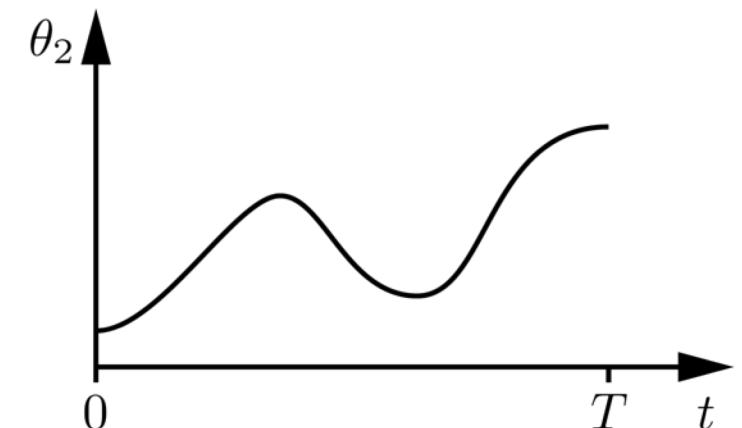
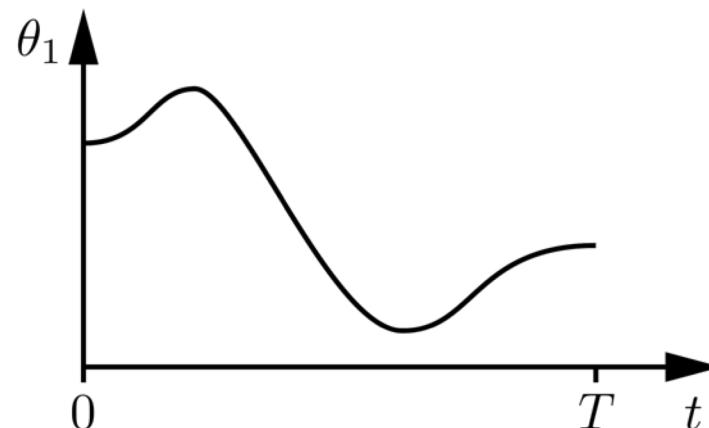
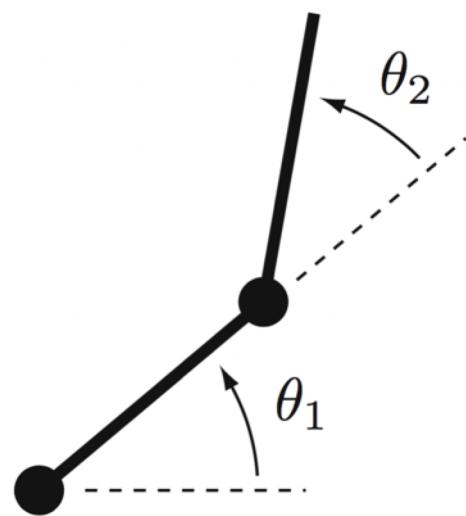


Chapter 2	Configuration Space
Chapter 3	Rigid-Body Motions
Chapter 4	Forward Kinematics
Chapter 5	Velocity Kinematics and Statics
Chapter 6	Inverse Kinematics
Chapter 7	Kinematics of Closed Chains
Chapter 8	Dynamics of Open Chains
Chapter 9	Trajectory Generation
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	9.3 Polynomial Via Point Trajectories
Chapter 10	Motion Planning
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Chapter 13	Wheeled Mobile Robots

