MATH 1508 (Laboratory) Engineering Applications of PreCalculus

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C O N N E X I O N S

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Chapter 1

Units[1]

1.1 Units

1.1.1 Introduction

Engineering is a field of study that involves a very high level of calculations. Thus students of engineering must become familiar with a wide range of formulas and computational methods. Virtually all the calculations that engineers perform involve the use of units. Because many calculations involve the use of multiple units, an engineer must become competent in the process of unit conversions. Unit conversions allow an engineer with the ability to convert units in one system of measurement (say, the British system of measure) to those of another system (say, the System Internationale or SI system of measure.)

Units and unit conversion are important not just to engineers, but all members of society. Everyday activities such as driving an automobile, shopping at a grocery store, or visiting an pharmacy illustrate situations where an individual experiences units and unit conversions. Let us consider driving an automobile. A simple glance at a vehicle's speedometer reveals that the speed of the vehicle can be expressed in the units miles/hour or kilometers/hour. Depending upon the country in which you reside, gasoline is sold in the units of gallons or liters. At the grocery store, the volume of a can of your favorite soda is often expressed in terms of ounces or milliliters. Likewise, the dosage of cough syrup that you obtain from your local pharmacy can be expressed in terms of the units ounces or milliliters. This list of examples from everyday life that involve units can be expanded without bound.

Whenever an engineer deals with a physical quantity, it is essential that units be included. Units are especially important to engineers for they provide the ability for engineers to express their thoughts precisely and to provide meaning to the numerical values that result from engineering calculations. Units provide a means for engineers to communicate results among other engineers as well as laymen.

Units are an integral part of what could be called the language of engineering. As a student of engineering, you should become accustomed to the inclusion of units with virtually all your answers to engineering problems. Failure to include units with your numerical results can lead to your having points deducted from your grades on assignments, laboratory exercises and examinations.

1.1.2 Metric Mishaps

Failing to include the proper units with the results of engineering calculations can lead to unanticipated failures in engineering systems. Serious errors that result from the dual usage of metric and non-metric units are often grouped under the heading of **metric mishaps**. Some common examples of **metric mishaps** include the following:

[1]This content is available online at <http://cnx.org/content/m38522/1.2/>.

- According to the National Transportation Safety Board, confusion surrounding the use of **pounds** and **kilograms** often results in aircraft being overloaded and unsuited for flight.
- The Institute for Safe Medication Practices has reported that confusion between the units **grains** and **grams** is a common reason for errors associated with the dosage of medication.

1.1.3 A Notable Engineering Failure: The NASA Mars Climate Orbiter

In 1999, NASA experienced the failure of its Mars Climate Orbitor spacecraft because a Lockheed Martin engineering team used English units of measurement while a NASA engineering team used the more conventional metric system for a key spacecraft operation. (NASA, 1999). This mismatch in units prevented the navigation information from transferring properly as in moved between the Mars Climate Orbiter spacecraft team in at a Lockheed Martin ground station in Denver and the flight team at NASA's Jet Propulsion Laboratory in Pasadena, California.

Working with NASA and other contractors, Lockheed Martin helped build, develop and operate the spacecraft for NASA. Its engineers provided navigation commands for Climate Orbiters thrusters in British units although NASA had been using the metric system predominantly since at least 1990.

After a 286 day journey, the spacecraft neared the planet Mars. As the spacecraft approached the surface of Mars, it fired its propulsion engine to push itself into orbit. Instead of the recommended 276 kilometer orbit, the spacecraft entered an orbit of approximately 57 kilometers. Because the spacecraft was not in the proper orbit, its propulsion system overheated and was subsequently disabled. This allowed the Mars Climate Orbiter to plow through the atmosphere out beyond Mars. It is theorized that it could now be orbiting the sun

The primary cause of this discrepancy was human error. Specifically, the flight system software on the **Mars Climate Orbiter** was written to calculate thruster performance using the metric unit Newtons (N), while the ground crew was entering course correction and thruster data using the Imperial measure Pound-force (lbf). This error has since been known as the *metric mixup* and has been carefully avoided in all missions since by NASA.

