa(i)}\boldsymbol{\beta}_{i|pa(i)}, \sigma^2_{i|pa(i)}),$$
 (10.1)

where i = 1, ..., p and  $\Phi(\mu, \sigma^2)$  denotes the cdf of the normal distribution. This binary situation is reduced to the continuous one by sampling a latent variable, a so-called utility,  $Z_i$  for each binary variable  $X_i$ . The general idea is found in Albert and Chib (1993). Here, we first focus on the situation that we are only interested in the main effects. I.e. we do not take any interactions into account although they now of course can occur as we do not longer consider the Gaussian case. The algorithm, which does not account for interactions, can be briefly summarized as:

- 1. For  $X_i$ , i = 0, ..., p-1, draw  $Z_i$  from its full conditional  $N(\boldsymbol{x}'_{pa(i)}\boldsymbol{\beta}_{i|pa(i)}, 1)$ , which is truncated at the left by 0 if  $x_i = 1$  and at the right if  $x_i = 0$ .
- 2. Add, delete, or switch a directed edge like in the Gaussian case, but take the utility  $Z_i$  instead of  $X_i$  as response in *i*th regression model; the covariables  $x_{pa(i)}$  of the *i*th model remain unchanged.
- 3. Update  $\beta_{i|pa(i)}$ , i = 0, ..., p 1.
- 4. Update  $\sigma_{i|pa(i)}^2$ , i = 0, ..., p 1.

To be able to take interactions into account, inside the algorithm interactions are treated as own variables. Due to the enormous complexity we restrict ourselves to two way interactions which seem sufficient for most situations in practice. For details, see Fronk (2002).

## Reversible Jump Algorithm for Mixed Case

For the mixed case, we assume the considered continuous and binary variables to follow a conditional Gaussian (CG) distribution. For a general introduction we refer to Lauritzen (1996). The univariate conditioned distribution of  $f(x_i \mid x_{pa(i)})$  are then CG regressions (see Lauritzen and Wermuth, 1989) and can be represented by a normal regression resp. a probit model with mixed covariables.

- 1. For all variables  $X_i$ ,  $i = 0, \ldots, p-1$ ,
  - If  $X_i$  is discrete,

For all observations  $X_{ki}$ , k = 1, ..., n, draw utility  $Z_{ki}$  from full conditional  $Z_{ki} \mid x_{ki}, \boldsymbol{x}_{kpa(i)} \boldsymbol{\beta}_{i|pa(i)}$ 

- 2. Update d, i.e. cancel, add, or switch the directed edge  $X_j \to X_i$ ; thereby distinguish
  - Response  $X_i$  is continuous:
    - (a) Take the algorithm for the Gaussian case, where now the covariables  $pa(X_i)$  can be continuous or binary
  - Response  $X_i$  is discrete
    - (a) Replace binary response  $X_i$  by continuous utility  $Z_i$
    - (b) Consider the new or vanishing interactions among the parents of  $X_i$  and possibly  $X_j$ , that can be pairwise discrete or mixed

- (c) Carry out birth, death or switch step
- 3. Update  $\beta_{i|pa(i)}$ , i = 0, ..., p 1.
- 4. Update  $\sigma_{i|pa(i)}^{2}$ , i = 0, ..., p 1.

For detailed explanations, see again Fronk (2002).

#### Remark about Markov-equivalence

Our algorithm does not take care about the so-called Markov equivalence, which describes the fact that different dags can represent the same statistical model. Equivalent dags can be summarized to equivalent classes which again can be represented by one single graph, the essential graph. Of course model selection could be done in a more effective way if only the space of those essential graphs would be considered. This will be a task of our research in future. For more details concerning Markov-equivalence we refer to papers of Andersson et al. (1997a, b) and Chickering (1995).

## 10.1.2 Syntax

The creation of objects has been described in general in subsection 2.5.1; in the context of dag models the corresponding dag object is created by:

dag objectname,

To perform a model selection as described above call:

objectname.estimate variables [if expression], [options] using dataset

Then the method estimate estimates the dag for the variables given in *variables* which have to be defined in *dataset*. The parameters of the via the dag defined regression models are also estimated. An if-statement may be specified to analyze only a part of the data set, i.e. only those observations where *expression* is true. There are several facultative *options* concerning the (start) parameters of the algorithm or the kind of output at the end. They are listed in the next paragraphs.

## 10.1.3 Options

## **Options for controlling MCMC simulations**

The following options correspond to those given on page 79 and are therefore only briefly explained.