## Important concepts, symbols, and equations

**Trajectory:** A specification of the configuration as a function of time.

$$\theta(t), t \in [0, T]$$

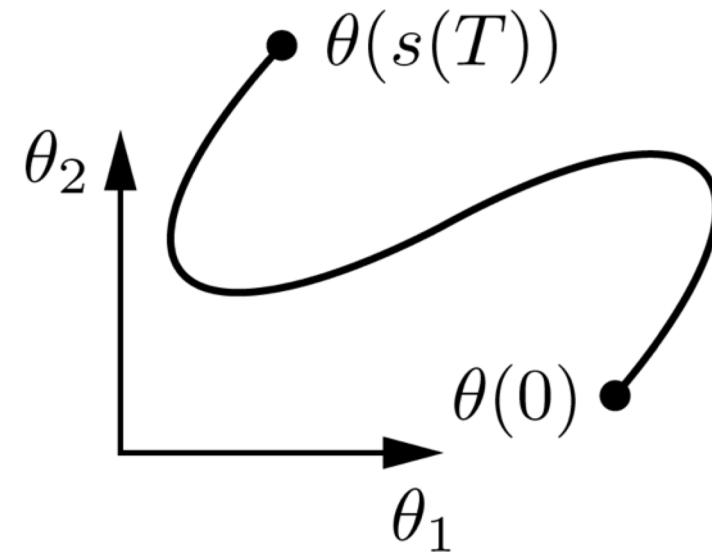
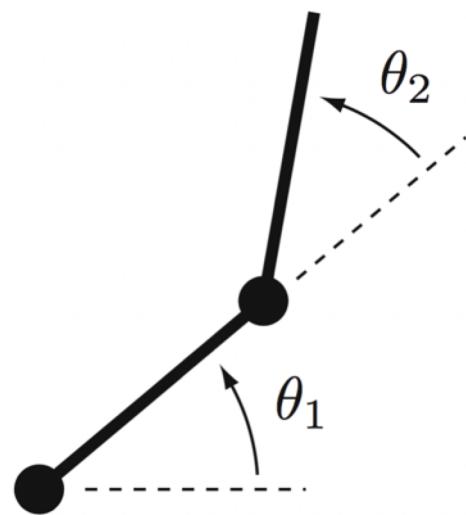


## Important concepts, symbols, and equations (cont.)

**Path:** A specification of the configuration as a function of a path parameter.

$$\theta(s), s \in [0, 1]$$

**Time scaling:** A mapping  $s(t): [0, T] \rightarrow [0, 1]$ , from time to the path parameter.



## Important concepts, symbols, and equations (cont.)

Motion as a function of  $\theta(s)$  and  $s(t)$ :

$$\dot{\theta} = \frac{d\theta}{ds} \dot{s},$$

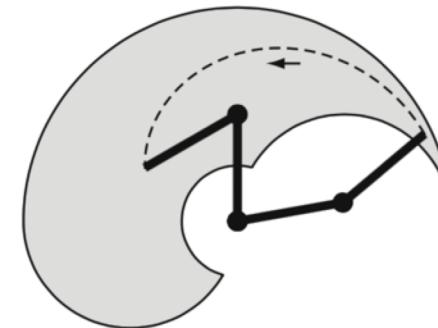
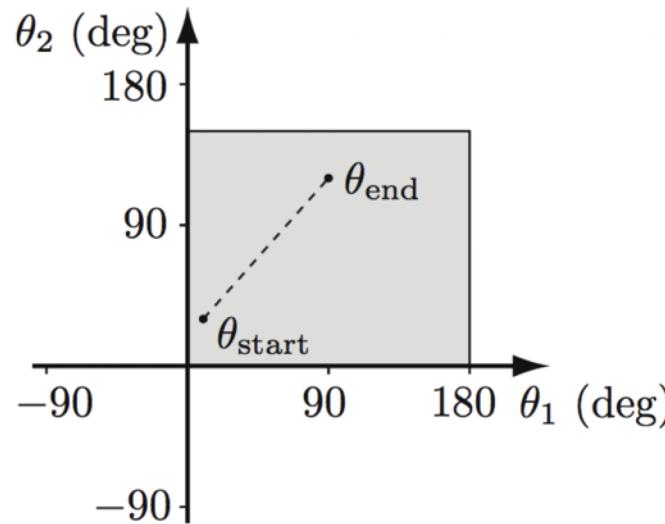
$$\ddot{\theta} = \frac{d\theta}{ds} \ddot{s} + \frac{d^2\theta}{ds^2} \dot{s}^2$$

Both  $\theta(s)$  and  $s(t)$  must be twice-differentiable.

