Chapter 2 Configuration Space
Chapter 3 Rigid-Body Motions
Chapter 4 Forward Kinematics
Chapter 5 Velocity Kinematics and Statics
Chapter 6 Inverse Kinematics
Chapter 7 Kinematics of Closed Chains
Chapter 8 Dynamics of Open Chains
Chapter 9 Trajectory Generation
9.1 Definitions
9.2 Point-to-Point Trajectories
9.3 Polynomial Via Point Trajectories
9.4 Time-Optimal Time Scaling

Chapter 10 Motion Planning
Chapter 11 Robot Control
Chapter 12 Grasping and Manipulation
Chapter 13 Wheeled Mobile Robots

Important concepts, symbols, and equations

Time-optimal time scaling: Find the time scaling $s(t)$ that moves the robot along a path from $\theta(0)$ to $\theta(1)$ in minimum time, given the robot's dynamics and actuator limits.

$$\boxed{\begin{aligned}\dot{\theta} &= \frac{d\theta}{ds}\dot{s}, \\ \ddot{\theta} &= \frac{d\theta}{ds}\ddot{s} + \frac{d^2\theta}{ds^2}\dot{s}^2\end{aligned}}$$

Dynamics:

$$M(\theta)\ddot{\theta} + \dot{\theta}^{\mathrm{T}}\Gamma(\theta)\dot{\theta} + g(\theta) = \tau$$

Dynamics along the path:

$$\underbrace{\left(M(\theta(s))\frac{d\theta}{ds}\right)}_{m(s)\in\mathbb{R}^n}\ddot{s} + \underbrace{\left(M(\theta(s))\frac{d^2\theta}{ds^2} + \left(\frac{d\theta}{ds}\right)^{\mathrm{T}}\Gamma(\theta(s))\frac{d\theta}{ds}\right)}_{c(s)\in\mathbb{R}^n}\dot{s}^2 + \underbrace{g(\theta(s))}_{g(s)\in\mathbb{R}^n} = \tau$$

note: the vector m may have negative components

$$m(s)\ddot{s} + c(s)\dot{s}^2 + g(s) = \tau$$

Important concepts, symbols, and equations (cont.)

Force/torque limits, joint i:

$$\tau_i^{\min}(s,\dot{s}) \leq \tau_i \leq \tau_i^{\max}(s,\dot{s})$$

Limits on $\ddot{s}$ as a function of $(s,\dot{s})$, joint i:

$$\tau_i^{\min}(s,\dot{s}) \leq m_i(s)\ddot{s} + c_i(s)\dot{s}^2 + g_i(s) \leq \tau_i^{\max}(s,\dot{s})$$

Important concepts, symbols, and equations (cont.)

Upper and lower bounds on $\ddot{s}$ due to joint i:

$$
\begin{aligned}
\text{if } m_i(s) > 0, \quad L_i(s,\dot{s}) &= \frac{\tau_i^{\min}(s,\dot{s}) - c(s)\dot{s}^2 - g(s)}{m_i(s)}, \\
U_i(s,\dot{s}) &= \frac{\tau_i^{\max}(s,\dot{s}) - c(s)\dot{s}^2 - g(s)}{m_i(s)} \\
\text{if } m_i(s) < 0, \quad L_i(s,\dot{s}) &= \frac{\tau_i^{\max}(s,\dot{s}) - c(s)\dot{s}^2 - g(s)}{m_i(s)}, \\
U_i(s,\dot{s}) &= \frac{\tau_i^{\min}(s,\dot{s}) - c(s)\dot{s}^2 - g(s)}{m_i(s)}
\end{aligned}
$$

Important concepts, symbols, and equations (cont.)

Upper and lower bounds on $\ddot{s}$ at $(s, \dot{s})$:

