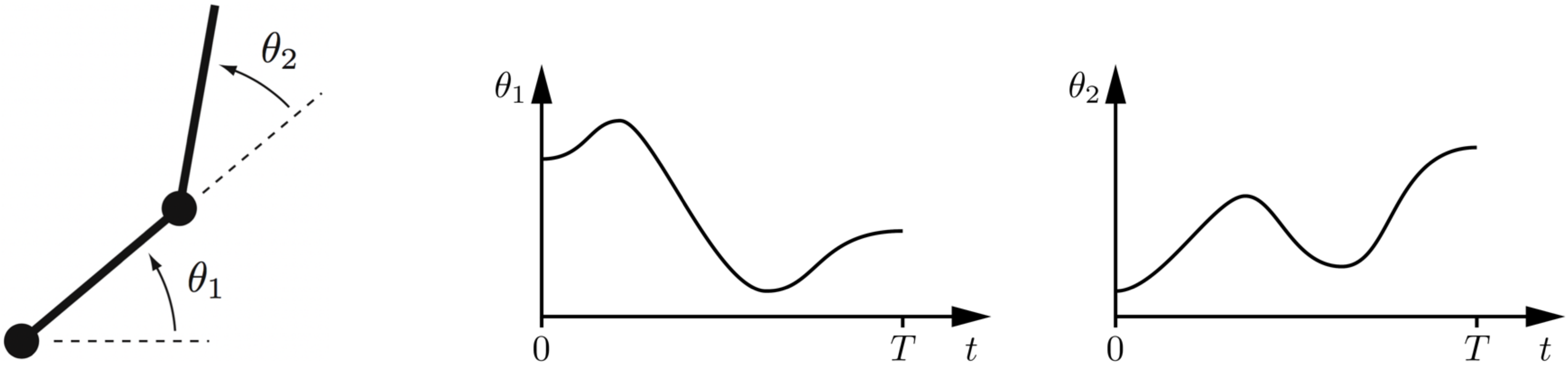
Important concepts, symbols, and equations

Trajectory: A specification of the configuration as a function of time.

$$\theta(t),\ t \in [0, T]$$

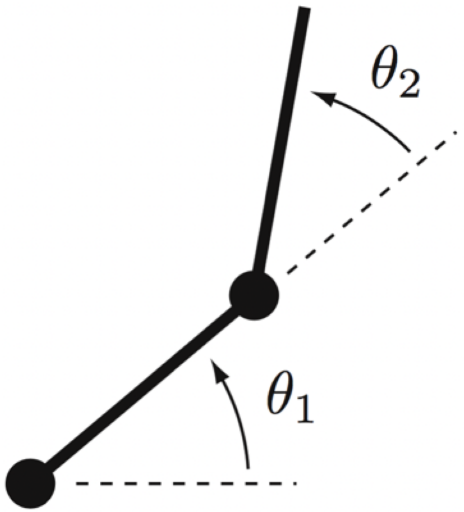


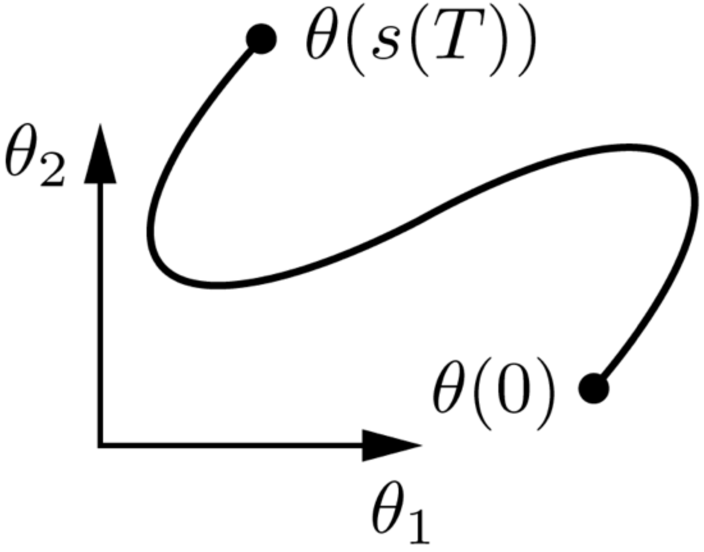
Important concepts, symbols, and equations (cont.)

