

# Analytics Assignment 3: GLM - Multinomial Regression

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## 1. Introduction

### 1.1 Generalized Linear Models

Generalized Linear Models (GLMs) are a class of statistical models that extend linear regression to handle a broader range of response variable distributions such as binomial, Poisson, and gamma, making them suitable for diverse types of data. Unlike traditional linear regression, GLMs are not constrained by the assumption of normality.

The Primary components of a GLM are:

1. **Response Variable:** with a distribution such as Binomial, Poisson etc.
2. **Linear Predictor:** A Linear combination of the predictor variables (similar to Linear Regression)
3. **Link Function:** to connect the linear predictor to the expected value of the response variable.

The General form of a GLM is:

$$g(\mu) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k$$

Where  $g(\mu)$  is the link function,  $\beta_i$  are the coefficients associated with the corresponding predictor variables  $X_i$

### 1.2 Link Function

A link function is used to connect the linear predictor to the expected value of the response variable i.e. it describes how the mean of the response variable is related to a linear combination of the predictor variables. The link function, denoted as  $g(\mu)$ , transforms the linear predictor into a scale appropriate for the response variable. The choice of the link function depends on the nature of the response variable and the distribution assumed for it some examples of link functions are **Logit**, **Inverse**, **Log** etc.

### 1.3 GLM: Multinomial Regression

Multinomial Regression is used when the response variable is categorical in nature and has more than 2 levels (categories). The link function used in Multinomial Regression is a **Logit** function which is defined as follows:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

some sample research questions which can be answered by multinomial regression include:

1. What Occupation are people most likely to chose based on their parents occupation and their own education
2. What food preferences will an animal have based on its size and habitat

## 2. Probability Distribution

### 2.1 Assumed Probability Distribution

Multinomial distribution assumes that the response variable has a multinomial distribution, which is a generalization of the binomial distribution for categorical variables with more than 2 categories.

The Probability Mass Function of a multinomial distribution is given by the equation:

$$Pr(X_1 = x_1 \text{ and } \dots \text{ and } X_k = x_k) = \frac{n!}{x_1! \dots x_k!} p_1^{x_1} \dots p_k^{x_k}$$

where n is the total number of observations or trials,  $x_i$  is the counts for each category and  $p_i$  are the probabilities for each category.

### 2.2 Support

The **Support** for multinomial regression is given by:

$$x_i \in \{0, \dots, n\}, i \in \{1, \dots, k\}, \text{ with } \sum_i x_i = n$$

This implies that the number of times each outcome  $x_i$  can occur is in the range 0 and the total number of observations n, and the sum of the count of all outcomes should add up to the total number of observations. k is the total number of possible categories of the outcome variable.

### 2.3 Parameters

The parameters for multinomial regression are that the number of trials “n” should be greater than 0 and the number of mutually exclusive events “k” should be greater than 0 with the probabilities of these events occurring  $p_i$  adding upto 1 ( $\sum_i p_i = 1$ )