## Important concepts, symbols, and equations (cont.)

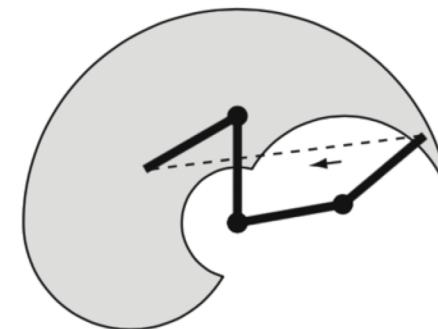
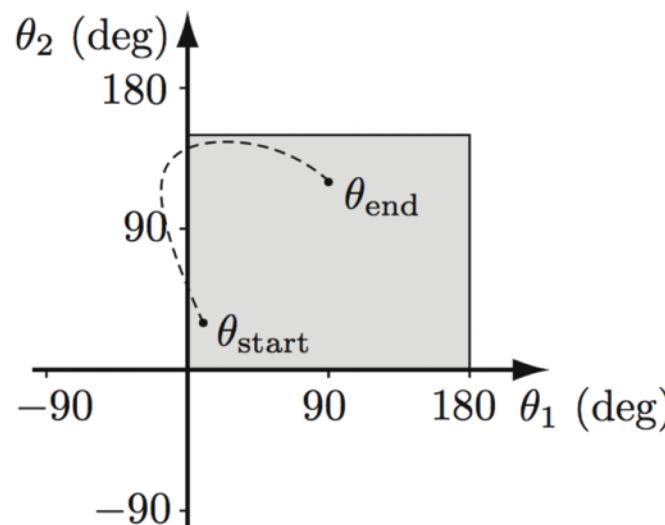
straight line in joint space

$$\theta(s) = \theta_{\text{start}} + s(\theta_{\text{end}} - \theta_{\text{start}})$$

