Lab 1: Harmonics and Intermodulation

EE160 section 3

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Introduction

The purpose for this lab was to familiarize ourselves with the spectrum analyzer and using it in combination with other electrical equipment to analyze harmonic generation with a diode, evaluate the input third order intercept point (IIP3) of an amplifier, and measure the spectra of periodic signals. Dividing the lab into five parts, we first constructed and verified the proper operation of our coupling-attenuation circuit (ATT-SA) which was essential to successfully completed the subsequent experiments. Next, experimental procedures involving the modification of our ATT-SA circuit to measure and record harmonics at different frequencies and voltage peak-to-peaks were done. Finally, the results were used to answer a few post-lab questions for analysis. Equipment used in this lab included: the spectrum analyzer, digital multimeter acting as a voltmeter and ohmmeter, breadboard, electrical components such resistors and capacitors, and an oscilloscope.

To understand how the spectrum analyzer aides us in achieving our purpose, Fourier Analysis must be explained. Fourier Analysis is the study of the way general functions may be represented or approximated by sums of simpler trigonometric functions. In engineering, Fourier Analysis is usually applied when processing signals such as audio or seismic waves. This technique can isolate narrowband components of a compound waveform, concentrating them for easier detection or removal. One branch of Fourier Analysis, Fourier series, can decompose periodic function or signal into a sum of oscillating functions (i.e. sine waves). A waveform, with period T, can be expressed as a Fourier series as shown below:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_O t}$$

---- **equation 1**

The Fourier coefficients (c_n) can be determined using the following formula:

$$c_n = \frac{1}{T}\int_0^T f(t) e^{-jn\omega_0 t\,dt}$$

---- **equation 2**

The power (p_n) in the n-th harmonic (when n is less than 0) relative to the Fourier coefficient in Figure I.2 are directly proportional to each other as seen below:

$$P_n = 2\,|c_n|^2$$

---- **equation 3**

Total Harmonic Distortion (THD), or the distortion produced by an amplifier which is measured in terms of the harmonics of the sinusoidal components of that signal, can be defined by a combination of cn and pn:

$$THD = \frac{\sum_{n=2}^{\infty} P_n}{P_1} = \frac{\sum_{n=2}^{\infty} |c_n|^2}{|c_1|^2}$$

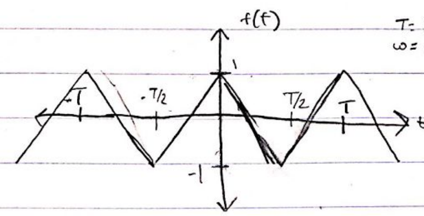
---- **equation 4**

Pn can be represented in a graph as a power line spectrum. The spectrum analyzer used in these experiments can display this spectrum with lines enlarged by the machine's variable frequency bandpass filter. When properly adjusted, p_n is synonymous with the amplitude of the spectral peak thus making it easier to determine THD and further understand how the harmonics are affected by a particular amplifier.

Prelab Questions

IV.1 Triangle Wave

$$f(t) = 1 - \frac{4|t|}{t}, \text{ if } |t| < \frac{T}{2}$$



$T = 1$

$\omega = 2\pi$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$X_n = |X_n| e^{j\phi_n}$; $X_{-n} = X_n^*$: $\phi_{-n} = -\phi_n$

$|X_n| = \frac{c_n}{2}$; $X_0 = C_0$

$$X_n = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-jn\omega_0 t} dt$$

$a_0 = c_0 = x_0$

$a_n = c_n \cos\phi_n$

$b_n = -c_n \sin\phi_n$

$x_n = \frac{1}{2}(a_n - jb_n)$

$$X_n = \frac{1}{T}\int_{-T}^{0} (4t+1) e^{-j\omega n t} dt + \frac{1}{T}\int_0^T (4t+1) e^{-j\omega n t} dt$$

$$= (4)\frac{1}{T}\int_{-T}^{0} t e^{-j\omega n t} dt + (1)\frac{1}{T}\int_{-T}^{0} e^{-j\omega n t} dt$$

$$+ (4)\frac{1}{T}\int_0^T t e^{-j\omega n t} dt + (1)\frac{1}{T}\int_0^T e^{-j\omega n t} dt$$

$$X_n = (4)\frac{e^{j\pi n}(-1+j\pi n)+1}{2\pi\omega n^2} + (1)\frac{j(e^{j\pi n}-1)}{2\pi n} + (4)\frac{e^{-j\pi n}(1+j\pi n)-1}{2\pi\omega n^2} + (1)\frac{j(e^{-j\pi n}-1)}{2\pi n}$$

$$= \frac{4e^{j\pi n}(-1+j\pi n)+1+4e^{-j\pi n}(1+j\pi n)-1}{2\pi\omega n^2} + \frac{j(e^{j\pi n}-1)+j(e^{-j\pi n}-1)}{2\pi n}$$

$$= \frac{4e^{j\pi n}j\pi n + 4e^{-j\pi n}j\pi n}{2\pi\omega n^2} + \frac{2j(e^{j\pi n}-1)}{2\pi n}$$

Experimental Measurements

Part 1: Coupling-attenuation Circuit (ATT-SA)

For this part, we constructed the circuit below on the breadboard:

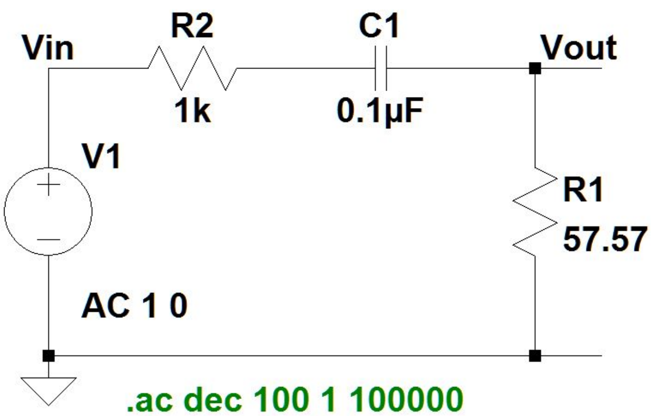


Figure 1: ATT-SA

With the spectrum analyzer disconnected from the ATT-SA circuit shown below, we measured the following values:

Table 1: nominal and measured values of ATT-SA components

component	Nominal value	Measured value
R1	56 □	57.57 □
R2	1k □	1.0001 k□
C1	0.1 µF	0.100 µF

Using these values, we can find the ATT-SA's theoretical cutoff frequency:

$$\frac{1}{\sqrt{2}}(-3dB) = \frac{R1}{R1+R2}\left(\frac{1}{2\pi(R1)(C)}\right) = \frac{57.57}{1000+57.57}\left(\frac{1}{2\pi(57.57)(0.1E-6)}\right) = 1.505\ kHz$$

Because the ATT-SA acts as a high-pass filter, the calculation above means that, as the input frequency approaches 1.505 kHz from the right-hand side of the spectrum, the output voltage will become attenuated by 3dB.

Next, we tested the circuit while under the 50□ load of the spectrum analyzer. Using the oscilloscope, we found the ATT-SA to have its max gain at 20kHz. The output read 440 mVrms. Using the equation below, we calculated 3dB below this voltage:

$$-3dB = 20log(\frac{Vf}{Vi})$$

$$Vf = Vi \cdot 10^{-3/20} = 440\ mVrms \cdot 10^{-3/20} = 311.5\ mVrms$$

Then, we decreased the input frequency until the output voltage reached this voltage. We measured the output voltage to drop to 310 mVrms once the frequency dropped to 1.11 kHz.

Next, we ran an AC signal analysis of the ATT-SA in figure 1, and obtained the frequency response shown in figure 2 below. The LTspice simulation showed that the gain dropped from a maximum of -25.27 dB by 3dB to -28.27 dB at a frequency of 1.512 kHz.

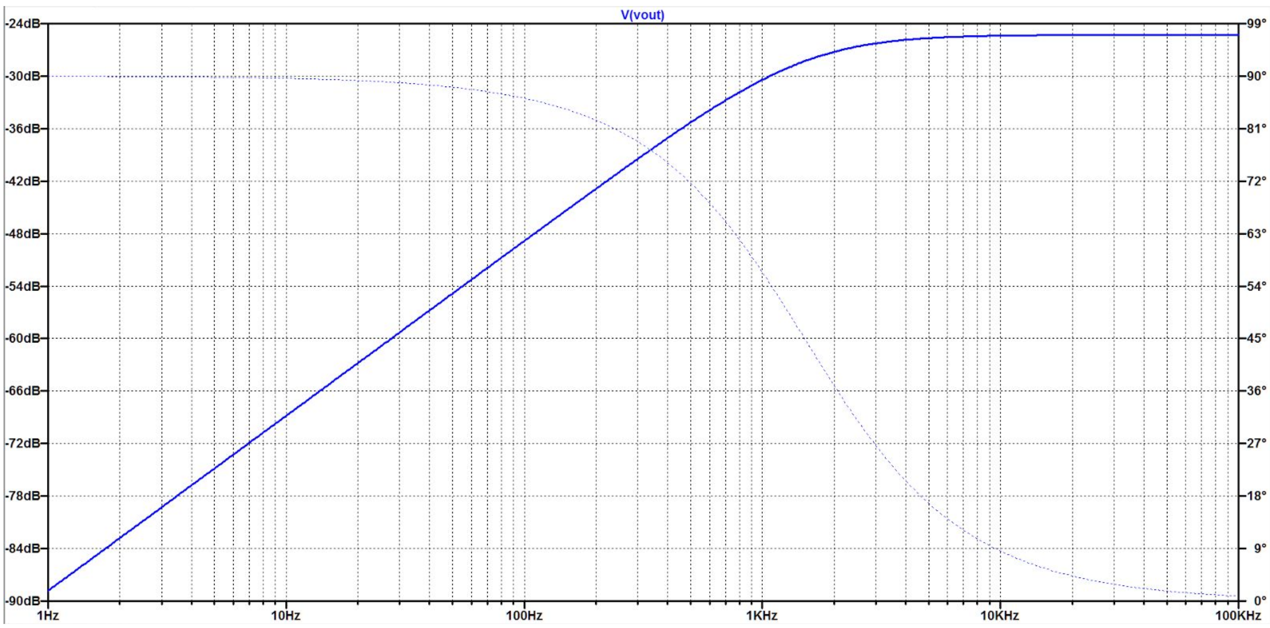


Figure 2: ATT-SA simulation in LTspice

Part 2: Making measurements using the spectrum analyzer

Next, we connected two function generators, each through its own 1k□ resistor, to the input of the ATT-SA.

