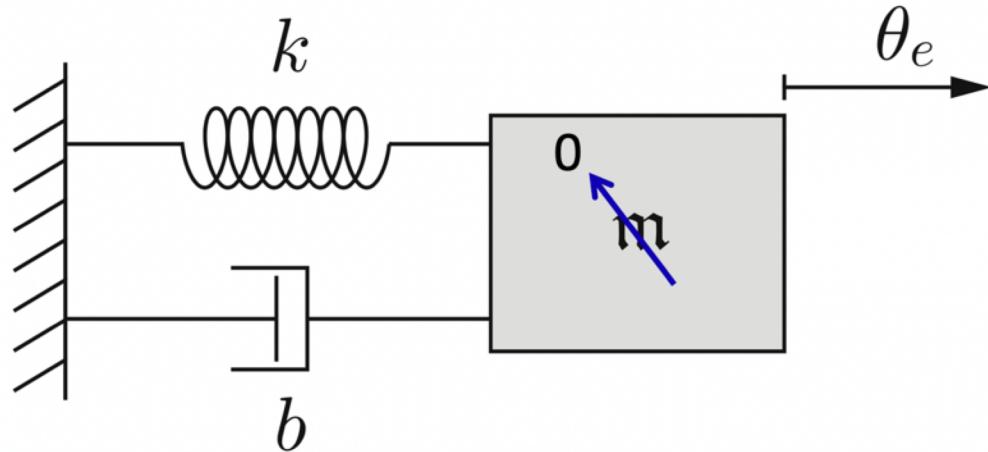


Chapter 2	Configuration Space
Chapter 3	Rigid-Body Motions
Chapter 4	Forward Kinematics
Chapter 5	Velocity Kinematics and Statics
Chapter 6	Inverse Kinematics
Chapter 7	Kinematics of Closed Chains
Chapter 8	Dynamics of Open Chains
Chapter 9	Trajectory Generation
Chapter 10	Motion Planning
Chapter 11	Robot Control
	11.1 Control System Overview
	11.2 Error Dynamics
Chapter 12	Grasping and Manipulation
Chapter 13	Wheeled Mobile Robots

Important concepts, symbols, and equations

First-order error dynamics



standard first-order form

$$m\ddot{\theta}_e + b\dot{\theta}_e + k\theta_e = 0$$

time constant

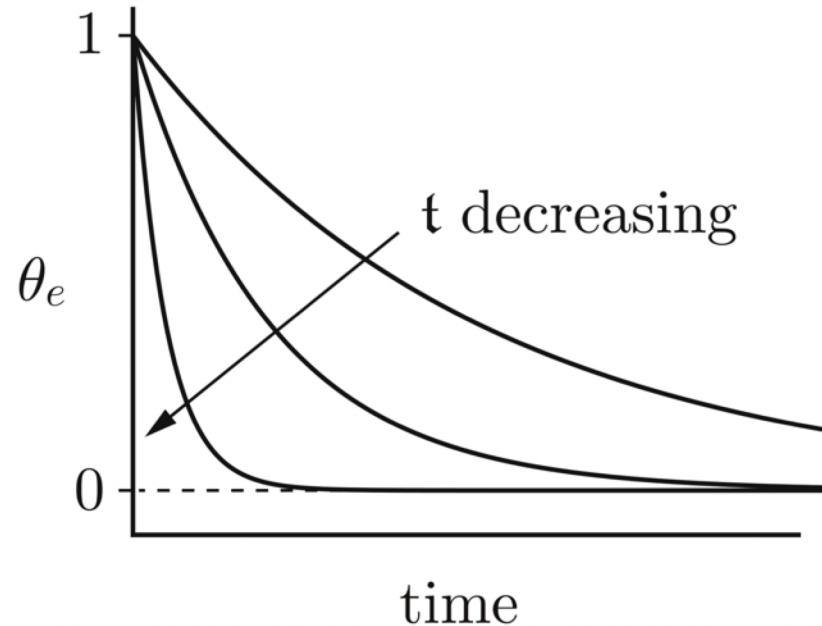
$$\tau = b/k$$

$$\dot{\theta}_e(t) + \frac{k}{b}\theta_e(t) = 0$$

$$\dot{\theta}_e(t) + \frac{1}{\tau}\theta_e(t) = 0$$

Important concepts, symbols, and equations (cont.)