Path: A specification of the configuration as a function of a path parameter.

$$\theta(s), \; s \in [0, 1]$$

Time scaling: A mapping $s(t)$: $[0, T] \to [0, 1]$, from time to the path parameter.





Important concepts, symbols, and equations (cont.)

Motion as a function of $\theta(s)$ and $s(t)$:

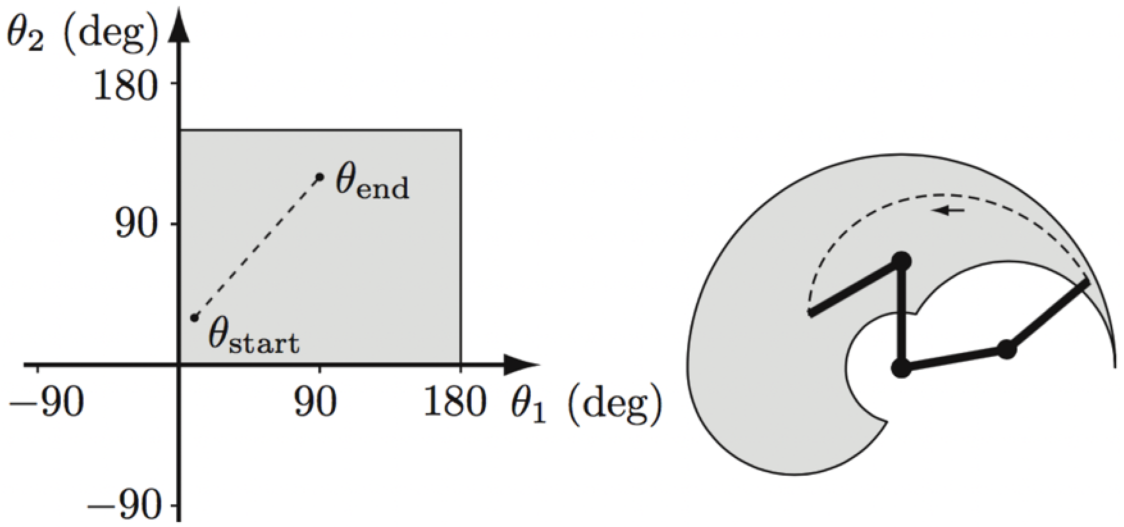
$$\dot{\theta} = \frac{d\theta}{ds}\dot{s},$$
$$\ddot{\theta} = \frac{d\theta}{ds}\ddot{s} + \frac{d^2\theta}{ds^2}\dot{s}^2$$

Both $\theta(s)$ and $s(t)$ must be twice-differentiable.

Important concepts, symbols, and equations (cont.)

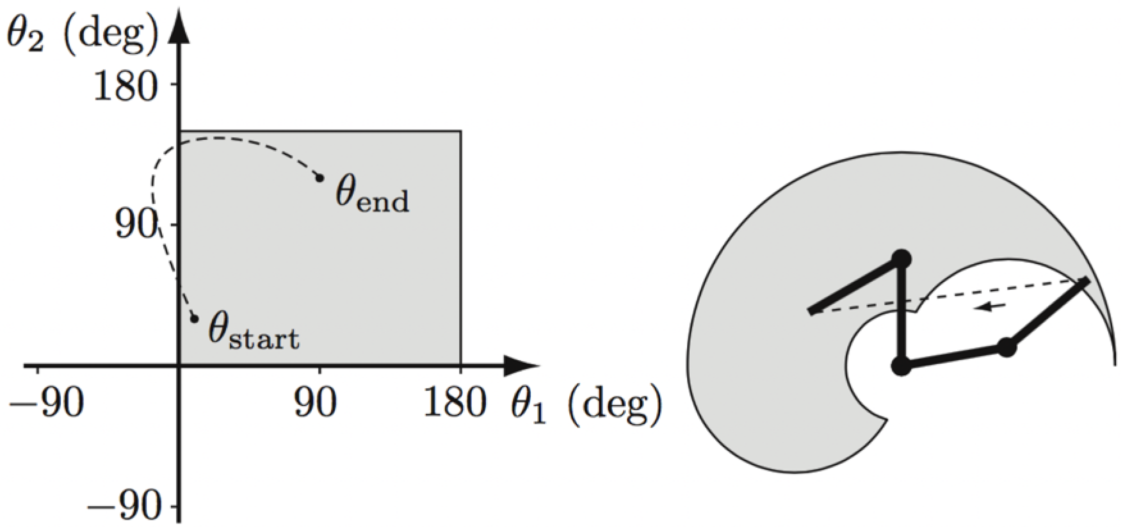
straight line in joint space

$$\theta(s) = \theta_{\text{start}} + s(\theta_{\text{end}} - \theta_{\text{start}})$$