• burnin = b

Changes the number of burnin iterations from 2000 to b; it is a positive integer number with 0 < b < 500001.

DEFAULT: burnin = 2000

• iterations = i

Changes the number of MCMC iterations from 52000 to i; it is a positive integer number with 0 < i < 10000000.

DEFAULT: iterations = 52000

#### $\bullet$ step = s

Changes the thinning parameter of MCMC iterations from 50 to s; it is a positive integer number with 0 < s < 1000.

DEFAULT: step = 50

## Options for initial values of algorithm

## • Changing hyperparameters of partial variances

As already mentioned we assume  $\sigma_{i|pa(i)}^2 \sim IG(\delta_{i|pa(i)}, \lambda_{i|pa(i)})$  for  $i = 0, \dots, p-1$ . By the following two commands the values of the two hyperparameters  $\delta_{i|pa(i)}$  and  $\lambda_{i|pa(i)}$  can be freely chosen. If this is not done the default values correspond to a non-informative gamma distribution.

#### - delta = c

Specifies the first parameter of the inverse gamma distribution of the partial variances,  $\delta_{i|pa(i)}$ , is set equal to d. Otherwise it is equal to 1. The value c has to be of type realvalue with 0 < c < 20.

DEFAULT: delta = 1

#### - lambda = l

Specifies the second parameter of the inverse gamma distribution of the partial variances,  $\lambda_{i|pa(i)}$ , is set equal to l. Otherwise it is equal to 0.005. The value d has to be of type real value with 0 < d < 20.

DEFAULT: lambda = 0.005

#### Choosing special graph to start from

Usually the algorithm starts from the independent model, that means from a dag without any edges. This can be changed by the command

type = 
$$0/1/2/3/4$$
  
DEFAULT: type = 0

where the different values have the following meanings:

## - type=0

Algorithm starts from an **independent** model with no edges.

#### - type=1

Algorithm starts from a **complete** model where all edges are directed from "lower" variables to "higher" ones, i.e.  $x_i \to x_j$ ,  $\forall i < j$ .

#### - type=2

Algorithm starts from a **complete** model where all edges are directed from "higher" variables to "lower" ones, i.e.  $x_i \to x_i$ ,  $\forall i < j$ .

#### - type=3

Algorithm starts from a model where there is an edge from each variable to the next "higher" one, like a **chain**, i.e.  $x_i \to x_j$ ,  $\forall i = j + 1$ .

#### - type=4

Algorithm starts from a model where there is an edge from each variable to the next "lower" one, like a **chain**, i.e.  $x_j \to x_i$ ,  $\forall i = j + 1$ .

## Options concerning the way of model selection

#### Kind of switch step

There are three ways how the switch step can be carried out in the rj-algorithm. The first one is similar to the performance of a birth or death step (i.e. adding or deleting an edge): A proposal is made and then accepted by its corresponding acceptance ratio. As it may be very complicate to calculate a good proposal, a simplification can be achieved by the consideration if the switch step leads to an equivalent dag model. If this holds true, the given and the proposed dag should be statistically indistinguishable. The proposed dag can therefore be accepted with probability 0.5. The kind of switch step can be chosen by the command

switch = normal/equi/mix,
DEFAULT: switch = normal

which differ in the following way:

#### - switch=normal

The switch step is carried out by proposing the new dag and accepting it with the corresponding acceptance probability. The transformation into equivalent model may occur very seldom and, consequently, the acceptance ratio very low.

## - switch=equi

The switch step is only allowed if it results into an equivalent model. In this case, it is performed with a probability of 0.5. Transformations into a non-equivalent model can only occur by a birth or death step.

#### - switch=mix

This command causes a mixture of both procedures described above: If the proposed switch step leads to an equivalent model it is accepted with probability 0.5. If it results into a non-equivalent model a proposal is made and accepted by the corresponding acceptance ratio.

#### Kind of distribution family / interactions

There are three different types of data sets as they can consists of continuous, binary, or mixed variables which results in the assumption of a Gaussian, a multinomial, or a conditional Gaussian distribution. Dependent on the kind of data set the rj-algorithm for the model selection changes as described above. This can be indicated by the optional command

In the case that the model selection for a binary data set shall be carried out accounting for interactions the command

is needed instead of family = discrete. In this case, a special option concerning the output is given by the command detail\_ia which is explained below.

