Introduction to Network Concepts

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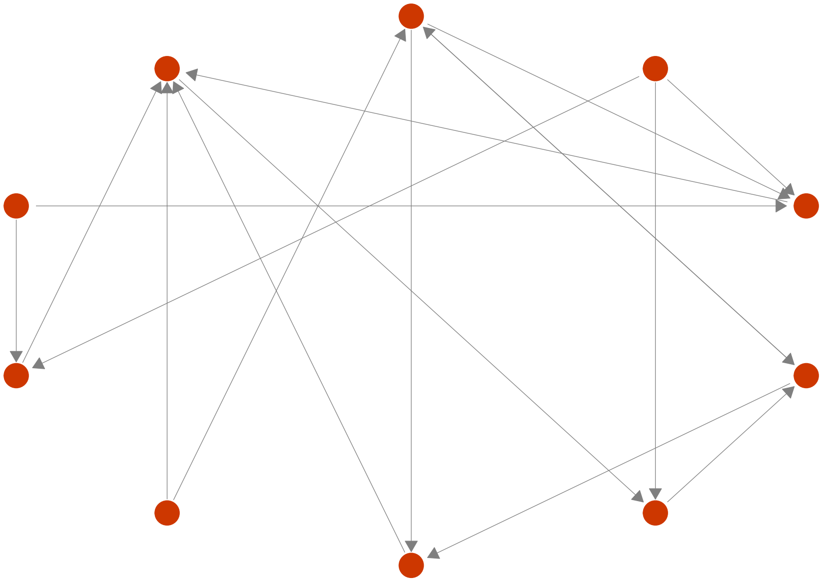
Today

- Introduction network terms
- The role of graph theory
- How matrices (ew math) help us

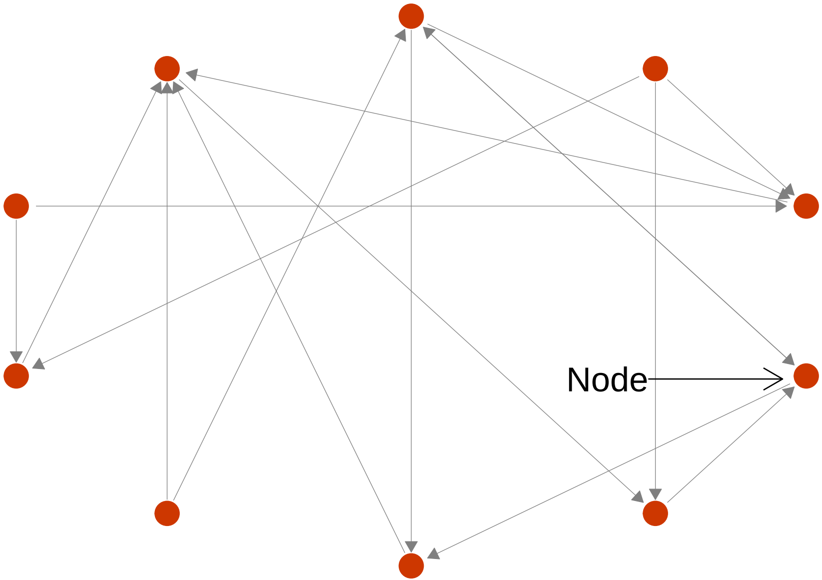
Nodes and Edges

Networks are made up of nodes that are connected with edges.

Network



Network



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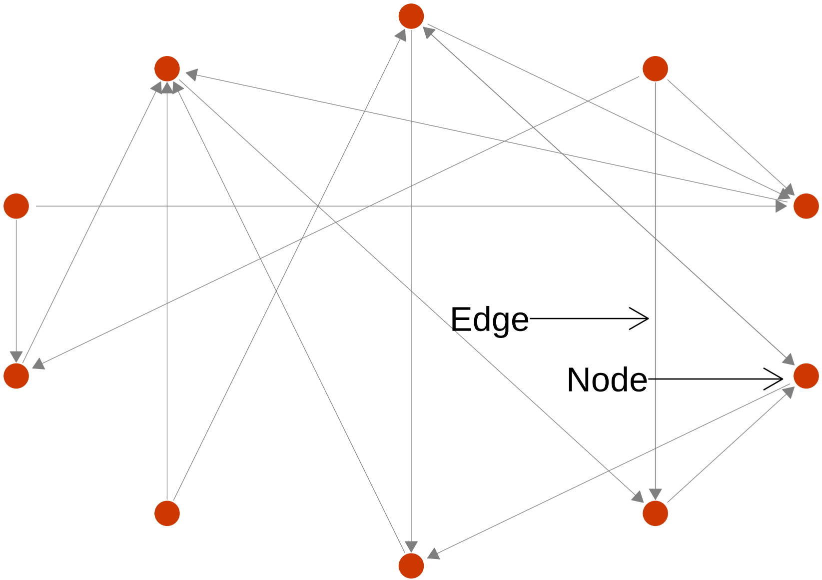

