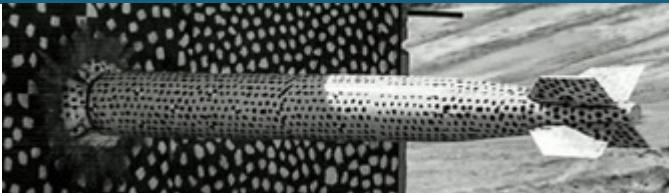
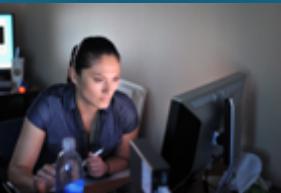




Application of Multifidelity Uncertainty Quantification Methods to a Subsurface Transport Model



SIAM CSE21

T. Portone, M.S. Eldred, G. Geraci, and L.P. Swiler

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2 UQ for geologic disposal safety assessment (GDSA)

Performance assessment for nuclear waste repository site.

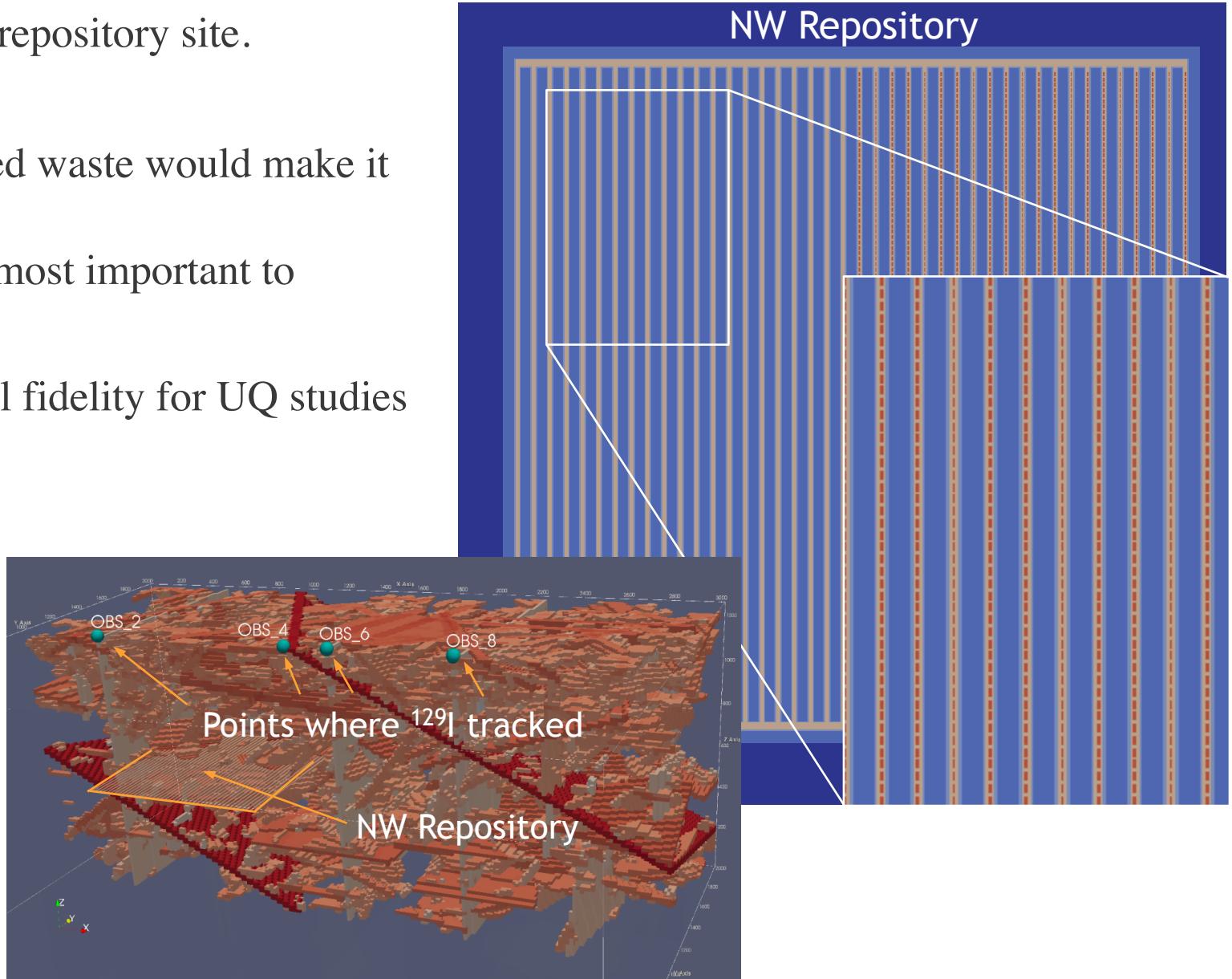
Want to understand things like:

- 1) probability dangerous levels of leaked waste would make it to a water supply;
- 2) subsurface/waste storage properties most important to repository performance.

Deploying an unprecedented level of model fidelity for UQ studies in this application area.