#### Restriction to the search space

It is possible to restrict the search space, i.e. to state an (missing) edge as fix or determinate the orientation of an edge. This is done by writing the restrictions into a file *restrict* which is then read by the command

The restriction is then given by a  $p \times p$  matrix that lies under the path  $path\_of\_restrict$ . The matrix is allowed to have three possible entries, namely 0,1, and 2 which have the following meaning: An entry of 2 corresponds to no restriction of the corresponding edge, it may occur or not. An entry of 1 indicates that this edge has to exist in each graph of the Markov chain, whereas 0 denotes that the corresponding edge must not occur.

## Options concerning the output

#### • Estimated regression coefficients

As already mentioned, the parameters of each regression model are estimated in every iteration. Because of the fact that the qualitative structure of the dag is usually of greater interest than the quantitative estimations of the regression coefficients, in the standard output these estimated parameters are omitted. Nevertheless

gives the mean, the 10%, 50% and the 90% quantile of every parameter of all regression models. As the model space for dags is very huge we abandon the possibility to store the estimated values for each dag. To perform the necessary calculation BayesX creates a temporary file under the device  $c:\backslash \ldots$  For this purpose, its important to ensure that a device with this name exists. Otherwise the user has to provide an alternative path for the storage file by the command

## Estimated coefficients of interactions

#### Criteria for the listed models

As model selection for dags is performed in an extremely huge search space one might not want to get a list of all models which have been visited during MCMC estimation regardless of the relative frequency of their appearance. The option

```
print_models = all/prob/limit/normal,
DEFAULT: print_models = normal
```

allows to focus on special criterions for the models printed in the output.

- print\_models = all
  - All models which have been visited by the Markov chain are printed.
- print\_models = prob

The most frequent models of the chain are printed except for those which are the less frequent ones and have altogether a probability of alpha=0.05. The value of alpha can be changed as it is explained a few lines below.

- print\_models = limit
  - Here, the first 10 models with the highest frequencies are printed. The number of listed models can be made different from 10 as it is explained a few lines below.
- print\_models = normal

The option normal is a mixture of limit and prob, as it chooses the one which produces less models. The default parameters are again alpha=0.05 and number=10.

Number of different days visited by the algorithm: 16

```
****** DIFFERENT MODELS sorted by frequencies *******
    ******* all models *************
          010
               000
                     000
                          1
                              3894
                                    0.3890
          000
               100
                     000
                          1
                              3814
                                    0.3810
          000
               100
                     100
                              806
                                    0.0806
```

Figure 10.2: Example for model listing in the output.

The default setting of print\_models is print\_models=normal.

#### $\bullet$ number = n

Changes the number of printed models in the option print\_models = limit to n. The variable number has to be of type realvalue with  $0 \le n \le 10000$ . DEFAULT = 10

## $\bullet$ alpha = a

Sets alpha in the option print\_models = prob equal to a. That means when using print\_models = prob the most frequent models which unify 1-a of the posterior probability are printed. The variable alpha has to be of type introduce with  $a \in [0,1]$ .

DEFAULT = 0.05

## $\bullet$ printit = p

Prints every p-th iteration in the output window instead of every 100-th. The printing of the iterations can be suppressed by setting p higher than the number of iterations. The variable printit has to be of type intvalue with 0 .

DEFAULT = 100

## 10.1.4 Estimation Output

The output can be written to a file by opening a logfile before using the estimation command. It has to be closed afterwards. The use of logfiles is described in detail in section 3.2. The output itself is structured as follows:

## • Listing of different dags:

The different models are listed by their adjacency matrices. In order to save space, the different rows are printed in one line with a blank indicating the beginning of a new one. The number of edges is given as well as the absolute and relative frequency of the model. For example the first line of the exemplifying output in Figure 10.2 gives the information

that the most frequent model is represented by the adjacency matrix 
$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and contains 1 edge. It occurred in the thinned out Markov chain for 3894 times which corresponds to a relative frequency of about 0.389. Notice, that different dags may represent the same statistical model as they may be Markov-equivalent.