First-order error dynamics



$$\theta_e(t) = e^{-t/\tau} \theta_e(0)$$

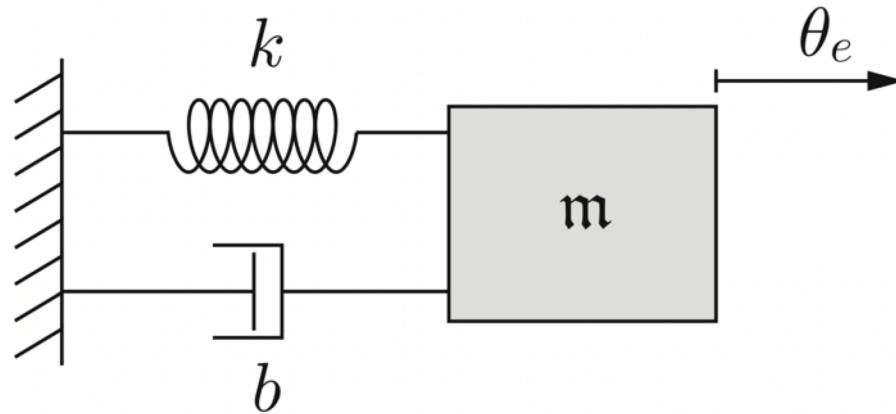
$$\theta_e(0) = 1$$

$$\frac{\theta_e(t)}{\theta_e(0)} = 0.02 = e^{-t/\tau}$$

$$\ln 0.02 = -t/\tau \rightarrow t = 3.91\tau$$

Important concepts, symbols, and equations (cont.)

Second-order error dynamics



$$\ddot{\theta}_e(t) + \frac{b}{m}\dot{\theta}_e(t) + \frac{k}{m}\theta_e(t) = 0$$

natural frequency damping ratio

$$\omega_n = \sqrt{k/m} \quad \zeta = b/(2\sqrt{km})$$
$$\ddot{\theta}_e(t) + 2\zeta\omega_n\dot{\theta}_e(t) + \omega_n^2\theta_e(t) = 0$$

standard second-order form

Important concepts, symbols, and equations (cont.)

Second-order error dynamics

$$\ddot{\theta}_e(t) + 2\zeta\omega_n\dot{\theta}_e(t) + \omega_n^2\theta_e(t) = 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$\zeta > 1$: **Overdamped**