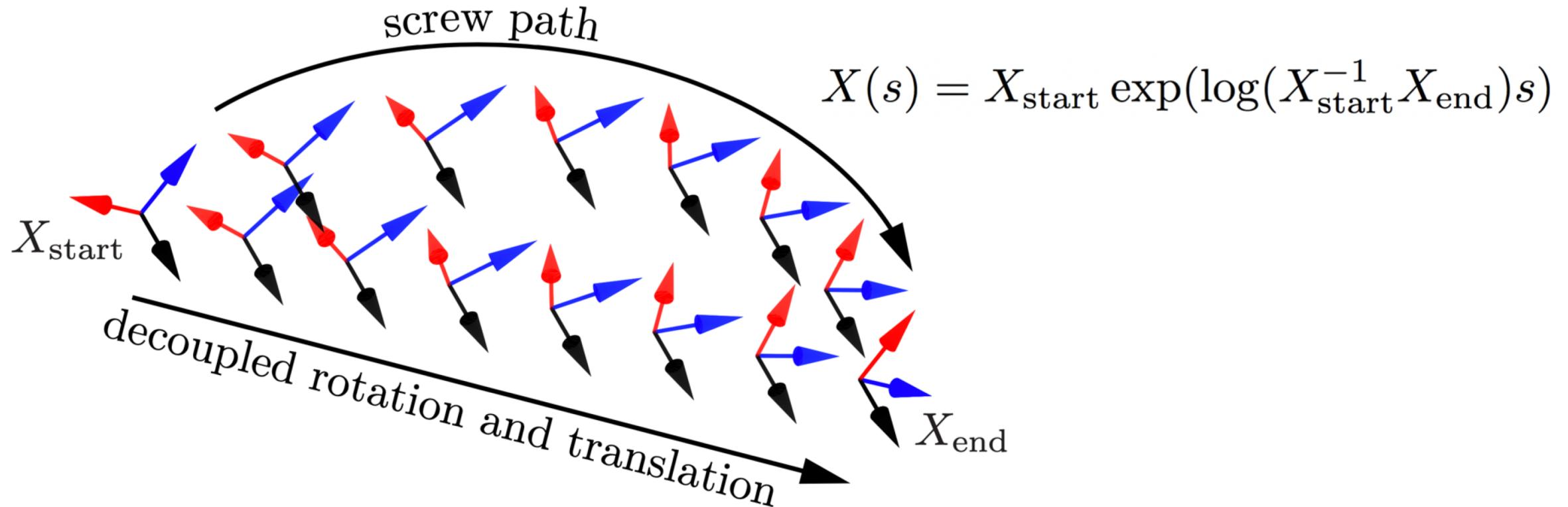


straight line in task space

$$X(s) = X_{\text{start}} + s(X_{\text{end}} - X_{\text{start}})$$



## Important concepts, symbols, and equations (cont.)



$$p(s) = p_{\text{start}} + s(p_{\text{end}} - p_{\text{start}}),$$

$$R(s) = R_{\text{start}} \exp(\log(R_{\text{start}}^T R_{\text{end}})s)$$

## Important concepts, symbols, and equations (cont.)

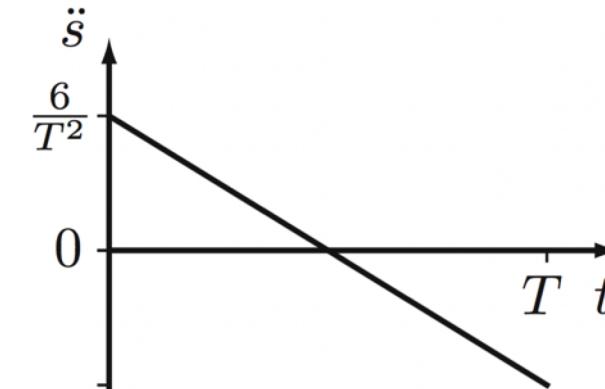
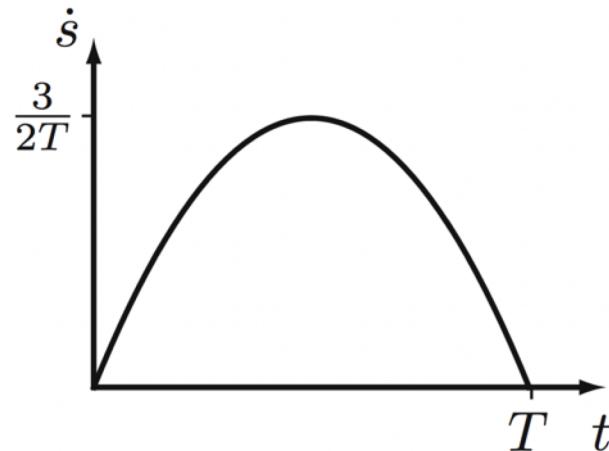
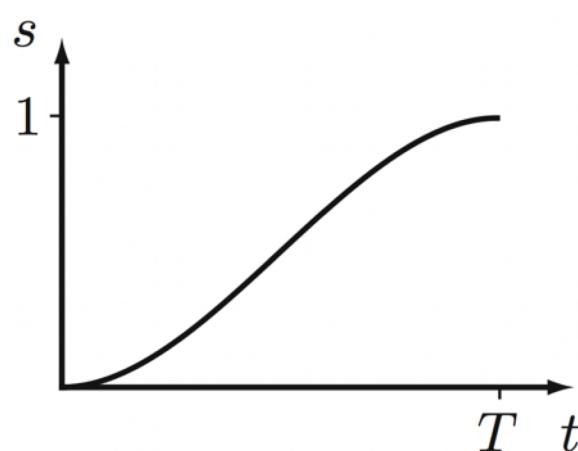
Third-order polynomial time scaling

$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{s}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = \frac{3}{T^2}, \quad a_3 = -\frac{2}{T^3}$$

$$\begin{array}{ll} s(0) = 0 & \dot{s}(0) = 0 \\ s(T) = 1 & \dot{s}(T) = 0 \end{array}$$