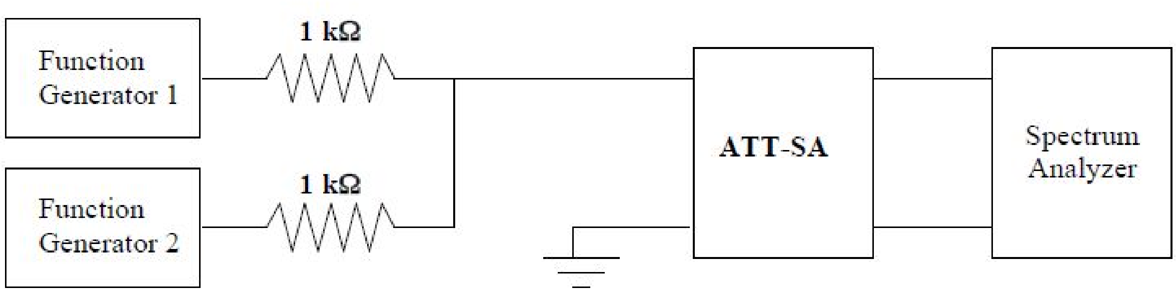


Figure 3: Dual-input circuit

We initially used frequencies of 1 MHz and 1.1MHz respectively, and used the spectrum analyzer to measure the amplitudes of the fundamental frequency and the first harmonics. We then increased each input signal by 1 MHz (through 10 MHz), maintaining the 1kHz difference. Our measurements are shown in table 2 below.

Table 2: Spectrum Analyzer Measurements (part V.2)

input (MHz)	input (Vpp)	units	-1st harm.	fund	1st harm.
1	5	f (MHz)	-1.086	0.012	1.123
1.1	5	dBm	-38.860	-27.700	-38.970
2	5	f (MHz)	-2.085	0.024	2.132
2.1	5	dBm	-33.710	-27.770	-33.900
3	5	f (MHz)	-3.092	0.000	3.112
3.1	5	dBm	-29.230	-27.560	-29.980
4	5	f (MHz)	-4.110	0.025	4.160
4.1	5	dBm	-26.900	-27.580	-27.910
5	5	f (MHz)	-5.110	0.029	5.170
5.1	5	dBm	-25.450	-27.580	-26.360
6	5	f (MHz)	-6.140	0.041	6.100
6.1	5	dBm	-24.010	-27.650	-24.220
7	5	f (MHz)	-7.170	0.041	7.130
7.1	5	dBm	-23.010	-27.650	-23.360
8	5	f (MHz)	-8.180	0.047	8.130
8.1	5	dBm	-22.300	-27.650	-22.180
9	5	f (MHz)	-9.060	0.058	9.180
9.1	5	dBm	-17.770	-27.650	-17.560
10	5	f (MHz)	-10.110	0.000	10.170
10.1	5	dBm	-17.050	-27.680	-16.640

<u>Part 3: Harmonic generation with a nonlinear device</u>

Next, we added a diode in series between the function generators and the ATT-SA input, and grounded the ATT-SA input through a 56□ resistor, according to figure 4 below.

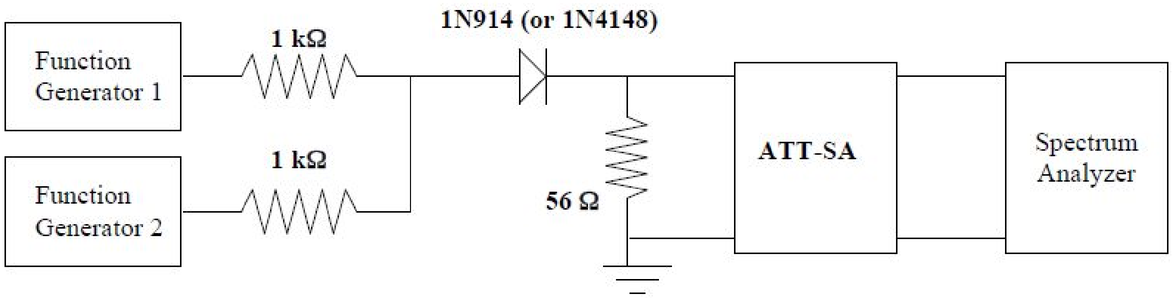


Figure 4: Dual-input nonlinear circuit

We then temporarily disconnected the second function generator, and only used a single 1 MHz input signal. We first used a 5Vpp input and measured the amplitudes of the fundamental frequency and the first and second harmonics. We then decreased the input by 3dB (5V - 3dB = 3.54V) and retook the measurements. We then decreased the input by an additional 3dB (3.54V - 3dB = 2.51V), and retook the measurements. The results are shown in the table below:

Table 3: Spectrum Analyzer Measurements (part V.3.a)

input (MHz)	input (Vpp)	units	-2nd harm.	-1st harm.	fund	1st harm.	2nd harm.
1	5	f (MHz)	-1.98	-0.98	0.016	1.03	2.04
		dBm	-65.30	-63.50	-27.650	-63.50	-65.50
1	3.54	f (MHz)	-1.98	-0.99	0.024	1.04	2.04
		dBm	-67.50	-66.50	-27.680	-67.00	-68.00
1	2.51	f (MHz)	-1.99	-0.98	0.027	1.04	2.04
		dBm	-71.50	-70.00	-27.700	-71.00	-71.00

Next, we reattached the second function generator, and set it to 1.1 MHz. With the input signals at 5 Vpp and 1 MHz and 1.1 MHz, respectively, the first and second harmonics were visible. We then increased the input frequencies by 1 MHz, and the third and fourth harmonics became visible. Then, we increased the input frequencies by another 1 MHz, and the fifth harmonics became visible.

