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Important concepts, symbols, and equations

For a single joint with the joint velocity as the control:

- **Open-loop (feedforward) control:** $\dot{\theta}(t) = \dot{\theta}_d(t)$
- **Closed-loop (feedback) control:** $\dot{\theta}(t) = f(\theta_d(t), \theta(t))$
- **FF + Proportional-Integral (PI) FB control:**

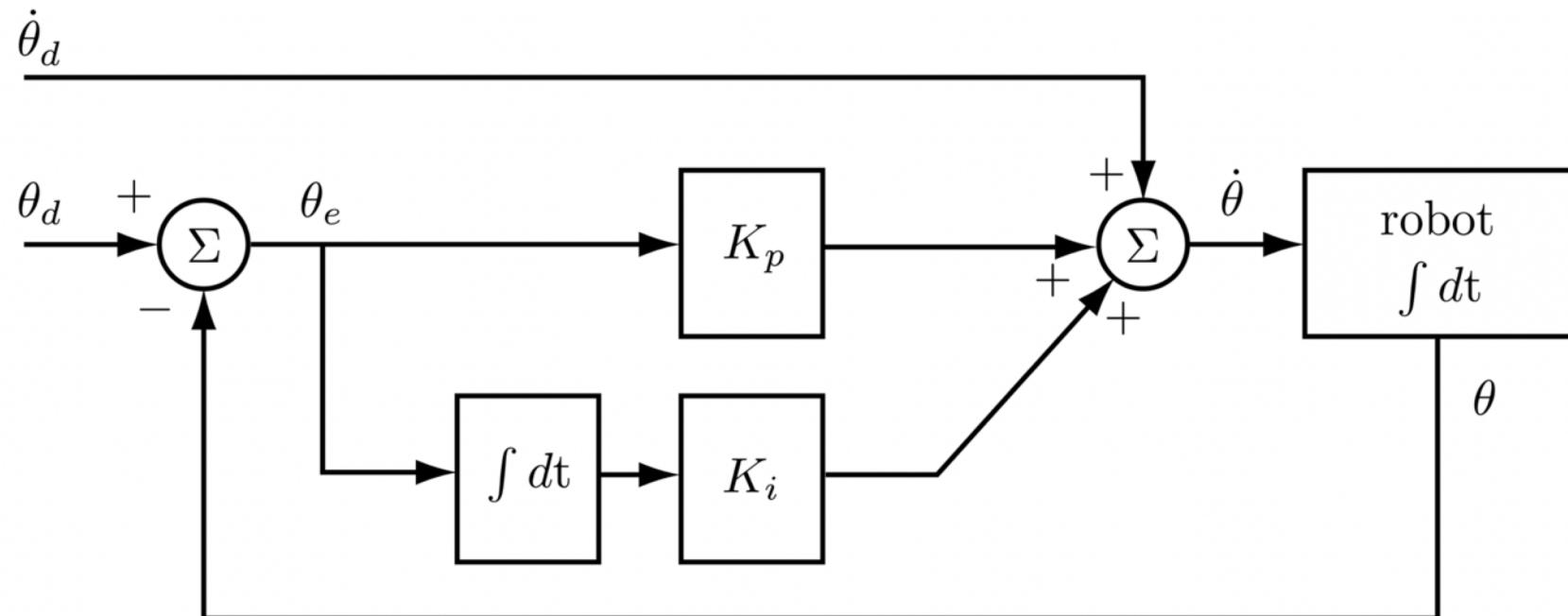
$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p \theta_e(t) + K_i \int_0^t \theta_e(t) dt, \quad K_p, K_i \geq 0$$

What is the point
of FF control in
this control law?

- reduces to FF control if $K_p, K_i = 0$
- if no FF term: **P control** when $K_i = 0$, **I control** when $K_p = 0$

Important concepts, symbols, and equations (cont.)

Block diagram



$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p\theta_e(t) + K_i \int_0^t \theta_e(t) dt$$

Important concepts, symbols, and equations (cont.)