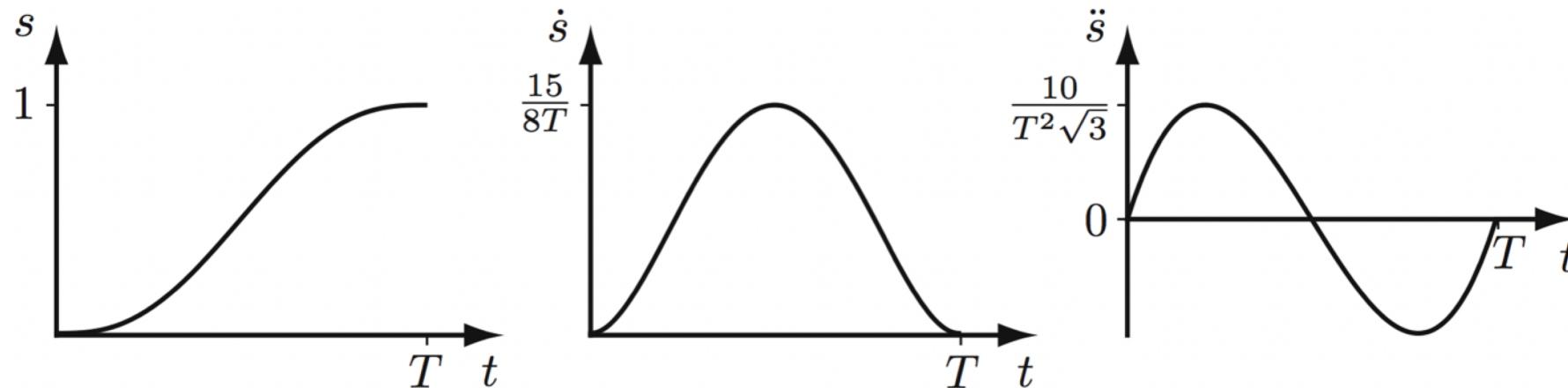


## Important concepts, symbols, and equations (cont.)

Fifth-order polynomial time scaling

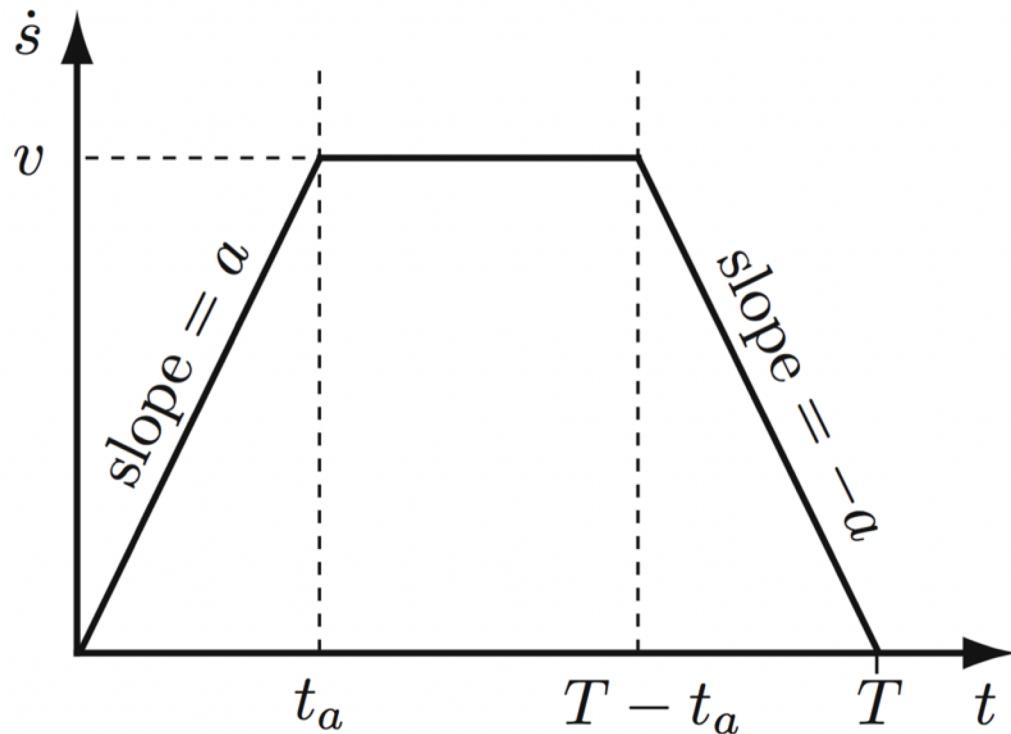
$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

$$\begin{aligned} s(0) &= 0 & \dot{s}(0) &= 0 & \ddot{s}(0) &= 0 \\ s(T) &= 1 & \dot{s}(T) &= 0 & \ddot{s}(T) &= 0 \end{aligned}$$



