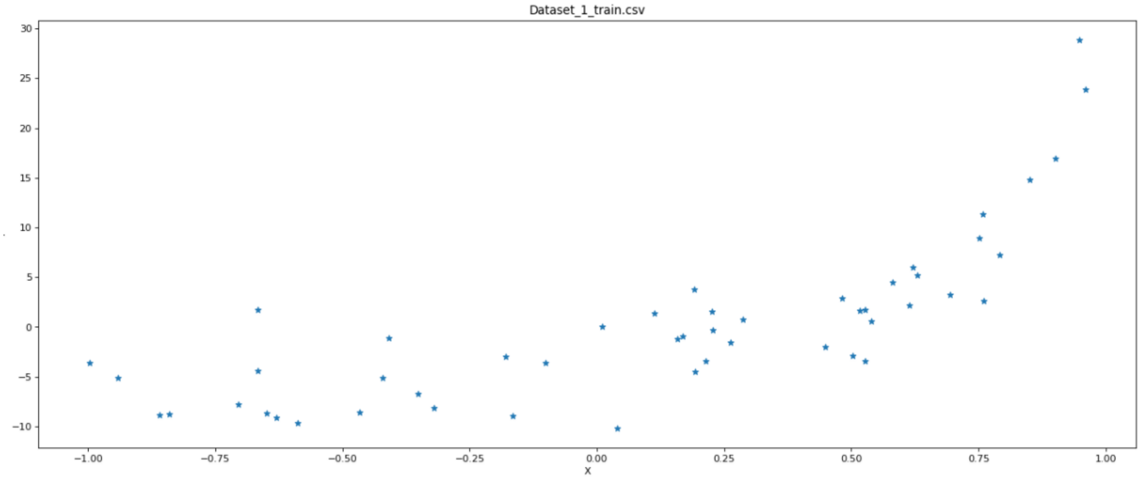
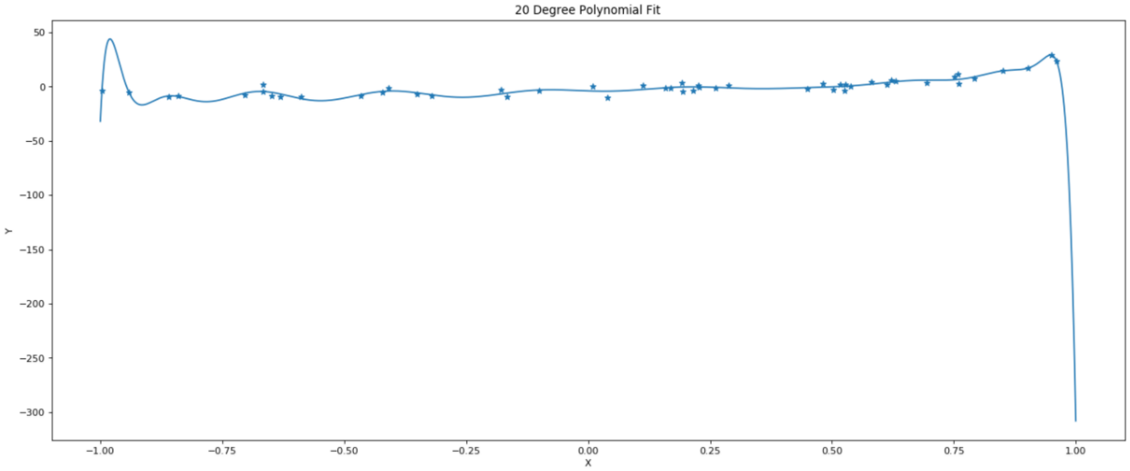
Question 1