1.1.4 Unit Conversion Procedure

The process of transforming from one unit of measure to another is called **unit conversion**. One can easily perform unit conversion using the procedure that will be presented in this section. You will soon discover that performing unit conversion can be reduced to multiplying one measurement by a carefully selected form of the integer 1 to produce the desired measurement.

Prior to presenting the procedure of unit conversion, it is important to understand a simple fact. Numbers with units such as 25.2 kilometers or 36.7 miles can be thought of and treated in exactly the same manner as coefficients that multiply variables, such as 25.2 x or 36.7 y. Of course here, x and y are variables.

From Algebra, we know that we can always multiply a quantity by 1 and retain its value. The key idea of unit conversion is to choose carefully the form of 1 that is used. We will illustrate this idea by means of an example.

Suppose that we wish to convert 25.2 kilometers to miles. In order to accomplish this conversion of units, it is important that one know the following information

$$1\text{km} = 0.621\text{mile} \tag{1.1}$$

Let us take this equation and divide each side by the term 1 km, as shown below

$$\frac{1\text{km}}{1\text{km}} = \frac{0.621\text{mile}}{1\text{km}} \tag{1.2}$$

Clearly the left hand side of this equation is equal to 1. That is

$$1 = \frac{0.621\text{mile}}{1\text{km}} \tag{1.3}$$

This will serve as our conversion factor to solve our problem at hand. It is important to remember that this conversion factor is nothing more than a carefully selected form of the number 1.

Let us return to the quantity 25.2 kilometers that we wish to convert to miles. We can apply the conversion factor as follows

$$25.2\text{km} \times \frac{0.621\text{mile}}{1\text{km}} = \frac{25.2\text{km} \times 0.621\text{mile}}{1\text{km}} \tag{1.4}$$

We observe that the unit (*km)* appears in both the numerator and denominator and can be removed from the from the fraction. So our result is

$$25.2\text{km} \times 0.621\text{mile} = 15.65\text{miles} \tag{1.5}$$

Thus we establish the result that 25.2 *km* is equivalent to 15.65 *miles*.

In obtaining the result, we developed a fraction that was equal to the integer 1. We then multiplied our original quantity by that fraction to give rise to our result. This is the basic idea behind unit conversion.

1.1.5 A Two-Step Procedure for Producing Correct Unit Conversion Factors

Here we will present a simple two-step procedure that produces the conversion factor that can be used to convert between a **given** unit and a **desired** unit. For the purpose of illustration, let us use the conversion between the **given** unit (**pounds**) and the (**desired**) unit of **kg**.

Step 1: We begin by writing an equation that relates the given unit and the required unit.

$$1\text{kg} = 2.205\text{lb} \tag{1.6}$$

Step 2: Convert the equation to fractional form with the desired units on top and the given units on the bottom.

$$\frac{1\text{kg}}{2.205\text{lb}} = \frac{2.205\text{lb}}{2.205\text{lb}} \tag{1.7}$$

$$\frac{0.454\text{kg}}{\text{lb}} = 1 \tag{1.8}$$

So to covert from **pounds** to **kg** we may use this as the proper conversion factor.

1.1.6 Another Notable Engineering Failure: The "Gimli Glider"

Like the NASA Mars Climate Orbiter, the "Gimli Glider" incident is an engineering failure that can be attributed directly to the errors involving the mismatch of units. The "Gimli Glider" is the nickname of the Air Canada commercial aircraft that was involved in an incident that took place on July 23, 1983. In the incident, a Boeing 767 passenger jet ran out of fuel at an altitude of 26,000 feet, about midway through its flight from Montreal to Edmundton via Ottawa. The aircraft safely landed at a former Canadian Air Force base in Gimli, Manitoba, thus contributing to the nickname associated with the aircraft. (New York Times, 1983) We will trace some of the steps that led to the incident while making use of data drawn from the website (Wikipedia).

Air Canada Flight 143 originated in Montreal. It safely arrived in Ottawa on its first leg. At that time, the pilot properly determined that the second leg of the flight (from Ottawa to Edmundton) would require 22,300 kilograms of jet fuel. The ground crew at the Ottawa airport, performed a dipstick check on the fuel tanks. They measured that there were 7,682 liters of fuel onboard the aircraft upon its arrival to Ottawa.