Graph Theory

This makes sense visually, but in order to analyze it we need to use math

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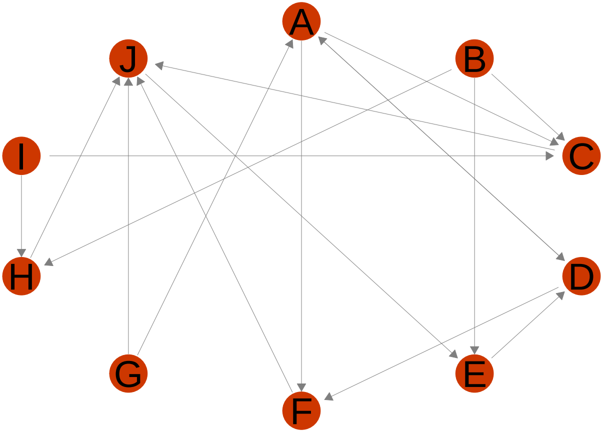
- E is the set of such connections: E={(A,C), (A,D), (B,C)}
    - Note: edges can have a direction: (A, C), is different from (C,A)

Network as a Graph

E={(A,C), (A,D), (A,F), (B,C), (B,E), (B,H), (C,J), (D,A), (D,F), (E,D), (F,J), (G,A), (G,J), (H,J), (I,C), (I,H), (J,E)}

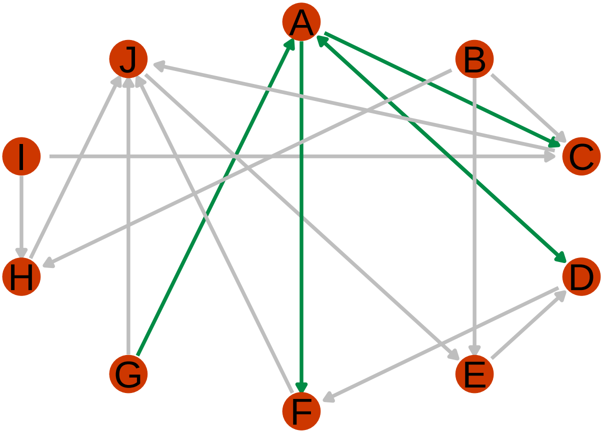


Adjacent, Incident, and Degree

- Two vertices that are connected by an edge are **adjacent**
- An edge that is connected to a vertice is said to be **incident** to it.
- The number of edges **incident** to a vertex is its **degree**

