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2a)

i. $T(n) = 2T(n/2) + \theta(1)$

ii. $T(n) = 7T(n/4) + \theta(1)$

As we don’t know the data structure holding the matrix, we can’t really assume the type of complexity it has

2b)

i. Leaves: n^2; Height: $\log_2 n$; Nodes: $((4^{\log_2 n + 1}) - 1) / 3$

ii. Leaves: $n^{\log_1 1} = n^0 = 1$; Height: n; Nodes: n

iii. Leaves: $n^{\log_5 3}$; Height: $\log_5 n$; Nodes: $((3^{\log_5 n + 1}) - 1) / 2$

iv. Leaves: $n^{n\log_1 1} + n^{n\log_1 1} = n^0 + n^0 = 1+1=2$; Height: 2n; Nodes: 2n

2c)

$T(n)=O(n)$

Assume: $T(k) \le ck$, for all $k<n$

Base Case: $T(1)$ is constant

Inductive Hypothesis: $T(k) \le ck$, for all $k<n$

Inductive Step: $T(n) \le T(n/3) + T(2n/3) + n^{1/2}$

$\le cn/3 + 2cn/3 + n^{1/2} = cn + n^{1/2}$

For any $c \ge 1$, cn dominates $n^{1/2}$, so $T(n) = O(n)$