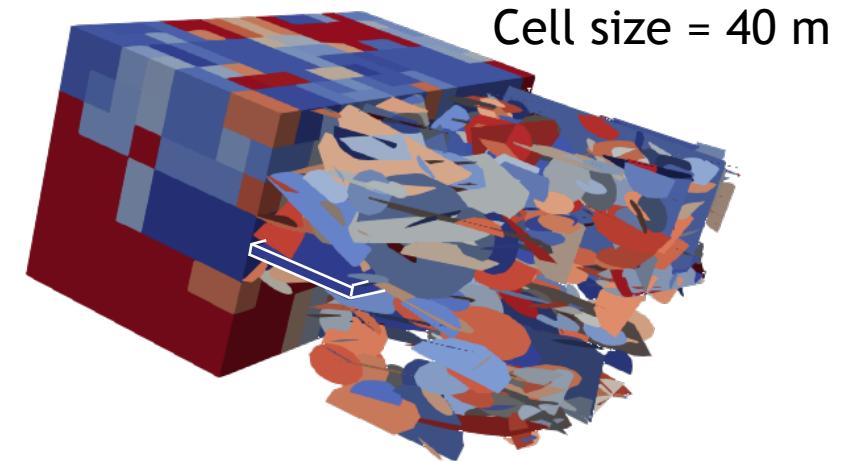
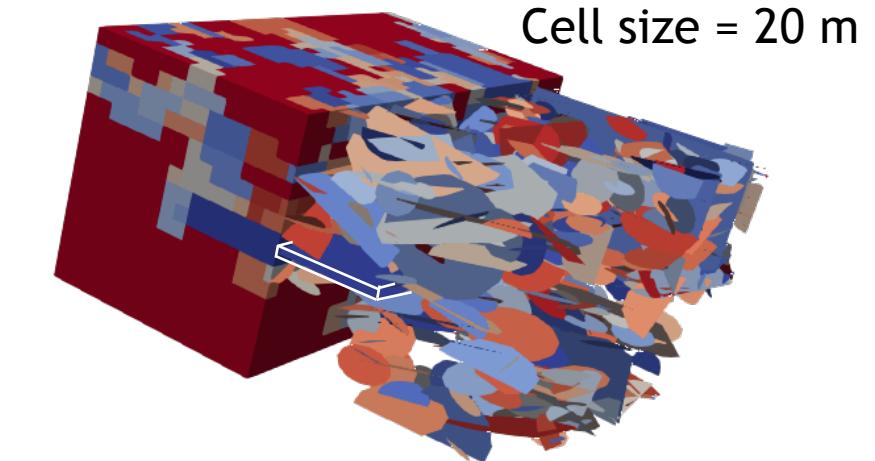
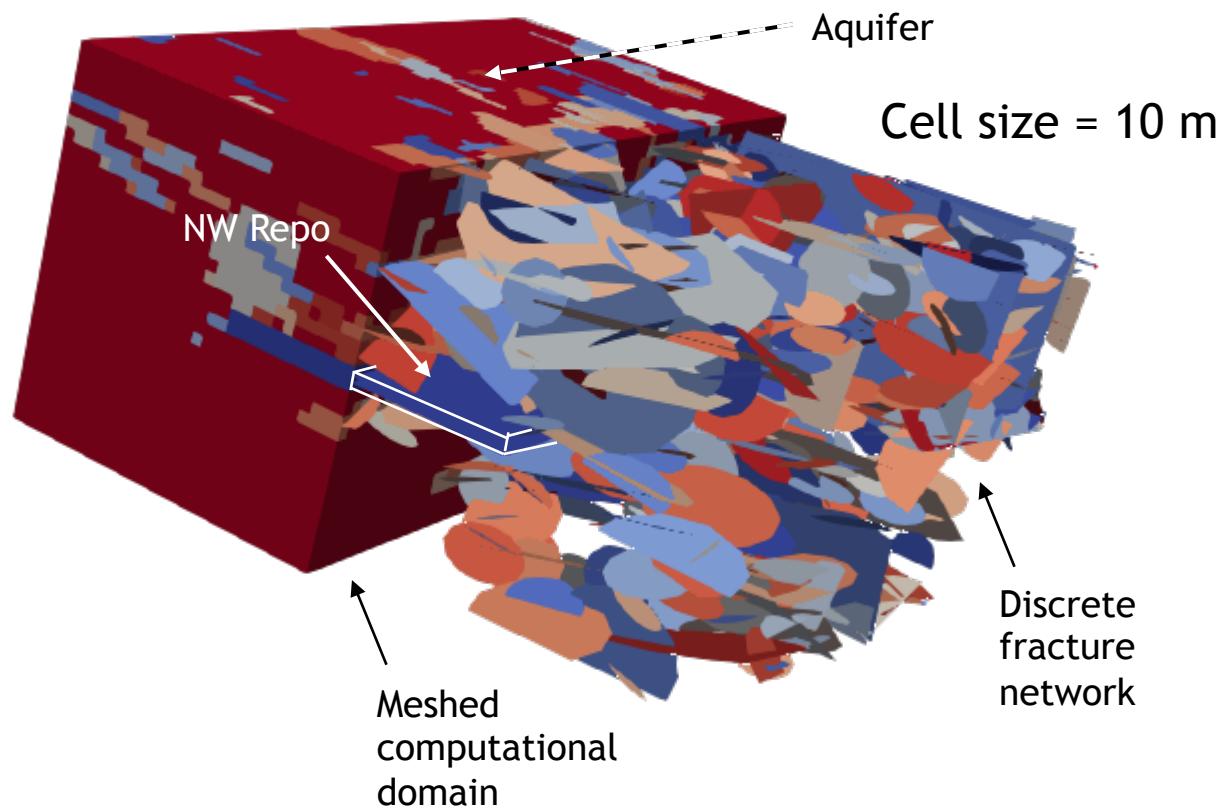
Simulations very costly (~ 1.5 hours on 512 cores per simulation): $\mathcal{O}(1000)$ model evaluations for production UQ studies.

Goal: explore feasibility of using multifidelity polynomial chaos expansion (PCEs) for global sensitivity analysis (GSA) in this problem.