Table 4: Spectrum Analyzer Measurements (part V.3.b)

input (MHz)	input (Vpp)		-5th	-4th	-3rd	-2nd	-1st	fund	1st	2nd	3rd	4th	5th
1	5	f (MHz)				-2.10	-1.09	0.000	1.09	2.10			
1.1	5	dBm				-62.00	-62.74	-27.510	-62.00	-63.00			
2	5	f (MHz)		-8.12	-6.10	-4.00	-2.00	0.057	2.16	4.20	6.20	8.29	
2.1	5	dBm		-64.50	-67.50	-54.50	-52.50	-27.550	-52.50	-55.30	-68.50	-64.80	
3	5	f (MHz)	-18.26	-12.10	-9.00	-6.05	-2.92	0.110	3.13	6.27	9.18	12.52	18.37

3.1	5	dBm	-63.00	-64.00	-67.00	-61.50	-48.30	-27.610	-48.80	-52.40	-66.50	-62.50	62.50

Then, returning to input frequencies of 1 MHz and 1.1 MHz, we decreased both input amplitudes so that only the fundamental was visible. We achieved this with a 1.5 Vpp input amplitude. We then decreased the amplitude of the second input signal by 10 dB (1.5V - 10dB = 0.47V).

Table 5: Spectrum Analyzer Measurements (part V.3.c)

input (MHz)	input (Vpp)	units	-2nd	-1st	fund	1st	2nd
1	1.5	f (MHz)			0.023		
1.1	1.5	dBm			-27.650		
1	1.5	f (MHz)			0.023		
1.1	0.47	dBm			-27.640		

We then increased both input amplitudes to an equal level that was high enough to make the fourth and fifth harmonics reappear. This occurred at input amplitudes of 10 Vpp. We then decreased the amplitude of the second input signal by 10 dB (10V - 10dB = 3.16V).

Table 6: Spectrum Analyzer Measurements (part V.3.d)

input (MHz)	input (Vpp)	units	-5th	-4th	-3rd	-2nd	-1st	fund	1st	2nd	3rd	4th	5th
1	10	f (MHz)	-8.31	-6.23	-4.16	-2.08	-1.03	0.072	1.13	2.15	4.27	6.35	8.43
1.1	10	dBm	-69.50	-66.50	-62.00	-53.50	-55.50	-27.55	-55.50	-56.20	-65.50	-67.00	-73.00
1	10	f (MHz)			-4.10	-1.98	-0.98	0.028	1.06	2.06	4.15		
1.1	3.16	dBm			-69.00	-59.40	-57.60	-27.45	-57.50	-59.30	-70.00		

<u>Part 4: Two-tone test of an amplifier</u>

Next, we constructed the dual-stage amplifier below, and connected its output to the input of the ATT-SA circuit in figure 1. Each op amp was powered by ±8V from the DC power supply. The input signals used were 50 kHz and 60 kHz at 26 mV. We then took measurements at 26 mV, 3dB below 26 mV (26mV - 3dB = 23V), and 3dB above 26 mV (26mV + 3dB = 29V).

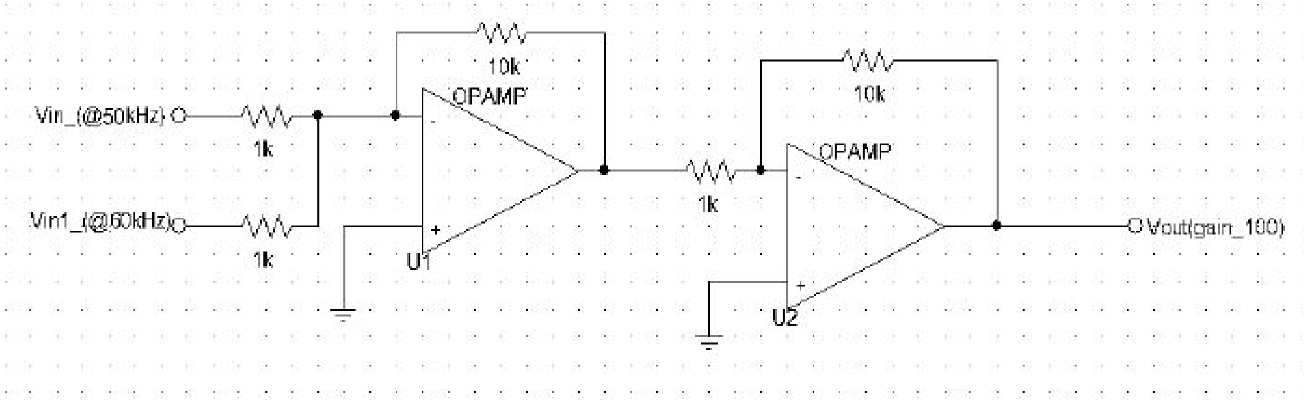


Figure 5: Dual-stage amplifier with gain = 100 [V/V]

Table 7: Spectrum Analyzer Measurements (part V.4)

input (kHz)	input (mVpp)	units	40 kHz	50 kHz	60 kHz	70 kHz	Δ dBm
50, 60	23	dBm	-92	-56	-56	-92	92 - 56 = 36 dBm
50, 60	26	dBm	-89	-55	-55	-89	89 - 55 = 34 dBm
50, 60	29	dBm	-88	-54	-54	-88	88 - 54 = 34 dBm