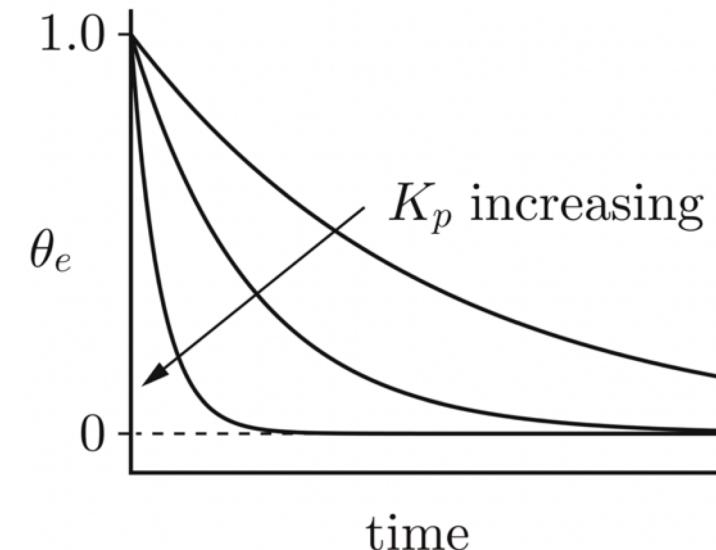
Setpoint control, $\theta_d(t) = c$, with a P controller

$$\dot{\theta}_e(t) = \overset{0}{\cancel{\dot{\theta}_d(t)}} - \dot{\theta}(t)$$

$$\dot{\theta}_e(t) = -K_p \theta_e(t) \rightarrow \dot{\theta}_e(t) + K_p \theta_e(t) = 0$$

$$t = 1/K_p$$

$$\theta_e(t) = e^{-t/t} \theta_e(0)$$



Important concepts, symbols, and equations (cont.)

Constant velocity control, $\theta_d(t) = ct + a$

P control

$$\dot{\theta}_e(t) = \dot{\theta}_d(t) - \dot{\theta}(t) = c - K_p \theta_e(t)$$

$$\dot{\theta}_e(t) + K_p \theta_e(t) = c$$

$$\theta_e(t) = \frac{c}{K_p} + \left(\theta_e(0) - \frac{c}{K_p} \right) e^{-K_p t}$$

PI control

$$\dot{\theta}(t) = K_p \theta_e(t) + K_i \int_0^t \theta_e(t) dt$$

$$\dot{\theta}_e(t) + K_p \theta_e(t) + K_i \int_0^t \theta_e(t) dt = c$$

$$\ddot{\theta}_e(t) + K_p \dot{\theta}_e(t) + K_i \theta_e(t) = 0$$

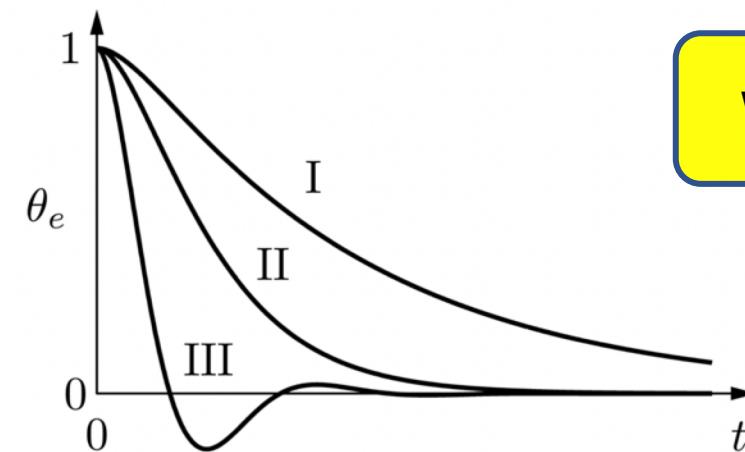
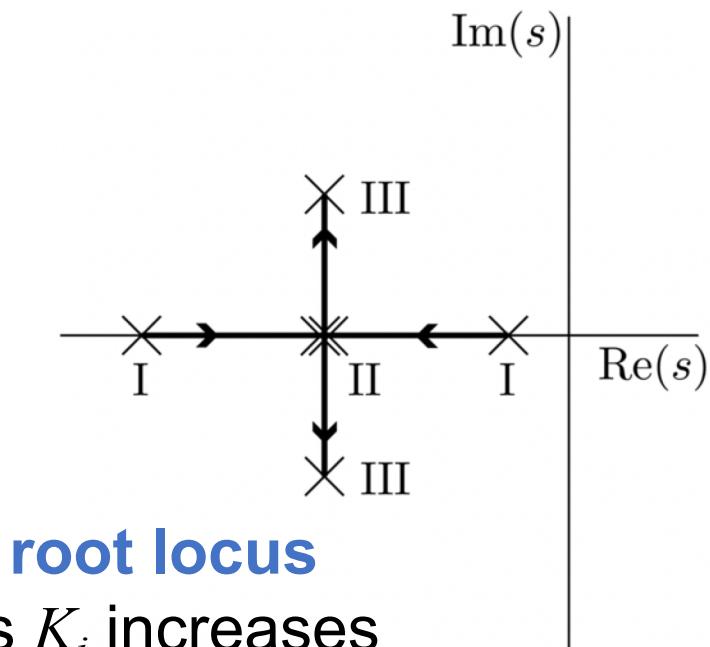
$$\zeta = K_p / (2\sqrt{K_i})$$

