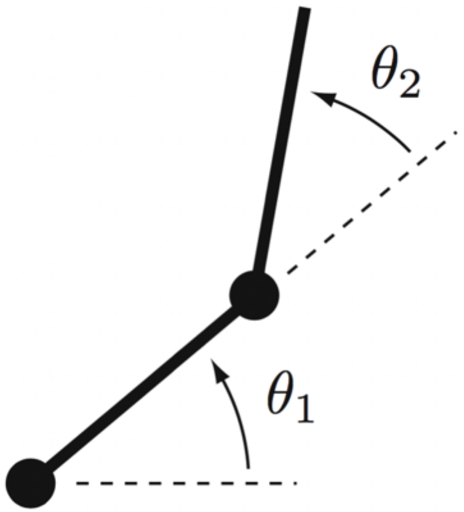
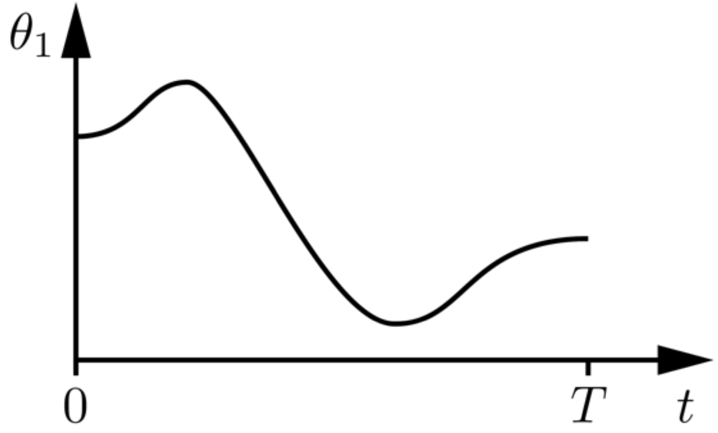
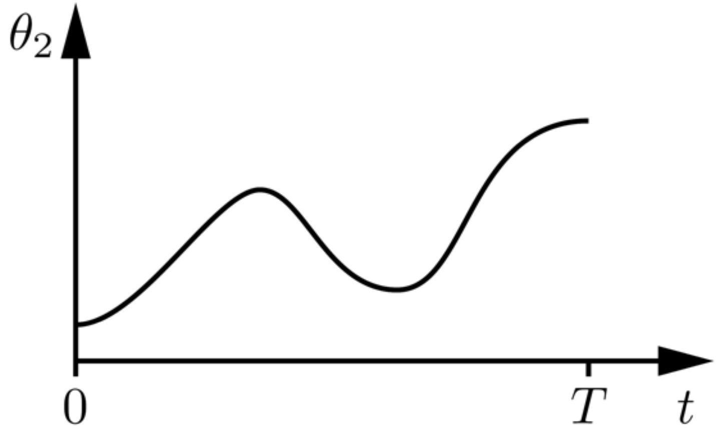
Important concepts, symbols, and equations

Trajectory: A specification of the configuration as a function of time.