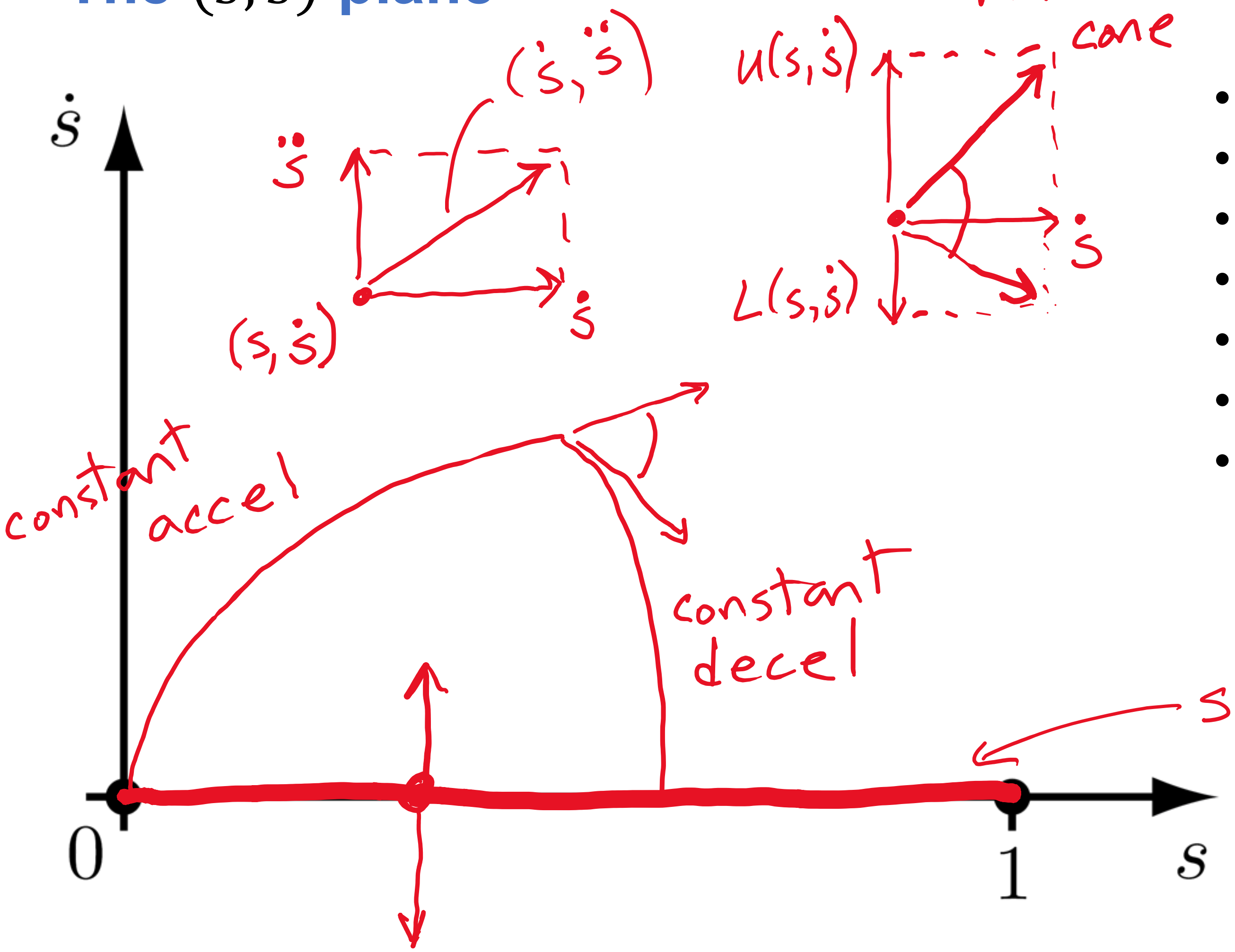
$$L(s,\dot{s}) = \max_i L_i(s,\dot{s}) \qquad \text{and} \qquad U(s,\dot{s}) = \min_i U_i(s,\dot{s})$$

$$\boxed{L(s,\dot{s}) \le \ddot{s} \le U(s,\dot{s})}$$

Important concepts, symbols, and equations (cont.)