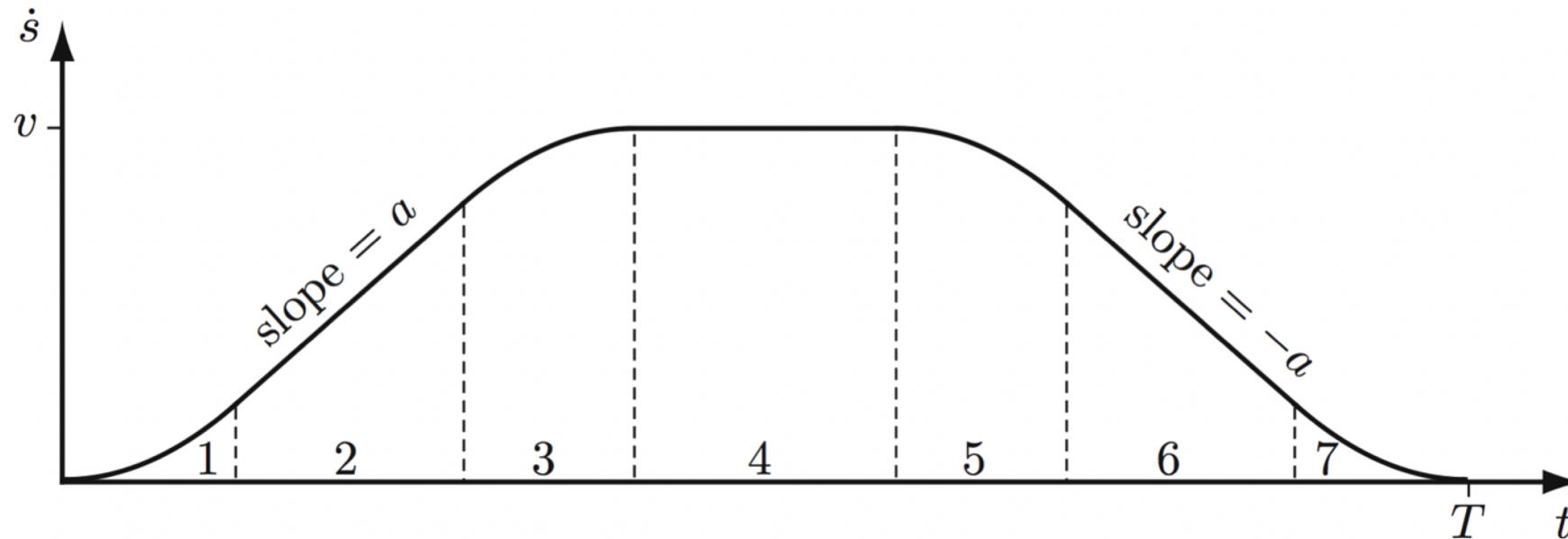
## Important concepts, symbols, and equations (cont.)