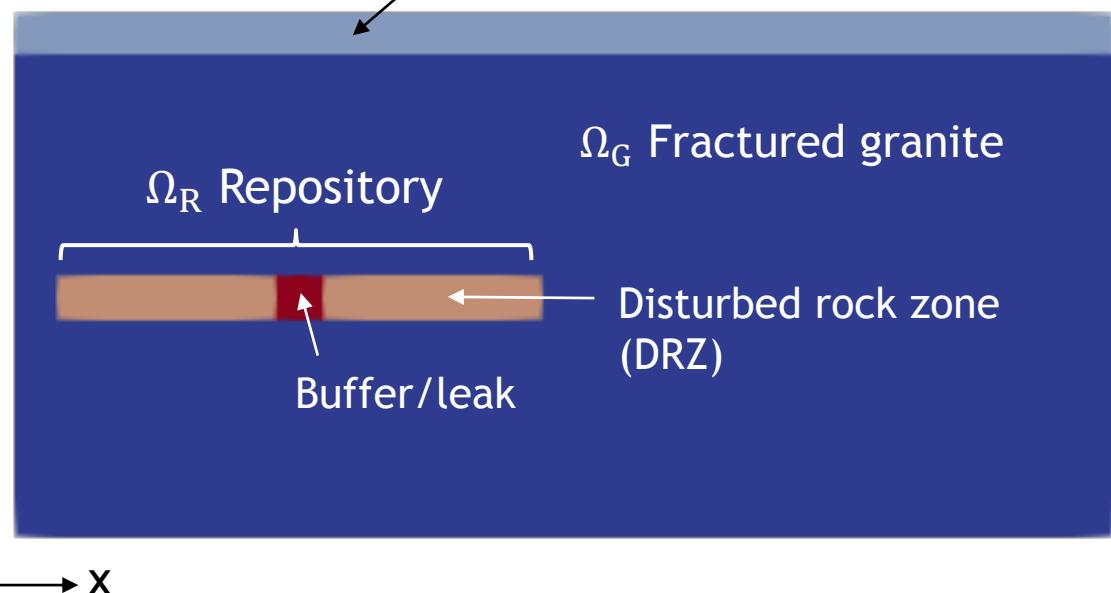
3 | Fractured subsurface uncertainty treatment and model hierarchy

- Can't observe all fractures in subsurface; represent uncertainty using random instantiations of discrete fracture networks (DFNs).
- DFNs mapped to equivalent continuous porous media (ECPMs).
 - Define grid, sweep through cells and map fracture permeability and porosity to equivalent continuum values.

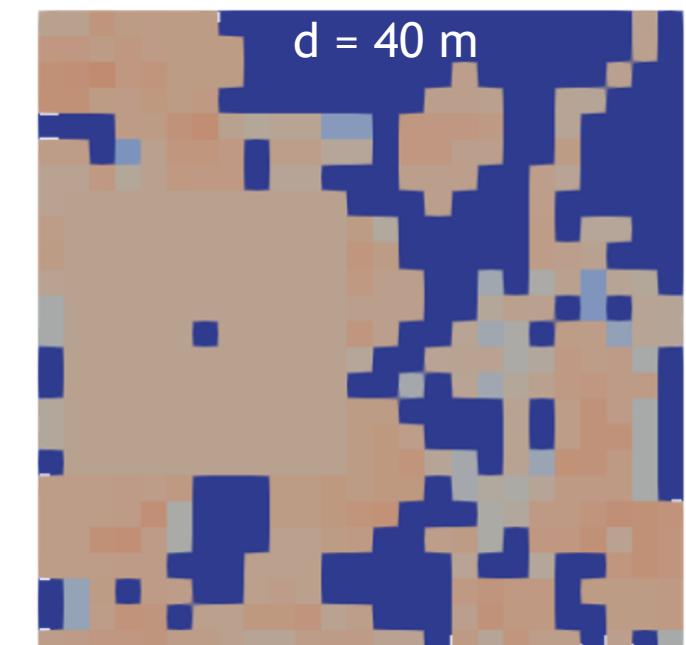
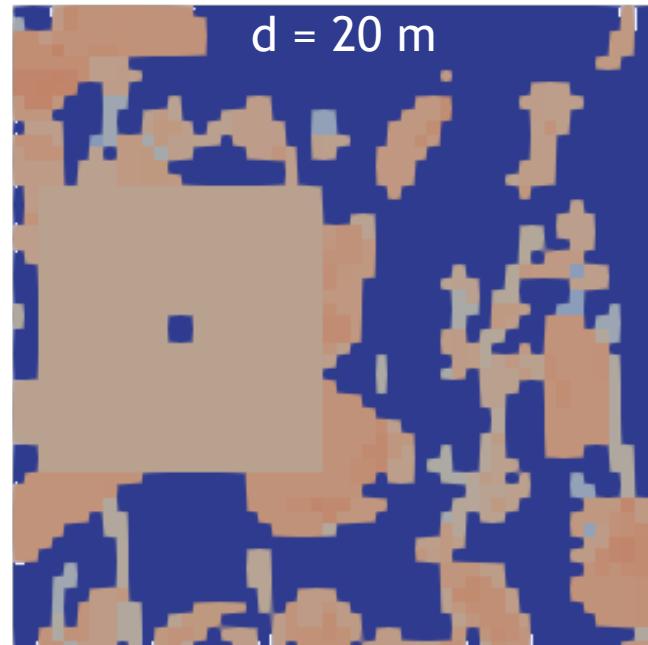
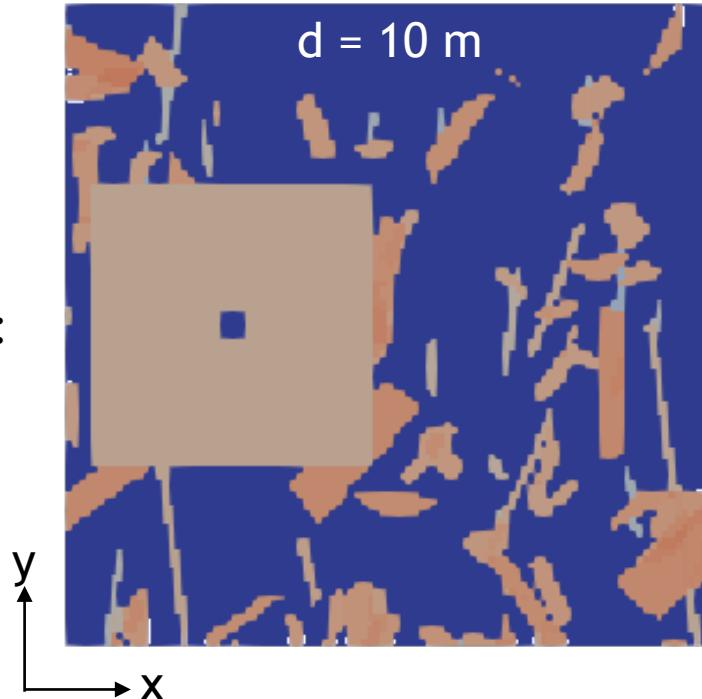