s goes from 0 to 1, and $\dot{s} \geq 0$ always

The $(s, \dot{s})$ plane

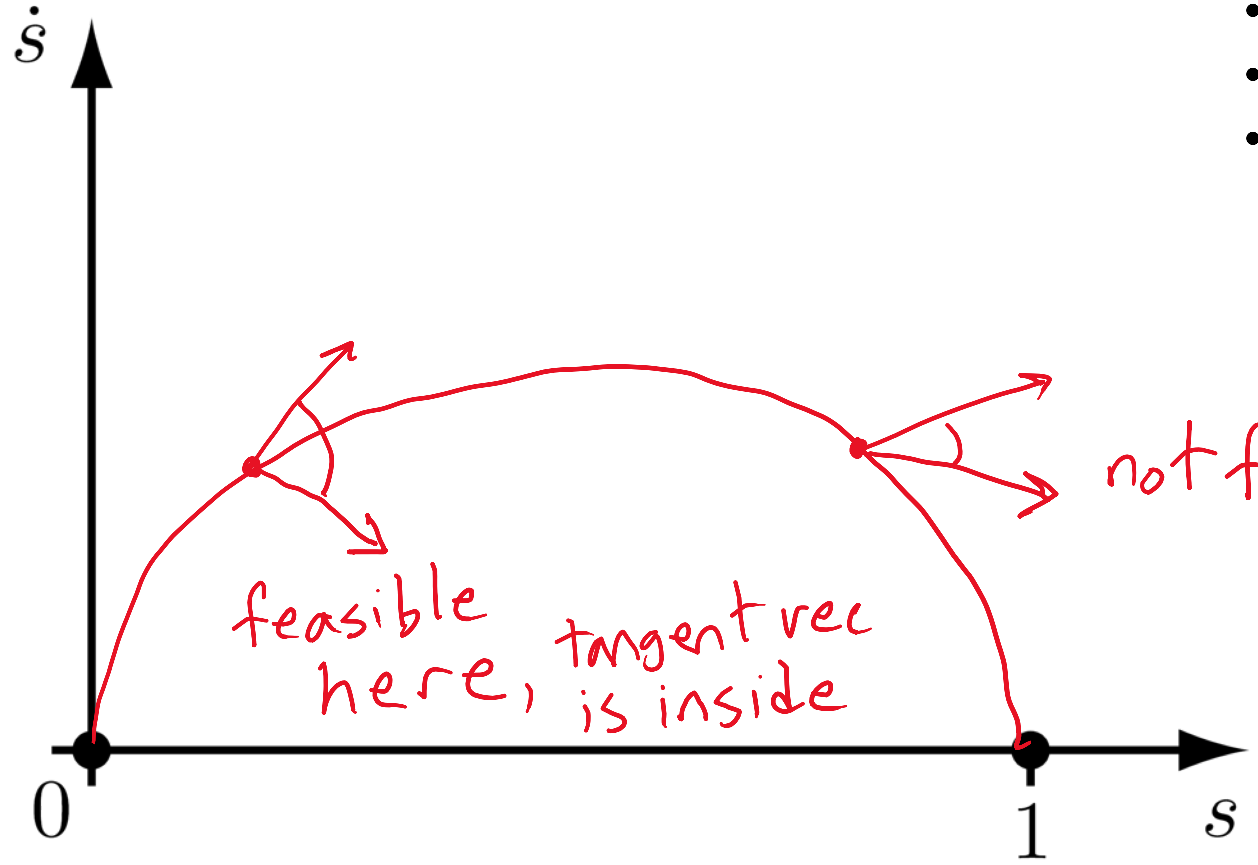


- Why we only consider quadrant I.
- A slow motion along the path.
- An example rate of change $(\dot{s}, \ddot{s})$ plotted at $(s, \dot{s})$.
- A **motion cone** defined by $L(s, \dot{s})$ and $U(s, \dot{s})$.
- Cones on the $\dot{s} = 0$ line.
- What does a constant accel curve look like?
- What does a constant decel curve look like?

slow motion has $\dot{s}$ infinetisimally above zero.

Important concepts, symbols, and equations (cont.)