$$\theta(t), \; t \in [0, T]$$





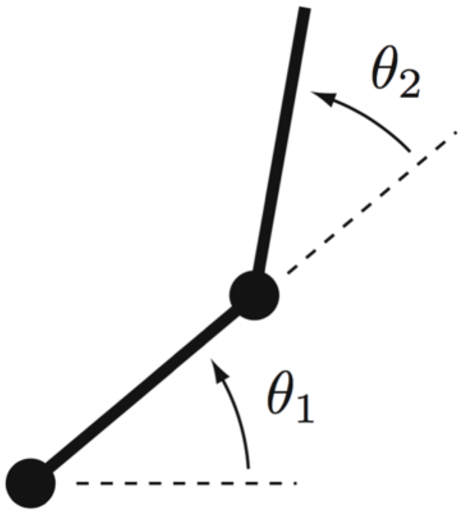


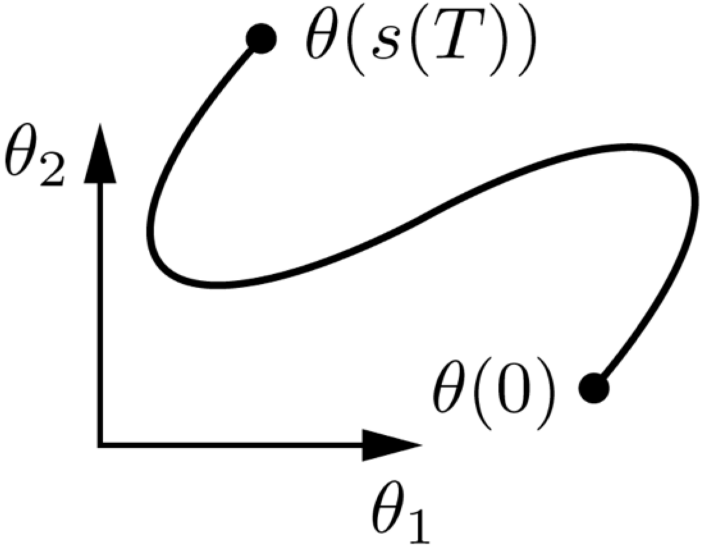
Important concepts, symbols, and equations (cont.)

Path: A specification of the configuration as a function of a path parameter.

$$\theta(s), \;\; s \in [0, 1]$$

Time scaling: A mapping $s(t)$: $[0, T] \rightarrow [0, 1]$, from time to the path parameter.





Important concepts, symbols, and equations (cont.)

Motion as a function of $\theta(s)$ and $s(t)$:

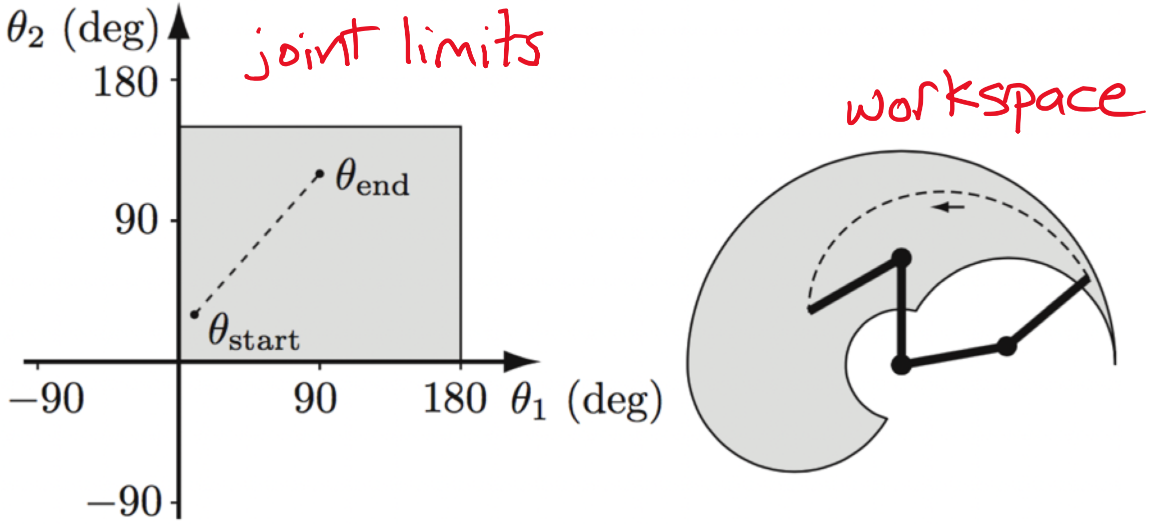
$$\dot{\theta} = \frac{d\theta}{ds}\dot{s},$$
$$\ddot{\theta} = \frac{d\theta}{ds}\ddot{s} + \frac{d^2\theta}{ds^2}\dot{s}^2$$

Both $\theta(s)$ and $s(t)$ must be twice-differentiable.

Important concepts, symbols, and equations (cont.)