$\zeta = 1$: **Critically damped**

$\zeta < 1$: **Underdamped**

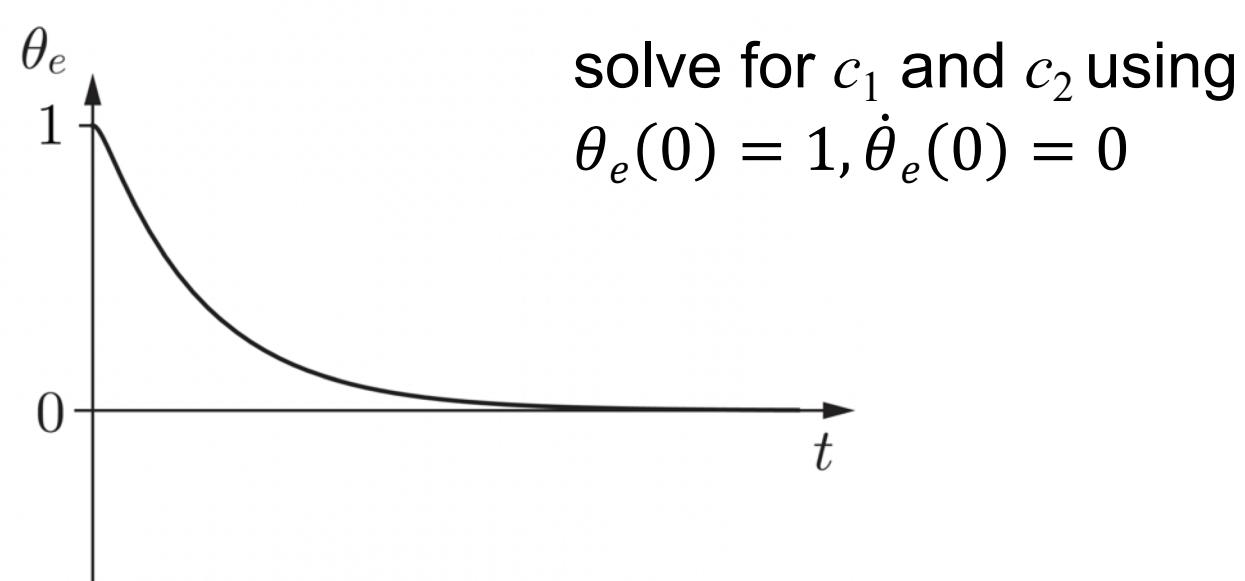
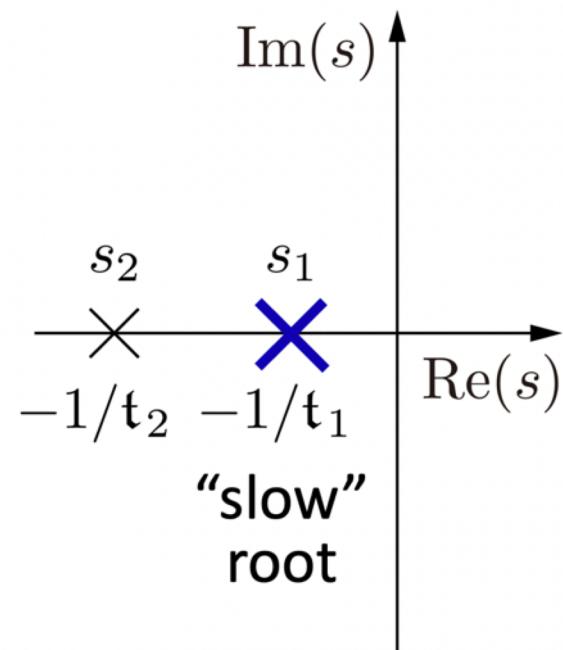
Important concepts, symbols, and equations (cont.)