Simplified crystalline problem

- 1000 x 1000 x 500 m
- 1 waste package in buffer region
- Same underlying DFN statistics as production problem.
- Same QoIs as production problem.



K_{xx} view
from above:

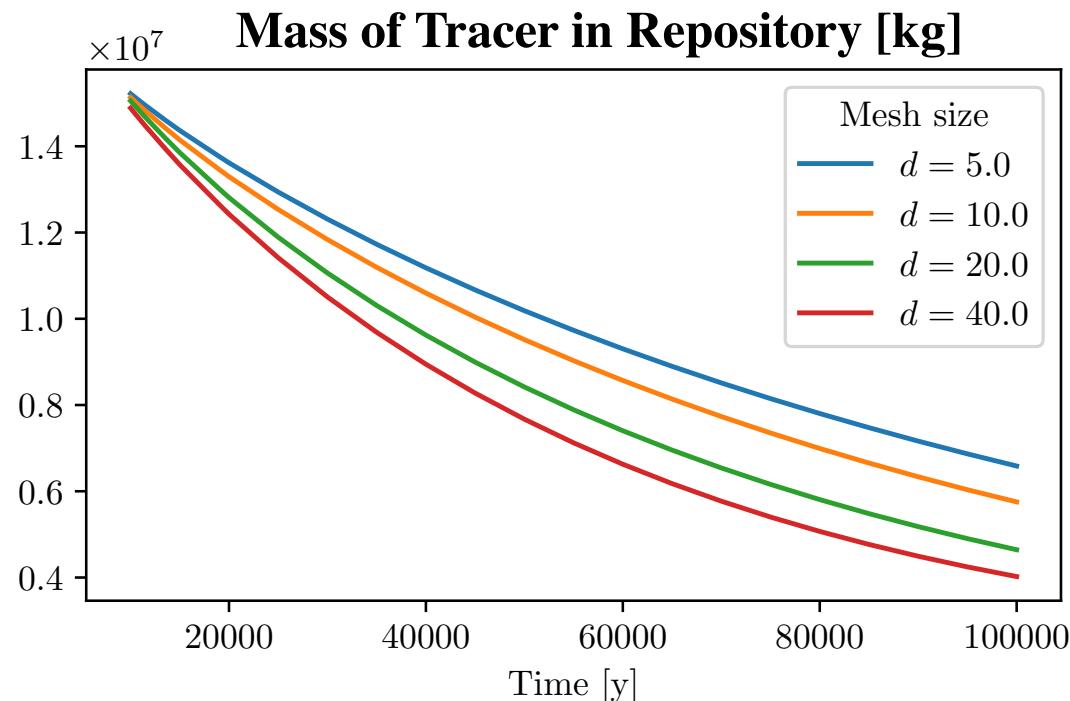
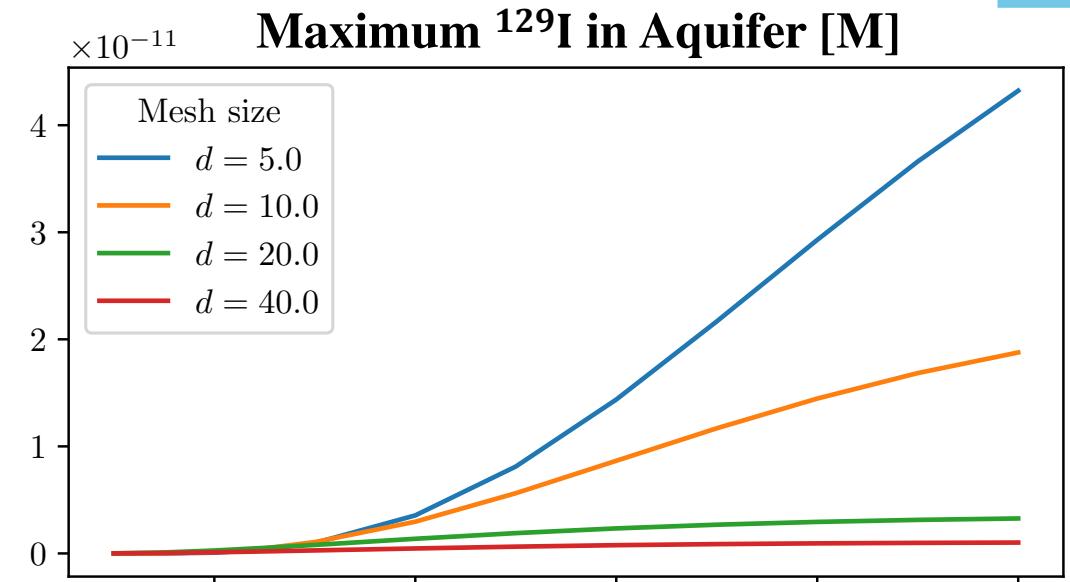


Challenge: mesh dependence of quantities of interest (QoIs)



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- Behavior of several QoIs converges as mesh is refined, but some do not.
- Max ^{129}I concentration in aquifer affected significantly by numerical diffusion/dispersion.
 - No correlation in location of max across meshes.
 - Some derived quantities depend on location of max.
- Mass in repository affected by increased flow from false connections in coarse mesh.
- Using the finest mesh we can afford.