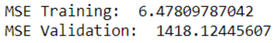
1.1

Visualizing the Dataset



Visualizing the 20-Degree Polynomial Fit





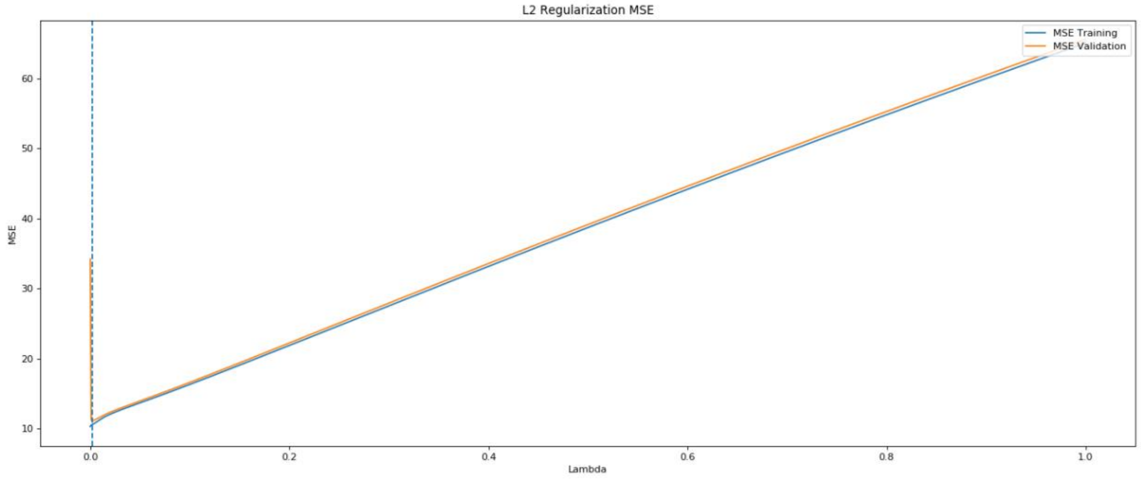
```
MSE Training:  6.47809787042
MSE Validation:  1418.12445607
```

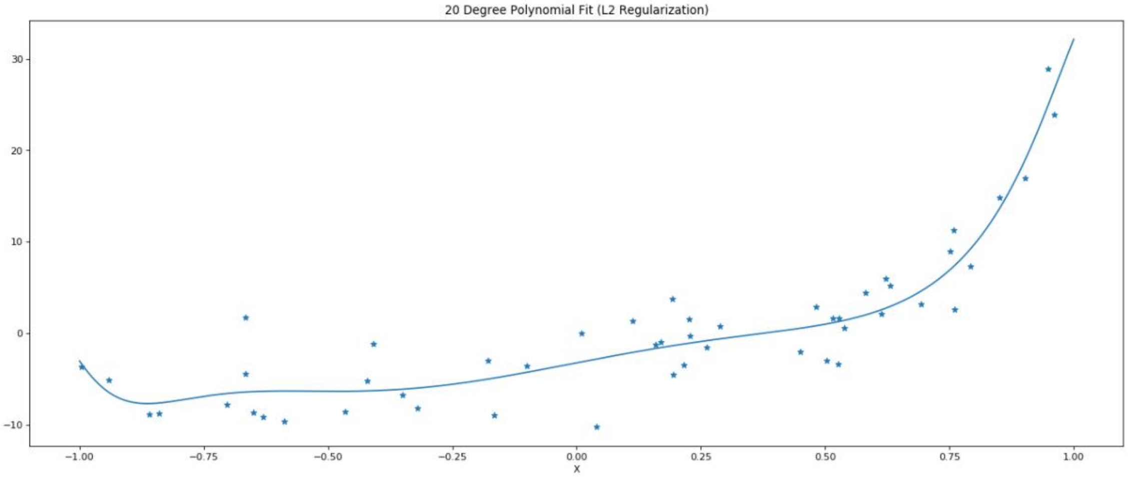
MSE Training = 6.47

MSE Validation = 1418

The quality of the fit looks fine if our target vector is between -0.8 and 0.8. However, there is clear overfitting. For any target Y greater than 0.8, the predicted value will be off significantly. We need to add regularization to the fit.

1.2



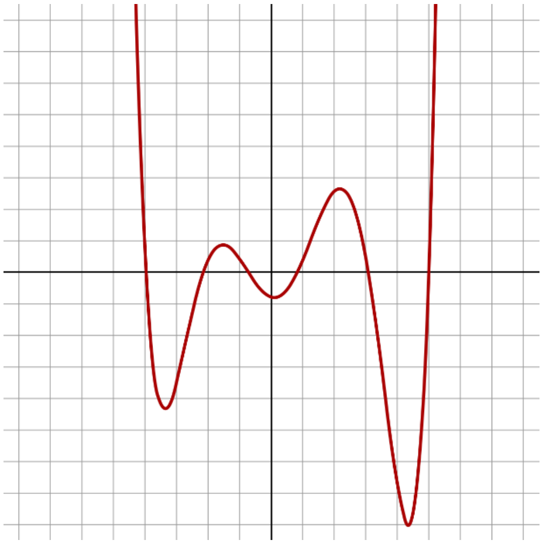


The best L2 regularization occurs at **Lambda = 0.002003**. At this point, the minimum MSE for training is **10.57**, and the minimum MSE for validation is **11.16**, and the minimum MSE for testing is **12.84.**

1.3

It looks like the fit with regularization is a 6th degree polynomial. I am inferring this from the number of highs and lows of the graph. The fit has 2 local maximum, and 3 global minimum. This would mean that in theory, this function would cross the X-axis 5 times, and therefore be a 6th degree polynomial.