Based on these data, the air and ground crew proceeded to calculate the amount of jet fuel that would need to be transferred to the fuel tanks in order to assure safe arrival in Edmundton. However, they used an incorrect conversion factor in their calculations. At the time of the incident Canada was converting from the Imperial system of measurement to the metric system. The new Boeing 767 aircraft were the first of the Air Canada fleet to calibrated to the new system, using **kilograms** and **liters** rather than **pounds** and **Imperial gallons**.

The crew wished to convert the 7,682 liters of fuel to its equivalent went in kilograms. In order to do so the crew applied an incorrect conversion factor (1 liter of fuel weighs 1.77 kg.) Actually, 1 liter of fuel weighs 0.803 pounds, but the crew used an improper weight.

The crew inaccurately calculated ed the weight of the fuel onboard the aircraft to be 13,597 kilograms. The erroneous calculation is shown below.

$$7,682l \times \frac{1.77\text{kg}}{1l} = 13,597\text{kg} \tag{1.9}$$

Next, the crew went about determining the weight of fuel that would need to be transferred to the fuel tanks. They found this to be 8,703 kilograms, as shown in the differencing operation below

$$22,300\text{kg} - 13,597\text{kg} = 8,703\text{kg} \tag{1.10}$$

Finally, the volume of fuel in liters that needed to be transferred to the fuel tanks before departure for Edmundton was calculated. Once again, the erroroneous conversion factor was used as shown below

$$8,703\text{kg} \times \frac{1l}{1.77\text{kg}} = 4,916l \tag{1.11}$$

As consequence of these steps, the ground crew transferred 4,916 liters of jet fuel into the fuel tanks. Both the air and ground crews incorrectly believed that this volume of jet fuel (4,916 liters) would be sufficient to insure a safe arrival in Edmundton. Unfortunately, the sequence of calculations contained errors and the aircraft was forced to glide to a safe landing well short of its desired target.

Let us now examine the steps of calculation that should have been performed and that would have enabled a safe landing of the aircraft in Edmundton. We first determine the correct weight of fuel that remained in the fuel tanks upon the aircraft's arrival in Ottawa. Here, we use the proper conversion information. That is one liter of jet fuel weighs 0.803 kilograms.

$$1l = 0.803\text{kg} \tag{1.12}$$

$$\frac{1l}{1l} = \frac{0.803\text{kg}}{1l} \tag{1.13}$$

$$1 = \frac{0.803\text{kg}}{1l} \tag{1.14}$$

$$7,682l \times 1 = 7,682l \times \frac{0.803\text{kg}}{1l} = 6,169\text{kg} \tag{1.15}$$

So only 6,169 kilograms of fuel remained in the fuel tank when the aircraft landed in Ottawa. The next step involves the determination of how much fuel needed to be transferred to the fuel tank in order to accomplish the flight to Edmundton. This is properly computed by differencing the weight of the fuel needed to accomplish the flight to Edmundton and the weight of the fuel remaining in the fuel tanks.

$$22,300\text{kg} - 6,169\text{kg} = 16,131\text{kg} \tag{1.16}$$

Thus a quantity of jet fuel weighing 16,131 needed to be transferred by the ground crew into the fuel tanks in order to insure the safe arrival of the aircraft in Edmundton. This weight of fuel (kilograms) can be converted to a volume (liters) as follows

$$1l = 0.803\text{kg} \tag{1.17}$$

$$\frac{1l}{0.803\text{kg}} = \frac{0.803\text{kg}}{0.803\text{kg}} \tag{1.18}$$

$$\frac{1.245l}{\text{kg}} = 1 \tag{1.19}$$

Now this conversion factor can be used to determine the number of liters of fuel that should have been transferred to the fuel tanks.

$$16,131\text{kg} \times 1 = 16,131\text{kg} \times \frac{1.245l}{\text{kg}} = 20,088l \tag{1.20}$$

We conclude that 20,088 liters of fuel needed to be transferred to the fuel tank to successfully complete the leg of the flight from Ottawa to Edmundton. This represents approximately 4 times as many liters of fuel as was incorrectly calculated by the air and ground crew in 1983. The inadequacy in the provisioning of fuel resulted in the "Gimli Glider" having to perform an emergency landing well short of its desired arrival location. Due to some skillful piloting of the aircraft, no one onboard was seriously injured.