#### • Listing of essential graphs

Additional to the dags, the essential graphs are printed, too. I.e. those dags, which are

Number of different equivalent classes visited by the algorithm: 6 \*\*\*\*\*\* DIFFERENT EQUIVALENCE CLASSES sorted by frequencies \*\*\*\*\*\*\* Skeleton: 010 000 000 No immoralities. Number of edges: 1 Abs.freq.: 7708 Rel. freq.: 0.771 Skeleton: 011 000 000 Immoralities: (0;1,2)Abs.freq.: 806 Number of edges: 2 Rel.freq.: 0.0806 Skeleton: 011 000 000 No immoralities. Number of edges: 2 Abs.freq.: 523 Rel.freq.: 0.0523

Figure 10.3: Example for listing of equivalence classes in the output.

equivalent to each other, are summarized and represented by their essential graph. The representation of the essential graph, which can contain undirected as well as directed edges, is as follows. First the underlying graph, the skeleton, is given by the adjacency matrix as described above. But now, the entries indicate always an undirected edge. (E.g. the undirected graph a-b of the two variables a and b is given by 01 00.) Then the immoralities of the essential graph are listed. Remember that within an essential graph an oriented edge can only occur as a part of an immorality  $b \to a \leftarrow c$  which is here represented by the triple (a;b,c). The example output of Figure Figure 10.3 shows that the first two dag models of Figure 10.2 which are equivalent have been summarized to their representing essential graph  $X_0 \to X_1$   $X_2$ . The next most frequent statistical model is represented by the essential graph  $X_0 \to X_2 \leftarrow X_1$  which is given by our representation as the skeleton matrix 011 000 000 and the immorality (0;1,2).

#### Averaged adjacency matrix:

The (i, j)-th element of the averaged adjacency matrix gives the estimated posterior probability of the presence of the edge  $i \to j$  in the true dag.

#### • Mean of skeletons:

The skeleton of a dag is defined as the same graph without regarding the directions of the edges. Equivalent dags have at least the same skeleton. So it may be helpful to have also a look at the averaged matrix of the skeletons, which is of course symmetric.

#### Correlation:

The marginal and the partial correlation matrices of the regarded data set is given, too.

#### • Ratios:

We give some short information about the acceptance ratios for the birth-, death- and switch-

steps which denotes the cases where an edge is added, dropped or switched. The first two cases imply a change in dimension and are therefore sampled by a reversible jump step.

#### • Estimated parameters:

If the option print\_dags is used, the estimated regression coefficients  $\beta_{i|-i}$ ,  $i=0,\ldots,p-1$ , are listed at the end. The notation -i denotes all variables except for i. Besides the mean of the sampled Markov chain for each parameter there is also the 10%, the 50% and the 90% quantile given. As in equivalent models the direction of edges and, thus, also the regression models vary, in most cases the estimated regression coefficients do not give a deeper insight into the model and have to be interpreted in a very careful way.

## 10.1.5 References

- Andersson, S. A., Madigan, D., and Perlman, M. D. (1997a). A Characterization of Markov equivalence Classes for Acyclic Digraphs. *The Annals of Statistics*, **25**, 505–541.
- Andersson, S. A., Madigan, D., and Perlman, M. D. (1997b). On the Markov equivalence of Chain Graphs, Undirected Graphs, and Acyclic Digraphs. *Scandinavian Journal of Statistics*, **24**, 81–102.
- CHICKERING, D. M. (1995). A Transformational Characterization of Equivalent Bayesian Network Structures. In P. Besnard and S. Hanks (Eds.), *Uncertainty in Artificial Intelligence, Proceedings of the Eleventh Conference*, San Francisco: Morgan Kaufmann, pp. 87 98
- BROOKS, S. P. (1998). Markov Chain Monte Carlo Method and its Application. *The Statistician*, **47**, 69–100.
- Fronk, E.-M. and Giudici, P. (2000). Markov Chain Monte Carlo model selection for DAG Models. *Discussion Paper*, Universität München.
- Geiger, D. and Heckerman, D. (1994). Learning Gaussian Networks. *Proceedings of the Tenth Conference on Uncertainty in Artificial Intelligence*. Morgan Kaufmann, 235–243.
- Geiger, D. and Heckerman, D. (1999). Parameter priors for directed acyclic graphical models and the characterisation of several probability distributions. Submitted for publication.
- GILKS, W. R., RICHARDSON, S., AND SPIEGELHALTER, D. J. (1996). Markov Chain Monte Carlo in Practice., Chapman and Hall, London.
- GIUDICI, P. AND GREEN, P. J. (1999). Decomposable Graphical Gaussian Model Determination. Biometrika, 86, 785–801.
- Green, P. J. (1995). Reversible Jump Markov Chain Monte Carlo Computation and Bayesian Model Determination. *Biometrika*, **82**, 711–32.
- Lauritzen, S. L. (1996). Graphical Models, Clarendon Press, Oxford.

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