$$\omega_n = \sqrt{K_i}$$

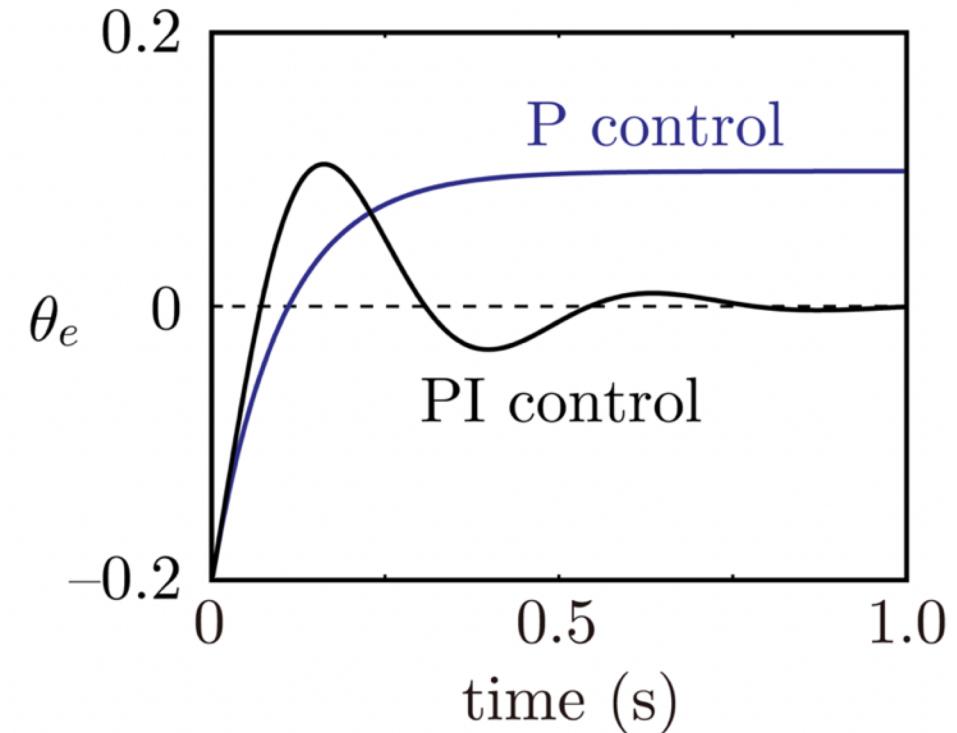
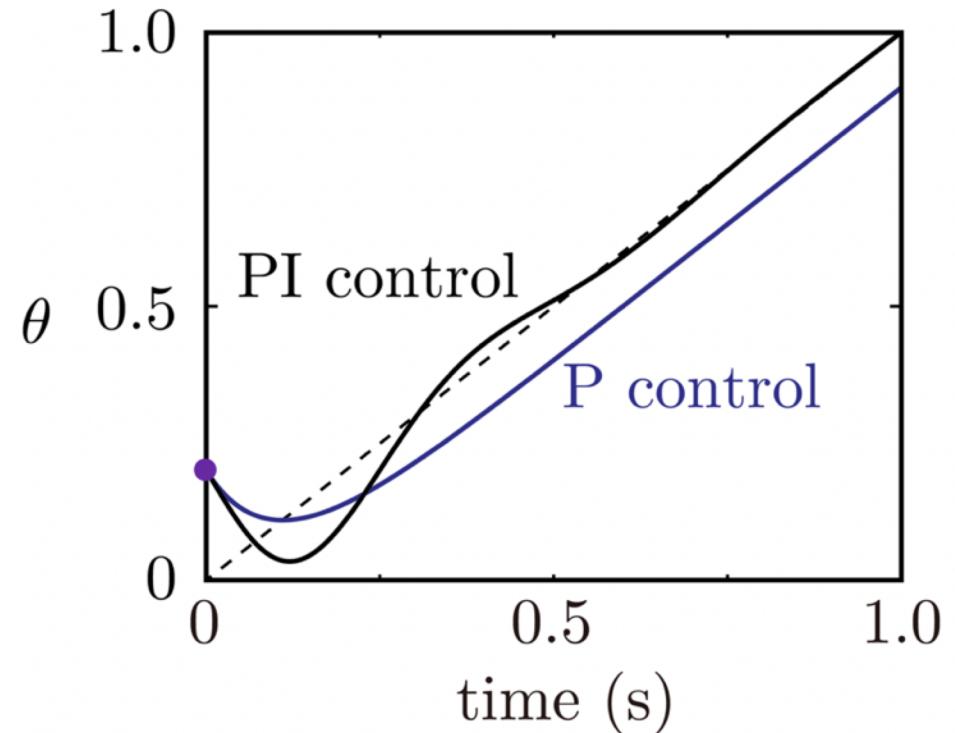
Important concepts, symbols, and equations (cont.)