1.1.7 Summary

This chapter has attempted to illustrate the level of importance that engineering students should assign to the topic of units. In addition, a procedure that allows for the conversion of a quantity expressed in one unit to another unit has been presented. This method is quite simple in that all it requires is that one multiply the quantity expressed in the original unit to be multiplied by a fractional form that is equal to the integer 1. A process for correctly determining the proper fractional form is presented also.

Engineering students should view mastering the topic of units as an importance step in their formal education as an engineer. They should keep in mind that virtually all problems in engineering courses will involve solutions that include units.

Events surrounding the NASA Mars Climate Orbiter and the "Gimli Glider" show how seemingly small mistakes involving units and conversion factors can cause failures to complex systems.

1.1.8 Exercises

1. Convert 1,052,832 feet to miles, meters, kilometers, and yards. Express each answer using 3 significant digits of accuracy.
2. How many millimeters, centimeters and meters are in 62.8 inches? Use 1 inch = 2.54 centimeters. Express each answer using 3 significant digits of accuracy.
3. Find the range of temperature in degrees Fahrenheit (^{0}F) for the following range of temperatures in degrees centigrade/Celsius (^{0}C): -15^{0}C to +25^{0}C .
4. "Normal" body temperature is said to be 98.6^{0}F $\pm$ 0.6^{0}F. Convert these values to Celsius and give the answer in terms of minimum and maximum values.
5. If a computer file is 8.2 gigabytes and the effective transfer rate is 41 megabits per second, how long does it take to transfer the file from one location to another? Assume that 1 byte = 8 bits.
6. Homeostasis is the condition of keeping our bodies alive by regulating its internal temperature and maintaining a stable environment. Approximately 2,000 calories per day are required to maintain the human body. It is known that 1 calorie is equivalent to 4.184 joules and that one watt is equivalent to one joule per second. Determine the number of watts that are equivalent to 2,000 calories/day.

Chapter 2

Significant Digits[1]

2.1 Significant Digits

2.1.1 Introduction

Though easy to comprehend, significant digits play an important role in engineering calculations. In this module, the rules are presented that govern how to determine which digits present in a number are significant. In addition, several applications are used to illustrate how significant digits can be used to express the results of engineering calculations.

2.1.2 Basic Rules

A number can be thought of as a string of digits. Significant digits represent those digits present in a number that carry significance or importance to the precision of the number.

Rule 1: All digits other than 0 are significant. For example, the number 46 has two significant digits and the number 25.8 has three significant digits.

Rule 2: Zeros appearing anywhere between two non-zero digits are significant. Let us consider the number 506.72. The 0 that occurs between the 5 and 6 is significant according to Rule 2. Thus the number 506.72 has five significant digits.

Rule 3: Leading zeros are not significant. For example, 0.00489 has threeee significant digits.

Rule 4: Trailing zeros in a number containing a decimal point are significant. For example, the number 36.500 has five significant digits.

Rule 5: Zeroes at the end of a number are significant only if they are behind a decimal point. Let us consider 4,600 as the number. It is not clear whether the zeros at the end of the number are significant. As a result, there could be two, three or four significant digits present. To avoid ambiguity, one may express the number by means of scientific notation. If the number is written as 4.6×10^3, then it has two significant digits. If the number is written as 4.60×10^3, then it has three significant digits. Lastly, if the number is written as 4.600×10^3, then it has four significant digits.

2.1.3 Rounding

The concept of significant digits is often used in connection with rounding. Rounding to n significant digits is a more general-purpose technique than rounding to n decimal places, since it handles numbers of different scales in a uniform way.