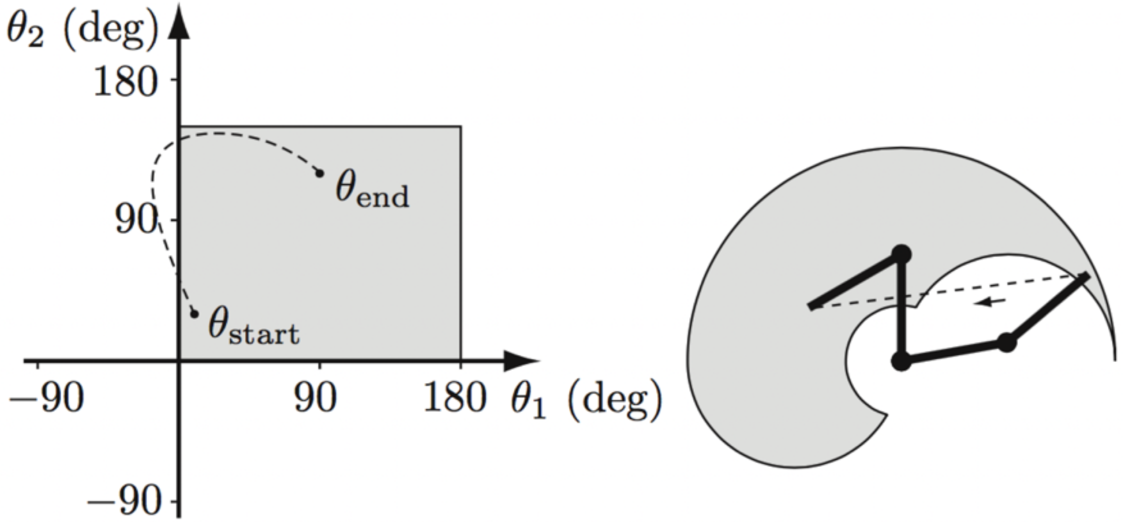
straight line in joint space

$$\theta(s) = \theta_{\text{start}} + s(\theta_{\text{end}} - \theta_{\text{start}})$$