The $(s, \dot{s})$ plane



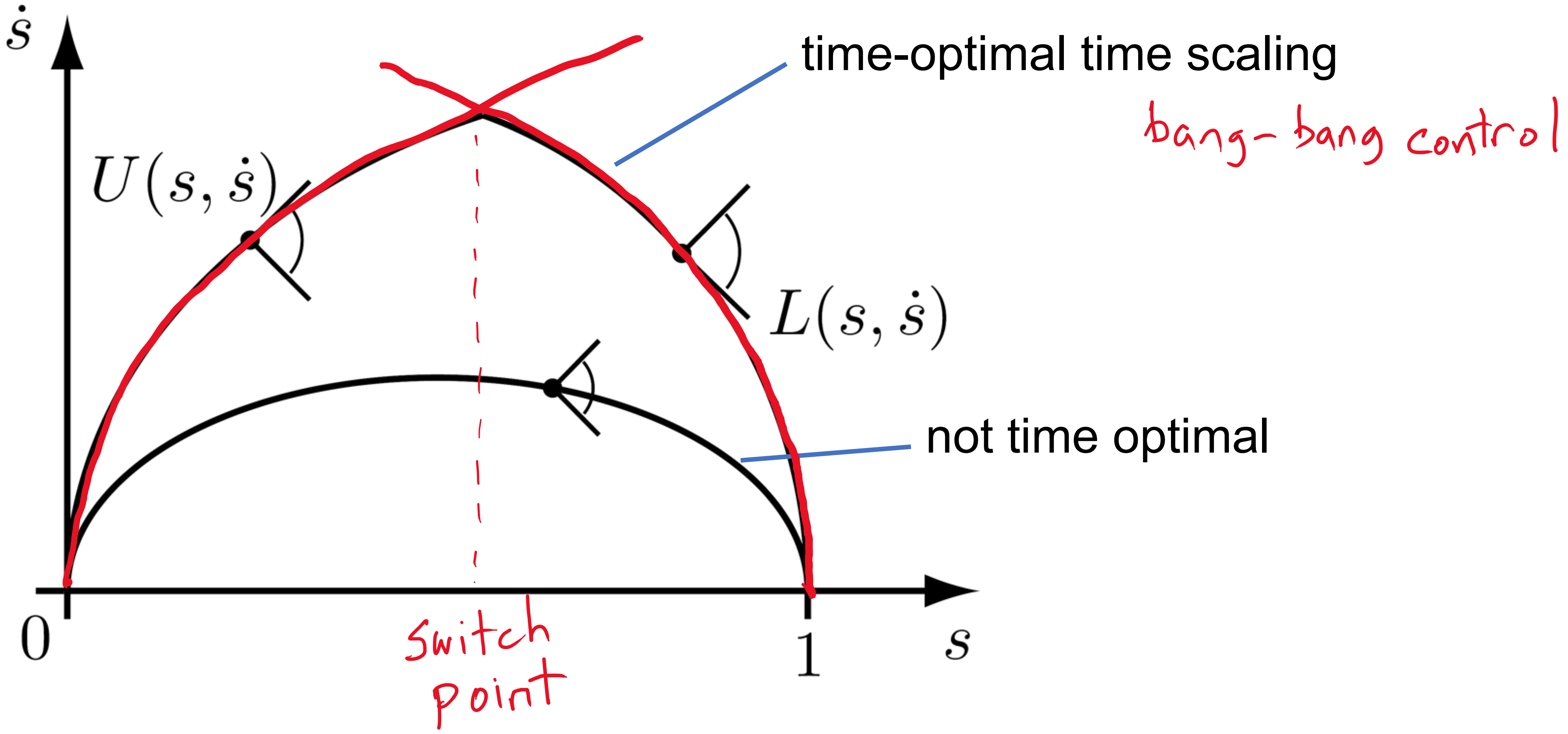
not feasible here,

tangent vec is not inside

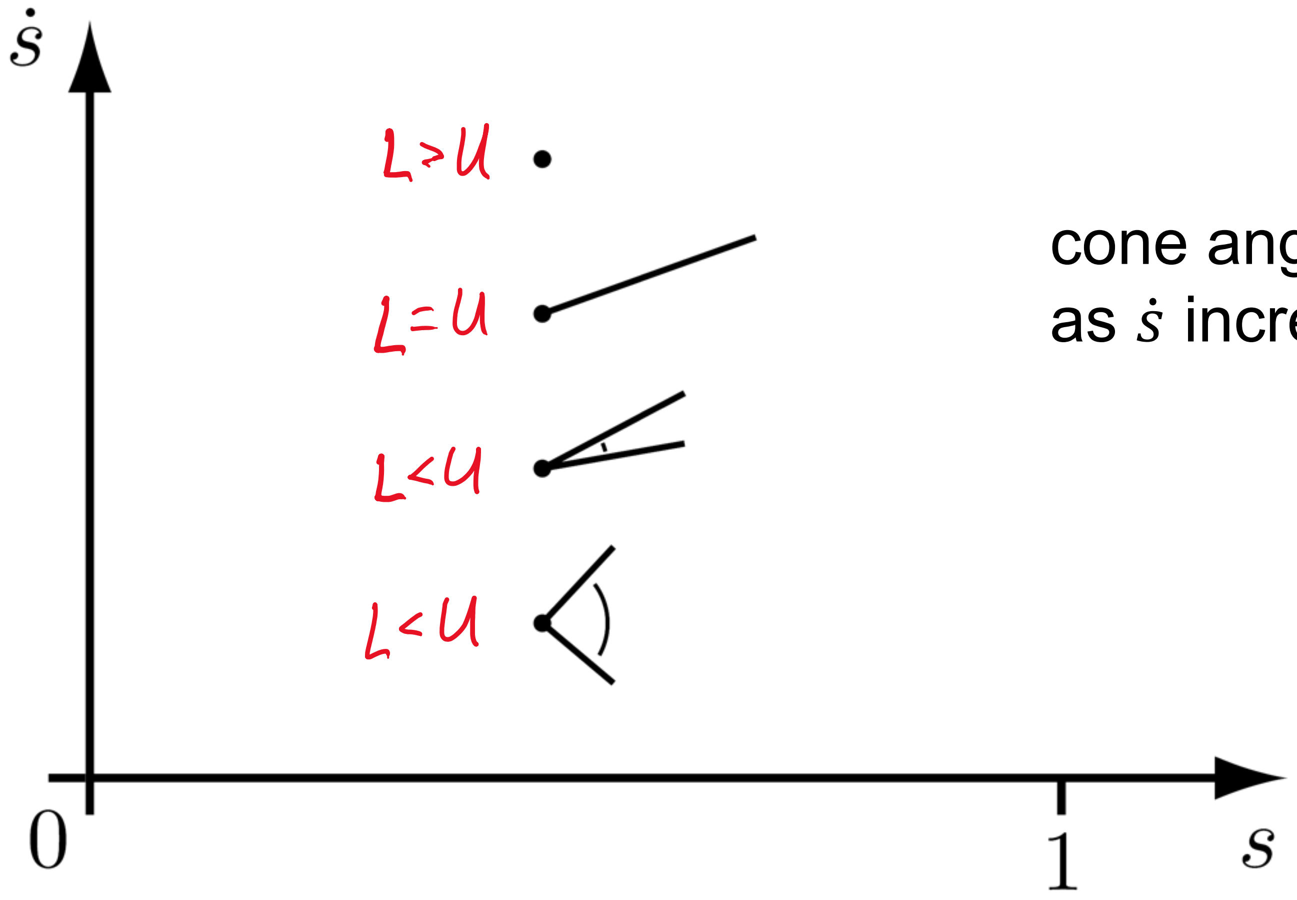
- Plot a complete time scaling.
- Check the feasibility of a time scaling.
- What happens at $(s, \dot{s})$ where $L(s, \dot{s}) > U(s, \dot{s})$?

if $L = U$, the motion cone collapses to a single tangent vector. If $L > U$, motion cone disappears. Robot cannot stay on the path.