Let us consider the population of a town. The population of the town might be known to the nearest thousand, say 12,000. Now let us consider the population of a state. The might be known only to the nearest

[1]This content is available online at <http://cnx.org/content/m38529/1.3/>.

million and might be stated as 12,000,000. The former number might be in error by hundreds while the latter number might be in error by hundreds of thousands of individuals. Despite this, the two numbers have the same significant digits. They are 5 and 2. This reflects the fact that the significance of the error (its likely size relative to the size of the quantity being measured) is the same in both cases.

The rules for rounding a number to n significant digits are:

Start with the leftmost non-zero digit (e.g. the "7" in 7400, or the "4" in 0.0456).

- Keep n digits. Replace the rest with zeros.
- Round up by one if appropriate. For example, if rounding 0.89 to 1 significant digit, the result would be 0.9.
- Or, round down by one if appropriate. For example, if rounding 0.042 to one significant digit, the result would be 0.04

2.1.4 Multiplication and Division

In a calculation involving multiplication, division, trigonometric functions, etc., the number of significant digits in an answer should equal the least number of significant digits in any one of the numbers being multiplied, divided etc.

The product, 4.56×3.456, involves the multiplication of a number with three significant digits and another with four significant digits. The product should be expressed as a number with three significant digits. Using a calculator the number, 15.75936, appears on the display of the device. It should then be rounded to three significant digits. So the product expressed using three significant digits is 15.8. Note that in this example, upward rounding was employed.

2.1.5 Addition and Subtraction

The results that one obtains through the operations of addition and subtraction follow the following rule. The result of addition and subtraction should have as many decimal places as the number with the smallest number of decimal places. For example the sum $13.678 + 2.59$ should be expressed as 16.27.

2.1.6 Something to Remember

When performing calculations, you are encouraged to keep as many digits as is practical. This practice will help you obtain more precise results by eliminating some of the error introduced by the rounding operation. Once you have a final answer, then one should go about applying the rules presented in this section to produce a result that is consistent from the standpoint of significant digits.

2.1.7 Example: Finding Current Using Ohm's Law

Let us put to work the ideas put forth earlier in the context of an example that involves calculations centering on Ohm's Law. Let us suppose that we are presented with the circuit shown in Fig. 1.

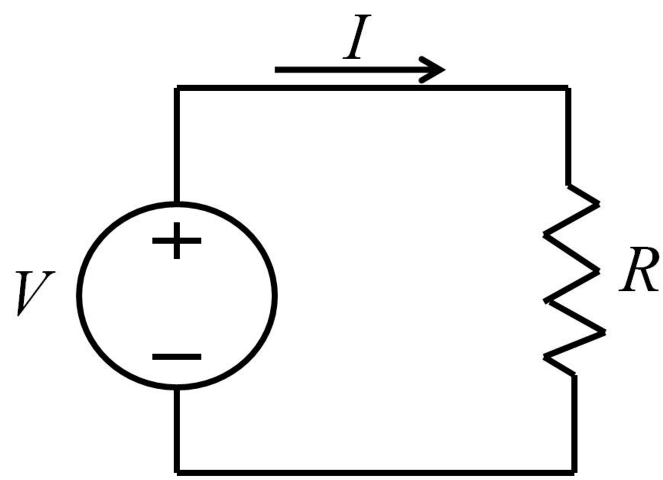

Figure 2.1: Electric circuit.

Suppose that the voltage (**V**) is 15 V and that the resistance is (**R**) 3.3 kΩ. What is the value of the current (**I**)?

Ohm's Law tells us that the current is the ratio of the voltage and the resistance.

$$I = \frac{V}{R} \tag{2.1}$$

Substituting the values for V and R into the equation yields the result

$$I = \frac{15V}{3,300\Omega} = 0.004545454A \tag{2.2}$$

Now, let us consider this result in the context of significant digits. Both the voltage and the resistance were expressed using two significant digits. We realize that the resulting value for the current (**I**) should contain two significant digits. We can accomplish this by expressing the result after rounding to two significant digit precision.

$$I = 0.0045A \tag{2.3}$$

Thus the answer to this example should be expressed as 0.0045 A.

We may choose to express this number using engineering notation and the result becomes

$$I = 4.5 \times 10^{-3}A = 4.5\text{mA} \tag{2.4}$$

This is the preferred manner for expressing the result.

