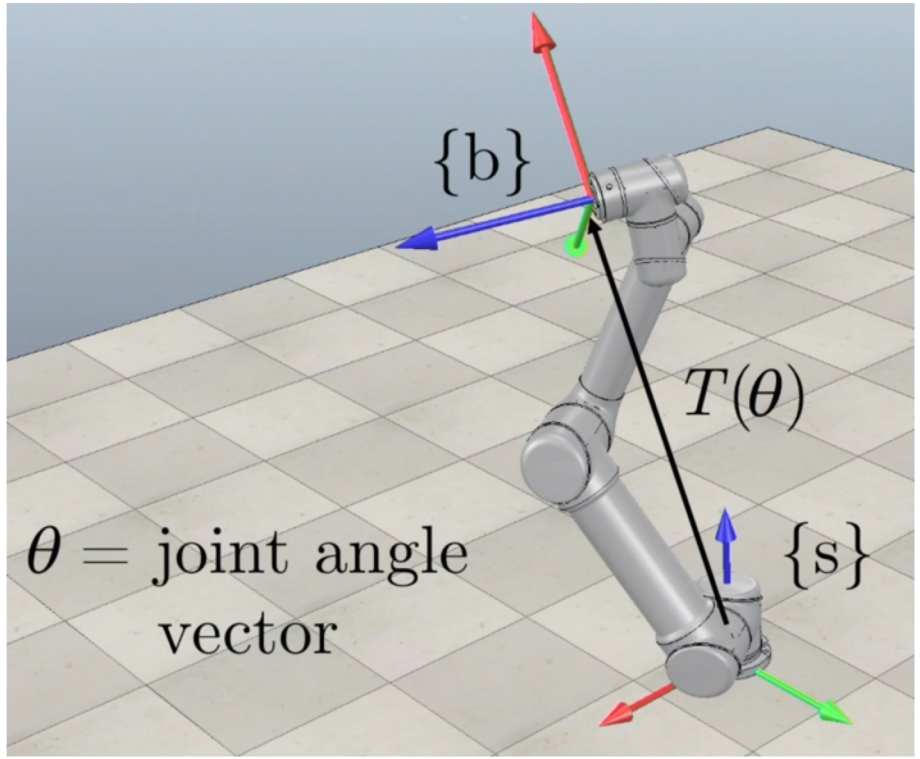
Important concepts, symbols, and equations

Forward kinematics of a serial chain:

Given

- $M = T_{sb}(0) \in SE(3)$, the configuration of the end-effector frame $\{\mathrm{b}\}$ at the home configuration $\theta = 0$,
- the screw axes for each joint at $\theta = 0$, and
- the joint vector θ,

find $T_{sb}(\theta) \in SE(3)$.



Important concepts, symbols, and equations (cont.)

- For screw axes $\mathcal{S}$ expressed in $\{\mathrm{s}\}$, the **product of exponentials** (PoE) in the space frame is

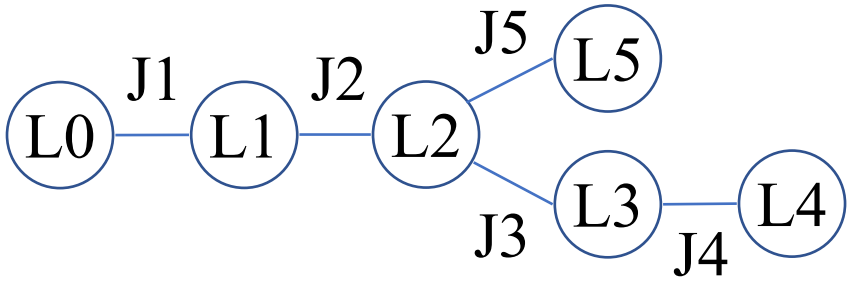
$$T(\theta) = e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_n]\theta_n} M$$

- For screw axes $\mathcal{B}$ expressed in $\{\mathrm{b}\}$, the PoE is

$$T(\theta) = M e^{[\mathcal{B}_1]\theta_1} \cdots e^{[\mathcal{B}_n]\theta_n}$$

Important concepts, symbols, and equations (cont.)