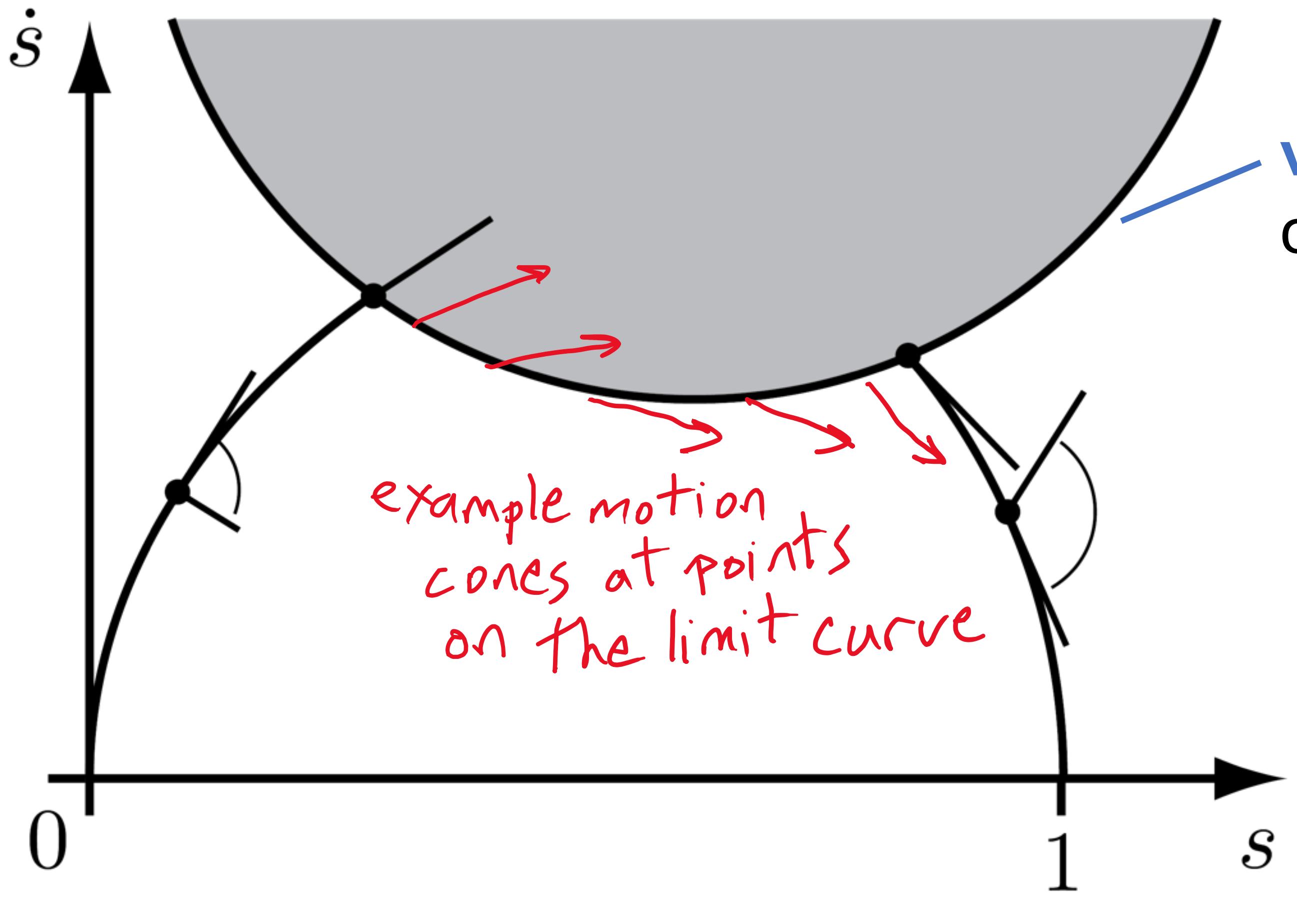
Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)



cone angle tends to shrink
as $\dot{s}$ increases



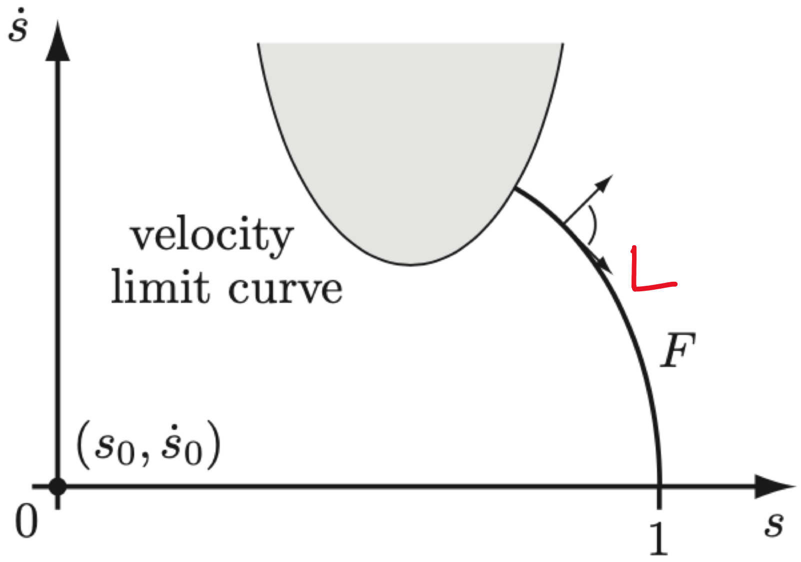
velocity limit curve where the cone is only a single tangent vector

Important concepts, symbols, and equations (cont.)