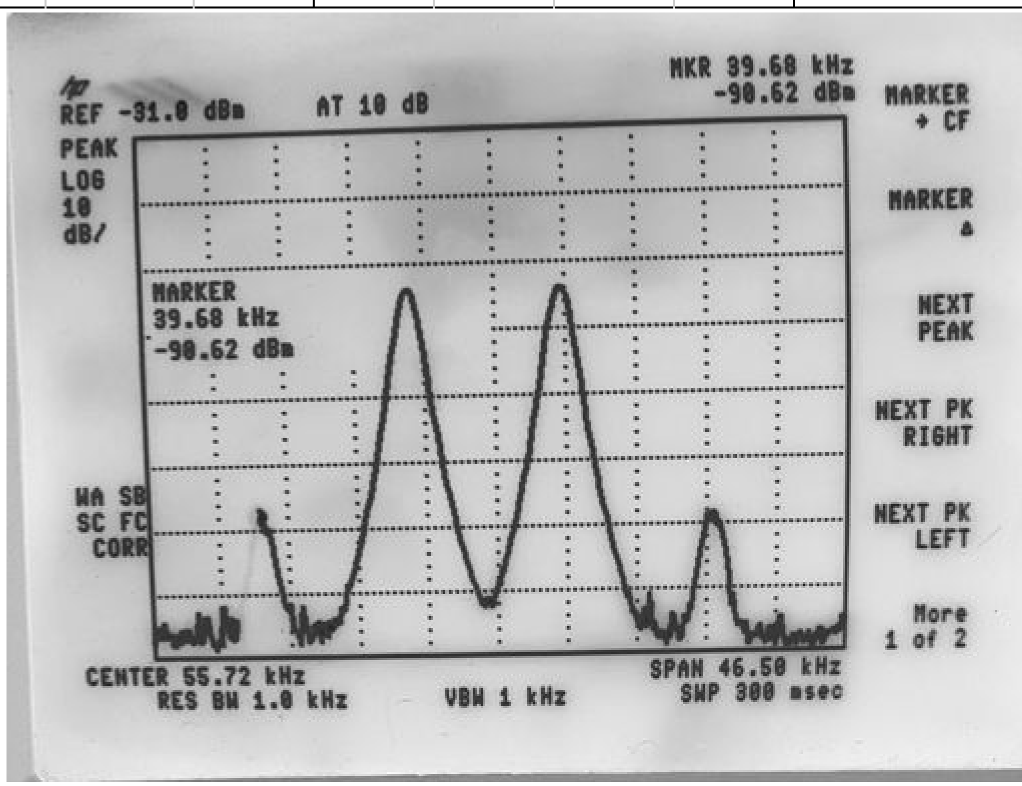


Figure 6: Spectrum analyzer screenshot of two-tone test

Part 5: Periodic waveform spectra

Part 5.1: Triangular Wave Spectrum

For this part, we applied a 1 Vpp 200 kHz triangle wave with 50% symmetry to the ATT-SA circuit. At 50% symmetry, the second and fourth harmonics nearly completely disappear.

Table 8: Spectrum Analyzer Measurements (part V.5.1)

input (kHz)	input (Vpp)	units	-5th	-4th	-3rd	-2nd	-1st	fund	1st	2nd	3rd	4th	5th
200	1	f (kHz)	-999.0		-596.0		-194.0	10.0	209.0		617.0		1020.0
triangle	50% symm	dBm	-68.8		-51.8		-33.1	-28.2	-32.7		-51.8		-60.5

Part 5.2: Rectangular Wave Spectrum

For this part, we applied a 1 Vpp 200 kHz square wave with 50%, 67%, and 75% duty cycles to the ATT-SA circuit. At 50% duty cycle, the second and fourth harmonics nearly completely disappear. At 67% duty cycle, the third harmonic nearly completely disappears. At 75% duty cycle, the fourth harmonic nearly completely disappears.

Table 9: Spectrum Analyzer Measurements (part V.5.2)

input (kHz)	input (Vpp)	units	-5th	-4th	-3rd	-2nd	-1st	fund	1st	2nd	3rd	4th	5th
200	1	f (kHz)	-984.0		-590.0		-189.0	11.0	216.0		617.0		1017.0
square	50% duty	dBm	-56.8		-54.5		-48.1	-28.1	-50.5		-55.5		-56.5
200	1	f (kHz)	-987.0	-788.0		-391.0	-187.0	11.0	216.0	414.0		817.0	1021.0
square	67% duty	dBm	-57.4	-57.5		-55.2	-51.8	-28.1	-51.9	-55.1		-57.5	-57.4
200	1	f (kHz)	-987.0		-590.0	-386.0	-187.0	11.0	216.0	414.0	618.0		1021.0
square	75% duty	dBm	-59.5		-57.5	-52.5	-50.6	-28.0	-50.5	-52.1	-57.1		-58.9

