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1a)

After looking at the entire grid, the algorithm splits it up into 4 quadrants. Then it looks for the one with the filled in square. An L-shaped piece is then placed at the center of the overall grid, covering the quadrants that don't yet have a filled piece. The algorithm is then recursively implemented until the entire board is filled.

1b)

Proof: After one square is filled, the rest can be filled with 3-block carpet pieces in the given 4 combinations. The board is of size $2^n \times 2^n$

Base Case: When $n = 1$, the board is of size 2×2 . When the specified square is filled, the other carpet piece will fill the rest of the board.

Inductive Hypothesis: Assume true for $k < n$ such that $2^k \times 2^k$, with one square filled in. This board can be filled by $(2^k - 1)/3$ L-shaped carpet pieces.

Inductive Step: A grid with the size of $2^{k+1} \times 2^{k+1}$ is made up of 4 $2^k \times 2^k$ quadrants. The filled square is in one of those quadrants. When an L-shaped piece is placed at the center of the $2^{k+1} \times 2^{k+1}$ board, each quadrant not containing the currently filled square are now "missing" a square as well, where each size is $2^k \times 2^k$. Thus, when applying the Inductive Hypothesis, the rest of the board is able to be filled with the previously mentioned L-shaped pieces.

1c)

$$f(n) = 1 = n^0 \quad n^{\log_2 4} = n^{1/2}$$

$$a/b^c = 4/2^0 = 4 > 1 \quad \Rightarrow \quad \text{Case 1}$$

$$\varepsilon = 1/2$$

$$n^0 = n^{1/2 - 1/2}$$

$$T(n) = \theta(n^{1/2})$$

1d)

import sys

```
def fill_carpet(y_topLeft, x_topLeft, size, y, x):
    if size == 2:
        if x_topLeft == x:
            if y_topLeft == y:
                print('{} {} {}'.format(y_topLeft + 1, x_topLeft + 1, 4))
            else:
                print('{} {} {}'.format(y_topLeft, x_topLeft + 1, 3))
        else:
            if y_topLeft == y:
                print('{} {} {}'.format(y_topLeft + 1, x_topLeft, 1))
```

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```

        else:
            print '{} {} {}'.format(y_topLeft, x_topLeft, 2)
    else:
        x_center = x_topLeft + (size/2)
        y_center = y_topLeft + (size/2)

        if x < x_center:
            x_lowR = x_upR = x_center
            y_lowR = y_center
            y_upR = y_center - 1
            if y < y_center:
                x_lowL = x_center - 1
                y_lowL = y_center
                x_upL = x
                y_upL = y
                print '{} {} {}'.format(y_center, x_center, 4)
            else:
                x_lowL = x
                y_lowL = y
                x_upL = x_center - 1
                y_upL = y_center - 1
                print '{} {} {}'.format(y_center - 1, x_center, 3)
        else:
            x_lowL = x_upL = x_center - 1
            y_lowL = y_center
            y_upL = y_center - 1
            if y < y_center:
                x_lowR = x_center
                y_lowR = y_center
                x_upR = x
                y_upR = y
                print '{} {} {}'.format(y_center, x_center - 1, 1)
            else:
                x_lowR = x
                y_lowR = y
                x_upR = x_center
                y_upR = y_center - 1
                print '{} {} {}'.format(y_center - 1, x_center - 1, 2)

    fill_carpet(y_topLeft + size/2, x_topLeft, size/2, y_lowL, x_lowL)
    fill_carpet(y_topLeft + size/2, x_topLeft + size/2, size/2, y_lowR, x_lowR)
    fill_carpet(y_topLeft, x_topLeft + size/2, size/2, y_upR, x_upR)
    fill_carpet(y_topLeft, x_topLeft, size/2, y_upL, x_upL)
```

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YOU DO NOT NEED TO CHANGE THE CODE BELOW THIS LINE

Read input

```
data = [int(x) for x in sys.stdin.readline().split()]
```

```
carpet_dimen = data[0]
```

```
y = data[1]
```

```
x = data[2]
```

```
fill_carpet(0, 0, carpet_dimen, y, x)
```