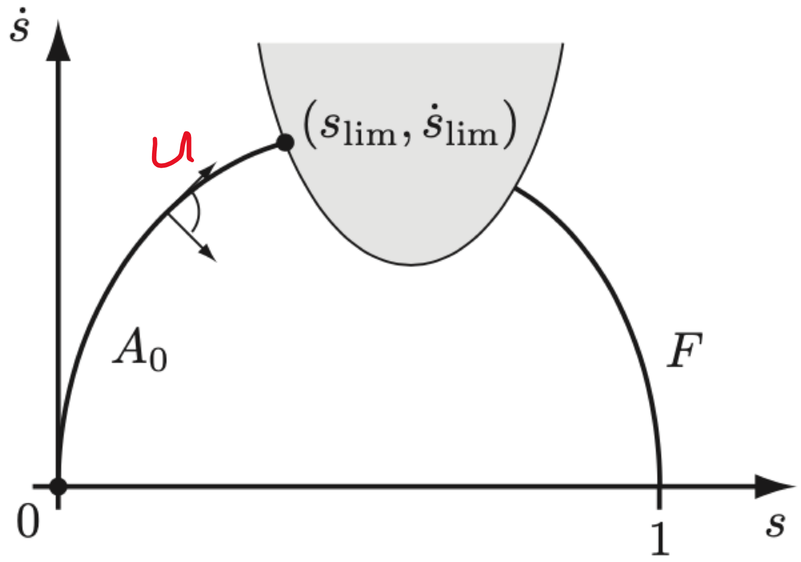
Time-optimal time-scaling algorithm

1. Initialize with $i = 0$ and $(s_0, \dot{s}_0) = (0, 0)$.
2. Integrate L backward from $(1, 0)$ to get the curve F.
3. Integrate U forward from $(s_i, \dot{s}_i)$ to get A_i. If it intersects F, finished.
4. Binary search to find how to start decelerating from A_i.
5. Increment i and integrate backwards to get A_i.
6. Update i and $(s_i, \dot{s}_i)$ and go back to step 3.

Important concepts, symbols, and equations (cont.)

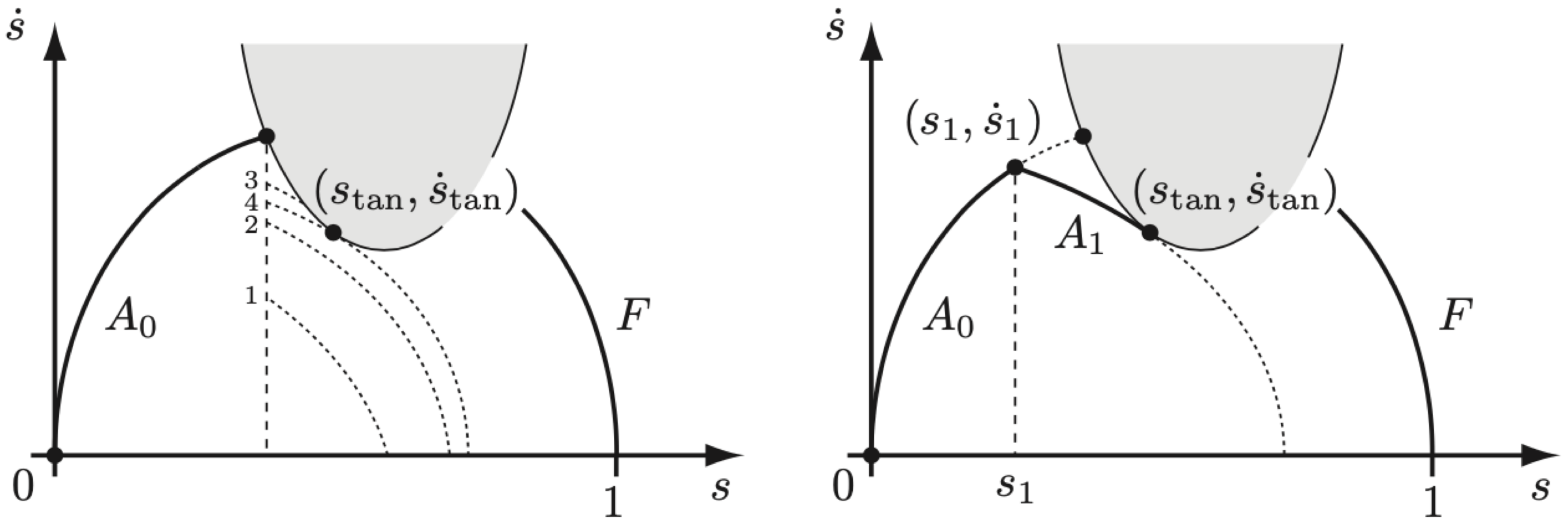


Step 2: $i = 0, \mathcal{S} = \{\}$



Step 3: $i = 0, \mathcal{S} = \{\}$

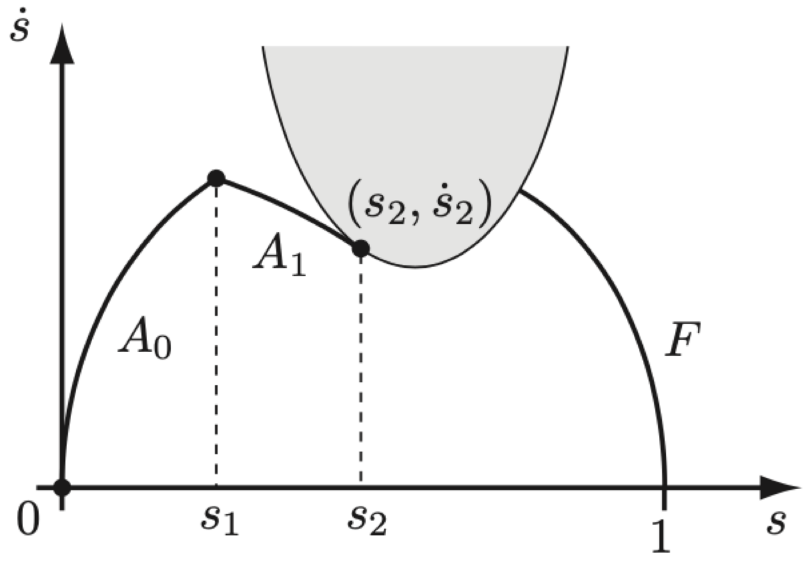
Important concepts, symbols, and equations (cont.)