### Trapezoidal time scaling



## Important concepts, symbols, and equations (cont.)

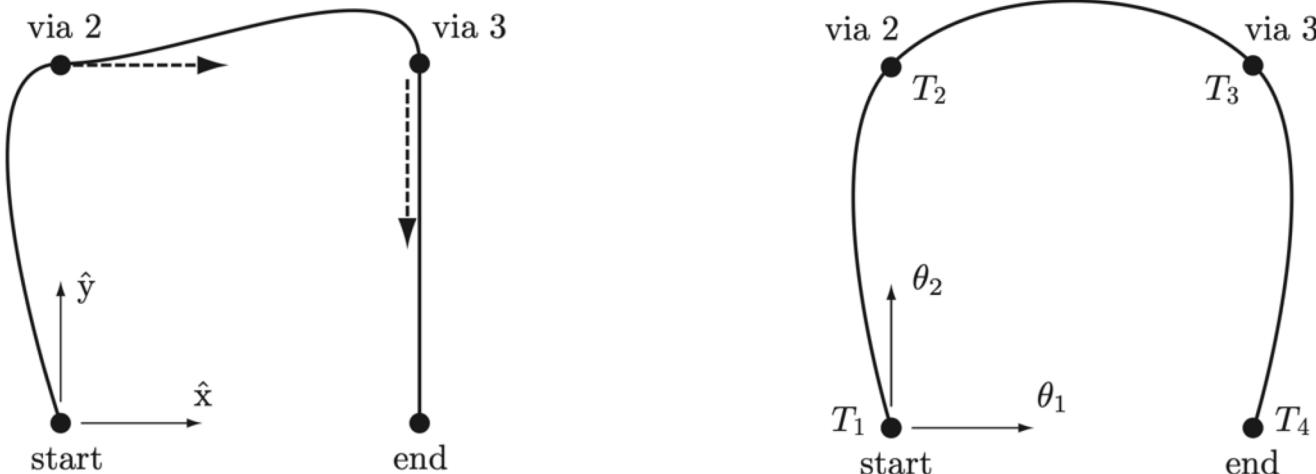
### S-curve time scaling



## Important concepts, symbols, and equations (cont.)

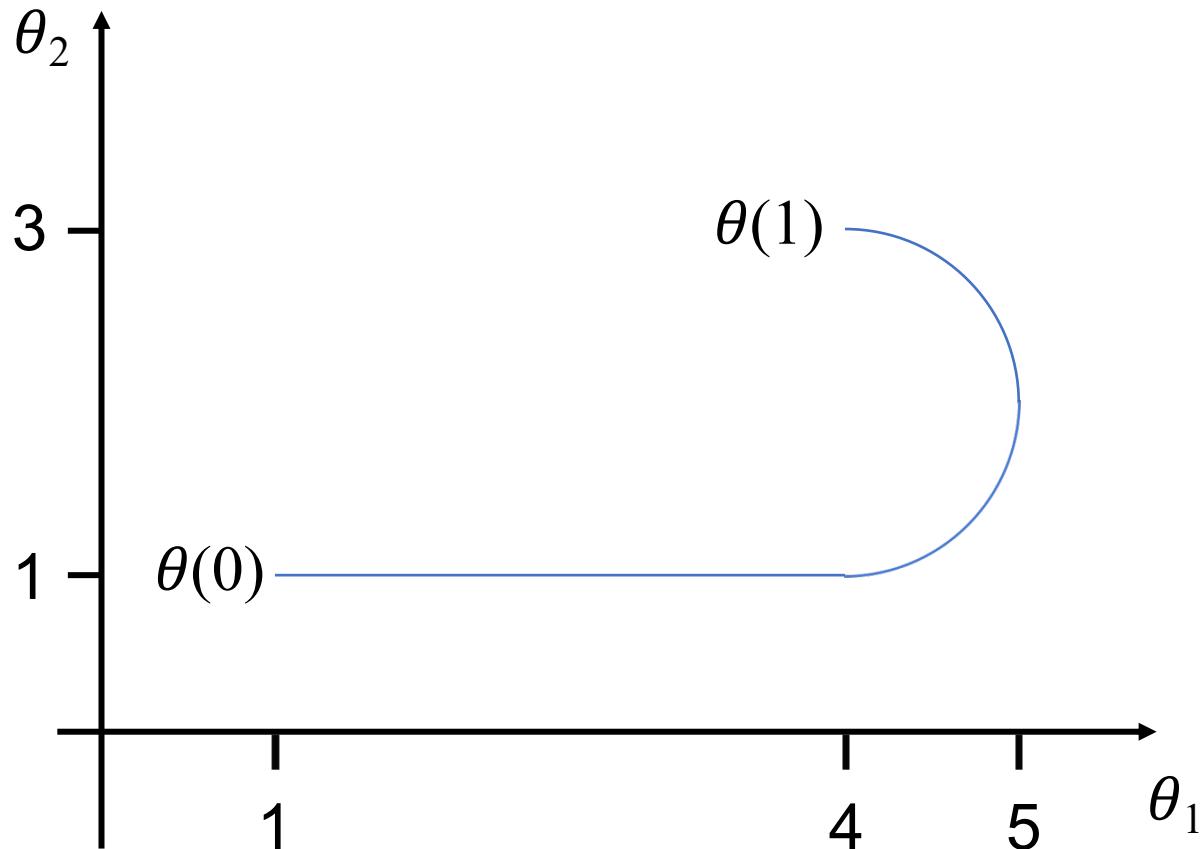
### Polynomial interpolation through via points

- third-order interpolation using via times, configurations, and velocities
- third-order interpolation using via times, configurations, and equal velocities and accelerations before and after vias



Many other methods, including **B-splines** (paths stay within convex hull of control points, but don't pass through them).

Give an expression for the path  $\theta(s)$ ,  $s \in [0,1]$ .



What kind of time scaling can be used to obtain a continuous jerk profile?

What is the maximum joint velocity obtained on a straight-line rest-to-rest trajectory with cubic polynomial time scaling?

Describe a circumstance under which the coast phase of the trapezoidal time scaling is not used.

Give an equation to implement a third-order polynomial time-scaled rest-to-rest motion following a screw axis.

A time scaling can be written as  $s(t)$  or  $\dot{s}(s)$ . If  $s(t) = at^2$ , what is  $\dot{s}(s)$ ?