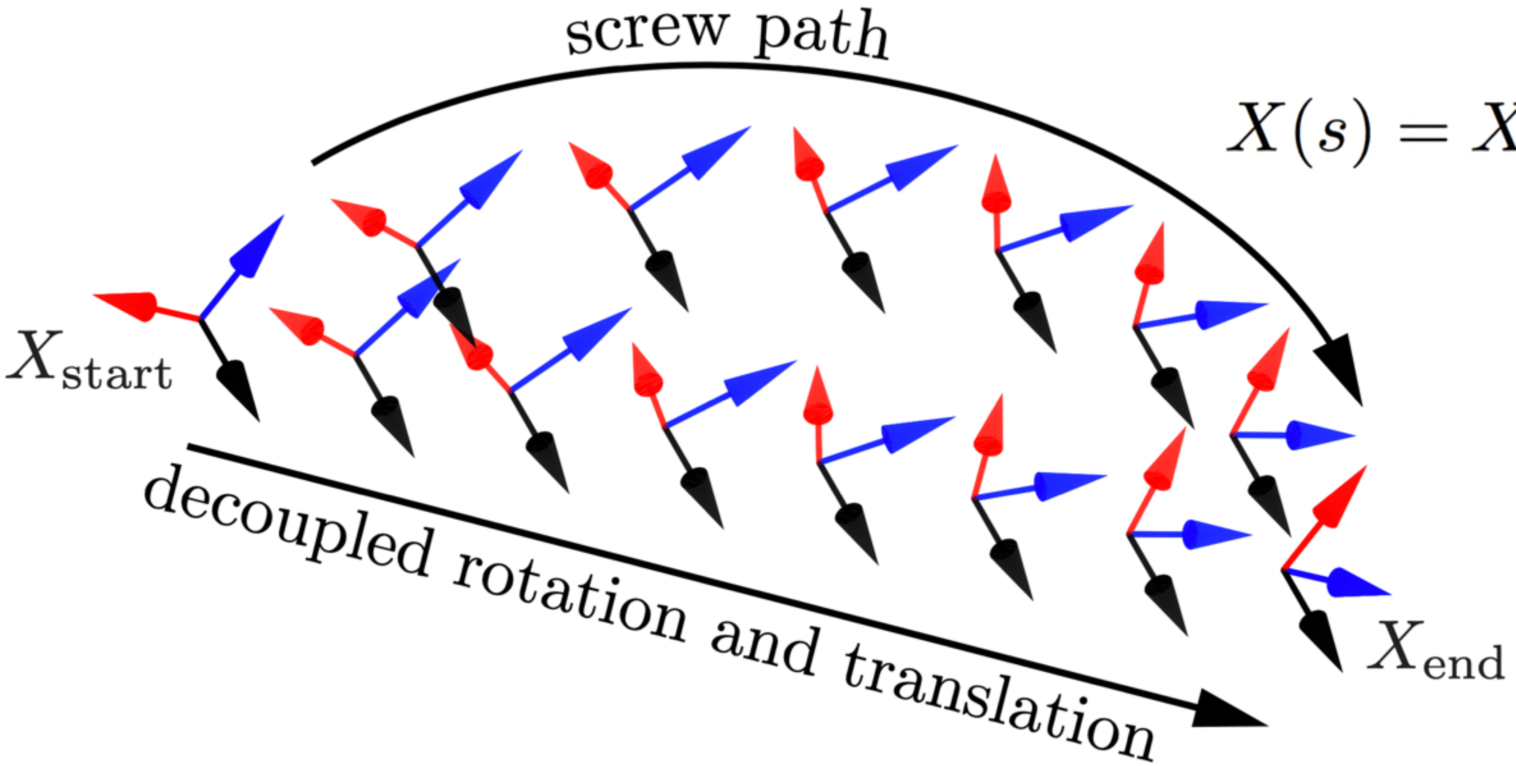


straight line in task space

$$X(s) = X_{\text{start}} + s(X_{\text{end}} - X_{\text{start}})$$



<u>Important concepts, symbols, and equations</u> (cont.)