characteristic equation of PI error dynamics: $s^2 + K_p s + K_i = 0$

$$s_{1,2} = -\frac{K_p}{2} \pm \sqrt{\frac{K_p^2}{4} - K_i}$$

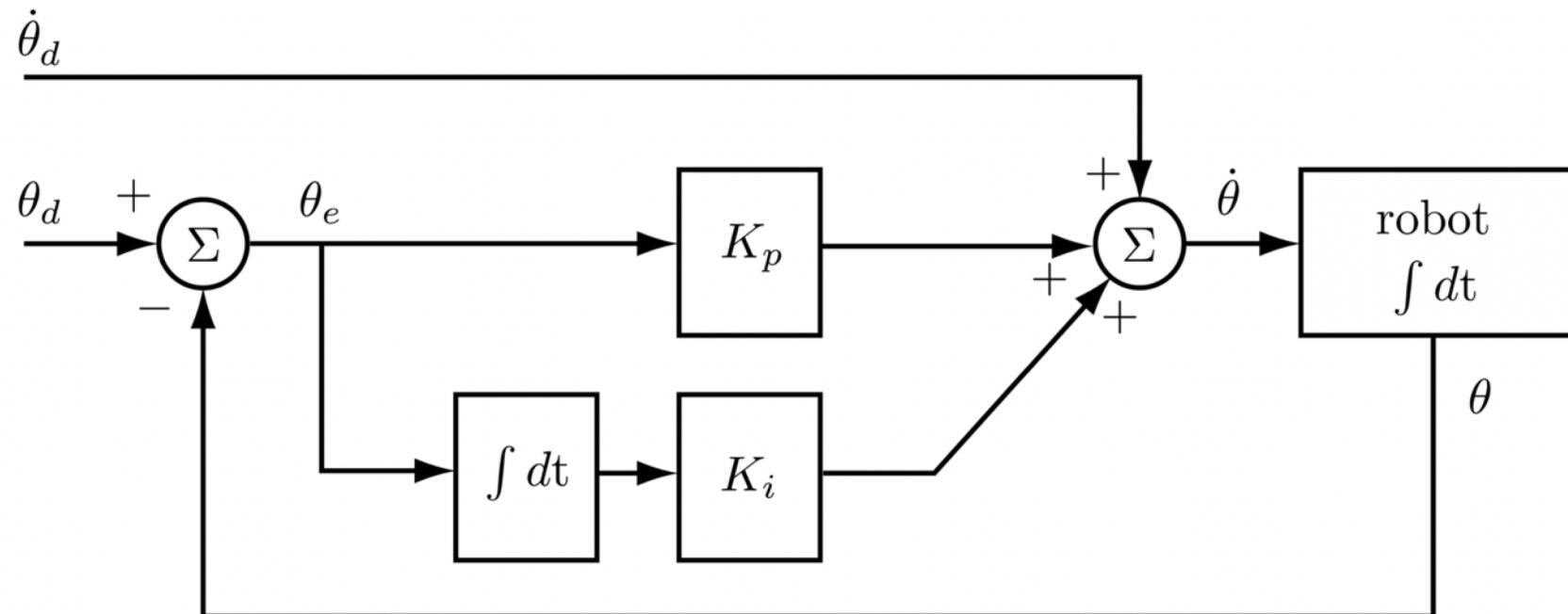


Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)

Multi-joint control: θ , θ_d , and θ_e are vectors and $K_p = k_p I$, $K_i = k_i I$



$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p \theta_e(t) + K_i \int_0^t \theta_e(t) dt$$

Important concepts, symbols, and equations (cont.)

$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p \theta_e(t) + K_i \int_0^t \theta_e(t) dt$$

Task-space motion control