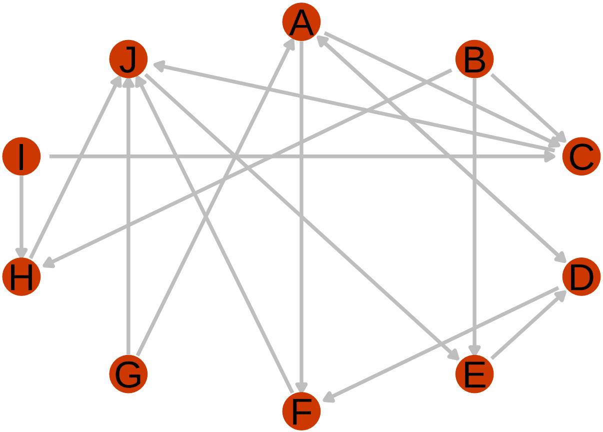
Degree

A has 4 edges incident to it, so it has degree of 4

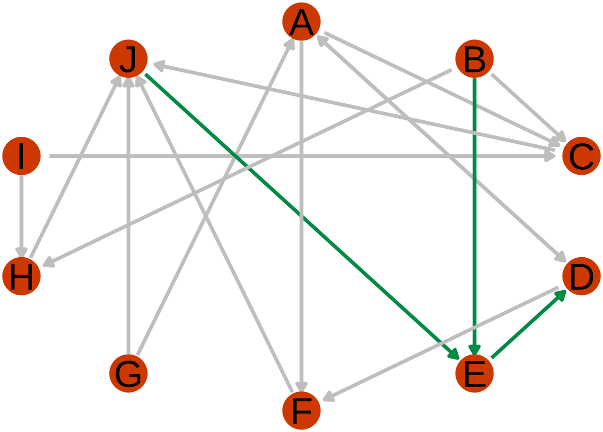


Degree for E

What is E’s degree?



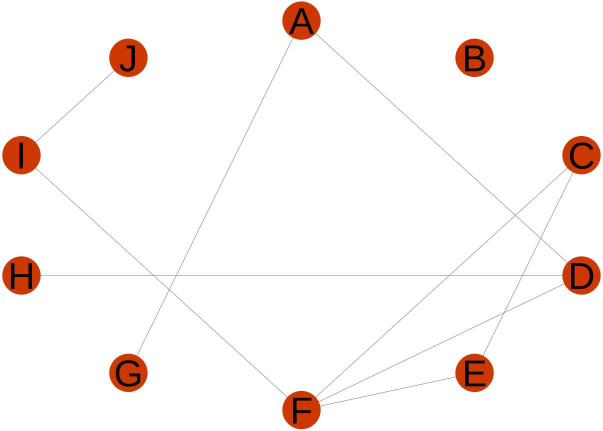
Degree for E



Undirected networks

It is also possible to have networks where edges do not have a direction:

E={(A,D), (A,G), (C,E), (C,F), (D,F), (D,H), (E,F), (F,I), (I,J)}



Directed vs Undirected Network

Whether a network is directed or undirected is dependent on the types of relationships:

Directed:

- Parent-Child
- Donor-Donee

Undirected

- Friend (hopefully)
- Classmate
- Ally

Characteristics for nodes

What are some things that might matter if you are at a node?

- Where can you move to?
- How fast can you get there?
- Who can you reach?

Moving along a network

We can move from one vertex (node) to another through edges.

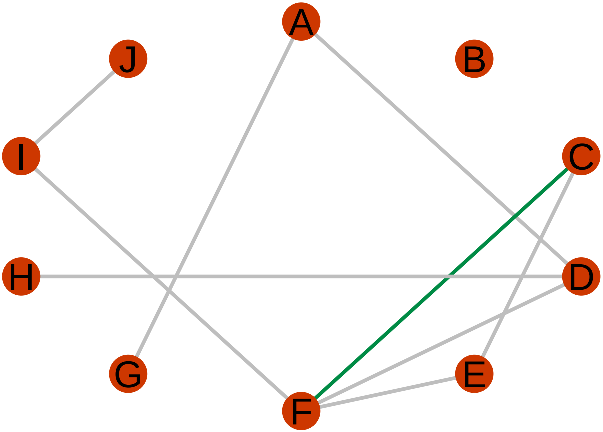
- A walk is a sequence of **adjacent** vertices:
  - G-A-D-H
  - F-C-E-F-I
  - H-D-A
- A path is a sequunce of **adjacent** vertices where none are repeated:
  - G-A-D-H
  - H-D-A
- All paths are walks, but not all walks are paths.

The length of paths and walks

- The **length** of a walk or paths is how many edges are in them.
- The shortest path is called a **geodesic**.
- The **(geodesic) distance** between two vertices is the length their **geodesic**