$$X(s) = X_{\text{start}} \exp(\log(X_{\text{start}}^{-1} X_{\text{end}}) s)$$

$$p(s) = p_{\text{start}} + s(p_{\text{end}} - p_{\text{start}}),$$

$$R(s) = R_{\text{start}} \exp(\log(R_{\text{start}}^{\mathrm{T}} R_{\text{end}}) s)$$

Important concepts, symbols, and equations (cont.)

Third-order polynomial time scaling

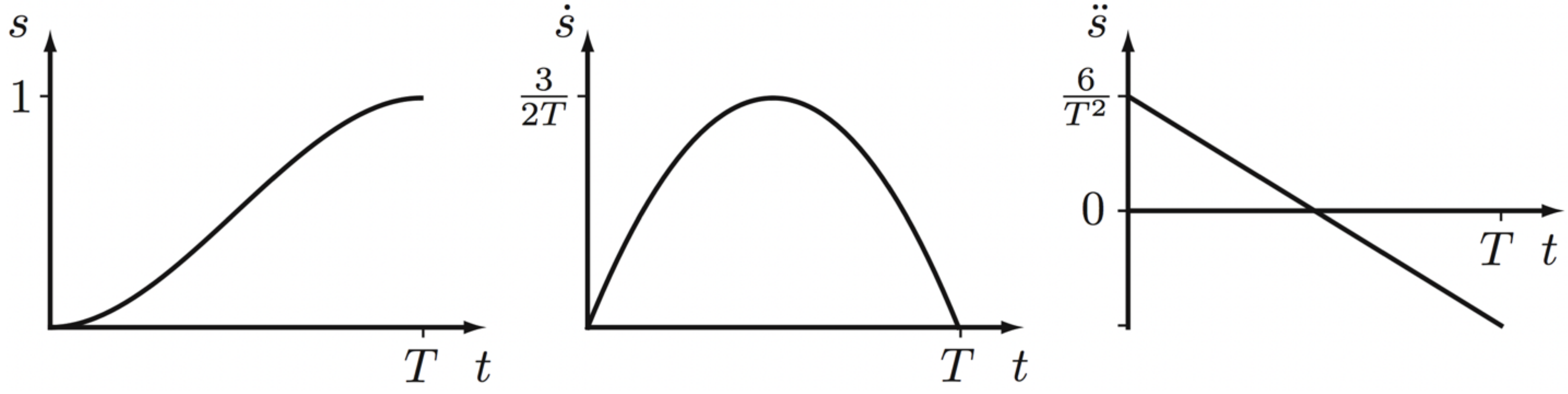
$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{s}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$s(0) = 0 \qquad \dot{s}(0) = 0$$