The **Universal Robot Description Format (URDF)** is an XML file describing the kinematics, inertial properties, and geometry of a tree-structured robot.

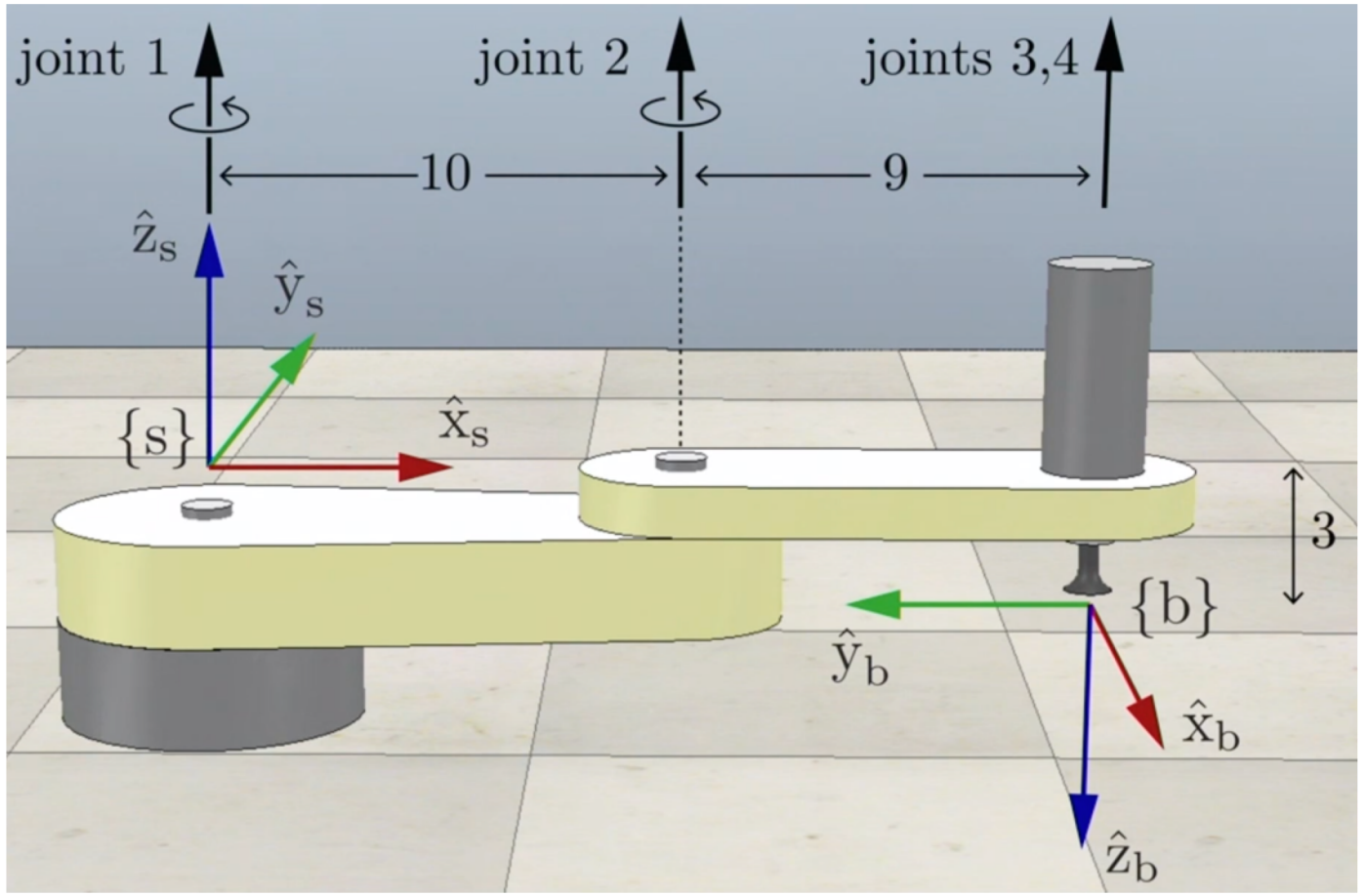


the **joint** element (kinematics)

```
<joint name="joint2" type="revolute">
  <parent link="link1"/>
  <child link="link2"/>
  <origin rpy="0 1.5708 0" xyz="0 0.5 0">
  <axis xyz="0 1 0">
  <limit lower="-3.0" upper="3.0">
</joint>
```

the **link** element (inertial properties)

