CS325 - Project 3

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Correctness

Equation 1

Given an array with an index starting at 0.

$$T[i,j] = \max \left\{ \begin{array}{l} A[i,j] \\ T[i-1,j] + A[i,j] \\ T[i,j-1] + A[i,j] \end{array} \right\}$$

Base Cases:

T[i,j] = NaN if i < 0 OR j < 0

$$X[i, j, n_{row}, n_{col}] = max \left\{ \begin{array}{l} T[i, j] \\ X[i - 1, j, n_{row}, n_{col}] \\ X[i, j - 1, n_{row}, n_{col}] \end{array} \right\}$$

Base Cases:

 $X[i,j,n_{row},n_{col}] = NaN \text{ if } i < n_{row} - 1 \text{ AND } j < n_{col} - 1; \forall n_{row},n_{col}$

$$Z[n_{row}, n_{col}] = X[n_{row}, n_{col}, n_{row}, n_{col}] \label{eq:Z}$$

 $Z[n_{row}, n_{col}]$ returns the score of the optimal solution given an $n_{col} \times n_{row}$ grid A.

Pseudocode

```
load A[x,y] with values on board
initialize ybest, xbest, T[x,y] with 0
initialize P[x,y] with pointer to A[x,y]
for i = 0 \dots y
    for j = 0 \dots x
        T[i,j] \leftarrow A[i,j]
        if i > 0
             if T[i-1,j] + A[i,j] > T[i,j]
                 T[i,j] \leftarrow T[i-1,j] + A[i,j]
                 P[i,j] <- pointer to A[i-1,j]
        if j > 0
             if T[i,j-1] + A[i,j] > T[i,j]
                 T[i,j] \leftarrow T[i,j-1] + A[i,j]
                 P[i,j] <- pointer to A[i,j-1]
         if T[i,j] > T[ybest,xbest]
             ybest <- i
             xbest <- j
point.y <- ybest</pre>
point.x <- xbest</pre>
while point.y !=-1 AND point.x !=-1
    concat point with path
    point <- P[point.y,point.x]</pre>
```

Running Time

 $\Theta(n_{col}*n_{row})$

Populating the DP table just requires looping over the length and width of the grid, or $n_{col} * n_{row}$.