6 Multifidelity Polynomial Chaos Expansion (MF PCE) for GSA



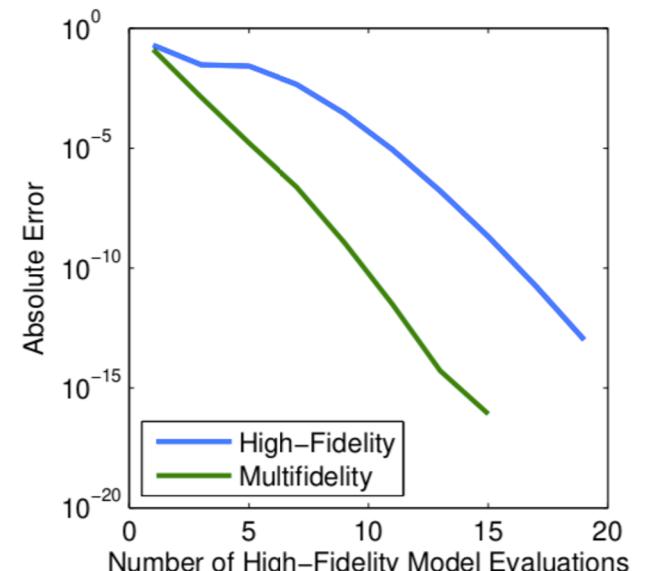
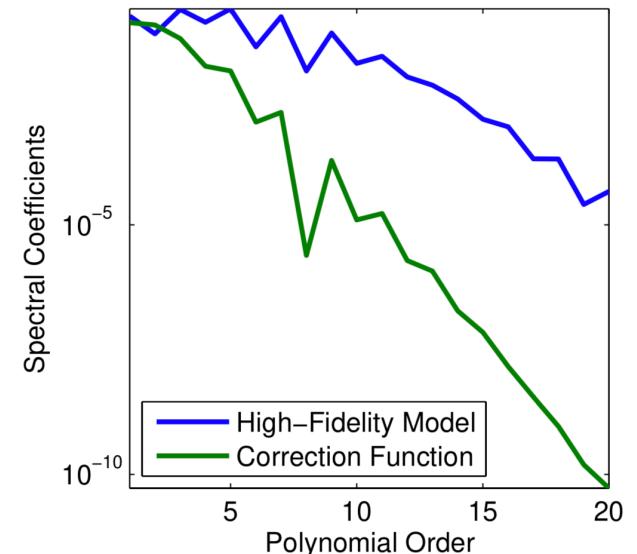
- Discrepancy-based, hierarchical multifidelity PCE (Ng, Eldred 2012).
- MF PCE coefficients can be recombined and used to compute Sobol indices in postprocessing (Sudret 2008).
- (R – model response; P – PCE expansion):

$$\begin{aligned} R_{10} &= R_{10} + (R_{20} - R_{20}) + (R_{40} - R_{40}) \\ &= R_{40} + (R_{20} - R_{40}) + (R_{10} - R_{20}) \\ &\approx P_{40} + (P_{20-40} + P_{10-20}) \end{aligned}$$

- If spectral content of $P_{10-20} \ll P_{10}$, need fewer 10 m evaluations to derive accurate expansion of R_{10} .
- In this case using a recursive approach to construct discrepancy PCE:

$$P_{20-40} \approx R_{20} - \textcolor{red}{P}_{40} \text{ rather than } P_{20-40} \approx R_{20} - \textcolor{red}{R}_{40}.$$

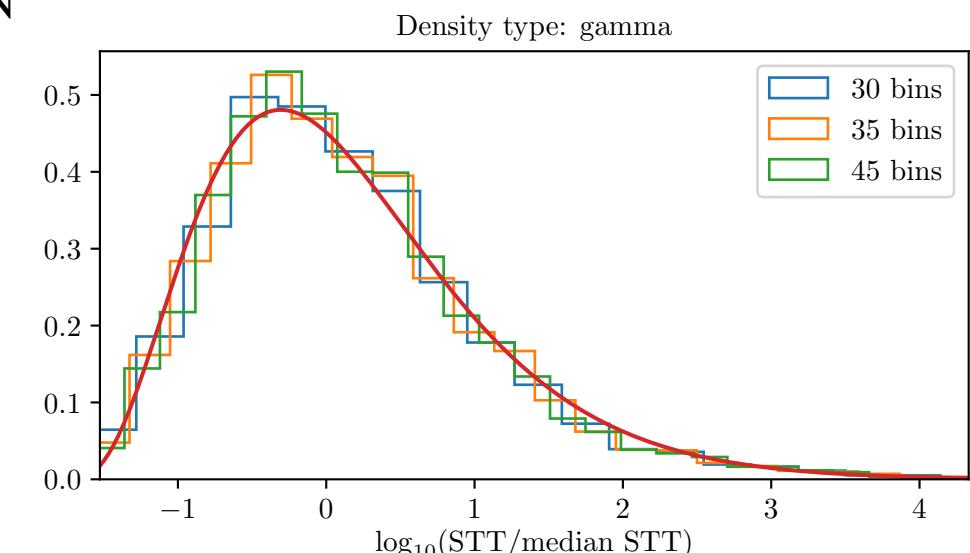
- Random (MC) sampling + coefficients estimated with regression + cross validation for PCE order.
- Sampling and PCE construction performed with Dakota.





