Hypothesis test

Cricket series example

The Captain always calls heads

👍	👎

1) **10-match series**
   Won 7 tosses

   Would you believe it is a fair coin?

2) **100-match series**
   Won 70 tosses

   Would you believe it is a fair coin?

3) **1000-match series**
   Won 700 tosses

   Would you believe it is a fair coin?

The important question we need to address is as follows:

What is the framework that can provide a quantifying metric to this intuition of ours?

Cricket series example

1) What is our default assumption?

The coin is fair

2) When shall we reject this assumption?

We shall reject only when we have enough data that makes us conclude otherwise

Judge in court

Assume that you are a judge in a court

A person is brought in front of you as a murder suspect

1) What is our default assumption?

The person is innocent

2) When shall we reject this assumption?

We shall reject only when we have enough data that makes us conclude that he is guilty

Machine Learning Deployment

A machine learning model (legacy) is in production for a few years, and is doing fairly well

You and your team have built a new model, and want to claim that it is better

1) What is the default assumption of the product owner?

The new model is not better than the legacy model

2) When shall we reject this assumption?

When enough data is given that the new model outperforms the legacy model significantly

Third umpire

Suppose you are the third umpire

The on-field umpire has called for your help, and given a soft signal

1) What is our default assumption?

The on-field umpire is correct

2) When shall we reject this assumption?

We shall reject only when we have enough data that makes us conclude that the on-field umpire’s decision can be changed

Fingerprint scanner

We unlock our phones using fingerprint scanner

A finger is now placed on the scanner

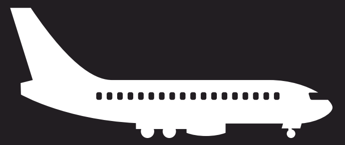
1) What should the default assumption be?

The fingerprint does not belong

2) When should the default assumption be rejected?

The default assumption should be rejected only when the data (fingerprint) is very conclusive that it belongs to the owner

Radar example



A radar has to detect a plane

1) What should the default assumption be?

There is no plane

2) When should the default assumption be rejected?

The default assumption should be rejected only when the data is very conclusive that there is a plane

Null Hypothesis H_0 The technical term for our default assumption

The coin is fair

The new model is not better than the legacy model

The person is innocent

The on-field umpire is correct

The fingerprint does not belong

There is no plane

All these are examples of setting up the Null Hypothesis

Terminologies

H_0 **Null Hypothesis**

Judge in court

H_0 **The person is innocent**

We shall reject only when we have enough data that makes us conclude that he is guilty

Data:

The person has a knife in his pocket	Innocent people can carry knife
The knife has blood stains	Maybe he is a cook/chef
Blood matches that of the victim	Ok, this is too much
His shirt has fingerprints of the victim	Highly unlikely that an innocent man has all these data points

Verdict: Guilty! (Reject the null hypothesis)

Probability of seeing data as extreme as what was observed, under the assumption that he is innocent, is very low

$P\left[\textbf{data}\,|\,\textbf{H0 is true}\right]$ is very low   This is called $p-$**value**