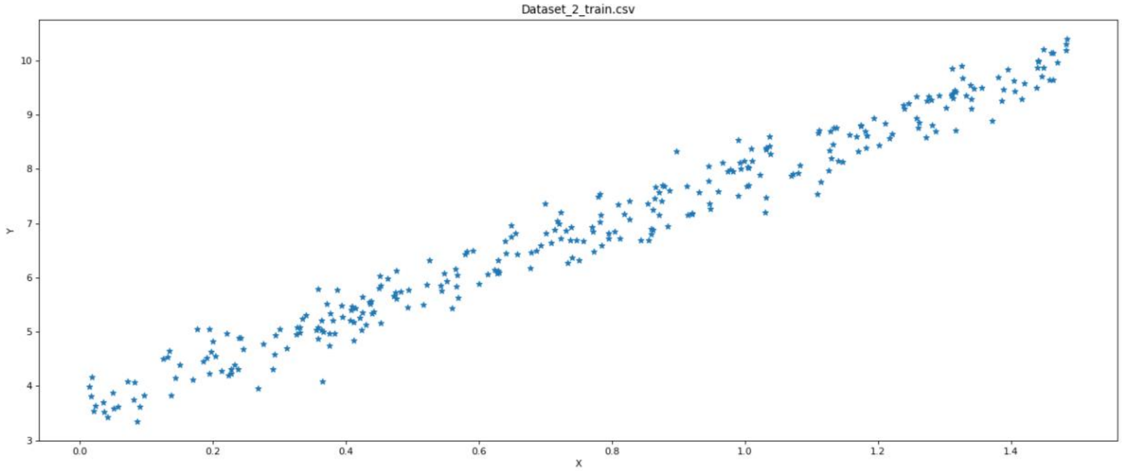
Shown below is an example of a 6th degree polynomial.



Question 2

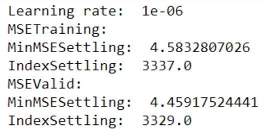
2.1

Visualizing the Data

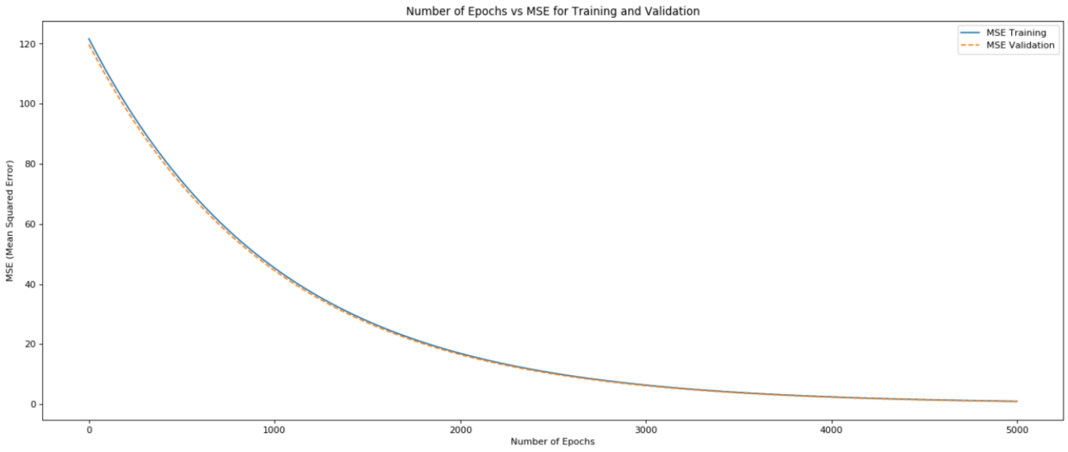


For this question, I was running 5k epochs. I declare that my MSE (therefore weights as well) have converged when the MSE has completed 5 half-lives. This corresponds to 97% of the final value.