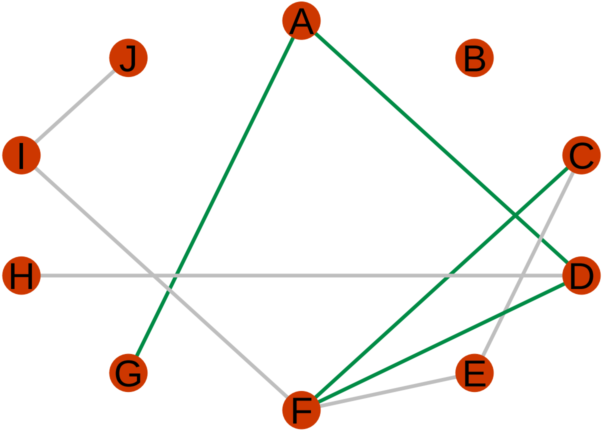
Geodesic Distance

The distance between F and C is 1



Geodesic Distance

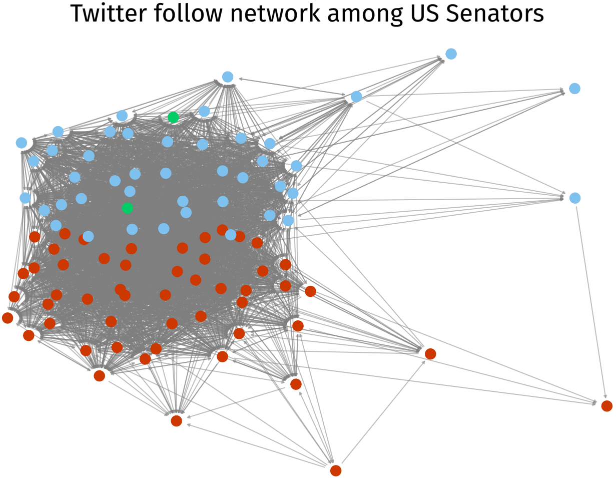
The distance between G and C is 4



Why care about distance?

- Distance is viewed as how ‘close’ two vertices are.
- Information spreads faster between two vertices that are closer

Looking at some real data: Twitter Senate Data



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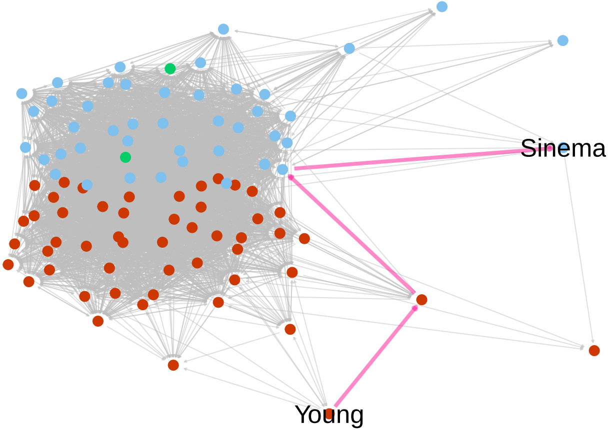

Young follows Portman, who follows Booker, who follows Sinema

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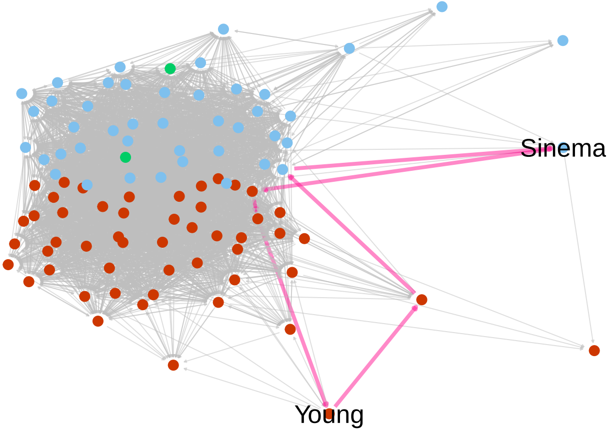


Sinema follows Scott, who follows Cornyn, who follows Young

What can we learn?

- The mean distance between any two senators is 1.45
- The median distance between any two senators is 1
- The median degree is 110.5

Degree for Directed Networks

Directed networks have both **outdegree** (how many edges start at a vertex) and **indegree** (how many edges point at a vertex):

- Median outdegree (how many each Senator follows): 51
- Median indegree (how many followers each Senator has): 57

Who is the most popular?