$\zeta > 1$: Overdamped

$$\theta_e(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$

$$s_1 = -\zeta \omega_n + \omega_n \sqrt{\zeta^2 - 1}$$

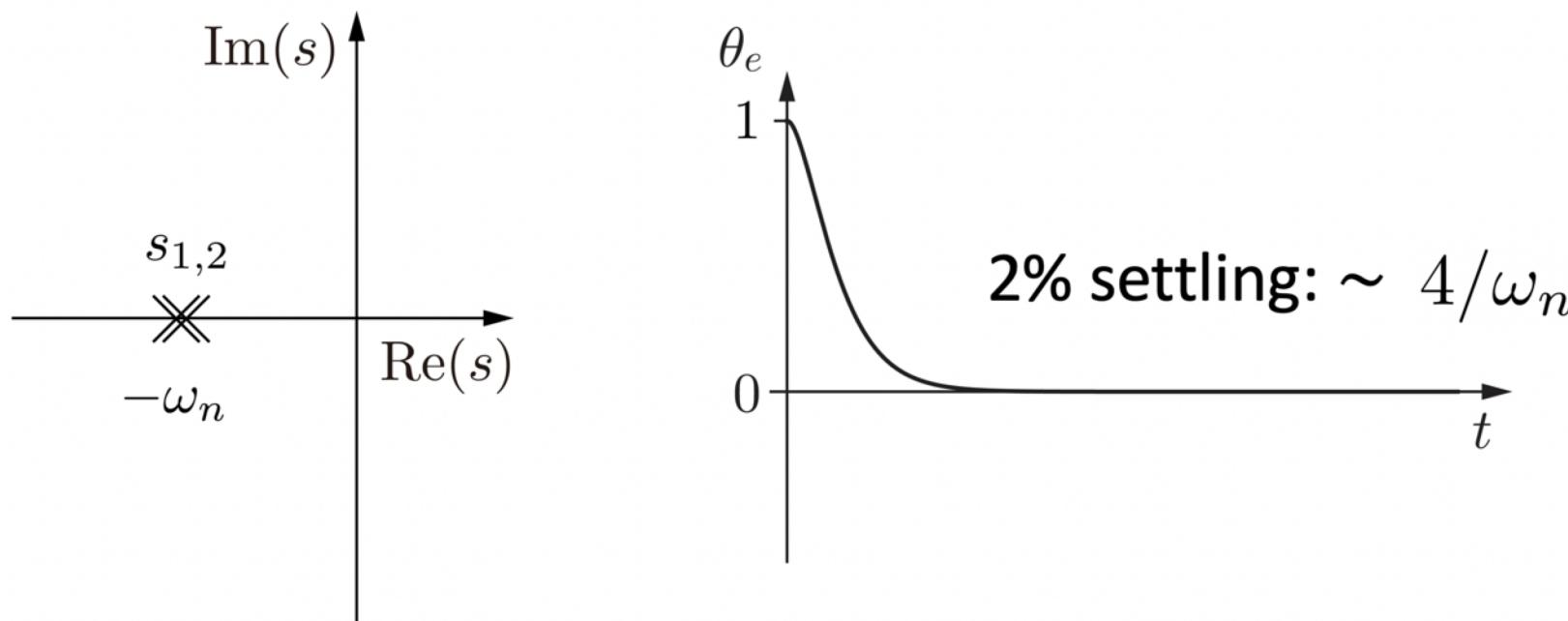
$$s_2 = -\zeta \omega_n - \omega_n \sqrt{\zeta^2 - 1}$$



Important concepts, symbols, and equations (cont.)

$\zeta = 1$: Critically damped

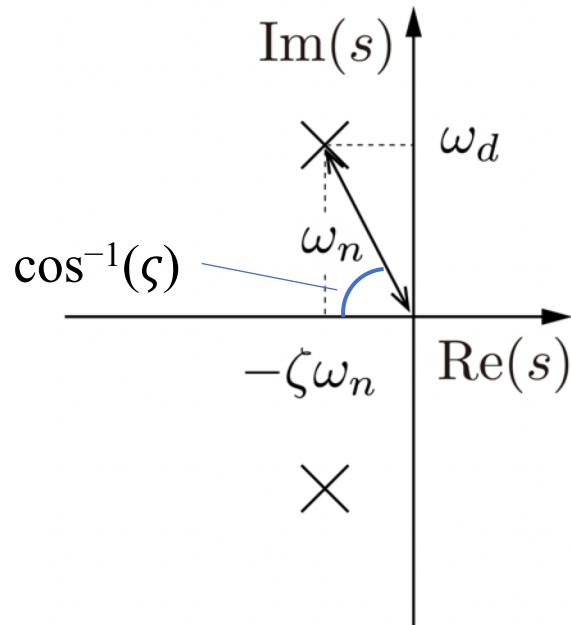
$$\theta_e(t) = (c_1 + c_2 t) e^{-\omega_n t} \quad s_{1,2} = -\omega_n$$



Important concepts, symbols, and equations (cont.)