Learning rate = 1e-06

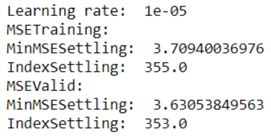


```
Learning rate:  1e-06
MSETraining:
MinMSESettling:  4.5832807026
IndexSettling:  3337.0
MSEValid:
MinMSESettling:  4.45917524441
IndexSettling:  3329.0
```

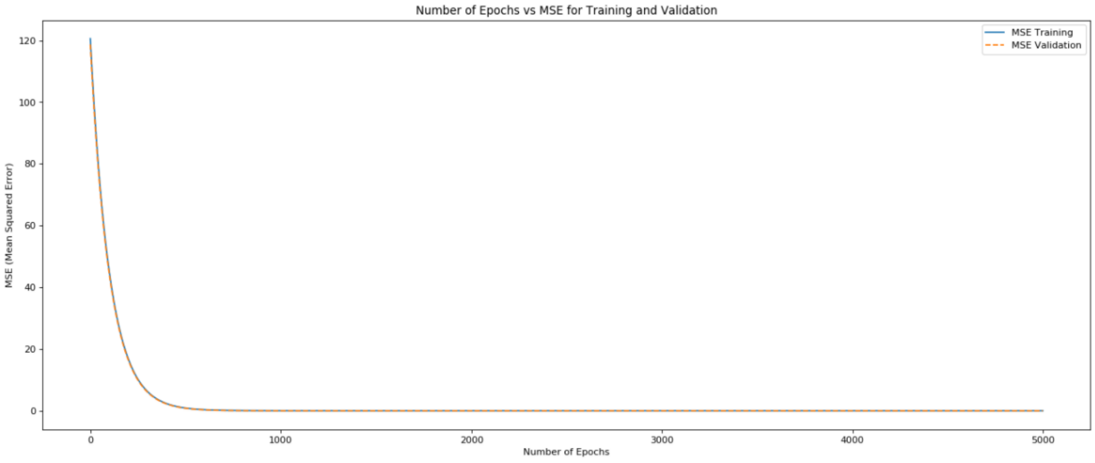


MSE converges at epoch number **3337**. The settling training MSE is **4.58**, and the settling validation MSE is **4.46**.

Learning rate = 1e-05

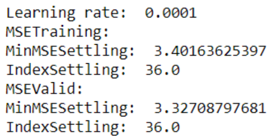


```
Learning rate:  1e-05
MSETraining:
MinMSESettling:  3.70940036976
IndexSettling:  355.0
MSEValid:
MinMSESettling:  3.63053849563
IndexSettling:  353.0
```

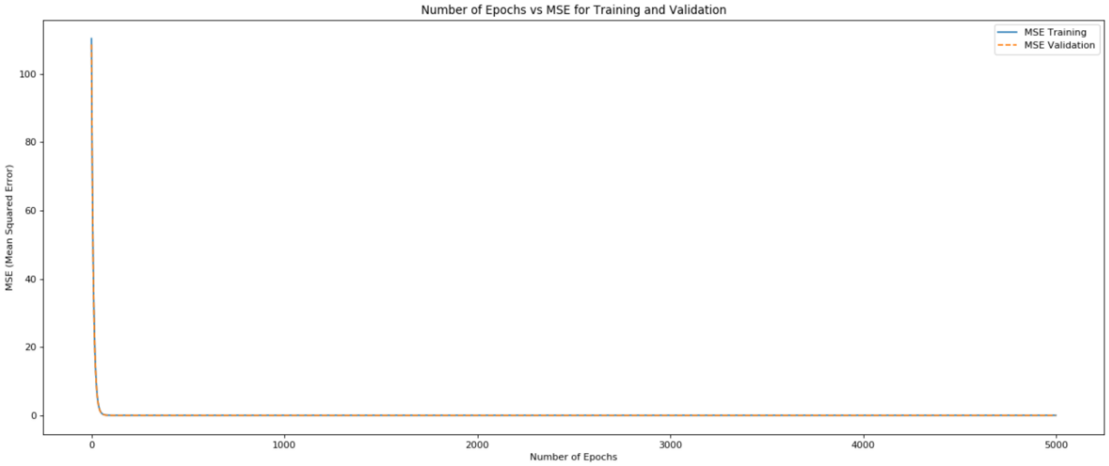


MSE converges at epoch number **335**. The settling training MSE is **3.71**, and the settling validation MSE is **3.63**.