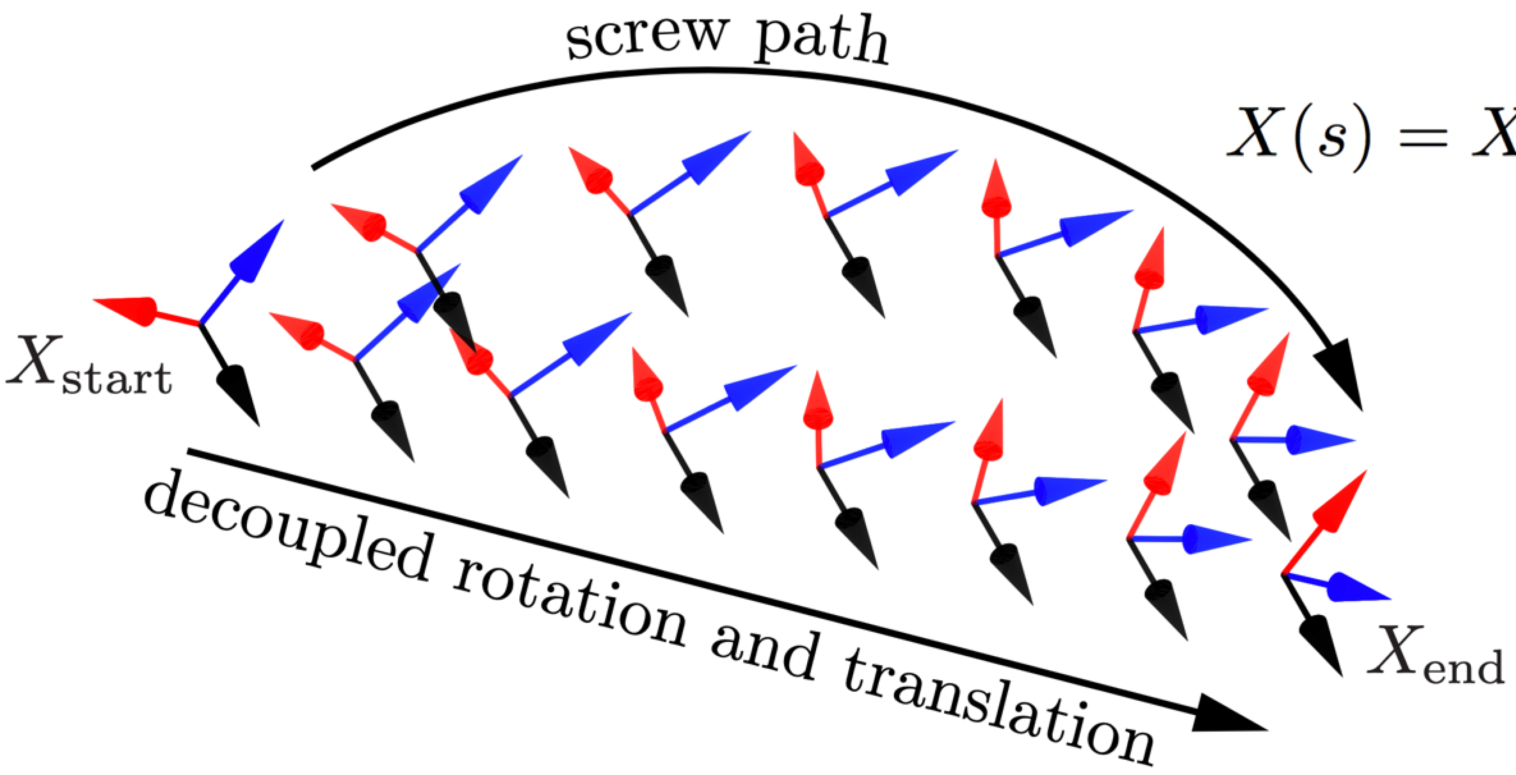


straight line in task space

$$X(s) = X_{\text{start}} + s(X_{\text{end}} - X_{\text{start}})$$



Important concepts, symbols, and equations (cont.)



$$X(s) = X_{\text{start}} \exp(\log(X_{\text{start}}^{-1} X_{\text{end}})s)$$

$$T_{s,\text{start}}^{-1} T_{s,\text{end}}$$

$$T_{\text{start},\cancel{s}} T_{\cancel{s},\text{end}}$$

$$T_{\text{start},\text{end}}$$

$$\log(T_{\text{start},\text{end}}) = [\mathcal{V}_{\text{start}}]$$

$$T(s) = T_{s,\text{start}} \exp([\mathcal{V}_{\text{start}}] s)$$

$$p(s) = p_{\text{start}} + s(p_{\text{end}} - p_{\text{start}}),$$

$$R(s) = R_{\text{start}} \exp(\log(R_{\text{start}}^{\text{T}} R_{\text{end}})s)$$

Important concepts, symbols, and equations (cont.)

Third-order polynomial time scaling

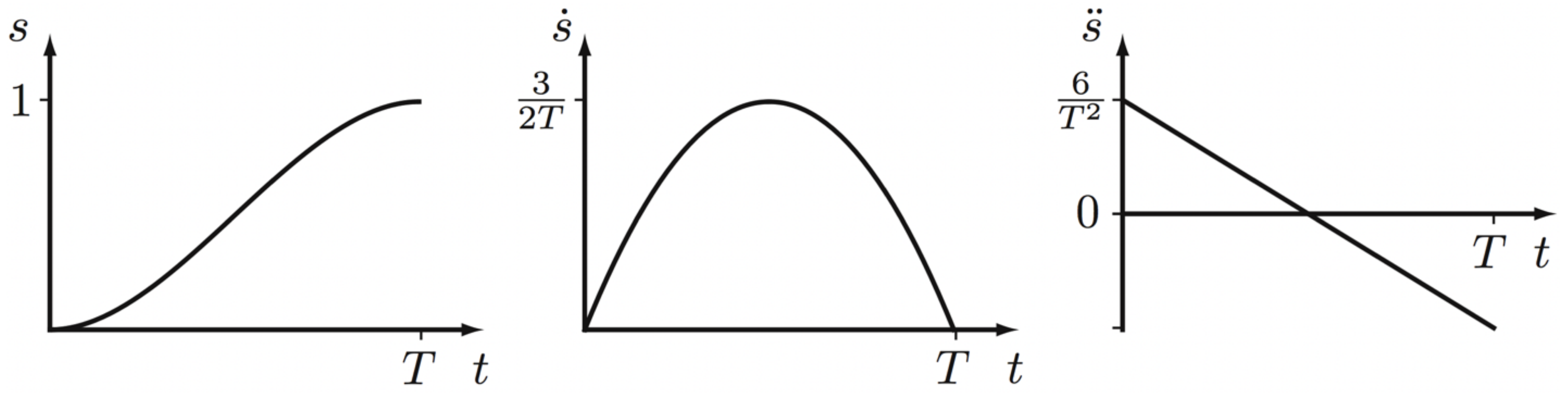
$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{s}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

