* All-Pair Shortest Path (Floyd-Warshall Algo)

- $V \cdot (E \log V)$

  $V \cdot V^2 \log V$

  $\boxed{V^3 \log V}$ — Worst Case

- What if we use Bellman-Ford's here?

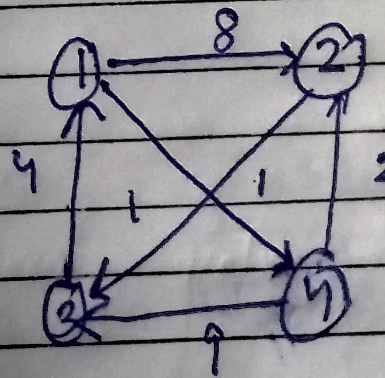
  $V \cdot E$

  V times,

  $V \cdot (VE)$

  $V^2 E \Rightarrow V^2 \cdot V^2$

  $\Rightarrow \boxed{V^4}$



- Directed, weighted graph

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- We'll make a distance matrix

$$D^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & \infty & 1 \\ 2 & \infty & 0 & 1 & \infty \\ 3 & 4 & \infty & 0 & \infty \\ 4 & \infty & 2 & 9 & 0 \end{array}$$

$$D^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & \infty & 1 \\ 2 & \infty & 0 & 1 & \infty \\ 3 & 4 & 12 & 0 & 5 \\ 4 & \infty & 2 & 9 & 0 \end{array}$$

3-2 ⇒ 3-1 & 1-2 = 4+8 = $\boxed{12}$

3-4 ⇒ 3-1 & 1-4 = 4+1 = $\boxed{5}$

$$D^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & 9 & 1 \\ 2 & \infty & 0 & 1 & \infty \\ 3 & 4 & 12 & 0 & 5 \\ 4 & \infty & 2 & 3 & 0 \end{array}$$

1-3 ⇒ 1-2 & 2-3 ⇒ 8+1 ⇒ $\boxed{9}$

4-3 ⇒ 4-2 & 2-3 ⇒ 2+1 ⇒ $\boxed{3}$

$$D^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & 9 & 1 \\ 2 & 5 & 0 & 1 & 6 \\ 3 & 4 & 12 & 0 & 5 \\ 4 & 7 & 2 & 3 & 0 \end{array}$$

2-1 ⇒ 2-3 & 3-1 ⇒ 1+4 ⇒ $\boxed{5}$

4-1 ⇒ 4-3 & 3-1 ⇒ 9+4 ⇒ $\boxed{13}$

2-4 ⇒ 2-3 & 3-1 & 1-4 ⇒ 1+4+1 ⇒ $\boxed{6}$

$\boxed{4-1}$ ⇒ 4-2 & 2-3 & 3-1
⇒ 2+1+4
⇒ $\boxed{7}$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 4 & 1 \\ 2 & 5 & 0 & 1 & 6 \\ 3 & 4 & 7 & 0 & 5 \\ 4 & 7 & 2 & 3 & 0 \end{array} = D^4$$

$1-2 \rightarrow 1-4 \;\&\; 4-2$
$\rightarrow 1+2 \rightarrow \boxed{3}$

$1-3 \rightarrow 1-4 \;\&\; 4-2 \;\&\; 2-3$
$\rightarrow 1+2+1$
$\rightarrow \boxed{4}$

$3-2 \rightarrow 3-1 \;\&\; 1-4 \;\&\; 4-2$
$\rightarrow 4+1+2$
$\rightarrow \boxed{7}$

Now, matrix has a size of $n \times n \rightarrow n^2$

No. of subproblems

$n^2 \times n$
$\rightarrow \boxed{O(n^3)}$