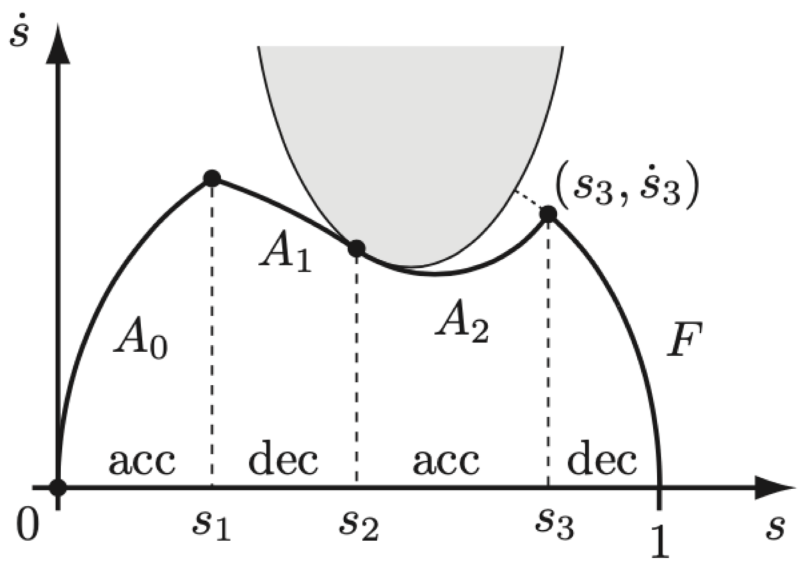
Step 4: $i = 0, \mathcal{S} = \{\}$

Step 5: $i = 1, \mathcal{S} = \{s_1\}$

<u>Important concepts, symbols, and equations</u> (cont.)