Learning rate = 1e-04

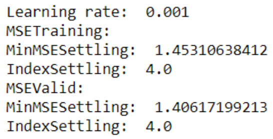


```
Learning rate:  0.0001
MSETraining:
MinMSESettling:  3.40163625397
IndexSettling:  36.0
MSEValid:
MinMSESettling:  3.32708797681
IndexSettling:  36.0
```

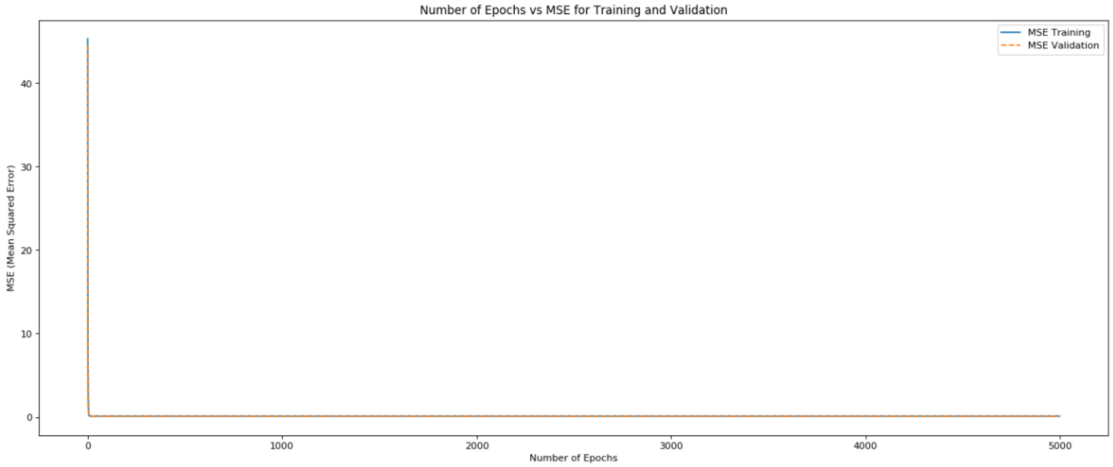


MSE converges at epoch number **36**. The settling training MSE is **3.4**, and the settling validation MSE is **3.33**.

Learning rate = 1e-03



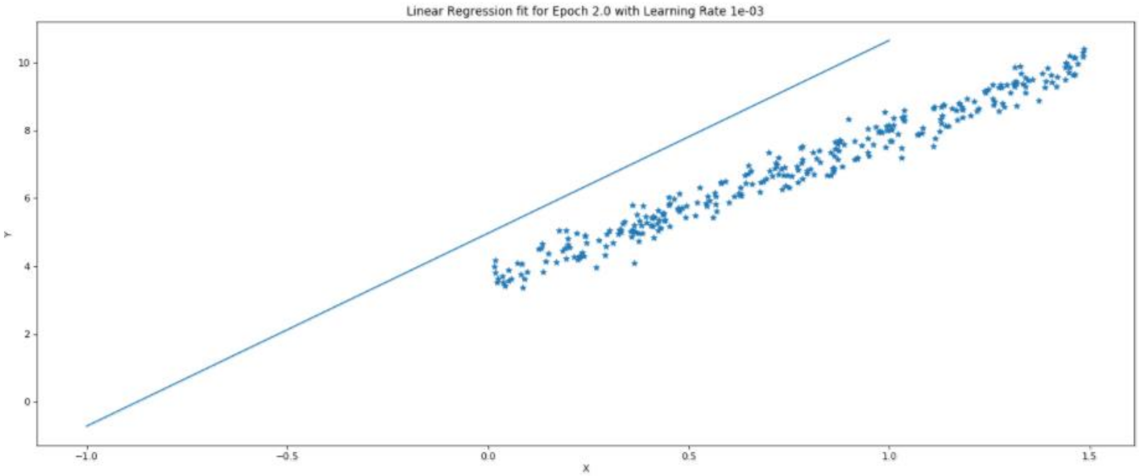
```
Learning rate:  0.001
MSETraining:
MinMSESettling:  1.45310638412
IndexSettling:  4.0
MSEValid:
MinMSESettling:  1.40617199213
IndexSettling:  4.0
```