2.1.8 Example: Finding Acceleration by means of Newton's Law of Motion

Sir Issac Newton was the first to formulate the relationship between force, mass and acceleration of an object. He found that a force (**F**) exerted on an object of mass (**m**) will produce an acceleration (**a**) according to the following equation

$$F = ma \tag{2.5}$$

Let us assume that we have a mass that weighs 120.6 kg. How much force is required to accelerate the mass at a rate of 26.1 m/s^2?

We recognize that the value for mass is expressed using four significant digits, while the value for the acceleration is expressed using only three significant digits. We substitute the values for mass and acceleration into Newton's Law of Motion

$$F = 120.6\text{kg} \times 26.2m/s^2 \tag{2.6}$$

$$F = 3147.66N \tag{2.7}$$

The number (3147.66) represents what is displayed on the calculator upon calculation of the product. This number has six significant digits. We wish our result to be expressed by the smaller of the number of significant digits contained in the two numbers that were multiplied. In this case the number should be expressed using just three significant digits. So the result expressed using three significant digits is

$$F = 3,150N \tag{2.8}$$

In order to remove any uncertainty relating to the trailing 0, the result can be written in engineering notation as

$$F = 3.15\text{kN} \tag{2.9}$$

This is the preferred form for the solution.

2.1.9 Exercises

1. Express the quantity 5.342 015 m using 3 significant digits.
2. Express the quantity 5.347 015 m using 3 significant digits.
3. How many significant digits are present in the number 0.1256? Repeat for 0.01256? Repeat for 0.012560?
4. A 5 V source is connected to a 22 Ω resistor. Express the current using 3 significant digits.
5. A 9 V source is connected to a resistor whose value is unknown. The current of the circuit is measured and found to be 125 mA. Express the value of the resistance using 3 significant digits.
6. A current measured to be 35.7 mA passes through a 2.36 MΩ resistor. Express the value of the voltage across the resistor using 3 significant digits.
7. The power delivered by an electrical source is equal to the product of the voltage of the source and the current that flows out of the source. Suppose that the voltage is 120 V and that the current is 856.7 mA. Express the power delivered by the source using 3 significant digits.
8. A mass of 5.00 kg undergoes an acceleration of 2.37 m/s^2. Express the force that is needed to produce this acceleration using 3 significant digits.
9. A force equal to 2.69 N is applied to a mass of 8.23 kg. Express the resulting acceleration using 3 significant digits.
10. A 78.9 kg object is accelerated at a rate of 2.10 m/s^2. How much force is required to produce this acceleration?

Chapter 3

Adding Fractions[1]

3.1 Adding Fractions

3.1.1 Introduction

In order to enjoy success as an engineer, it is important to learn how to add fractions. In this module, you will learn to add fractions using the lowest common denominator (LCD) method. Also, you will learn the role that the addition of fractions plays in determining the equivalent resistance of resistors connected in parallel.

3.1.2 Lowest Common Denominator (LCD) Method

In the course of working algebraic problems, one often encounters situations that require the addition of fractions with unequal denominators. For example, let us consider the following

$$\frac{3}{10}+\frac{1}{12} \tag{3.1}$$

In order to add the two fractions, it is important to rewrite each fraction with the same denominator. In order to accomplish this, we begin by expressing the denominator of the first fraction in terms of a product of its factors

$$10=2\times 5 \tag{3.2}$$

We do the same with the denominator of the second fraction

$$12=2\times 2\times 3=2^2\times 5 \tag{3.3}$$

We can express the lowest common denominator as

$$\mathrm{LCD}=2^2\times 3\times 5=60 \tag{3.4}$$

We proceed to express the sum of fractions using the lowest common denominator just found

$$\frac{3}{10}+\frac{1}{12}=\frac{3\times 6}{10\times 6}+\frac{1\times 5}{12\times 5}=\frac{18}{60}+\frac{5}{60}=\frac{23}{60} \tag{3.5}$$

Thus we obtain the result of the addition as 23/60.

[1] This content is available online at <http://cnx.org/content/m38554/1.3/>.