Characterizing subsurface uncertainty

- How do subsurface properties affect quantities of interest through GSA?
 - Can't impose subsurface properties directly—have to compute them after generating a DFN.
- Compute proxy variables based on graph/network representation of each DFN (fractures = nodes, intersections = edges)
- Looked for proxy variables that were correlated with QoIs.
 - Number intersections with repository (NIwR)
 - Average degree (average number of intersections a fracture is part of)
 - Shortest travel time (STT) between repo and aquifer (rough approx. based on fracture area and pressure differentials)
- **These are derived quantities and therefore can't be prescribed by adaptive algorithms;** use regression.
 - Fitted analytical distributions for proxy variables from ~3000 sample DFNs.
 - Augment build point input space with proxy variables for each DFN
- **This is the first time we have been able to account for subsurface variability in GSA**
 - However, these are only proxies and cannot completely capture the induced variance from subsurface.

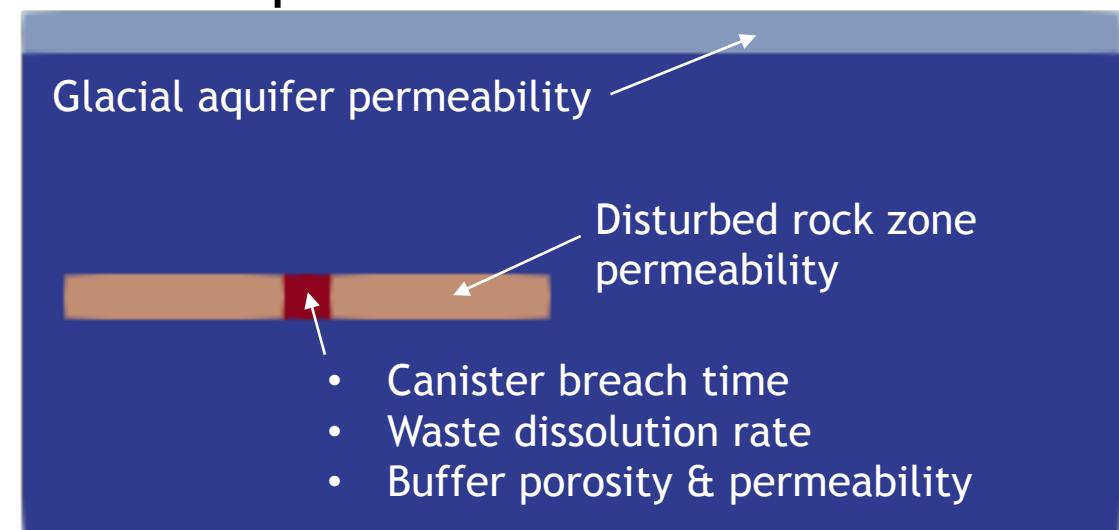


8 Preliminary multifidelity PCE study with subsurface uncertainty

- Each sample model evaluation was generated with a different DFN (different subsurface instantiation)
 - Input parameter space augmented with proxy variables
 - Constructed regression PCE over augmented input space
- Constructed PCE with 828 samples from 10 m model for comparison
- Hand-selected number of samples at $d = 10, 20$, and 40 for preliminary studies.
- Interested in potential benefits of augmenting small number of high-fidelity (HF) samples with samples from coarser meshes.
- Generated single-fidelity PCE with 18 HF samples.
- Generated multifidelity PCE augmenting HF samples with 828 samples from $d = 20, d = 40$.

	$d = 10$	$d = 20$	$d = 40$
N samples	18	828	828
Relative costs	1	0.02	0.006

Uncertain parameters:

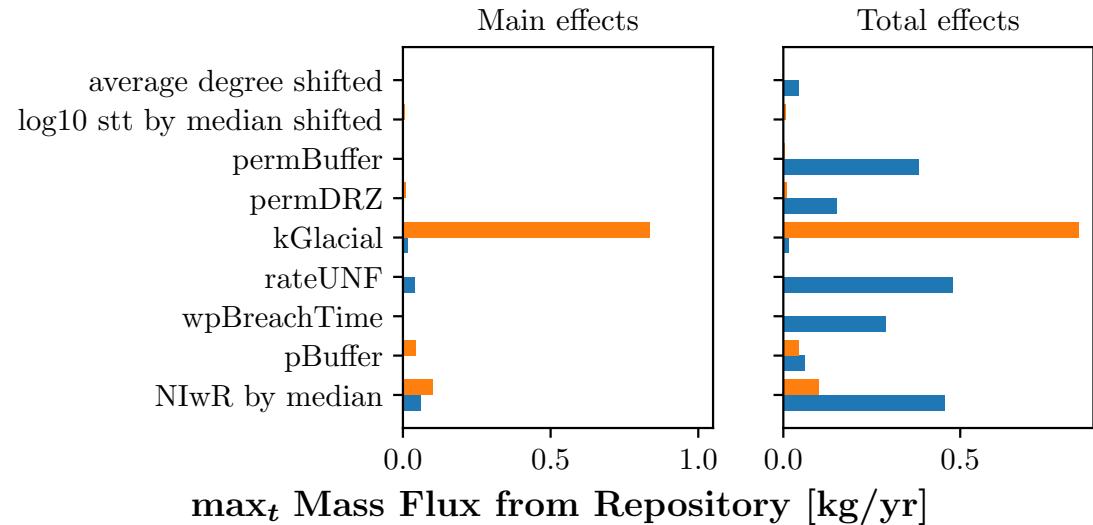


For all but the QOIs depending on location of peak ^{129}I , estimation of Sobol indices was drastically improved with the multifidelity PCE.