desired motion $X_d \in SE(3)$

$\frac{[\mathcal{V}_d] = X_d^{-1} \dot{X}_d}{X \in SE(3)}$

actual motion $[\mathcal{V}_b] = X^{-1} \dot{X}$

error: $[X_e] = \log X_{bd} = \log(X^{-1} X_d)$

$$\begin{aligned}\mathcal{V}_b(t) &= [\text{Ad}_{X_{bd}}] \mathcal{V}_d(t) + K_p X_e(t) + K_i \int_0^t X_e(t) dt \\ \dot{\theta} &= J_b^\dagger(\theta) \mathcal{V}_b\end{aligned}$$

Important concepts, symbols, and equations (cont.)

Decoupled task-space motion control

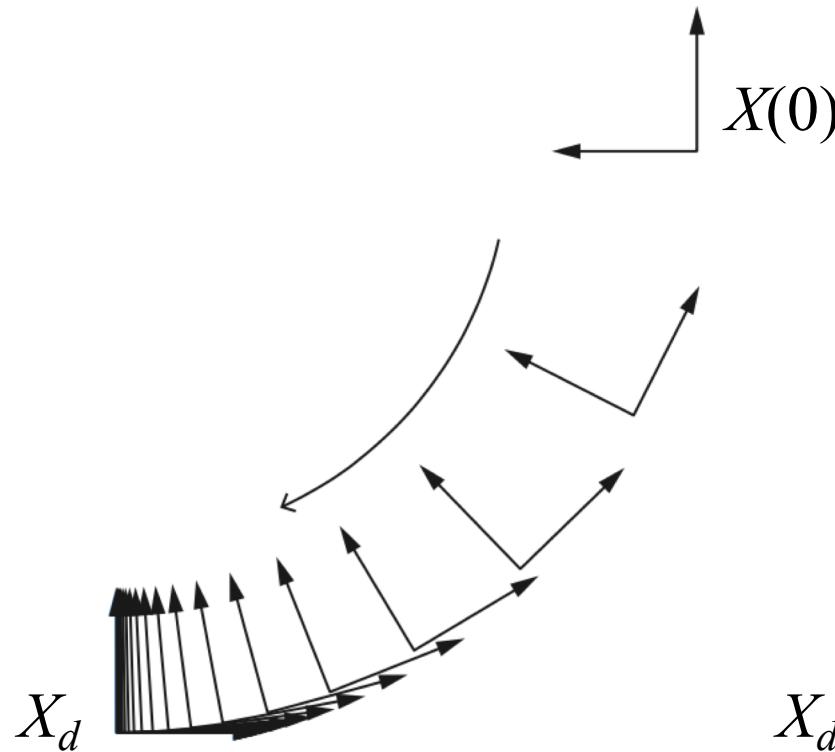
$$X_e(t) = \begin{bmatrix} \omega_e(t) \\ p_d(t) - p(t) \end{bmatrix} \quad [\omega_e] = \log(R^T R_d)$$