	Following	Followers
Daniel Sullivan	59	55
Lisa Murkowski	77	74
Gordon Jones	52	42
Richard Shelby	58	65
John Boozman	36	63

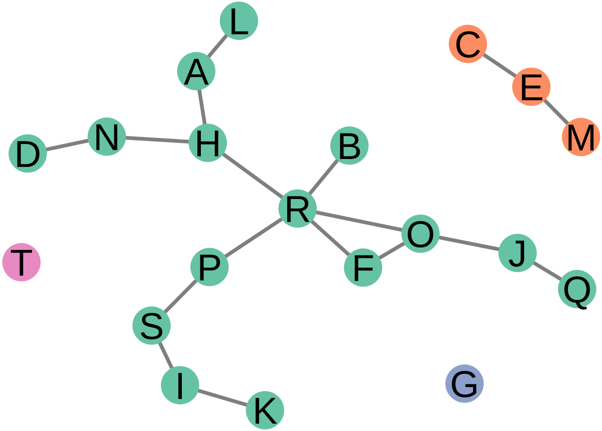
- Mark Warner follows the most (follows 87)
- Susan Collins is followed by the most (follows 78)

Who can reach who?

- A vertex with no incident edges is known as an **isolate**
- We can also group vertices by who they can reach.

A **component** is the maximal (largest) group of vertices where every vertex within it can reach every other vertex.

Some Components



This network has 4 components, the largest has 15 vertices in it.

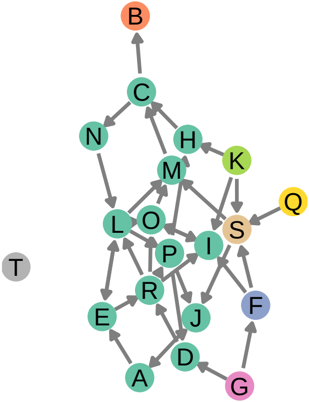
Components with Directed Graphs

When a network is directed, then we need to think about direction.

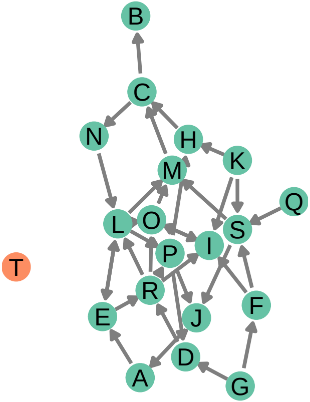
- **Weak Component:** Is a component if we disregard direction of edges.
- **Strong Component:** Is a component if we follow direction of edges

Strong/Weak Components:

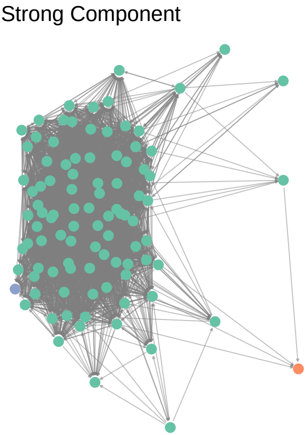
Strong Component



Weak Component



Real Life Data



Bring the math back in

How do we organize this data? An **adjacency matrix**

Matrix Referesher

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

- A matrix is a rectangular collection of numbers ($n \times m$ in size).
- Each number (element) is given a position by what row and column it is.
- The 1 above is in row 2, column 3.

Adjacency matrix

- For now our matrices will be square: $n \times n$ where n is the number of vertices.
- A 1 means an edge exists from that row to that column, a 0 means no edge exists.

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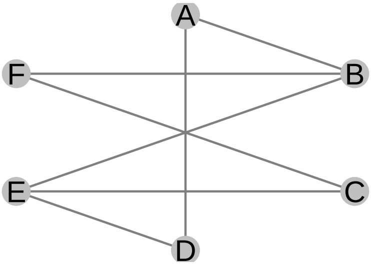
- In $\mathbf{A}$ there is an edge from node 3 to node 2, and from node 1 to node 3 (directed).

Adjacency Matrix vs Network Graph



	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	0	0	1	1
C	0	0	0	0	1	1
D	1	0	0	0	1	0
E	0	1	1	1	0	0
F	0	1	1	0	0	0

Undirected Adjacency Matrix

- For undirected graphs the adjacency matrix is mirrored across the diagonal
- Meaning that the entry for $a_{i,j}$ is the same as for $a_{j,i}$

Senator Adjacency Matrix

	Daniel Sullivan	Lisa Murkowski	Doug Jones
Daniel Sullivan	0	1	0
Lisa Murkowski	1	0	0
Doug Jones	0	1	0
Richard Shelby	0	1	1
John Boozman	1	1	0