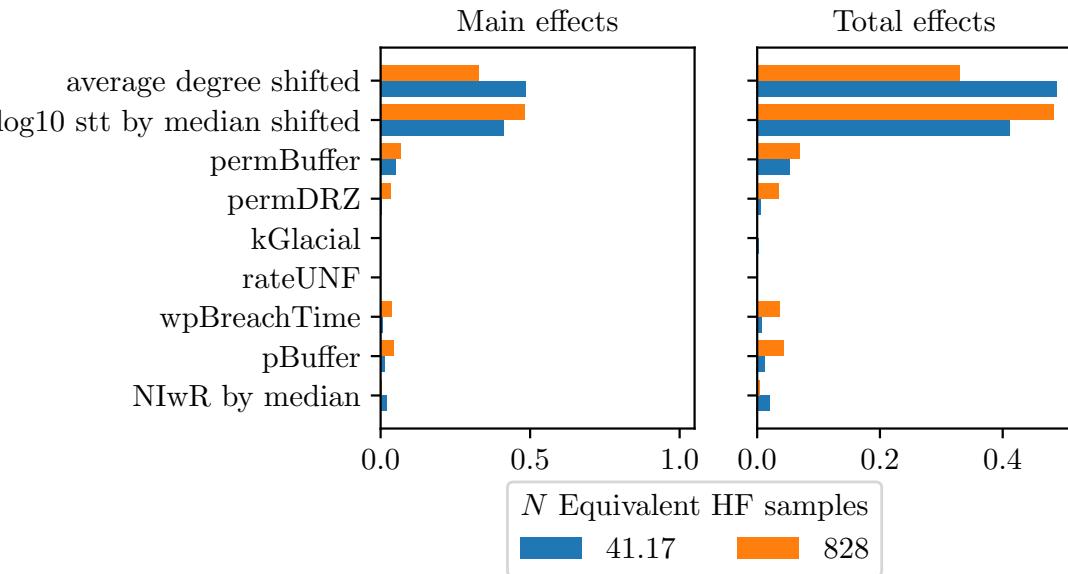
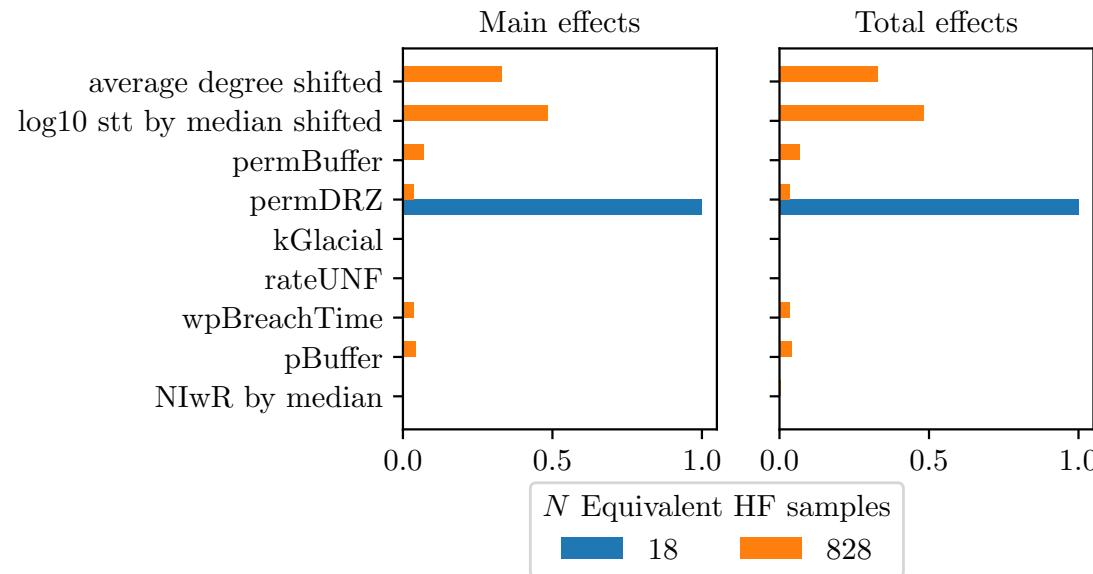
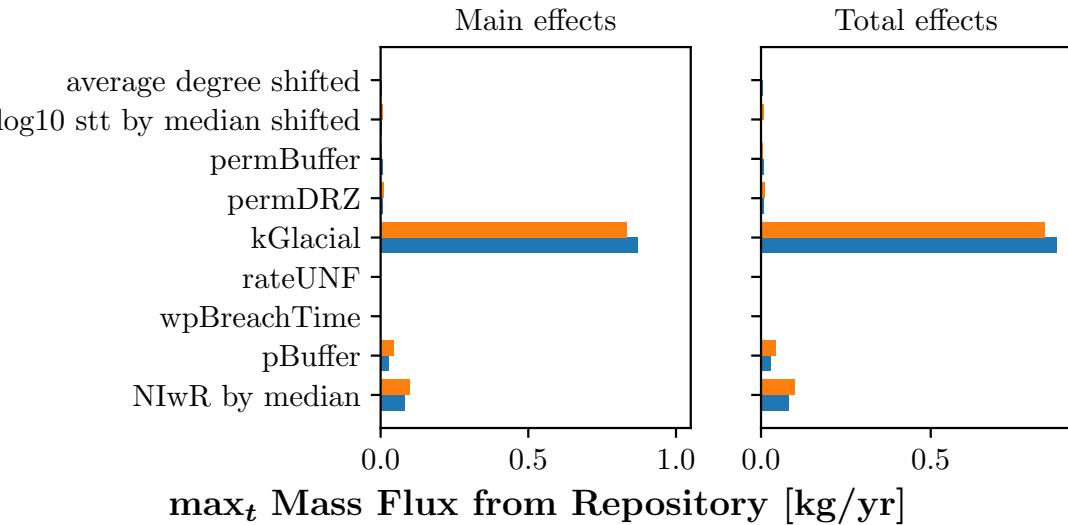
Single-fidelity PCE, HF model

$$\text{Peak } ^{129}\text{I} \equiv \max_{t, \underline{x} \in \Omega_A} ^{129}\text{I} [\text{M}]$$



