CS325 - Project 1

Group 6
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Proof of Claim 1

> Claim 1: y_i is not visible iff $\exists j, k$ such that $j < i < k$ and $y* > m_i x * + b_i$ where $(x*, y*)$ is the intersection of y_j and j_k.

$A \equiv y_i$ is not visible
$B \equiv \exists j, k$ such that $j < i < k$ and $y* < m_i x * + b_i$ where $(x*, y*)$ is the intersection of y_j and y_k.
$A \Leftrightarrow B$

First Prove $A \Rightarrow B$

Direct Proof:

Let y_i be a line that is not visible.
Then $l < i < n$ because y_l and y_n are always visible.
Let k be the smallest index greater than i such that y_k is visible.
e.g. $y_1, y_2, ..., y_k, y_{k+1}, ..., y_{n-1}, y_n$
Let $(x*, y*)$ be the left most point on y_k that is visible.
Let j be the greatest index such that y_i intersects y_k at $(x*, y*)$ is visible.
Because y_i through y_{k-1} are not visible (by definition of k_j) $j < i < k$.
Since $x*, y*$ is visible and y_i is not visible, $m_i x + b_i < y*$.

Prove $B \Rightarrow A$

Direct Proof:

Since $m_i < m_k$, the intersection point of y_i and y_k is left of $x*$.
Since $m_i < m_k$, $m_i x + b_i < m_k x + b_k \; \forall x > \bar{x}$.
Likewise since $m_i > m_j$, y_i and y_j intersect at $(\bar{\bar{x}}, \bar{\bar{y}})$ right of $x*$ $(\bar{\bar{x}} > x*)$.
$\therefore m_i x + b_i < m_j x + b_j; \; \forall x < \bar{\bar{x}}$.
$\therefore y_i$ is not visible.
$y_k + y_j$ intersect at $m_k x + b_k = m_j x + b_j$
$x = \dfrac{(b_j - b_k)}{(m_k - m_j)}$
Is $m_j \left(\dfrac{b_j - b_k}{m_k - m_j} \right) + b_j > m_i \left(\dfrac{b_j - b_k}{m_k - m_j} \right) + b_i$
If $m_k > m_j$ then instead compare $m_j(b_j - b_k) + b_j(m_k - m_j) > m_i(b_j + b_k)$

Proof of Claim 2

> Claim 2: If $\{y_{j_1}, y_{j_2}, ..., y_{j_t}\}$ is the visible subset of $\{y_1, y_2, ..., y_{i-1}\}(t \leq i-1)$ then $\{y_{j_1}, y_{j_2}, ..., y_{j_k}, y_i\}$ is the visible subset of $\{y_1, y_2, ..., y_i\}$ where y_{j_k} is the last line such that $y_{j_k}(x*) > y_i(x*)$ where $(x*, y_{j_k}(x*))$ is the point of intersection of lines y_{j_k} and $y_{j_{k-1}}$.

Direct Proof

0.1 Prove that $y_i \in V^+k$

Let $A^+ = A\{y_i\}$.
Because $m_i > m_n, n < i, y_i$ is visible by the Claim 1 proof in the "Visible Line Notes" handout.
Since y_i is visible and $y_i \in A^+, y_i$ must also be in V^+, the visible subset of A^+.

0.2 Prove that $y_{j_k} \in V^+k$

Let $(x^*, y_{j_k}(x^*))$ be the point of intersection of the lines y_{j_k} and $y_{j_{k-1}}$.
Since $y_{j_k}(x^*)y_i(x^*)$ by definition, y_{j_k} is visible with respect to y_i.
Since y_{j_k} was already in V, it is defined to be visible with respect to all other elements.
$\therefore y_{j_k} \in V^+$.

0.3 Prove that $y_{j_n} \in V^+, 0 < n < k$

Because $y_{j_n} \in V$, it is defined to be visible with all other elements.
So we must show that y_{j_n} is visible with respect to y_i as well.
Let $(x_n^*, y_{j_n}(x_n^*))$ be the point of intersection of the lines y_{j_n} and $y_{j_{n+1}}$.
By definition, $m_{j_n} < m_{j_{n+1}}$, so $\forall x_n < x_n^*, y_{j_n}(x_n) > y_{j_{n+1}}(x_n)$.
$\therefore\ y_{j_1}(x_{1,2}^*) = y_{j_2}(x_{1,2}^*) \geq y_{j_3}(x_{1,2}^*), y_{j_2}(x_{2,3}^*) = y_{j_3}(x_{2,3}^*) \geq y_{j_4}(x_{2,3}^*), ..., y_{j_{n-1}}(x_{n-1,n}^*) = y_{j_n}(x_{n-1,n}^*) \geq y_{j_{n+1}}(x_{n-1,n}^*), ..., y_{j_{k-1}}(x_{k-1,k}^*) = y_{j_k}(x_{k-1,k}^*) \geq y_{j_i}(x_{k-1,k}^*)$
Since y_{j_n} was already in V, it is defined to be visible with respect to all other elements.
$\therefore y_{j_n} \in V^+$.

0.4 Prove that $y_{j_n} \in V^+, 0 < n < k$

Because $y_{j_n} \in V$, it is defined to be visible with all other elements.
So we must show that y_{j_n} is visible with respect to y_i as well.
Let $(x_n^*, y_{j_n}(x_n^*))$ be the point of intersection of the lines y_{j_n} and $y_{j_{n+1}}$.
By definition, $m_{j_n} < m_{j_{n+1}}$, so $\forall x_n < x_n^*, y_{j_n}(x_n) > y_{j_{n+1}}(x_n)$.
$\therefore\ y_{j_1}(x_{1,2}^*) = y_{j_2}(x_{1,2}^*) \geq y_{j_3}(x_{1,2}^*), y_{j_2}(x_{2,3}^*) = y_{j_3}(x_{2,3}^*) \geq y_{j_4}(x_{2,3}^*), ..., y_{j_{n-1}}(x_{n-1,n}^*) = y_{j_n}(x_{n-1,n}^*) \geq y_{j_{n+1}}(x_{n-1,n}^*), ..., y_{j_{k-1}}(x_{k-1,k}^*) = y_{j_k}(x_{k-1,k}^*) \geq y_{j_i}(x_{k-1,k}^*)$
Since y_{j_n} was already in V, it is defined to be visible with respect to all other elements.
$\therefore y_{j_n} \in V^+$.