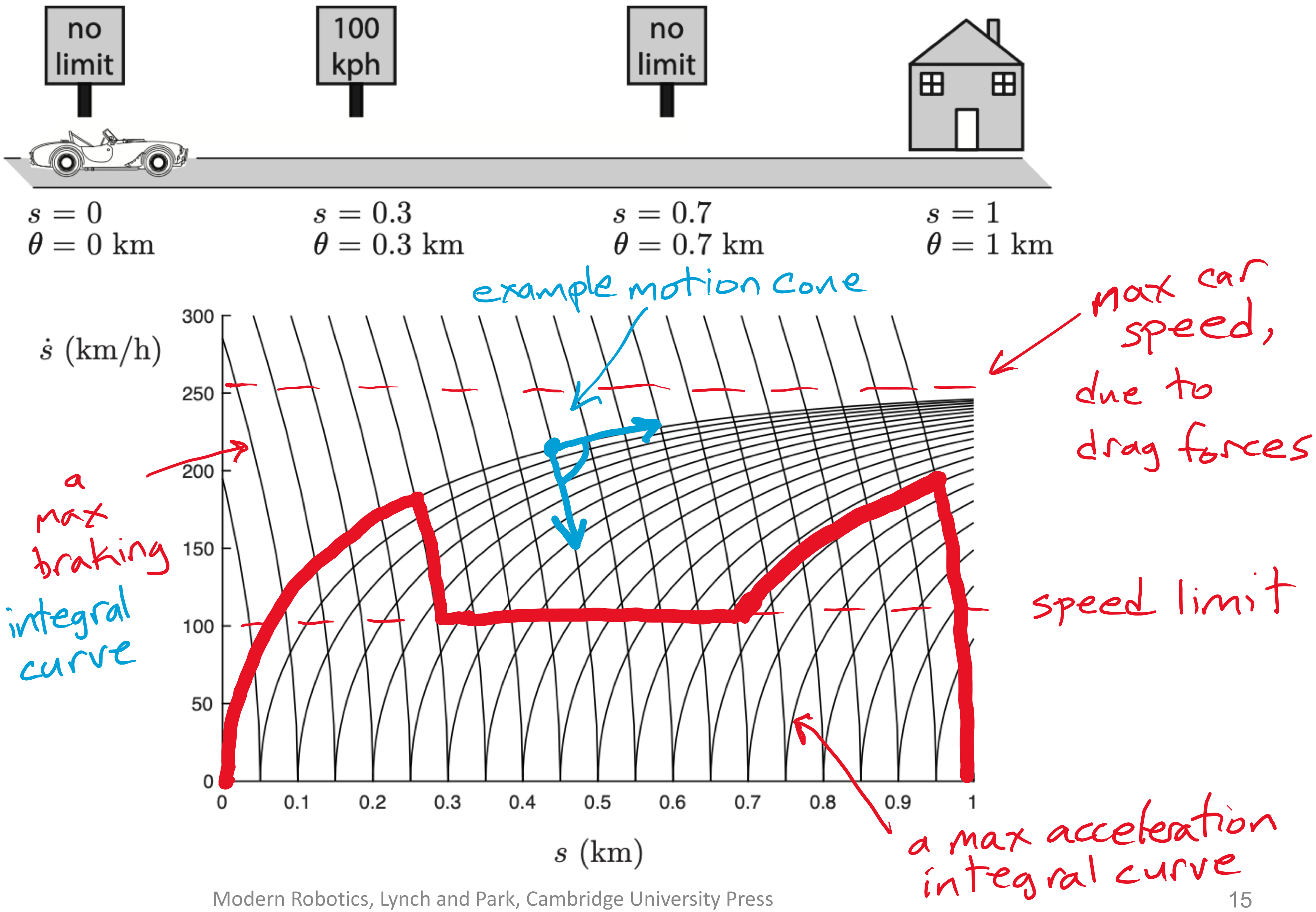


Step 6: $i = 2, \mathcal{S} = \{s_1, s_2\}$

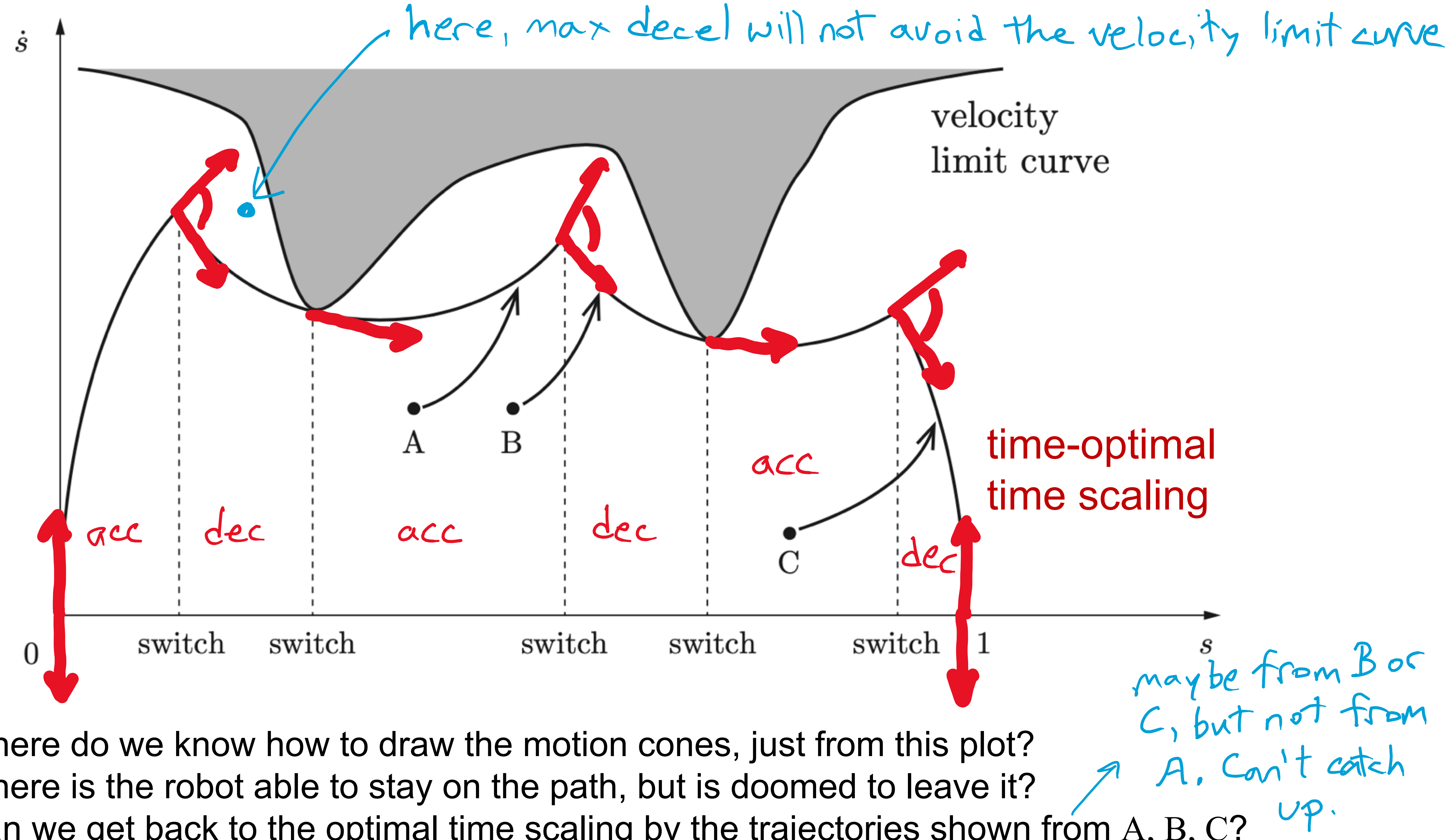


Step 3: $i = 3, \mathcal{S} = \{s_1, s_2, s_3\}$

Draw the **legal** time-optimal time scaling for a driver rushing home with the max braking and max acceleration **integral curves** shown.



Modern Robotics, Lynch and Park, Cambridge University Press



1. Where do we know how to draw the motion cones, just from this plot?
2. Where is the robot able to stay on the path, but is doomed to leave it?
3. Can we get back to the optimal time scaling by the trajectories shown from A, B, C?