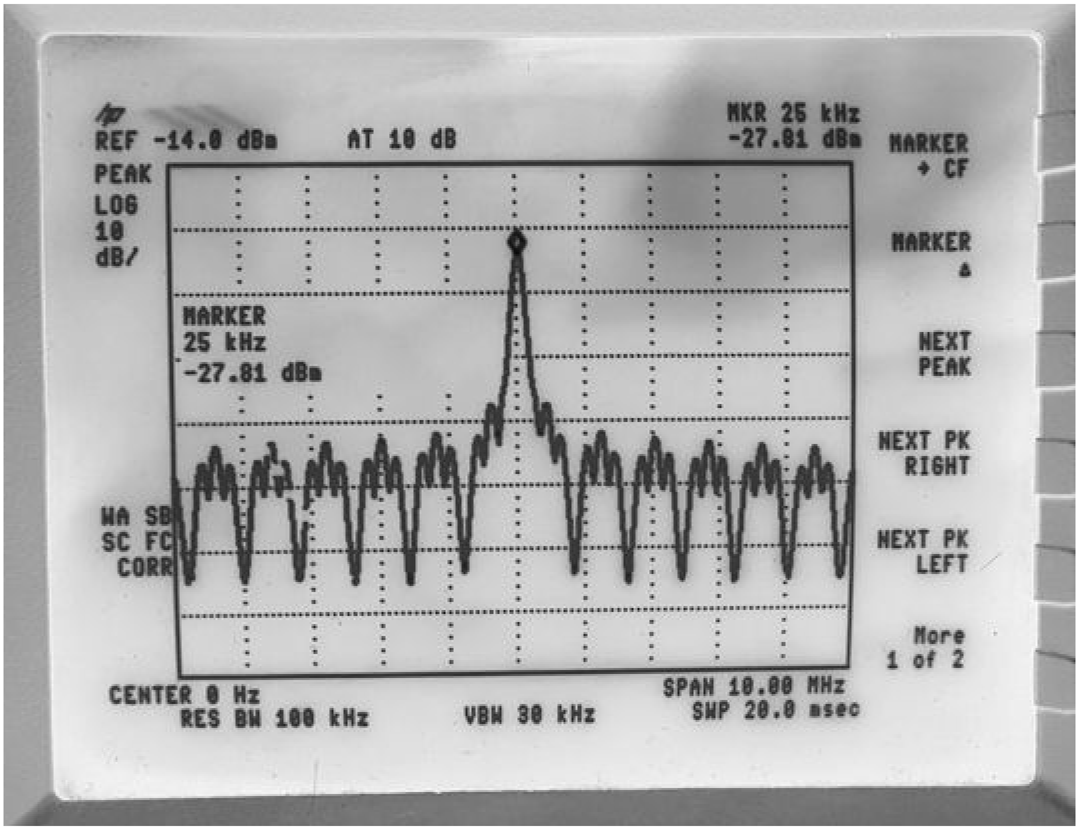


Figure 7: 10 MHz span on spectrum analyzer (part V.5.2.4)

Part 5.3: Truncated Cosine Wave

We built the implemented the clipper circuit shown below between the function generator input and the ATT-SA. We connected the oscilloscope to the anode and cathode of the diode. We then applied a 5 Vpp 200 kHz sine wave to the clipper circuit, and used a range of ±2.5V for the DC offset. Then, for each harmonic, we adjusted the DC offset to find the local minimum and maximum values of magnitude. At these values, we recorded the value of τ in order to calculate the conduction angle between the local minimum and maximum values.

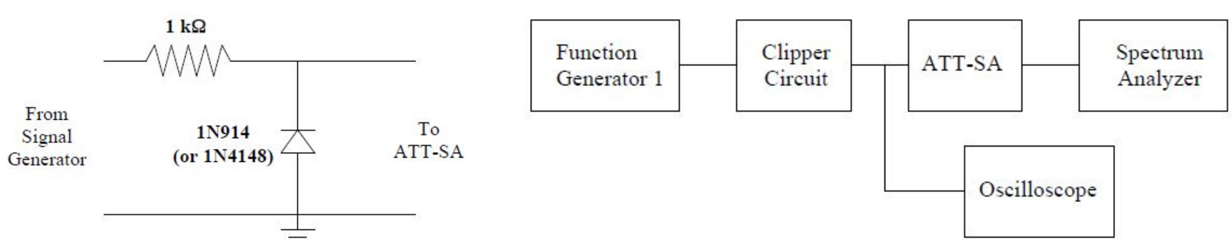


Figure 8: Clipper circuit diagram and measurement setup

Referring to figure 9, we used the following equation to calculate the conduction angle θ:

$$2\theta = \tau / T_o \cdot 2\pi$$

$$\theta = \tau \cdot f_o \cdot \pi$$

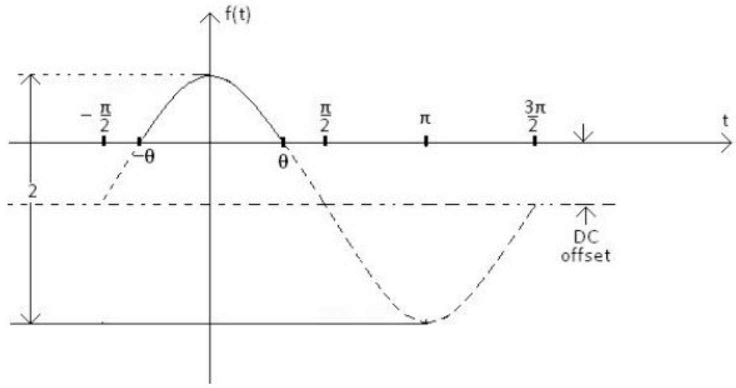


Figure 9: Clipped cosine wave

Table 10: Measurements and calculations of conduction angle θ

harmonic		offset [V]	gain [dBm]	τ [μs]	θ [rad]	$\Delta\,\theta$
	min	-2.5	-40.9	0	0	
1	max	2.5	-15.72	5	π	π
	min	1.1	-80	5	π	
2	max	-0.7	-29.6	2.44	1.533	1.609
	min	1.1	-80	5	π	
3	max	0.2	-39.7	3.3	2.073	1.069
	min	1.1	-80	5	π	
4	max	-0.7	-44.2	2.4	1.508	1.634
	min	1.1	-80	5	π	
5	min	-0.1	-49.1	3	1.885	1.257

