Important concepts, symbols, and equations

Relationship between planar and spatial twist:

$$\mathcal{V}_{b6} = \left[\begin{array}{c} 0 \\ 0 \\ \boxed{\mathcal{V}_b} \\ 0 \end{array}\right] \begin{array}{l} \omega_{bx} \\ \omega_{by} \\ \boxed{\begin{array}{l} \omega_{bz} \\ v_{bx} \\ v_{by} \end{array}} \\ v_{bz} \end{array}$$

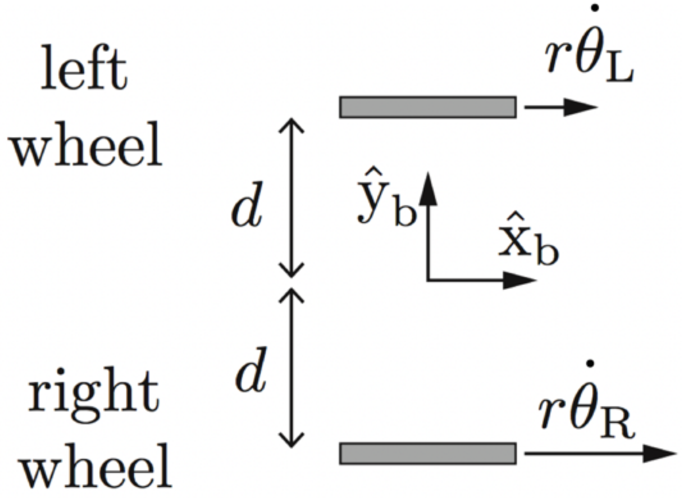
Important concepts, symbols, and equations (cont.)

Odometry (or **dead reckoning**)

1. Measure the wheel displacements, $\Delta\theta$.
2. Assume constant wheel speeds, so $\dot{\theta} = \Delta\theta/\Delta t$, $\Delta t = 1$.
3. $\boxed{\text{Find } \mathcal{V}_b = F\dot{\theta} = F\Delta\theta.}$
4. Integrate $\mathcal{V}_{b6}$ for $\Delta t = 1$, $T_{b_k b_{k+1}} = e^{[\mathcal{V}_{b6}]}$.
5. $T_{sb_{k+1}} = T_{sb_k} T_{b_k b_{k+1}}$ (or express as q_{k+1}). where $q = \begin{bmatrix} \phi \\ x \\ y \end{bmatrix}$

Important concepts, symbols, and equations (cont.)

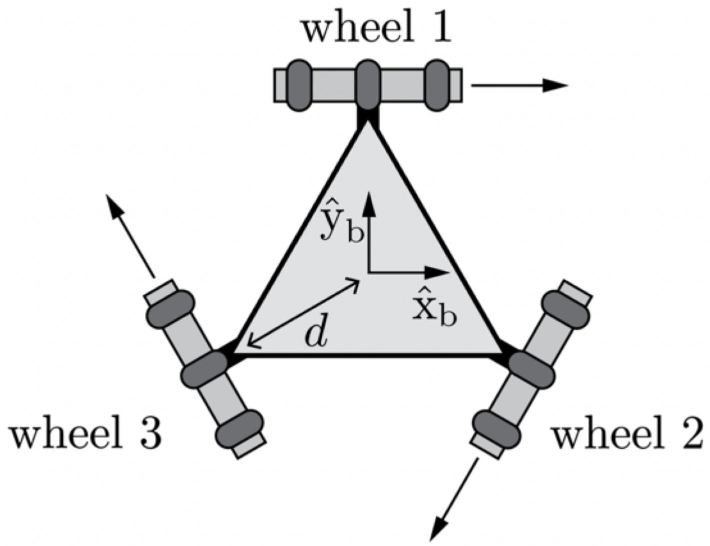
Diff-drive



$$\mathcal{V}_b = F\overset{\dot{\theta}}{\Delta\theta} = r\begin{bmatrix} -1/(2d) & 1/(2d) \\ 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} \overset{\dot{\theta}}{\begin{bmatrix} \Delta\theta_L \\ \Delta\theta_R \end{bmatrix}}$$

Important concepts, symbols, and equations (cont.)

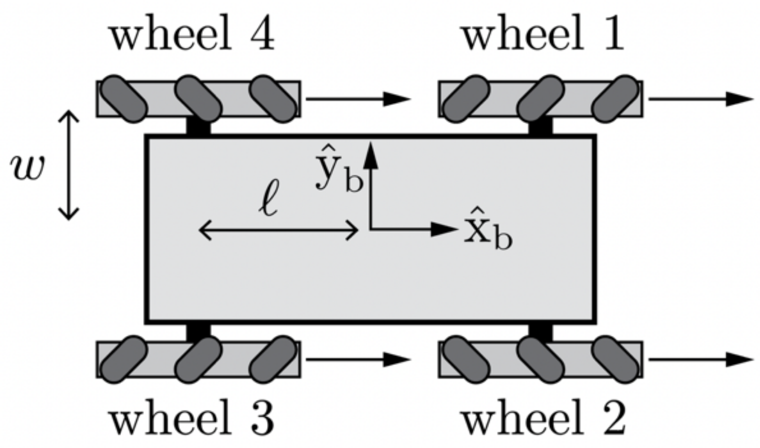
$$\dot{\theta} = H(0)\mathcal{V}_b \rightarrow \mathcal{V}_b = H^{\dagger}(0)\dot{\theta} = F\dot{\theta} = F\Delta\theta$$



$$\mathcal{V}_b = F\Delta\theta = r\begin{bmatrix} -1/(3d) & -1/(3d) & -1/(3d) \\ 2/3 & -1/3 & -1/3 \\ 0 & -1/(2\sin(\pi/3)) & 1/(2\sin(\pi/3)) \end{bmatrix}\Delta\theta$$