4 constraints to solve for 4 constants

$$\boxed{\begin{array}{ll} s(0) = 0 & \dot{s}(0) = 0 \\ s(T) = 1 & \dot{s}(T) = 0 \end{array}}$$

$$a_0 = 0, \; a_1 = 0, \; a_2 = \frac{3}{T^2}, \; a_3 = -\frac{2}{T^3}$$



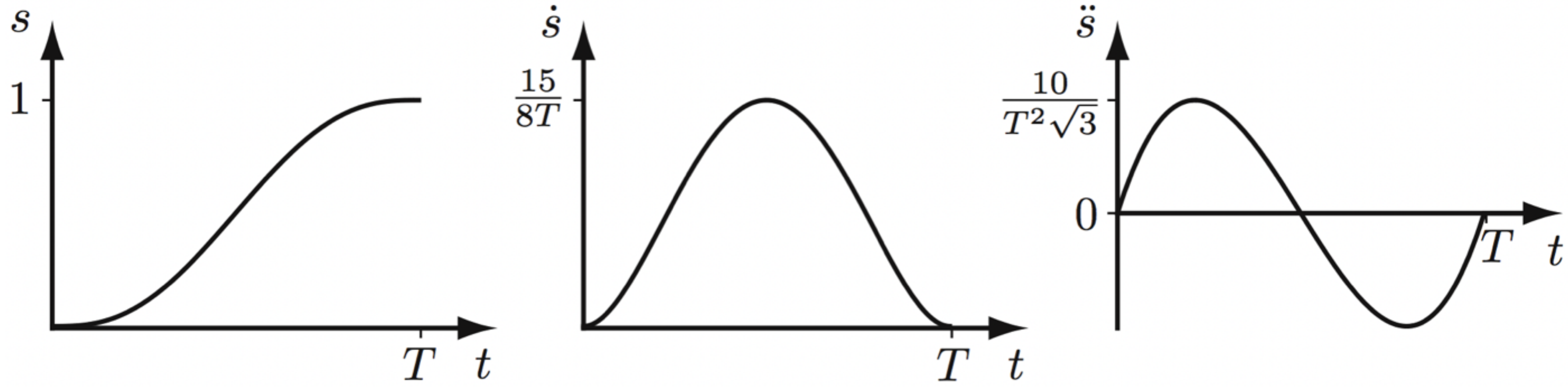
Important concepts, symbols, and equations (cont.)