$$s(T) = 1 \qquad \dot{s}(T) = 0$$

$$a_0 = 0,\ a_1 = 0,\ a_2 = \frac{3}{T^2},\ a_3 = -\frac{2}{T^3}$$



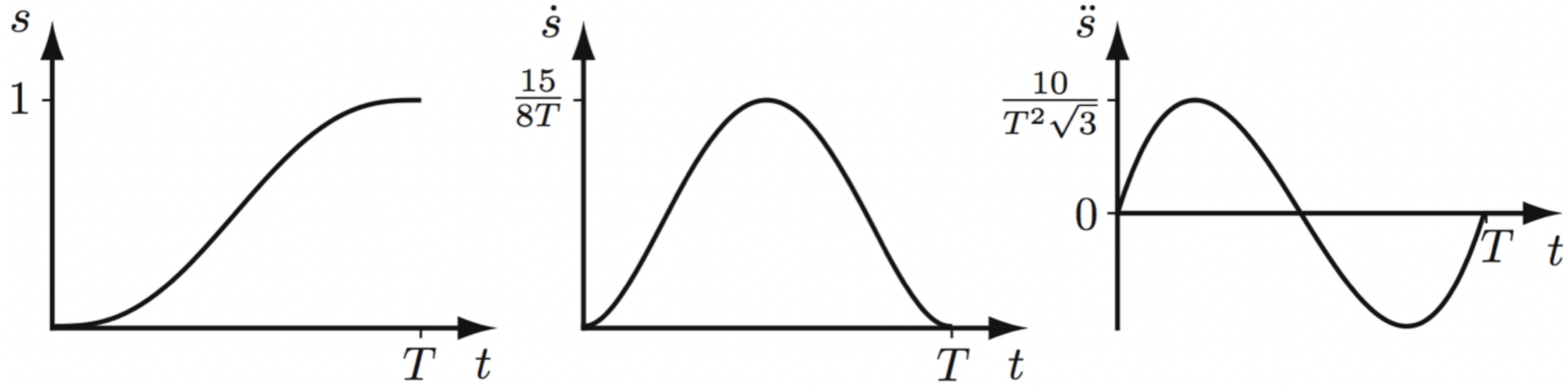
Important concepts, symbols, and equations (cont.)

Fifth-order polynomial time scaling

$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

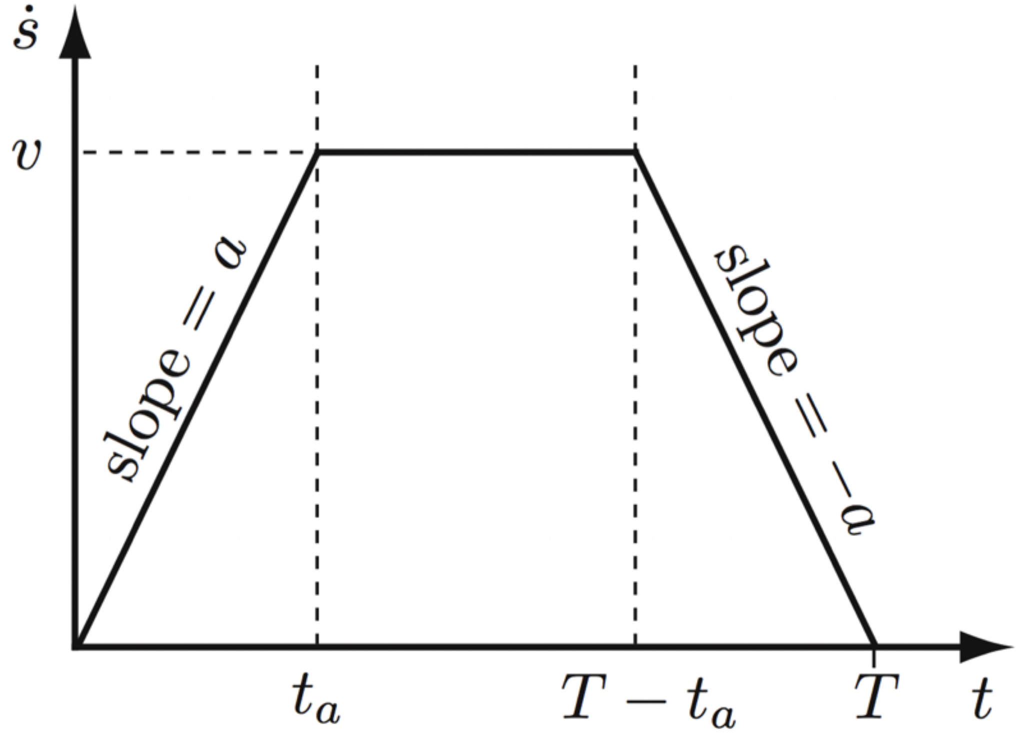
$$s(0) = 0 \quad \dot{s}(0) = 0 \quad \ddot{s}(0) = 0$$