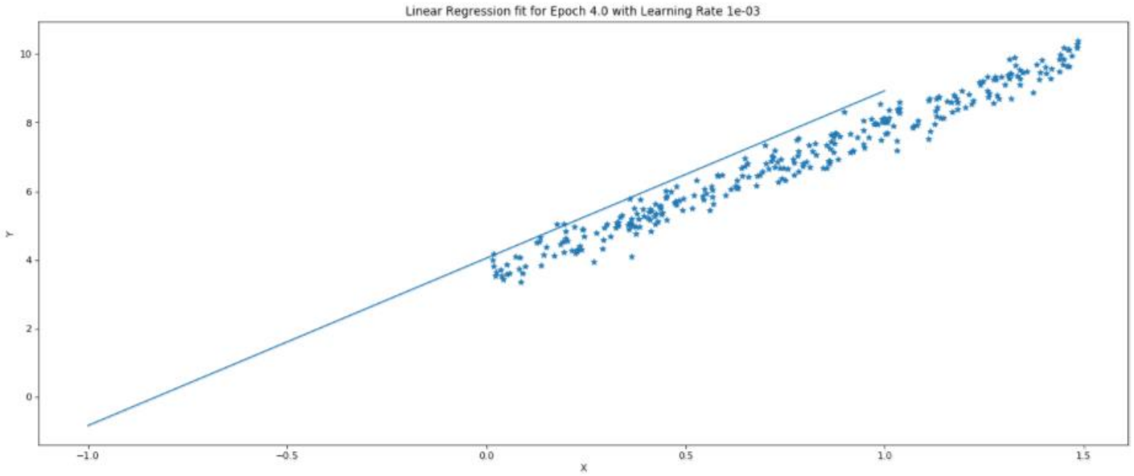
MSE converges at epoch number **4**. The settling training MSE is **1.45**, and the settling validation MSE is **1.406**.

It seems as if the best learning rate is 1e-03. Any higher than this and the function converges in less than 1 epoch, which is dangerous as it could diverge. Here are 5 epochs for how the fit evolves.