Fifth-order polynomial time scaling

$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

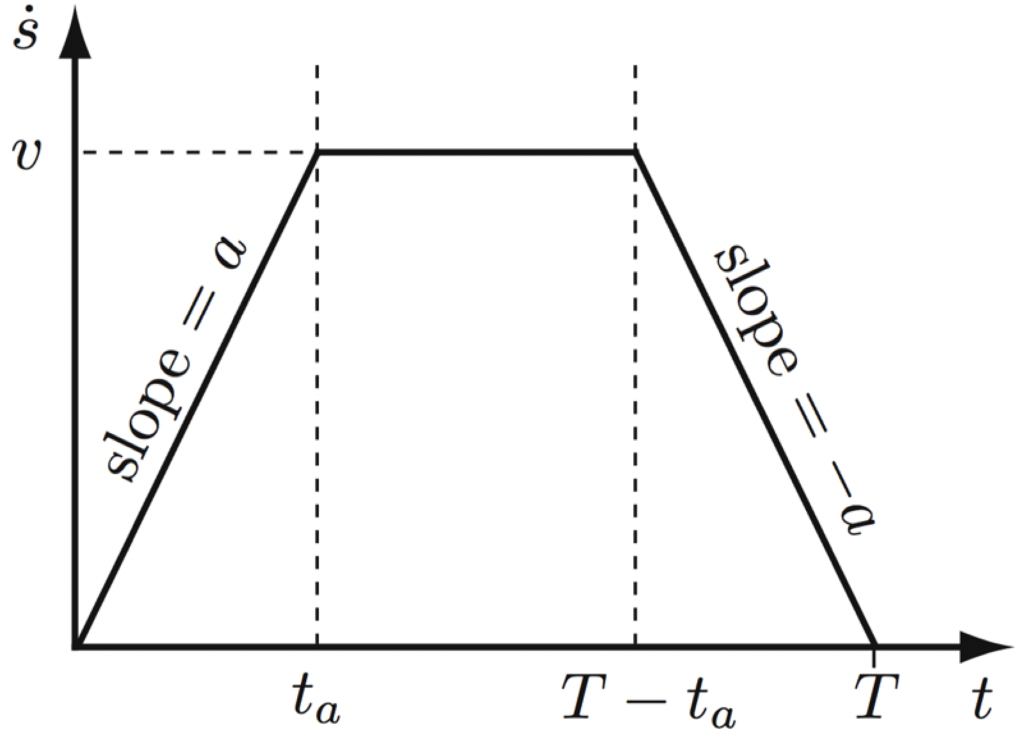
6 constraints to solve for 6 constants

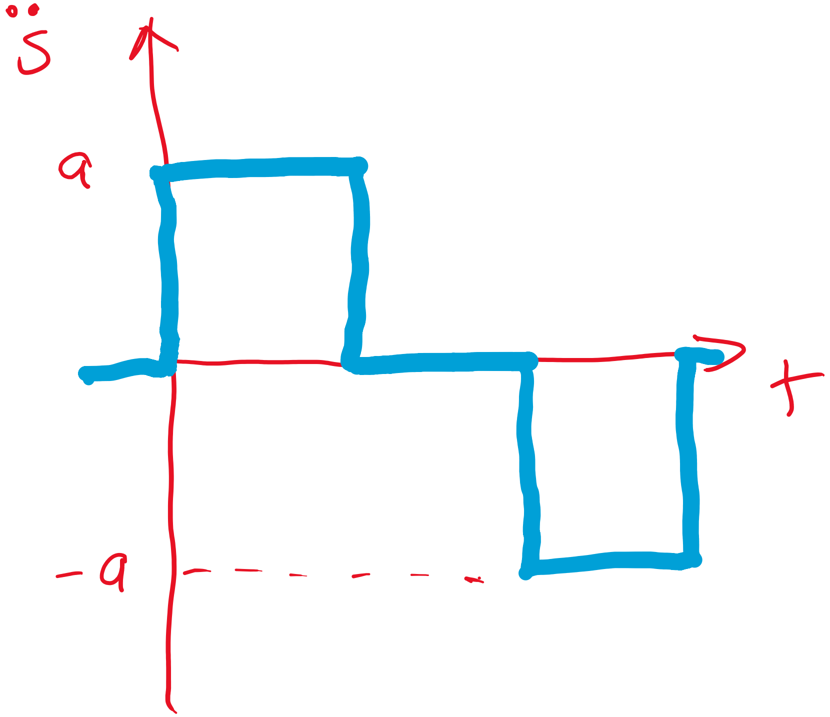
$$s(0) = 0 \qquad \dot{s}(0) = 0 \qquad \ddot{s}(0) = 0$$
$$s(T) = 1 \qquad \dot{s}(T) = 0 \qquad \ddot{s}(T) = 0$$



Important concepts, symbols, and equations (cont.)