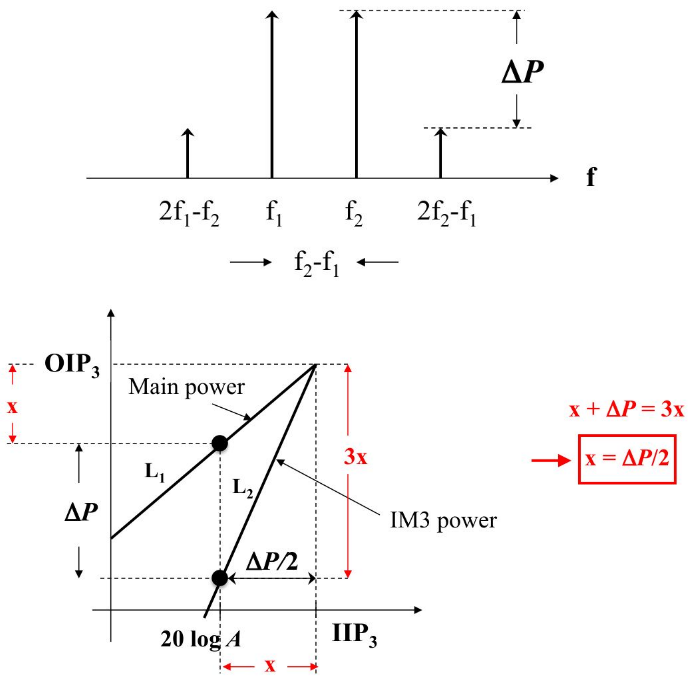
Analysis and Discussion of Results

1. Discuss how the two frequencies are resolved with the aid of the spectrum analyzer. Using the data collected in lab, compare the measurements with the theoretical expressions presented in class, using a relationship between input waveform voltage and the output power measured by the spectrum analyzer.

The spectrum analyzer helps break down the two input signals into their component frequencies. By using the spectrum analyzer, we can see the magnitudes of the 50 kHz and 60 kHz sine wave inputs, as well as their harmonics in the frequency domain.

2. Discuss how the two-tone test helps in measuring the third order non-linearity of an amplifier. Using the measure data sets, determine the average IIP3 value of the circuit amplifier in the experiment.

In the two-tone test, we use two sine wave input signals with 10kHz difference. We apply the output power supply at –8V and +8V in both opamps. Then increasing the input amplitudes when 40 kHz and 70 kHz are visible in the spectrum analyzer. We pick the values at 60 kHz and 70 kHz for IIP3 Calculation.



The IIP3 in dBm can then be approximated as

IIP3(dBm) =(ΔP/2)(dB) + 20logA(dBm)

From the data we got in table 7

36/2 + 20log23 = 45.23dbm

34/2 + 20log26 = 45.3dBm

34/2 + 20log29 = 46.24dbm

Avg IIP3(dBm) = (45.23+45.3+45.24)/3 = <u>45.9dBm</u>

3. Verify the attenuation and cutoff frequency values measured as described in section V.0. In the same graph, compare the Bode amplitude plot obtained from the LTspice model with the laboratory measurements.

Simulation in Figure 2.

4. In the lab you measured six periodic waveforms: 2 sine waves, 1 triangular wave, and 3 rectangular waves. For each of these waveforms you are asked to:

a. Compute the amplitude of each harmonic in dB relative to the fundamental (which will be the 0dB reference). You may already have done this in the lab as you made your measurements.

b. Compute the percent THD. Ignore the power in all harmonics above the fifth.

c. Compare the theoretical power spectrum with the measured spectrum. You may use tables(s) or graph(s) (or both) to present the results of your comparisons.

The formula of THD calculation

$$Px=\sum_{n=-\infty}^{\infty}|xn|^2$$

$$THD=\frac{\frac{1}{2}Px-|x1|^2}{|x1|^2}$$

Triangular Wave Spectrum

input (kHz)	input (Vpp)	units	-5th	-4th	-3rd	-2nd	-1st	fund	1st	2nd	3rd	4th	5th
200	1	f (kHz)	-999.0		-596.0		-194.0	10.0	209.0		617.0		1020.0
triangle	50% symm	dBm	-68.8		-51.8		-33.1	-28.2	-32.7		-51.8		-60.5

amplitude of harmonics:

1^{st} harmonic	3^{rd} harmonic	5^{th} harmonic
P1= 10^(-32.7/10)=0.000537mw	P 2= 0.000007mW	P3=0.000001mW

Px = P1^2+ P2 ^2+P3^2= 2.884E-7

THD = (Px-|x1|^2)/(|x1|^2) = 99%

Rectangular Wave Spectrum(amplitude of harmonics)

input (kHz)	input (Vpp)	units	-5th	-4th	-3rd	-2nd	-1st	fund	1st	2nd	3rd	4th	5th
200	1	f (kHz)	-984.0		-590.0		-189.0	11.0	216.0		617.0		1017.0
square	50% duty	dBm	-56.8		-54.5		-48.1	-28.1	-50.5		-55.5		-56.5

1^{st} harmonic	3^{rd} harmonic	5^{th} harmonic

P1=0.000009mW	*P2 = 0.000003mW*	*P3 = 0.000002mW*

Px = 9.2388E-11
THD = 99%

input (kHz)	*input (Vpp)*	*units*	*-5th*	*-4th*	*-3rd*	*-2nd*	*-1st*	*fund*	*1st*	*2nd*	*3rd*	*4th*	*5th*
200	*1*	*f (kHz)*	*-987.0*	*-788.0*		*-391.0*	*-187.0*	*11.0*	*216.0*	*414.0*		*817.0*	*1021.0*
square	*67% duty*	*dBm*	*-57.4*	*-57.5*		*-55.2*	*-51.8*	*-28.1*	*-51.9*	*-55.1*		*-57.5*	*-57.4*