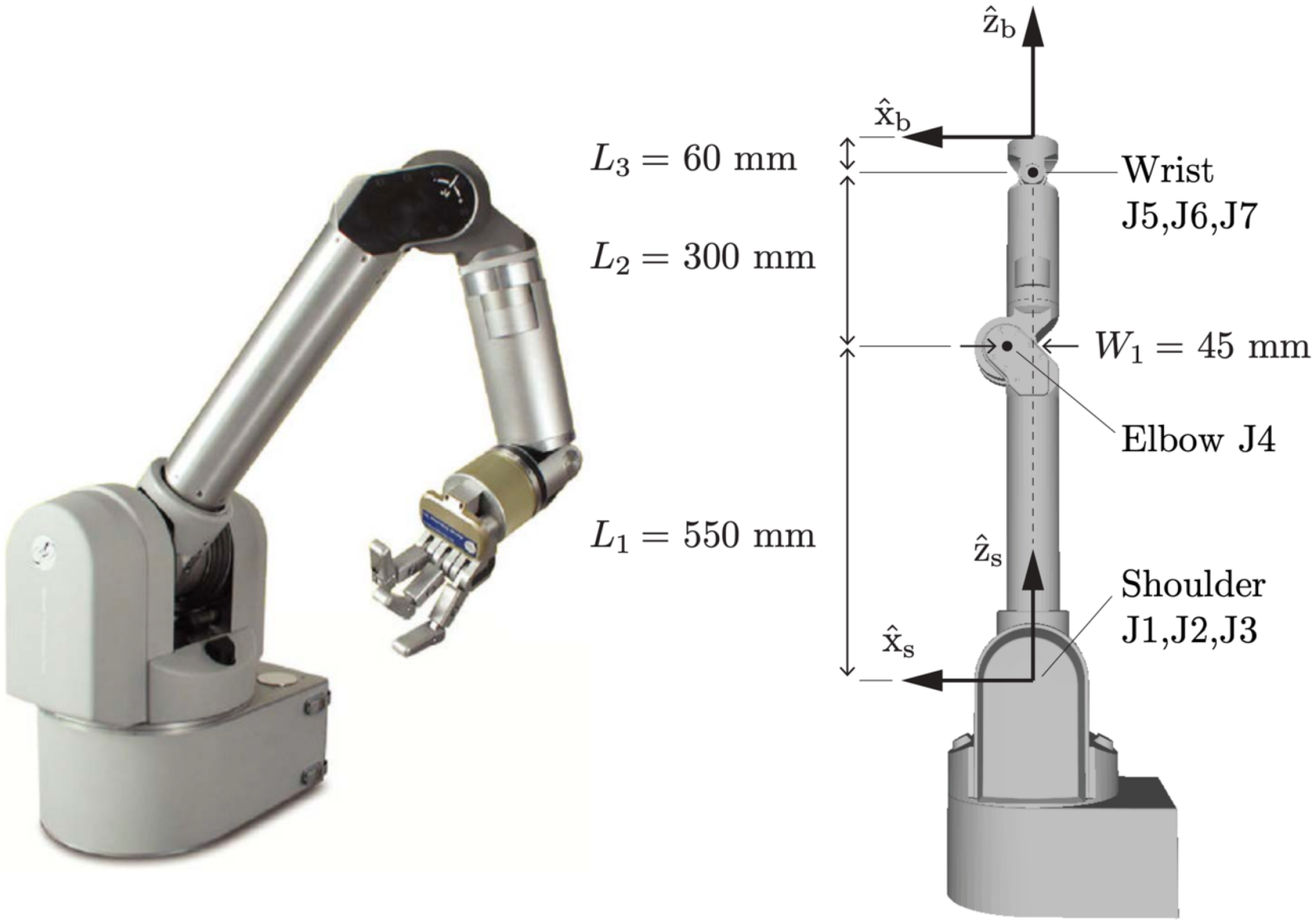
```
<link name="link2">
  <inertial>
    <origin rpy="0 0 0" xyz="0 0 0.5">
    <mass value="2.25">
    <inertia ixx="1.0" ixy="0" ixz="0"
             iyy="2.0" iyz="0" izz="3.0"/>
  </inertial>
</link>
```