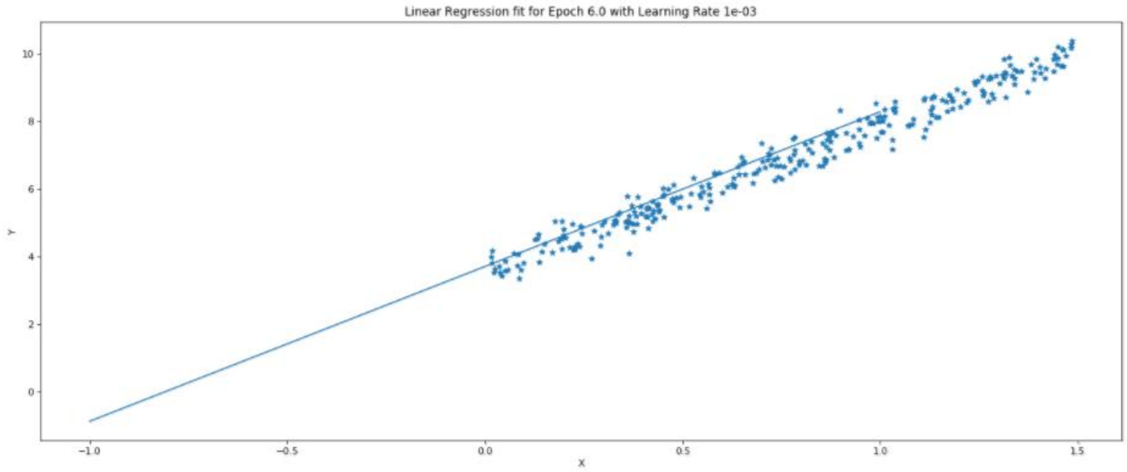
Epoch 2



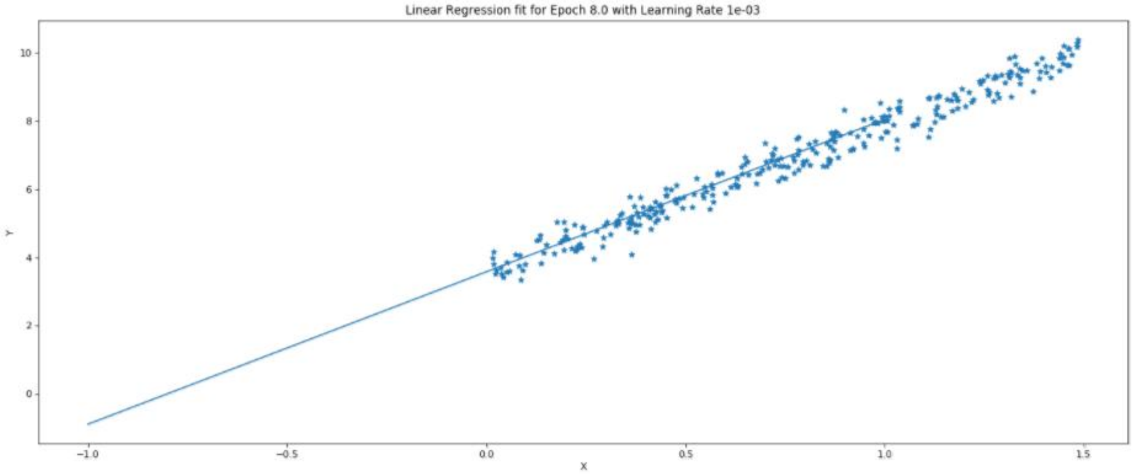
Epoch 4



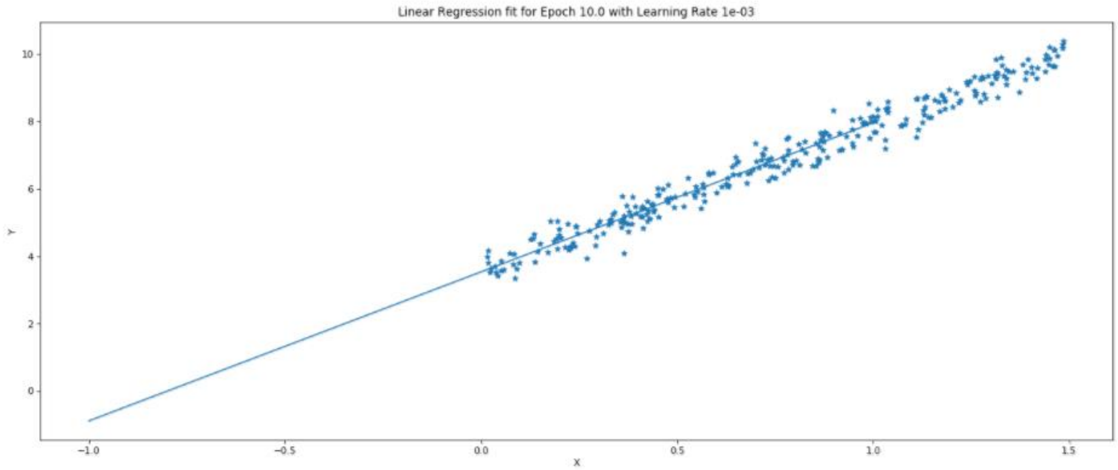
Epoch 6



Epoch 8



Epoch 10



As is clear from the visualizations above, the fit evolves quite rapidly for Learning rate = 1e-03. After 6 epochs we are very close to the final value, and after 8 we have practically found the best fit possible.

Question 3

3.1

Files are saved as follows upon being shuffled:

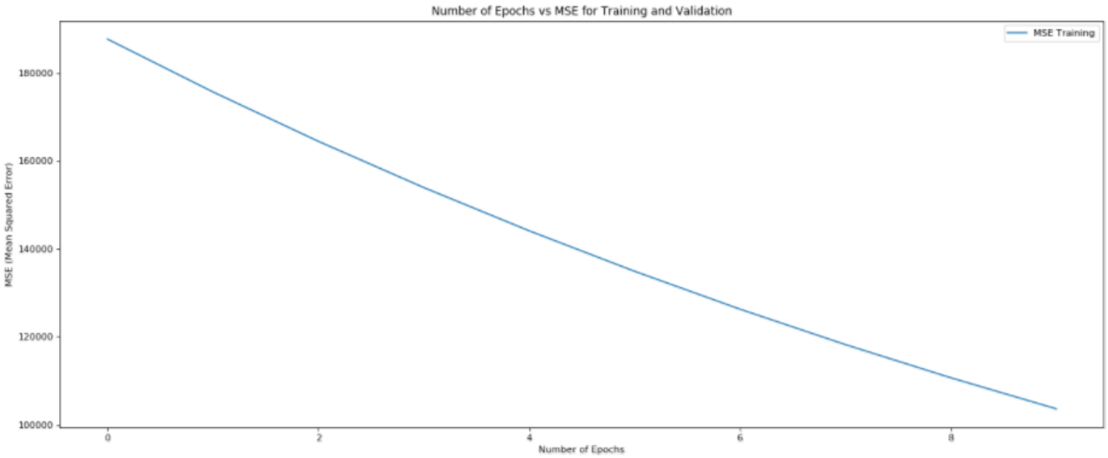
Dataset_CandC-train0.csv

Dataset_CandC-test0.csv

The number varies from 0 to 4 for a total of 5 datasets.

The NaN values are filled within a uniform distribution of 1 standard deviation away from the mean for each column. That way the new values do not bias the data.