Trapezoidal time scaling





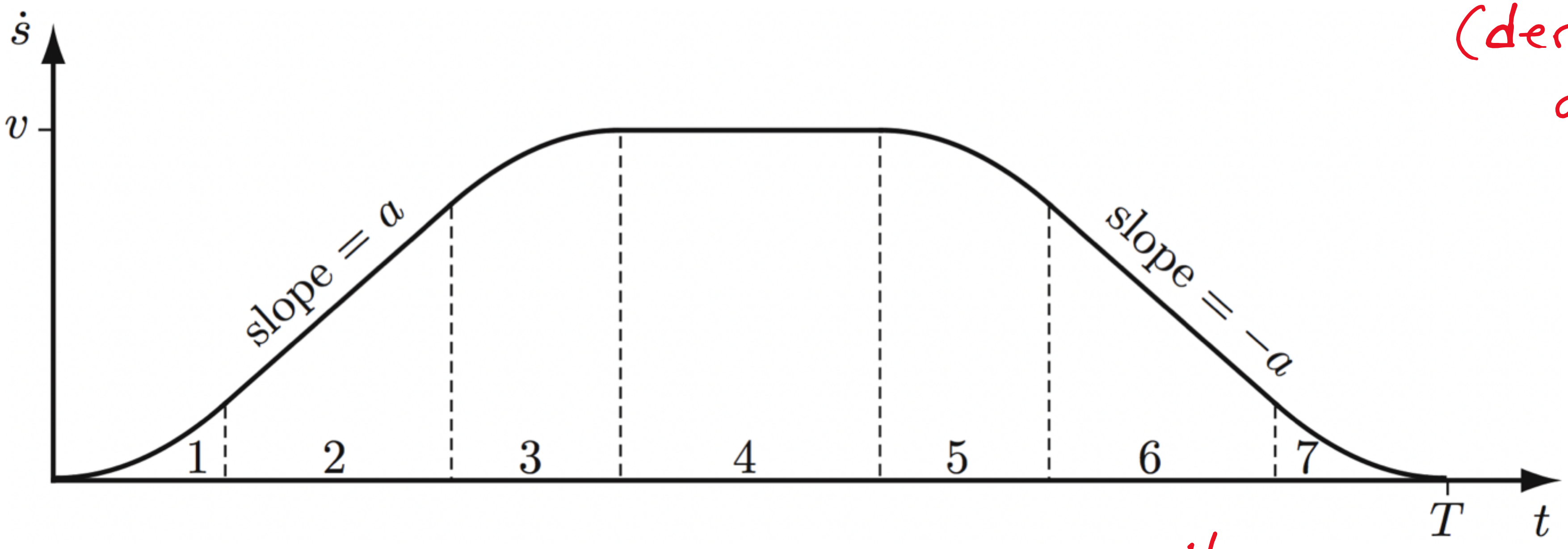
commonly used in motor controllers

Important concepts, symbols, and equations (cont.)

S-curve time scaling

segments 1, 3, 5, + 7: constant jerk

(derivative of acceleration)

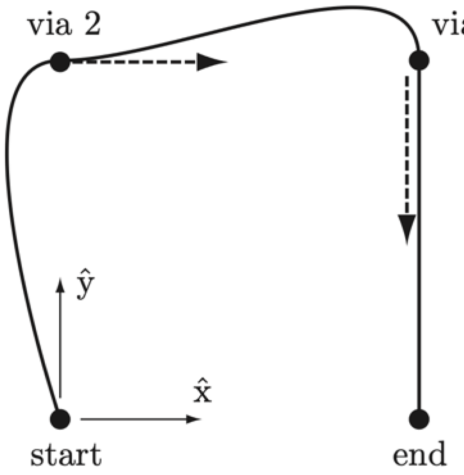


commonly used in motor controllers

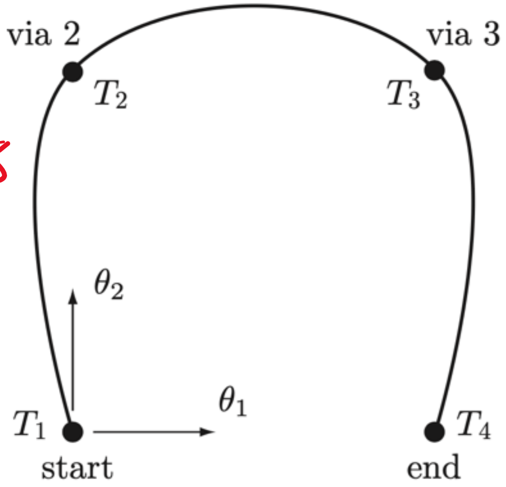
Important concepts, symbols, and equations (cont.)

Polynomial interpolation through via points

- third-order interpolation using via times, configurations, and velocities
- third-order interpolation using via times, configurations, and equal velocities and accelerations before and after vias

