CS325 - Project 1

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Proof of Claim 1

Claim 1: y_i is not visible iff $\exists j, k$ such that $j < i < k$ and $y* > m_i x * + b_i$ where $(x*, y*)$ is the intersection of y_j and j_k.

$A \equiv y_i$ is not visible
$B \equiv \exists j, k$ such that $j < i < k$ and $y* < m_i x * + b_i$ where $(x*, y*)$ is the intersection of y_j and y_k.
$A \Leftrightarrow B$

First Prove $A \Rightarrow B$

Direct Proof:
Let y_i be a line that is not visible.
Then $l < i < n$ because y_i and y_n are always visible.
Let k be the smallest index greater than i such that y_k is visible.
e.g. $y_1, y_2, ..., y_k, y_{k+1}, ..., y_{n-1}, y_n$
Let $(x*, y*)$ be the left most point on y_k that is visible.
Let j be the greatest index such that y_i intersects y_k at $(x*, y*)$ is visible.
Because y_i through y_{k-1} are not visible (by definition of k_j) $j < i < k$.
Since $x*, y*$ is visible and y_i is not visible, $m_i x + b_i < y*$.

Prove $B \Rightarrow A$

Direct Proof:
Since $m_i < m_k$, the intersection point of y_i and y_k is left of $x*$.
Since $m_i < m_k$, $m_i x + b_i < m_k x + b_k \; \forall x > \bar{x}$.
Likewise since $m_i > m_j$, y_i and y_j intersect at $(\bar{\bar{x}}, \bar{\bar{y}})$ right of $x*$ $(\bar{\bar{x}} > x*)$.
$\therefore m_i x + b_i < m_j x + b_j; \; \forall x < \bar{\bar{x}}$.
$\therefore y_i$ is not visible.
$y_k + y_j$ intersect at $m_k x + b_k = m_j x + b_j$
$x = \dfrac{(b_j - b_k)}{(m_k - m_j)}$
Is $m_j \left(\dfrac{b_j - b_k}{m_k - m_j} \right) + b_j > m_i \left(\dfrac{b_j - b_k}{m_k - m_j} \right) + b_i$
If $m_k > m_j$ then instead compare $m_j(b_j - b_k) + b_j(m_k - m_j) > m_i(b_j + b_k)$