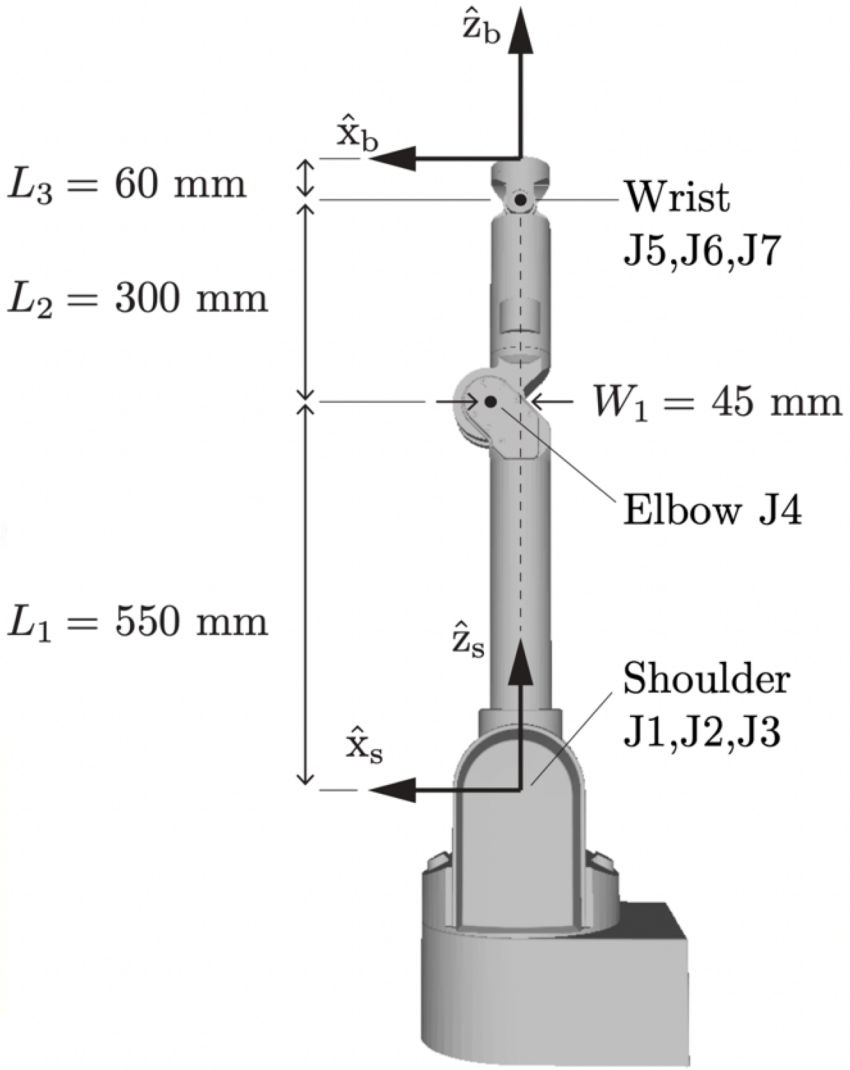
RRRP robot

$$M = \begin{bmatrix} 0 & -1 & 0 & 19 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{S}_1 = (0, 0, 1, 0, 0, 0)$$
$$\mathcal{S}_2 = (0, 0, 1, 0, -10, 0)$$
$$\mathcal{S}_3 = (0, 0, 1, 0, -19, 0)$$
$$\mathcal{S}_4 = (0, 0, 0, 0, 0, 1)$$



