Book of Proofs

# Fundamentals

## Sets

A set is often expressed by listing its element between commas enclosed by braces. For example, is a set with four elements which are 2, 4, 6, and 8.

A set can be an infinite set like a collection of all integers

#### Uppercase Alphabet set denotation

A set can be denoted using a uppercase alphabets like

#### Special Symbols for sets

- Natural number set symbol, - set of all integers symbol, - set of all real numbers, - empty set, - rational number.

#### Infinite set

A set with infinite number of elements in it. Special sets like , , and other specific infinite sets like

#### Finite Set

A set with a finite number of elements in it.

– set with 4 elements – 2, 4, 6, 8.

#### Element of a set symbol -

In the above example, 2 is an element of the set A. This can be denoted as . Similarly, 7 is not an element of the set A. It can be denoted as .

#### Equality of sets

Two sets can be equal if all the elements of the two sets are equal irrespective of its order.

#### Various elements in a set

A set can have numbers, coordinates, other sets, alphabets and other mathematical things in it.

Alphabetical lowercase letters can used to denote a set’s element. For example,

. Here a is representing a two-by-two matrix.

#### Cardinality or size of a finite set

It is denoted for example by . In this example cardinality of set A is 4;

#### Empty set denoted by .

It represents an empty set . A set with no elements within it.

#### Set builder notation

A set builder notation is used to denote complex sets in a simpler form.

For example, a set of all even integers can denote as , which holds the results .

Here the braces can be read as “a set of all things of the form” and colon as “such that”. Together, the above set builder notation can be read as E is equal to a set of all elements of the form *2n* such that *n is an element of integer set*.

Some authors interchange colon () with a bar (|).

#### Distinction of

If A is a number like -1, 1, etc., then means absolute value of A. If A is a set then represents the cardinality or size of the set A.

#### Intervals on a number line

In a number line, an infinite set interval can be by a dark segmented line. A solid circle denote an element is included in the interval. A hallow circle denote an elements is not included in the interval.

##### Closed Interval

##### Open Interval

##### Half-open Interval

##### Infinite Interval

The notation can denote both an open interval on a line and a point on a plane. But the difference is usually clear from the context.

### Exercises

1. Write each of the following sets by listing elements between braces.

. A set of Integers.

A set of Integers.

Other questions yet to complete.

## The Cartesian Product

The cartesian product of two sets is another set which is denoted as and defined as

Thus is a set of ordered pairs of elements of A and B.

#### Cardinality of a cartesian product

#### Ordered Pairs

A list of two things x and y enclosed with in parenthesis and separated by comma.

#### Cartesian Power

#### Exercises for Section 1.2

1. Write out the indicated sets by listing their elements between braces.
2. Suppose and

## Subsets

Suppose A and B are two sets. If every element of A is also an element of B, then A is a subset of B denoted as . If every element of A in not in B then A is not an subset of B denoted as .

An important fact .

If a set consists of n elements in it then the total possible subset of a set is determine using the equation .

## Power Sets

If A is a set, the power set of A is another set, denoted as and defined to be a set of all subsets of A. In symbols, .

If A is finite set, then .

## Union, Intersection, Difference

Just like number operations like addition, subtraction and multiplication. They are various operations on sets like Cartesian product, Union, Intersection and Difference.

Suppose A and B are two sets.

#### Union

The Union of A and B is a set

#### Intersection

The Intersection of A and B is a set

#### Difference

The difference of A and B is a set

## Complement

#### Universal Set

When dealing with set, we almost always regard it as a subset of some larger set. This larger set is called the universal set. For example, prime number set is a subset of . Denoted as

Let A be a set with a Universal set of U. The complement of .

## Venn Diagrams

# Note

Considering my short-term goals at this time (3/5/2020), the path of focused and deep learn of Book of Proof may be a deviation to my goal achievement. But it is important to learn this material to better comprehend the theory of algorithms. Come back and continue this material after achieving your goal.