3.2



This is the graph of MSE vs number of epochs for training set and test sets “Dataset_CandC-train0.csv” and “Dataset_CandC-test0.csv”. Even at 10 epochs, the program took 20 minutes to run. Hence, it would be impossible to run 5k epochs without a much powerful computer. The MSE is shown to be decreasing, and if given enough time to run, should decrease to at least 3 digit numbers.

The current parameters learnt are shown in a 123 x 1 vector:

[-2.2110612359479471e+58, 5.6131592685503863e+56, -3.4520358447900248e+57, -1.1995627289040644e+58, 7.8404641934841262e+57, -3.1862670015588582e+57, 5.3046831932668113e+56, -1.7560878298645836e+57, 4.2455869735876663e+58, 1.0181548236524958e+58, 1.5085534267857655e+58, 9.4866670603062665e+56, 4.1988914651045971e+58, -1.0857582171932625e+58, -3.9918481033716562e+56, -7.3284934721806389e+57, -4.576934162734759e+58, -1.4989103870662064e+58, 8.1158466507651524e+57, 9.1920774984549106e+57, 1.0032800479405474e+58, 1.0679189390351476e+58, 1.035184468221502e+58, -1.2765181499109856e+58, 1.8714601624276659e+57, -2.6398016341672946e+57, 2.713087838677778e+57, 1.0559535594926575e+57, -8.62709558339133e+56, -4.6127163818292844e+57, -6.5188491607692707e+57, -1.6146126588174351e+58, 3.8441179308404464e+57, 3.0452635581857659e+57, 9.722977226151969e+57, 6.4019019043490177e+57, 3.9084673404870646e+57, 8.2165549687273536e+57, -4.3955727520185461e+57, -1.0164419718080775e+58, 2.550992102530322e+58, -2.1312440830845314e+58, -8.3655775996817217e+57, 2.055107167846005e+58, -1.6198254636947755e+57, -2.1376056429454059e+58, -1.2863569333446943e+58, -2.6584092752103644e+58, -1.8526603735979748e+58, 2.7048068418073749e+58, -8.6423329185622029e+56, -1.8504807885151898e+57, 3.6371107769090111e+56, -1.7254165829122914e+56, -1.9704834171068009e+58, 1.6788797475116458e+58, -1.1689363370116805e+58, 7.5687839287945561e+56, 1.2105704511567027e+57, 5.2553905699674465e+57, -5.3349363624178694e+57, -1.2854254335171776e+58, 3.6535949903341504e+57, 1.0067327517541021e+58, -1.0151537205886126e+58, 1.5692443080440949e+58, -1.0889284485046146e+58, 1.1581806796410007e+58, -7.7853185983945528e+56, -1.1375266643505856e+58, 9.4550269487405841e+57, -5.2238646706500588e+57, -1.3457610001748452e+57, 1.3230466847153691e+58, 1.5247559941837622e+58, 4.574399685462755e+57, -1.3892474104314147e+58, -1.0537670120744126e+58, -1.1892193475346963e+58, -1.4149899516881874e+57, -3.9392695655521756e+57, -2.1672657891451978e+57, -2.4059764412002491e+57, 2.3486153687092678e+58, -5.8357843421788917e+57, 1.3543712969063585e+58, -7.5595061918852325e+56, -4.761936352792218e+58, -3.7178184801292963e+57, -1.8528471494273816e+58, 7.3306189391616139e+55, -6.1999071790232347e+55, 1.3438431638448256e+57, -4.0162274613828509e+56, -1.3480838896962651e+58, 2.9698039461088188e+58, 2.0258888264312145e+58, 1.7742565987191e+57, 6.6801144193462943e+55, 1.6817832655404575e+58, -9.5516165719123502e+57, -4.0589512409756921e+57, 2.1676524597001373e+57, -5.4298446912148068e+57, 8.1222131074559226e+56, 1.7315938014179858e+58, -2.5902864915995224e+58, -1.9577058472748907e+58, -1.595645295392898e+56, -8.9148194029302596e+56, 1.5422953766826657e+58, -8.1289493576770244e+55, 5.4758748648029424e+57, 5.5239575671703243e+56, 2.8404169991175577e+56, 1.6184513971993542e+57, -3.2335509230867466e+57, 4.982338123282793e+57, -2.8769202496394164e+57, 5.421826121982496e+57, -1.3911624873886753e+57, -2.4397088182992268e+56, 3.0705750365132001e+57, 10]

Note: Above can be seen as the values for [w0, w1, w2, …, w123]

Each weight is separated by a comma. The original weights array was initialized with every weight being equal to 10.