Barrett Technology's WAM 7R robot

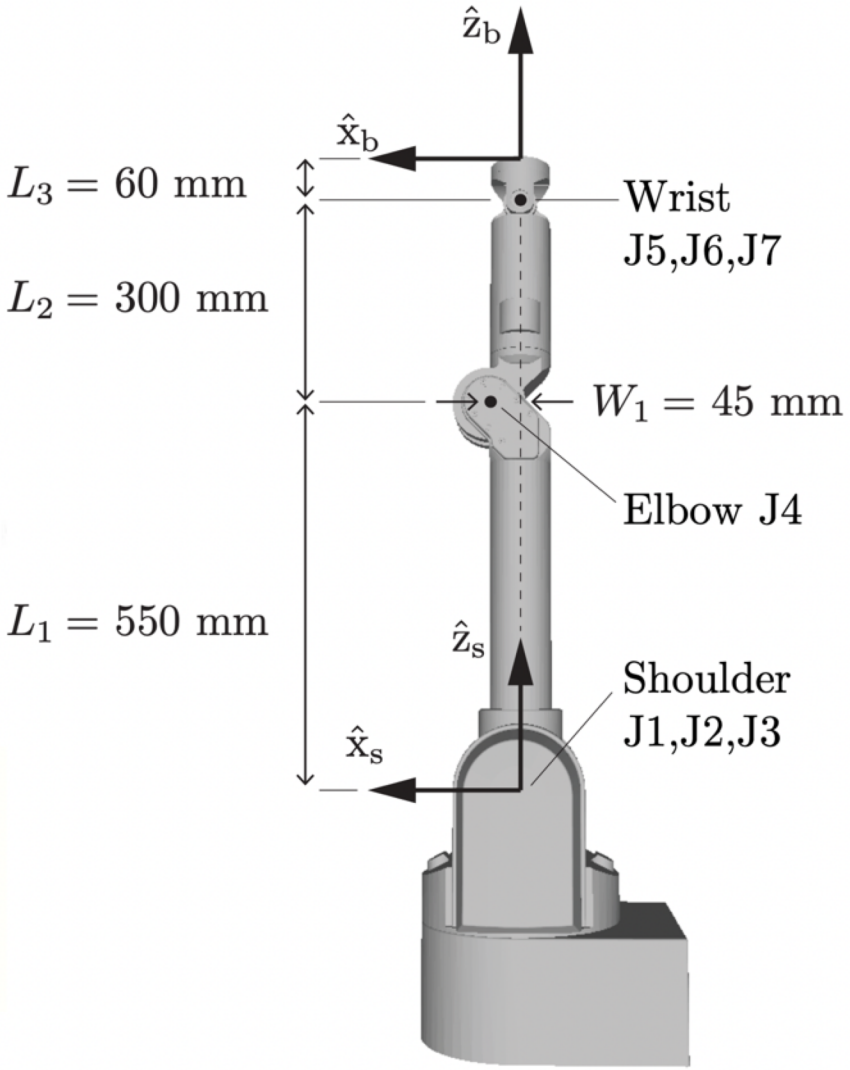


Joint axes 1, 3, 5, and 7 are aligned with $\hat{z}_s$.
Joint axes 2, 4, and 6 are out of the page.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 + L_2 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

screw axes $\mathcal{B}_i$

i	ω_i	v_i
1	$(0, 0, 1)$	$(0, 0, 0)$
2	$(0, 1, 0)$	$(L_1 + L_2 + L_3, 0, 0)$
3	$(0, 0, 1)$	$(0, 0, 0)$
4	$(0, 1, 0)$	$(L_2 + L_3, 0, W_1)$
5	$(0, 0, 1)$	$(0, 0, 0)$
6	$(0, 1, 0)$	$(L_3, 0, 0)$
7	$(0, 0, 1)$	$(0, 0, 0)$



Joint axes 1, 3, 5, and 7 are aligned with $\hat{z}_s$.
Joint axes 2, 4, and 6 are out of the page.

screw axes S_i

i	ω_i	v_i
1	0, 0, 1	0, 0, 0
2	0, 1, 0	0, 0, 0
3	0, 0, 1	0, 0, 0
4	0, 1, 0	$-L_1$, 0, W_1
5	0, 0, 1	0, 0, 0
6	0, 1, 0	$-L_1$, $-L_2$, 0, 0
7	0, 0, 1	0, 0, 0