$$\begin{bmatrix} \omega_b(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} R^T(t)R_d(t) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega_d(t) \\ \dot{p}_d(t) \end{bmatrix} + K_p X_e(t) + K_i \int_0^t X_e(t) dt$$

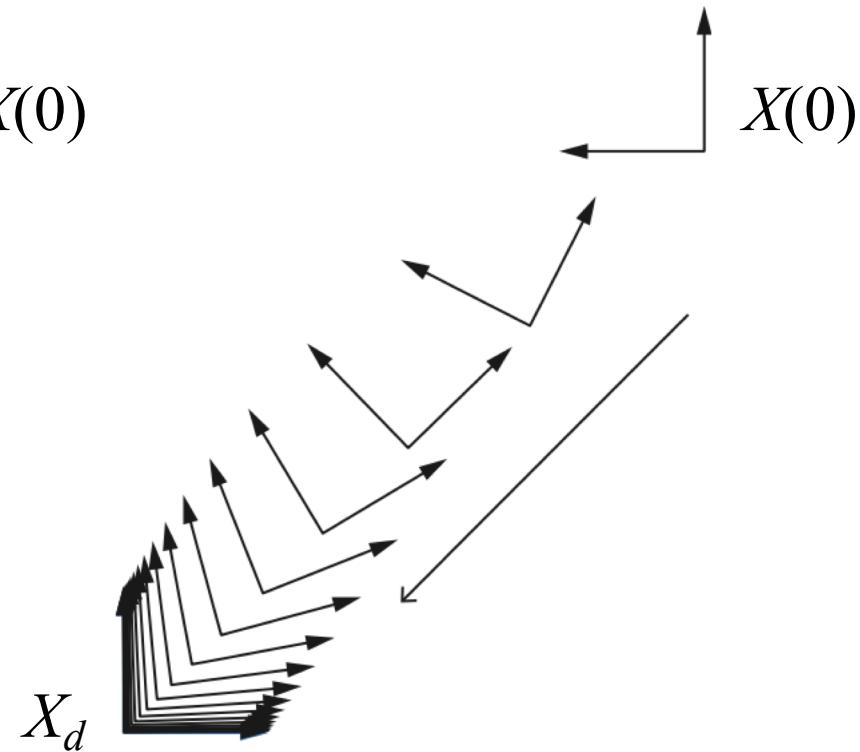
$$\dot{\theta} = J^\dagger(\theta) \begin{bmatrix} \omega_b(t) \\ \dot{p}(t) \end{bmatrix}$$

What is this Jacobian?

Important concepts, symbols, and equations (cont.)



task-space P controller



decoupled task-space P controller

You are designing a P controller to track a joint reference trajectory that is moving at a constant rate of 3 radians/s. What is the smallest gain K_p that ensures a steady-state position error of no more than 0.1 radians? Give units.

To eliminate steady-state error, you decide to use a PI controller. What gains K_p and K_i should you choose to achieve critical damping and a settling time of 0.1 s? Give units.

Explain how to estimate the error integral if the controller's frequency is $1/T$.

Why not choose arbitrarily large K_p and K_i to achieve arbitrarily fast settling?

How well would a PI controller track a quadratic joint trajectory? (Not a ramp.)