1st harmonic	*2nd harmonic*	*4th harmonic*	*5th harmonic*
P1=0.000007mW	*P2=0.000003mW*	*P4=0.000002mW*	*p5=0.000002mW*

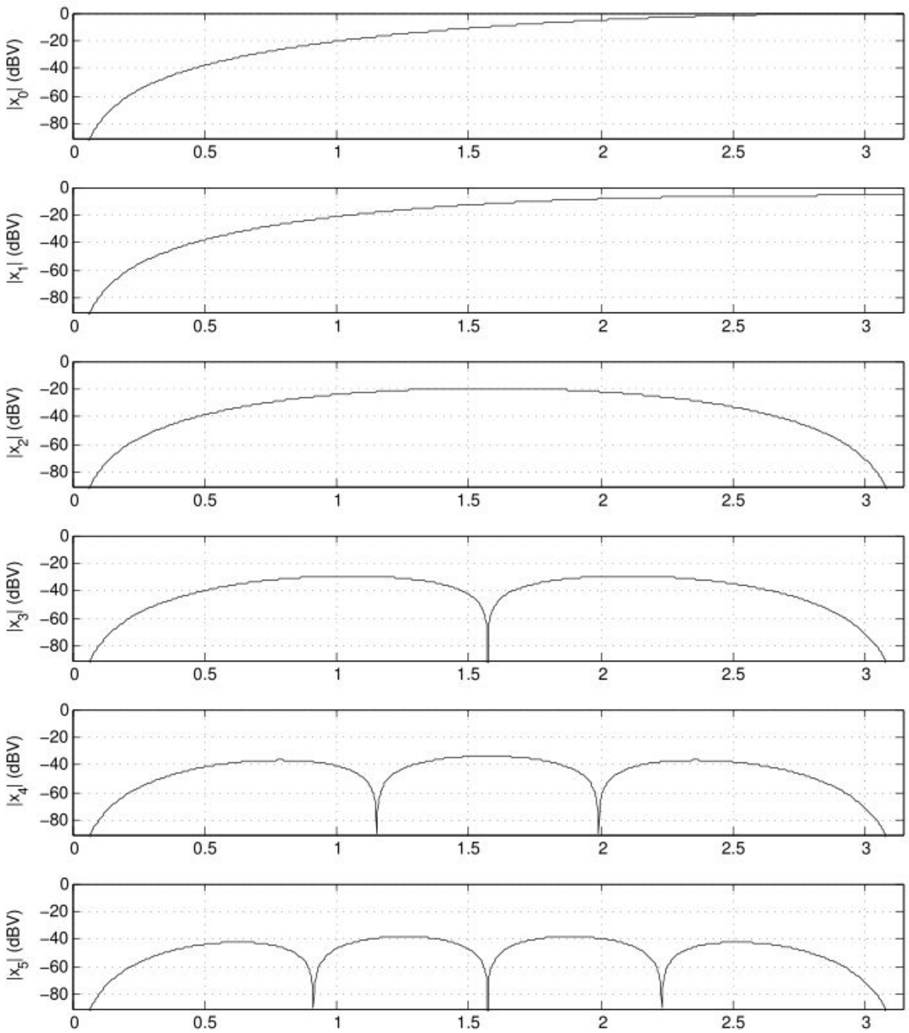
Px =5.77105E-11
THD = 99%

input (kHz)	*input (Vpp)*	*units*	*-5th*	*-4th*	*-3rd*	*-2nd*	*-1st*	*fund*	*1st*	*2nd*	*3rd*	*4th*	*5th*
200	*1*	*f (kHz)*	*-987.0*		*-590.0*	*-386.0*	*-187.0*	*11.0*	*216.0*	*414.0*	*618.0*		*1021.0*
square	*75% duty*	*dBm*	*-59.5*		*-57.5*	*-52.5*	*-50.6*	*-28.0*	*-50.5*	*-52.1*	*-57.1*		*-58.9*

1st harmonic	*2nd harmonic*	*3rd harmonic*	*5th harmonic*
P1=0.000009mw	*P2=0.000006mW*	*p3=0.000002mW*	*p5=0.000001mW*

Px= 1.22913E-10
THD = 99%

6. Prepare a table comparing the theoretical and measured values of the conduction angles at which each harmonic (n = 1 to 5) reaches its local maximum and minimum values.



Harmonic	Theoretical	Mesured	%error
N=1			0
N=2	1.6	1.609	0.56
N=3	1.1	1.069	2.82
N=4	1.6	1.634	2.13
N=5	1.3	1,257	3.3

7. If the theoretical and measured results in Section V.5.3 differ by more than 10%, discuss *possible reasons for these differences. In particular, what effect would you expect from the non-ideal diode characteristics?*

The theoretical and measured results in Section V5.3 differ by less than 10%. (see table above)

Conclusion

In conclusion, we familiarized ourselves with the spectrum analyzer and used it in combination with other electrical equipment to analyze harmonic generation with a diode, evaluate the input third order intercept point (IIP3) of an amplifier, and measure the spectra of periodic signals. After collecting the data, we had a better understanding of how nonlinear devices affect the fourier series. Difficulties faced in this lab were learning how to properly use the spectrum analyzer and determining what we wanted to analyze for the Two-tone test procedure. We solved these problems by asking for assistance from our TA and our professor in a span of two weeks. Percent error in our experiments may have been due to the precision of our measuring technique. I would recommend this lab to future 160 students as it is a great way to refresh on topics related to Fourier analysis.