If $p-$**value** is very low, we reject H0

Terminologies

H_0 **Null Hypothesis**

$p-$**value**

Deep dive: coin toss

Put a quantitative metric on our suspicion that coin is biased

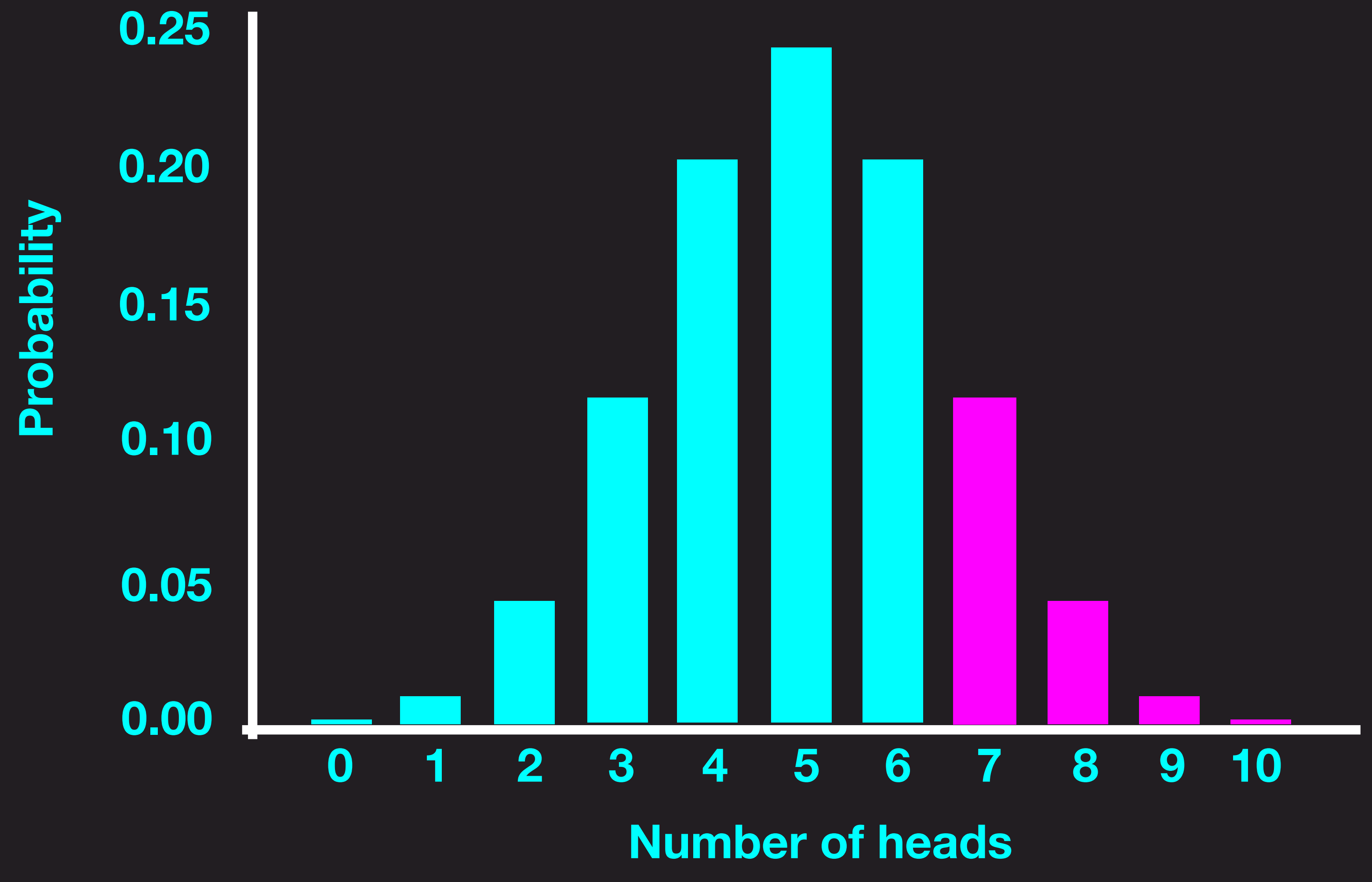
Deep dive: coin toss

Put a quantitative metric on our suspicion that coin is biased

H_0: coin is fair. Probability of heads = 0.5

1) 10-match series **7 Heads** **Would you believe it is a fair coin?**

Let T = number of heads	Test Statistic
Is T a random variable?	Yes
What is its distribution?	Binomial
What is the observed value of T ?	T_{obs} = 7



What is $p-$value?

$p-$value is the probability of observing data as extreme the observed test statistic under the assumption that the null hypothesis is true

The $p-$value is therefore given by $P\left[T \geq 7 \middle| H_0 \text{ is true}\right]$

How do we compute P-value here?

```
binom.pmf(k=7, n=10, p=0.5) + binom.pmf(k=8, n=10, p=0.5) + binom.pmf(k=9, n=10, p=0.5) + binom.pmf(k=10, n=10, p=0.5)
```

```
1 - binom.cdf(k=6, n=10, p=0.5)
```

$= 0.172$

Deep dive: coin toss

Put a quantitative metric on our suspicion that coin is biased

H_0: coin is fair. Probability of heads = 0.5

Test Statistic

Let T = number of heads

1) 10-match series

7 Heads

$P\left[T \geq 7 \middle| H_0 \text{ is true}\right] =$ `1 - binom.cdf(k=6, n=10, p=0.5)` $= 0.172$

2) 100-match series

70 Heads

$P\left[T \geq 70 \middle| H_0 \text{ is true}\right] =$ `1 - binom.cdf(k=69, n=100, p=0.5)` $= 0.000039$

3) 1000-match series

700 Heads

$P\left[T \geq 700 \middle| H_0 \text{ is true}\right] =$ `1 - binom.cdf(k=699, n=1000, p=0.5)` $= 0$

When do we reject the null hypothesis H0?