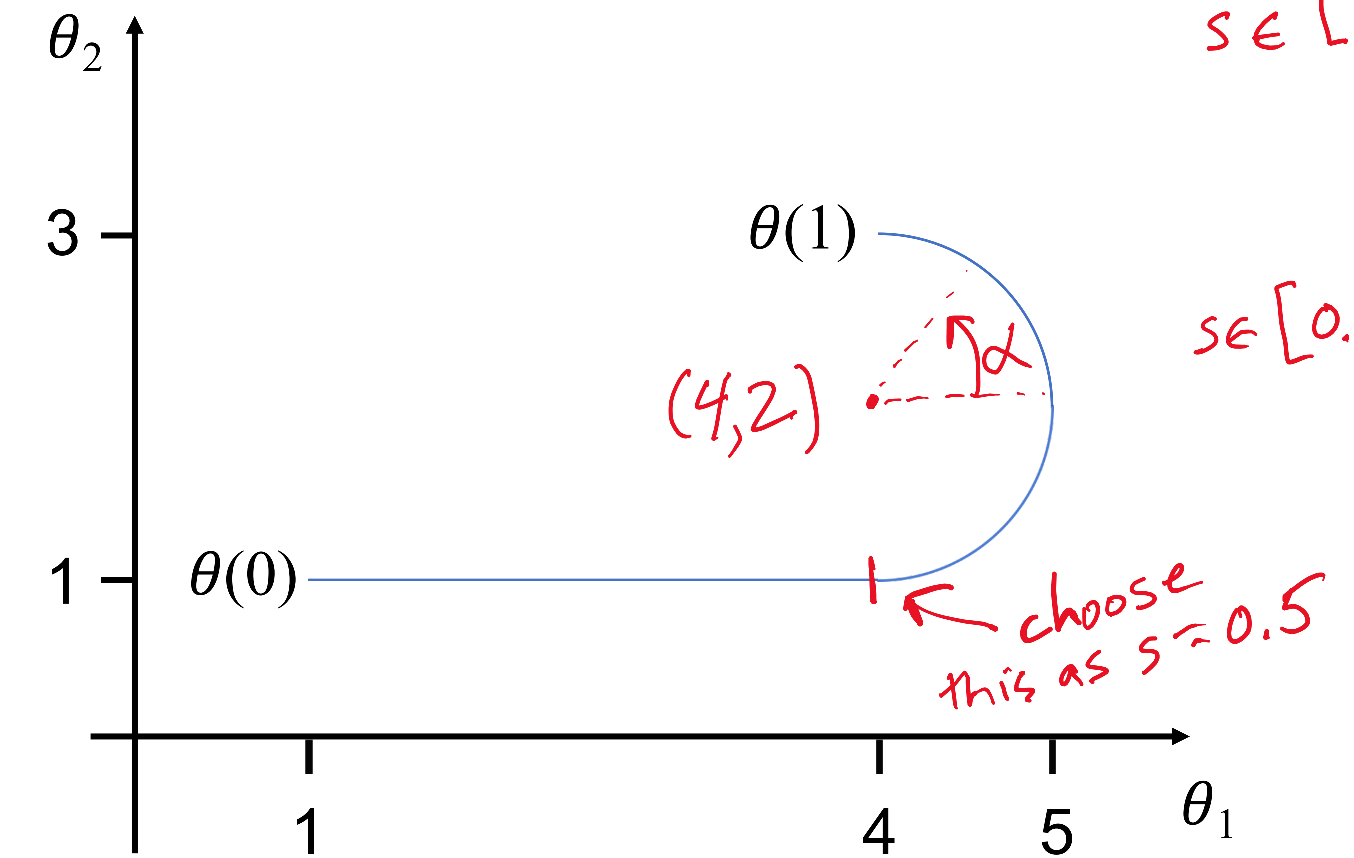


both have 12 constraints on 12 coefficients for each of 2 axes



Many other methods, including **B-splines** (paths stay within convex hull of control points, but don't pass through them).

Give an expression for the path $\theta(s)$, $s \in [0,1]$.



$s \in [0, 0.5)$: $\begin{bmatrix}\theta_1 \\ \theta_2\end{bmatrix}(s) = \begin{bmatrix}1 + 6s \\ 1\end{bmatrix}$

$s \in [0.5, 1]$: α goes from $-\pi/2$ to $\pi/2$

$\alpha = 2\pi(s - 0.75)$

so at $s = 0.5$, $\alpha = -\pi/2$

at $s = 1$, $\alpha = \pi/2$

$$\begin{bmatrix}\theta_1 \\ \theta_2\end{bmatrix}(s) = \begin{bmatrix}4 + \cos(2\pi(s-0.75)) \\ 2 + \sin(2\pi(s-0.75))\end{bmatrix}$$

What kind of time scaling can be used to obtain a continuous jerk profile?

Choose $\dddot{s}(0) = \dddot{s}(T) = 0$, so now 8 constraints. 7^{th}-order polynomial.

What is the maximum joint velocity obtained on a straight-line rest-to-rest trajectory with cubic polynomial time scaling?

$\dot{\theta}_{max} = \frac{d\theta}{ds}\dot{s}_{max}$, $\frac{d\theta}{ds} = (\theta_{end} - \theta_{start})$ $\dot{s}_{max} = \frac{3}{2T}$

Describe a circumstance under which the coast phase of the trapezoidal time scaling is not used.

The motion is not long enough to reach max velocity before deceleration must begin.

Give an equation to implement a third-order polynomial time-scaled rest-to-rest motion following a screw axis.

$X(t) = X_{start} \exp(\log(X_{start}^{-1} X_{end})(a_0 + a_1 t + a_2 t^2 + a_3 t^3))$

A time scaling can be written as $s(t)$ or $\dot{s}(s)$. If $s(t) = at^2$, what is $\dot{s}(s)$?

$\dot{s} = 2at$

$s = at^2$

$t = \sqrt{\frac{s}{a}}$

so $\dot{s}(s) = 2\sqrt{as}$