Multifidelity PCE

$$\text{Peak } ^{129}\text{I} \equiv \max_{t, \underline{x} \in \Omega_A} ^{129}\text{I} [\text{M}]$$

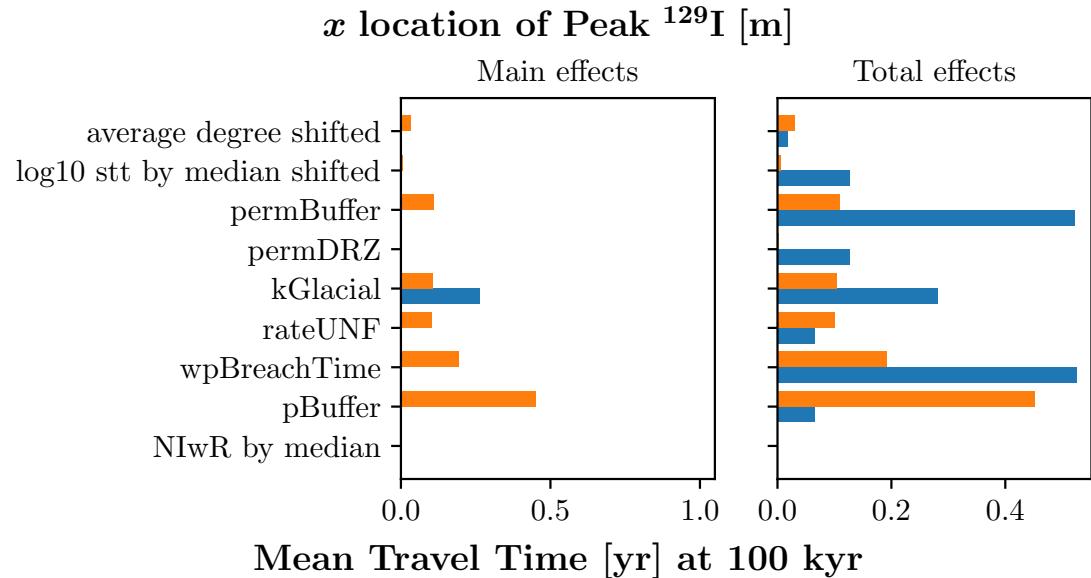


For QOIs depending on location of peak ^{129}I , multifidelity PCE Sobol indices were not significantly improved.

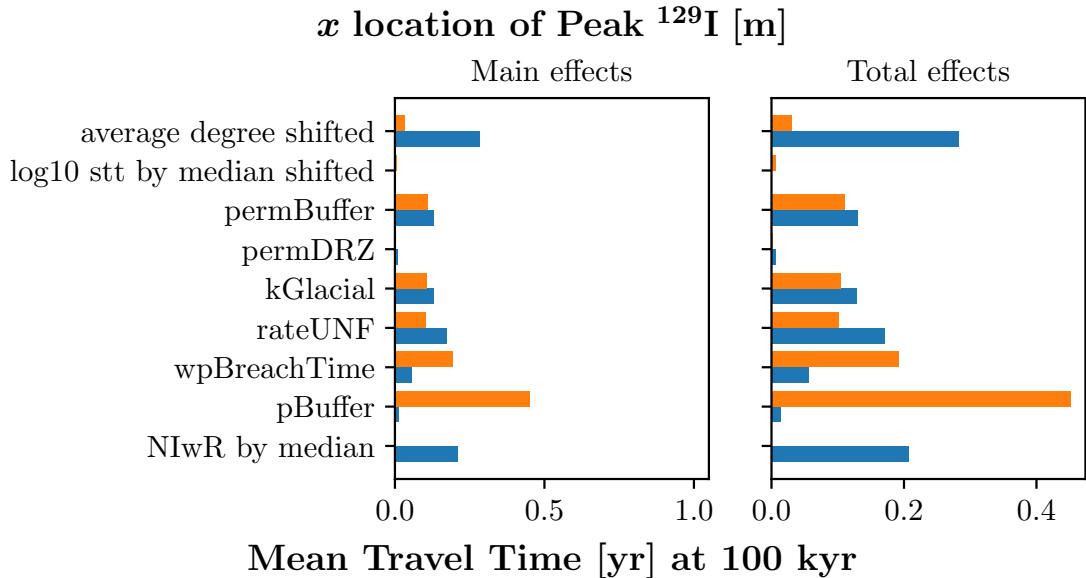


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Single-fidelity PCE, HF model



Multifidelity PCE



N Equivalent HF samples
18 828

N Equivalent HF samples
41.17 828



Takeaways/future directions

- Overall, these results show there is promise in pursuing MF PCE for GSA in this problem.
- However, performance for QoIs depending significantly on ^{129}I location was poor.
 - Most important QoIs for performance assessment not fully settled yet; some of these QoIs may not factor into eventual performance assessment.
- Need to determine reasonable model hierarchy and reasonable QoIs.
- Challenge: how to take advantage of adaptive algorithms/optimize sample profiles in presence of proxy variables?
 - Optimal sample allocations for multifidelity approximation of Sobol indices with PCE – see Michael Merritt's talk later in the session!