$$s(T) = 1 \quad \dot{s}(T) = 0 \quad \ddot{s}(T) = 0$$



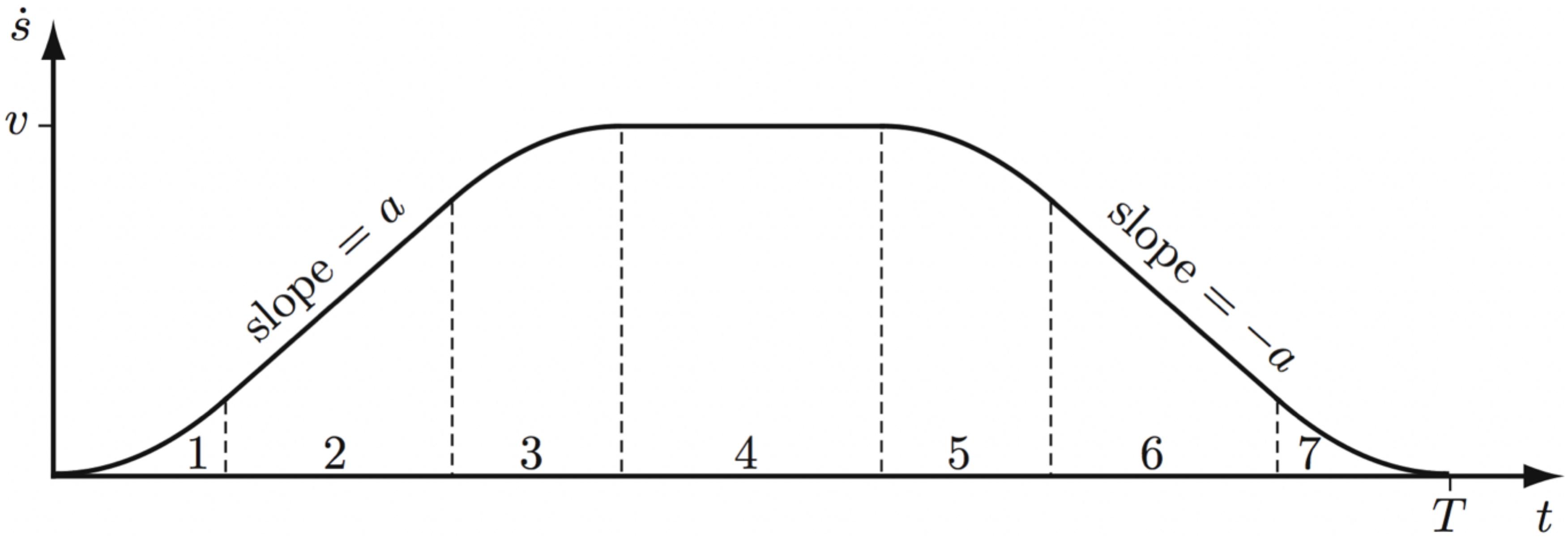
Important concepts, symbols, and equations (cont.)

Trapezoidal time scaling



Important concepts, symbols, and equations (cont.)

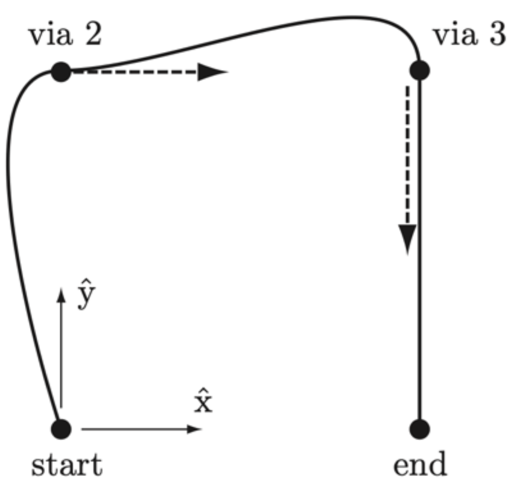
S-curve time scaling

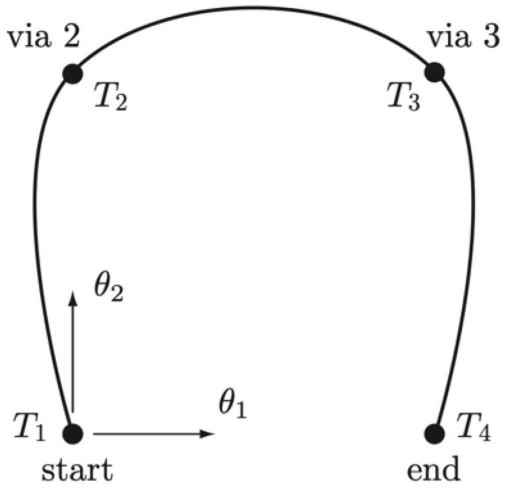


Important concepts, symbols, and equations (cont.)

