CSCI 3155: Lab 2 (due February 9)

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1. **Grammars: Synthetic Examples**

   (a) Describe the language defined by the following grammar:

   $$\begin{aligned} S &::= A\ B\ A \\ A &::= \texttt{a} \mid \texttt{a}\ A \\ B &::= \epsilon \mid \texttt{b}\ B\ \texttt{c} \mid B\ B \end{aligned}$$

   The sentences in this language begin with one or more a's, followed by 0 or more b's and c's, where the number of b's equals the number of c's and, further, this substring of b's and c's acts like a string of balanced parentheses with each "opening" b having a paired "closing" c to somewhere to the right of it in the string.

   (b) Consider the following grammar:

   $$\begin{aligned} S &::= A\ \texttt{a}\ B\ \texttt{b} \\ A &::= A\ \texttt{b} \mid \texttt{b} \\ B &::= \texttt{a}\ B \mid \texttt{a} \end{aligned}$$

   Which of the following sentences are in the language generated by this grammar? For the sentences that are described by this grammar, demonstrate that they are by giving derivations.

   i. `baab` - is in this language

   $$\begin{aligned} S &\Rightarrow A\ \texttt{a}\ B\ \texttt{b} \\ &\Rightarrow \texttt{b}\ \texttt{a}\ B\ \texttt{b} \\ &\Rightarrow \texttt{b}\ \texttt{a}\ \texttt{a}\ \texttt{b} \end{aligned}$$

   ii. `bbbab` - **not** in this language because B will produce at least one `a` and as there is already a terminal `a` in first step derivation of start symbol S, there must be at least 2 `a`'s in the sentences described by this grammar.

   iii. `bbaaaaa` - **not** in this language because all sentences in this language must end in a `b` since there is a terminal `b` in any derivation from start symbol S.

iv. `bbaab` - is in this language

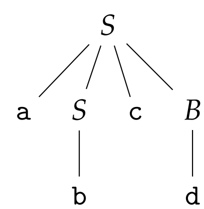
$$\begin{aligned} S &\Rightarrow A\,\mathtt{a}\,B\,\mathtt{b} \\ &\Rightarrow A\,\mathtt{b}\,\mathtt{a}\,B\,\mathtt{b} \\ &\Rightarrow \mathtt{b}\,\mathtt{b}\,\mathtt{a}\,B\,\mathtt{b} \\ &\Rightarrow \mathtt{b}\,\mathtt{b}\,\mathtt{a}\,\mathtt{a}\,\mathtt{b} \end{aligned}$$

(c) Consider the following grammar:

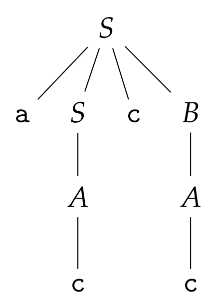
$$\begin{aligned} S &::= A\,S\,\mathtt{c}\,B \mid A \mid \mathtt{b} \\ A &::= \mathtt{c}\,A \mid \mathtt{c} \\ B &::= \mathtt{d} \mid A \end{aligned}$$

Which of the following sentences are in the language generated by this grammar? For the sentences that are described by this grammar, demonstrate that they are by giving parse trees.

1. `abcd` - is in the language



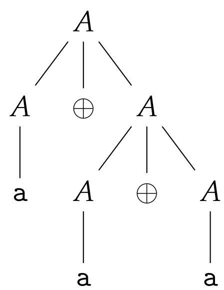
2. `acccbd` - **not** in the language because S may produce AS`c`B, A or `b`. A can only produce one or more `c`'s. `b` is terminal and the B derived from S can only generate a substring of one or more `c`'s or a single `d` neither of which will give an end of `bd`.
3. `acccbd` - **not** in the language because there is no way to produce an internal `b` without producing another `a` and there is only one `a` in `acccbd`.
4. `acd` - **not** in the language because there is no production in which S can go to an empty string, which would be necesarry to produce just the 3 terminal symbols.
5. `accc` - is in the language



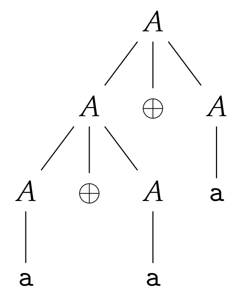
(d) Consider the following grammar:

$$A ::= \texttt{a} \mid \texttt{b} \mid A \oplus A$$

Show that this grammar is ambiguous.
I show that this grammar is ambiguous by giving two distinct parse trees for the sentence, $\texttt{a}\oplus\texttt{a}\oplus\texttt{a}$:



distinct parse trees, left and right



(e) Let us write

$$A \Downarrow n$$

for the judgment form that should mean A has a total n $\texttt{a}$ symbols where n is the meta-variable for natural numbers. Define this judgment form via a set of inference rules. You may rely upon arithmetic operators over natural numbers. Hint: There should be one inference rule for each production of the non-terminal A (called a syntax-directed judgment form).
We begin with two base case judgments that follow directly from our definition, these are axioms:

$$\frac{}{\texttt{a} \Downarrow 1}\ (1) \quad \text{and} \quad \frac{}{\texttt{b} \Downarrow 0}\ (2)$$

From these two rules, we can construct our third inference rule, for the third production of the non-terminal A.
Given a sentence of the form $x \oplus y$, where x and y are productions of A, and each x and y have n and m $\texttt{a}$ symbols, respectively, we can write our last rule:

$$\frac{x \Downarrow n \quad y \Downarrow m}{x \oplus y \Downarrow n + m}\ (3)$$

2. **Grammars: Understanding a Language**