When the $p-$value is very low

Typically used threshold is 0.05 (This can change based on business needs)

This threshold is denoted by α and is called Significance Level

Terminologies

H_0 Null Hypothesis

$p-$value

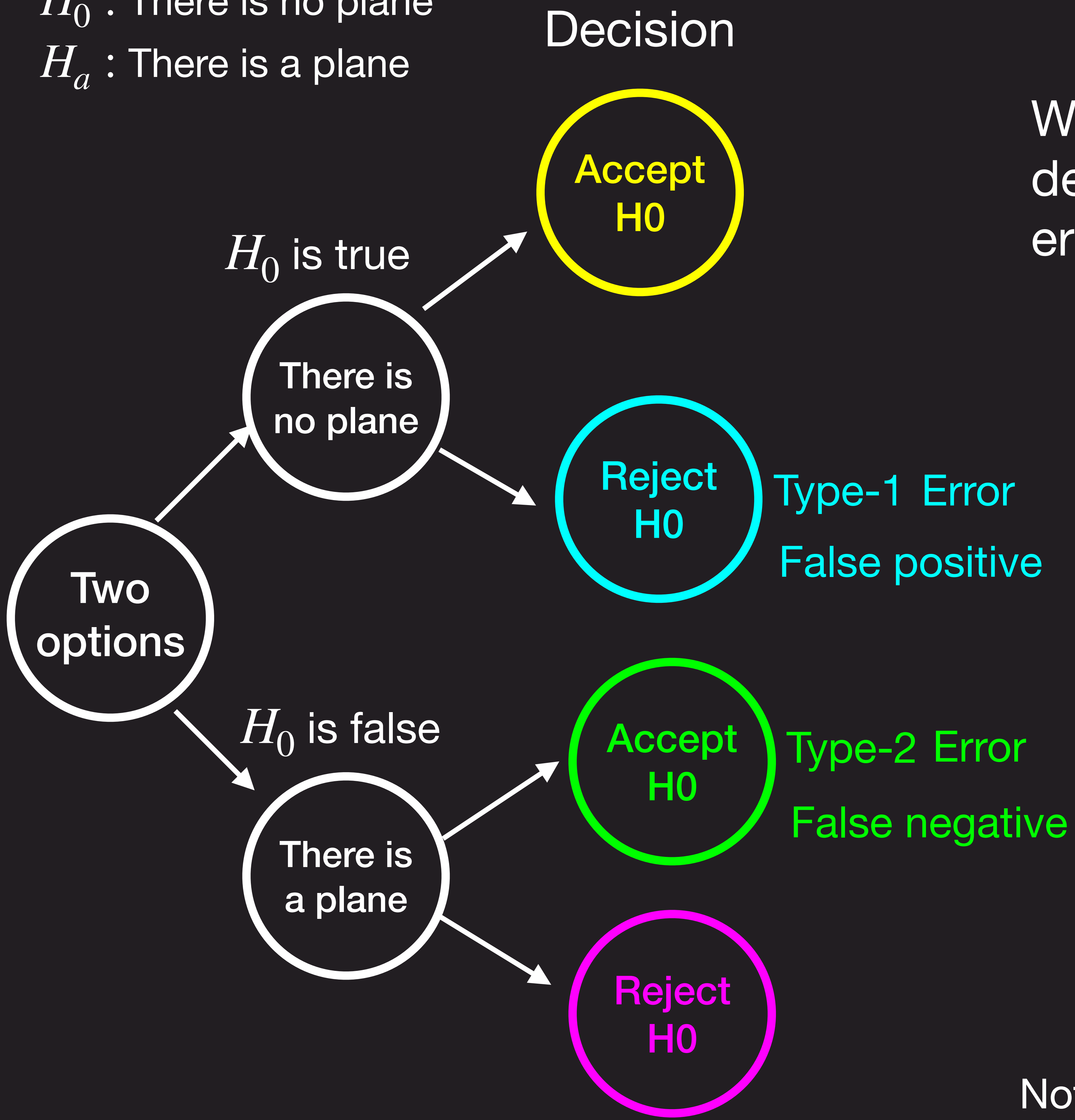
Test Statistic

Significance Level

Radar example

H_0 : There is no plane

H_a : There is a plane



Decision

Two options

H_0 is true

There is no plane

Accept H0

Reject H0

Type-1 Error

False positive

H_0 is false

There is a plane

Accept H0

Type-2 Error

False negative

Reject H0

Which of these decisions are errors?

		Decision	
		Accept	Reject
H_0	True	**True negative**	**False positive**
	False	**False negative**	**True positive**

Note: Statisticians do not say “Accept”. They say “fail to reject”

Null Vs Alternate

H0 indicated “null” hypothesis

In the event of rejection of H0, we need the right “alternate hypothesis”

Court example

H_0 “person is innocent”

H_a “person is guilty”

Coin toss with 70% heads

H_0 “coin is fair”

H_a “coin is biased towards heads”

ML deployment

H_0 “The new model is not better than the legacy model”

H_a “The new model is better than the legacy model”