Polynomial interpolation through via points

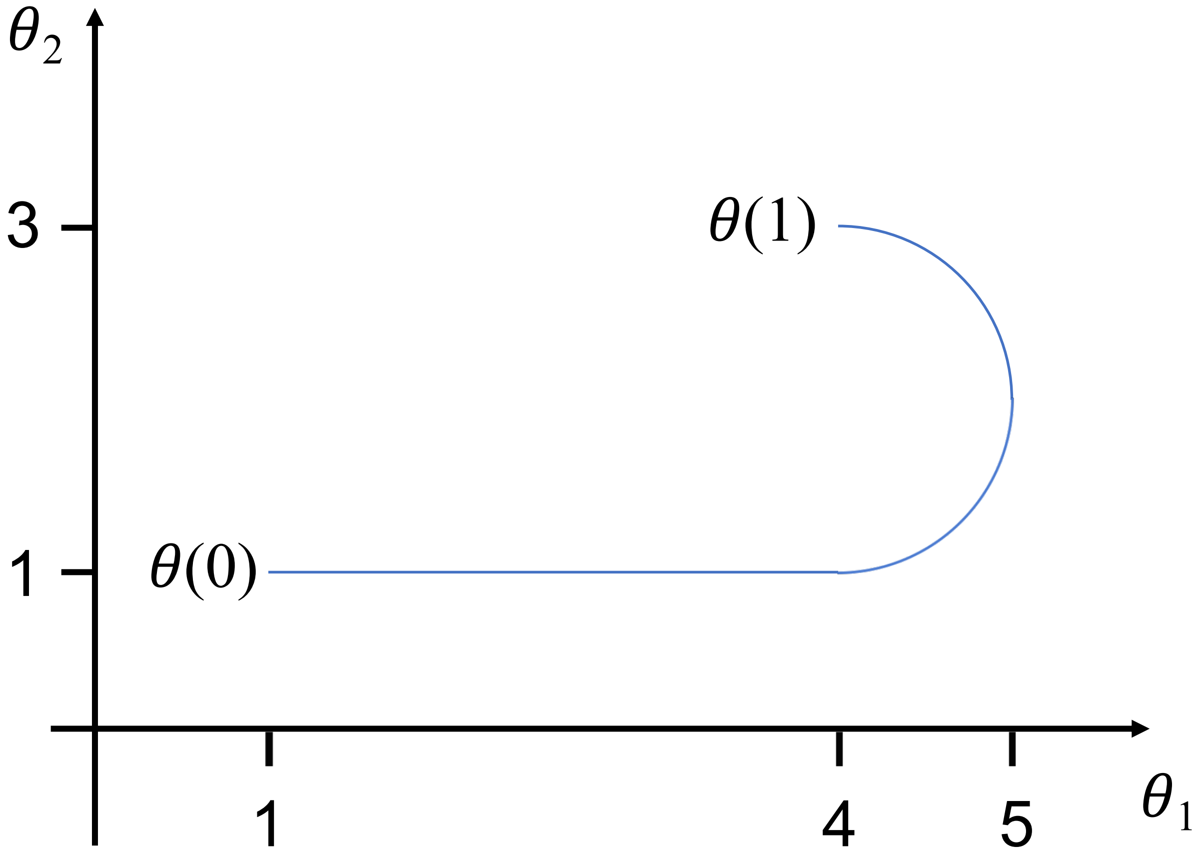
- third-order interpolation using via times, configurations, and velocities
- third-order interpolation using via times, configurations, and equal velocities and accelerations before and after vias





Many other methods, including **B-splines** (paths stay within convex hull of control points, but don't pass through them).

Give an expression for the path $\theta(s)$, $s \in [0,1]$.



What kind of time scaling can be used to obtain a continuous jerk profile?

What is the maximum joint velocity obtained on a straight-line rest-to-rest trajectory with cubic polynomial time scaling?

Describe a circumstance under which the coast phase of the trapezoidal time scaling is not used.

Give an equation to implement a third-order polynomial time-scaled rest-to-rest motion following a screw axis.

A time scaling can be written as $s(t)$ or $\dot{s}(s)$. If $s(t) = at^2$, what is $\dot{s}(s)$?