$\zeta < 1$: Underdamped

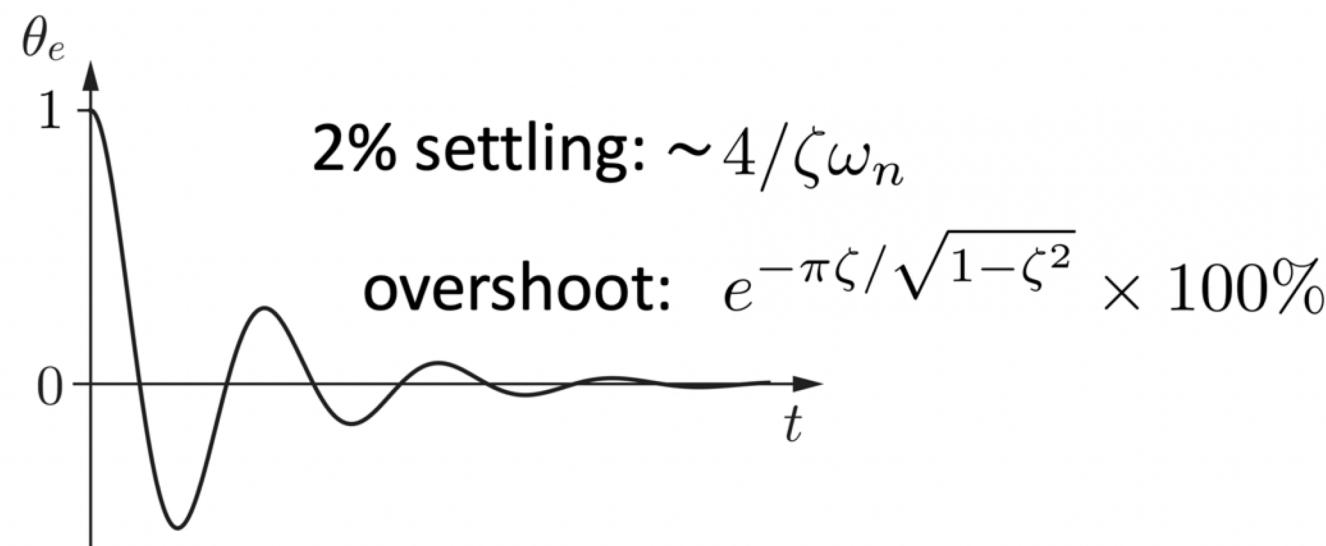
$$\theta_e(t) = (c_1 \cos \omega_d t + c_2 \sin \omega_d t) e^{-\zeta \omega_n t}$$