Important concepts, symbols, and equations (cont.)

$$\dot{\theta} = H(0)\mathcal{V}_b \rightarrow \mathcal{V}_b = H^{\dagger}(0)\dot{\theta} = F\dot{\theta} = F\Delta\theta$$



$$\mathcal{V}_b = F\Delta\theta = \frac{r}{4}\begin{bmatrix} -1/(\ell+w) & 1/(\ell+w) & 1/(\ell+w) & -1/(\ell+w) \\ 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix}\Delta\theta$$

Important concepts, symbols, and equations (cont.)

$$T_{b_k b_{k+1}} = e^{[\mathcal{V}_{b6}]}$$

$$T_{sb_{k+1}} = T_{sb_k} T_{b_k b_{k+1}} = T_{sb_k} e^{[\mathcal{V}_{b6}]}$$

$$\rightarrow q_{k+1}$$

or

$$T_{b_k b_{k+1}} = e^{[\mathcal{V}_{b6}]}$$

$$\rightarrow \Delta q_b \rightarrow \Delta q \rightarrow q_{k+1} = q_k + \Delta q$$

rotate the
linear
component

Could instead use $SE(2)$ representations and use a matrix exponential for $se(2)$.

Important concepts, symbols, and equations (cont.)

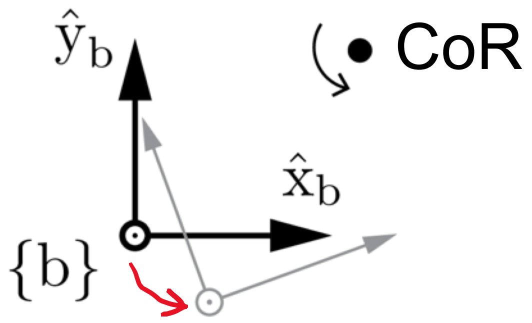
"Matrix exponential" for $se(2)$ using **center of rotation (CoR)** visualization

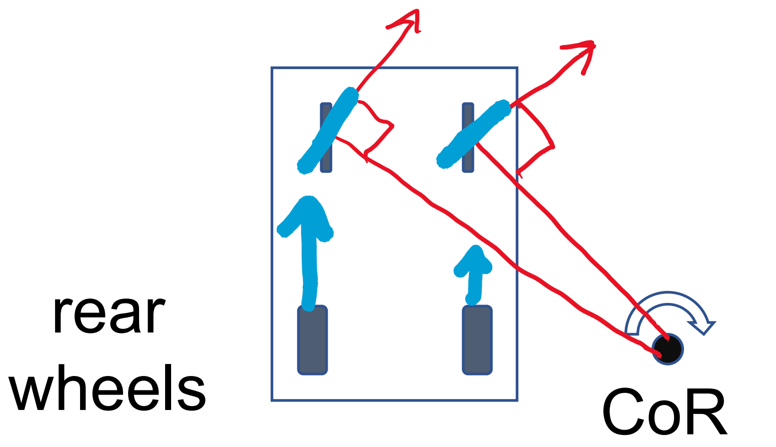
$$\mathcal{V}_b = \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix}$$

$$\text{if } \omega_{bz} = 0, \qquad \Delta q_b = \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix} = \begin{bmatrix} 0 \\ v_{bx} \\ v_{by} \end{bmatrix};$$

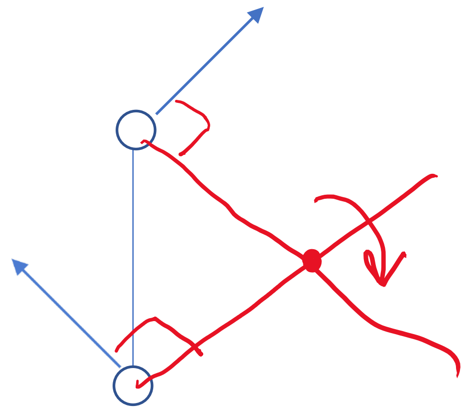
$$\text{if } \omega_{bz} \neq 0, \qquad \Delta q_b = \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix} = \begin{bmatrix} \omega_{bz} \\ (v_{bx}\sin\omega_{bz} + v_{by}(\cos\omega_{bz} - 1))/\omega_{bz} \\ (v_{by}\sin\omega_{bz} + v_{bx}(1 - \cos\omega_{bz}))/\omega_{bz} \end{bmatrix}$$

$$(-v_{by}/\omega_{bz}\,,\, v_{bx}/\omega_{bz})$$





Draw the proper angles of the front wheels of the car-like mobile robot for the CoR shown. (Ackermann steering.) How do the rolling speeds of the rear wheels compare?



Your mobile robot is equipped with two mouse sensors for odometry. They report the velocity vectors shown. Where is the CoR?

The 2 velocity vectors are described by 4 components, whereas a CoR and rotation speed are defined by 3, so in general the 2 velocity vectors won't *exactly* agree on a CoR.