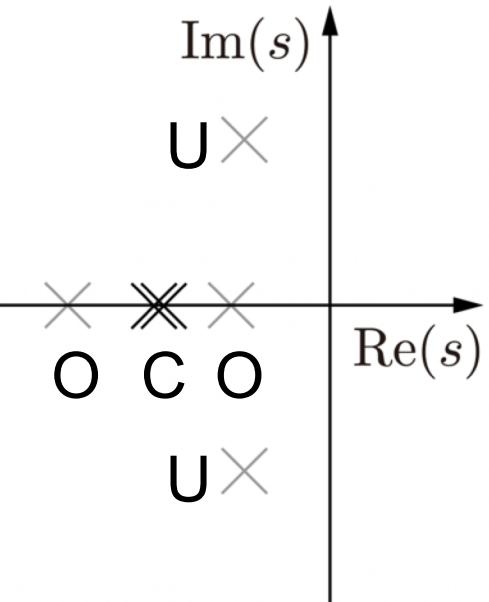
damped natural frequency

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

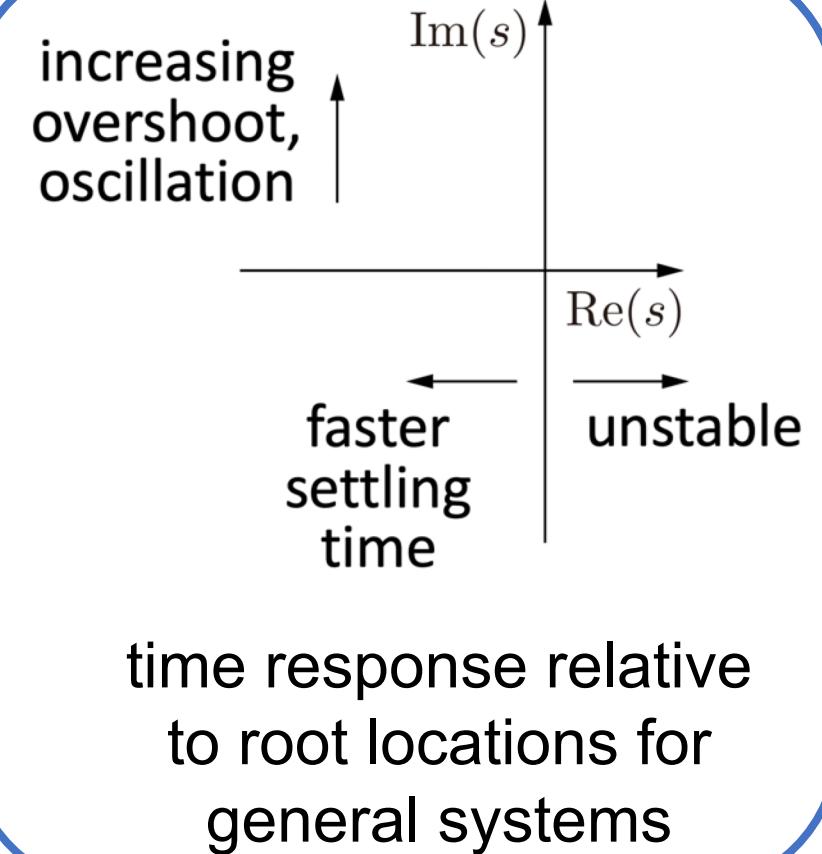
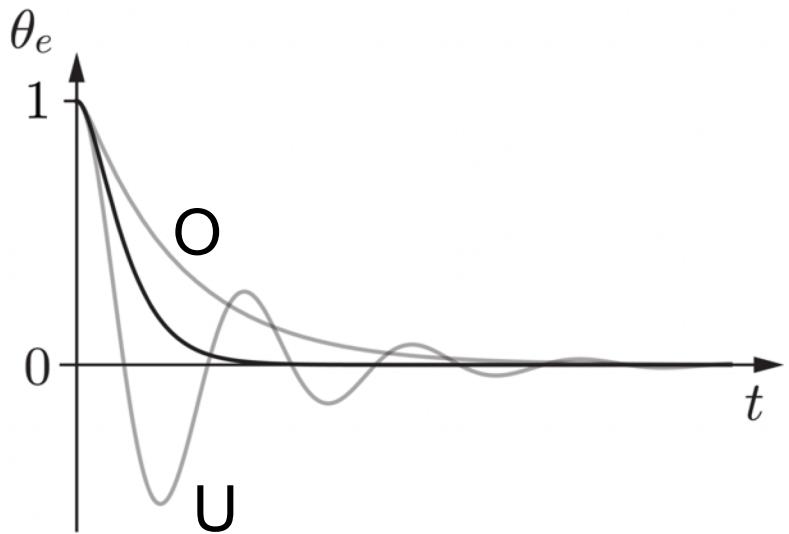
$$s_{1,2} = -\zeta \omega_n \pm j\omega_d$$



Important concepts, symbols, and equations (cont.)



second-order systems



$$\omega_n = \sqrt{k/m} \quad \zeta = b/(2\sqrt{km}) \quad \text{2% settling: } \sim 4/\zeta\omega_n$$

$$\ddot{\theta}_e(t) + 2\zeta\omega_n\dot{\theta}_e(t) + \omega_n^2\theta_e(t) = 0 \quad \text{overshoot: } e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\%$$

When controlling a robot joint, what do b , k , and m usually correspond to?

How do you change m to decrease settling time? k, b ?

How do you change m to decrease overshoot? k, b ?

Shown are one of the roots of five different second-order systems, A, B, C, D, and E.
List them in the following orders:

1. Natural frequency, highest to lowest.
2. Damped natural frequency, highest to lowest.
3. Damping ratio, highest to lowest.
4. Overshoot in unit step error response, highest to lowest.
5. Settling time, longest to shortest.

Which has the “best” transient response?