Let us consider another example in which three fractions are to be added

$$\frac{2}{3}+\frac{3}{2}+\frac{5}{7} \tag{3.6}$$

The denominator of each fraction is a prime number, so the lowest common denominator is their product

$$\mathrm{LCD}=3\times 2\times 7=42 \tag{3.7}$$

We must rewrite each fraction as an equivalent fraction with a denominator of 42

$$\frac{2}{3}=\frac{2\times 2\times 7}{3\times 2\times 7}=\frac{28}{42} \tag{3.8}$$

$$\frac{3}{2}=\frac{3\times 3\times 7}{3\times 2\times 7}=\frac{63}{42} \tag{3.9}$$

$$\frac{5}{7}=\frac{5\times 3\times 2}{3\times 2\times 7}=\frac{30}{42} \tag{3.10}$$

Next, we express the sum of fractions using those equivalent fractions just determined

$$\frac{2}{3}+\frac{3}{2}+\frac{5}{7}=\frac{28}{42}+\frac{63}{42}+\frac{30}{42}=\frac{28+63+30}{42}=\frac{121}{42} \tag{3.11}$$

So 121/42 is the desired result.

3.1.3 Application: Combining Resistors in Parallel

Figure 1 depicts a physical device known as a resistor. A resistor is often used in an electrical circuit to control the amount of current that flows throughout the circuit. The relationship between voltage, current and resistance in an electric circuit is governed by a fundamental law of Physics known as Ohm's Law. Stated in words, Ohm's Law tells us that the potential difference (**V**) measured in Volts across a resistor is directly proportional the current (**I**) measured in Amps that flows through the resistor. Additionally, the constant of proportionality is the value of the resistance (**R**), measured in Ohms. Ohm's Law can be stated mathematically as

$$V=IR \tag{3.12}$$

Photograph of a resistor.

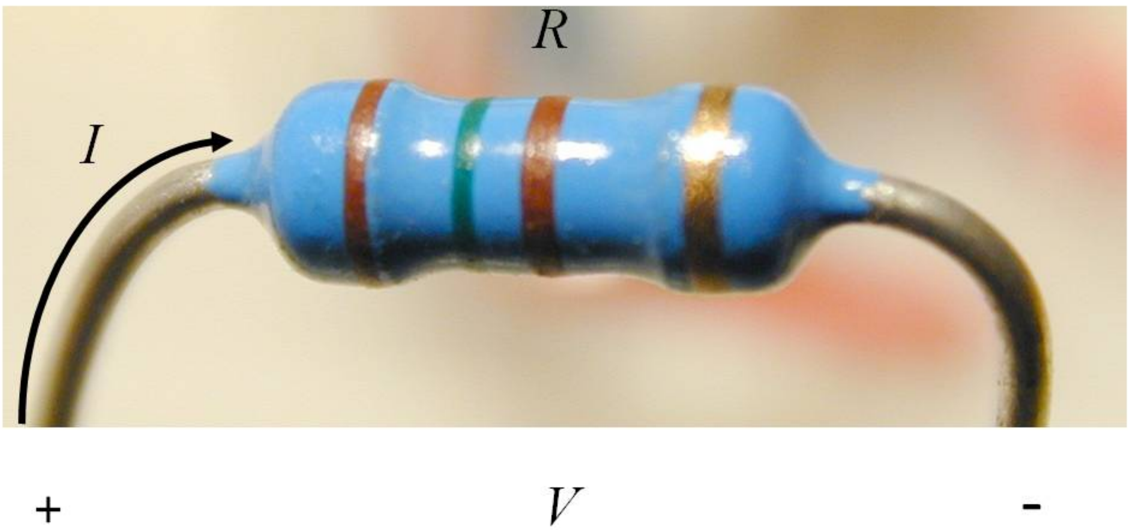

Figure 3.1

Instead of a single resistor, electric circuits often employ multiple resistors. Let us consider the case where we have two resistors that are denoted as $\mathbf{R}_1$ and $\mathbf{R}_2$.

The two resistors can be connected in an end-to-end manner as shown in Figure 2 (a).

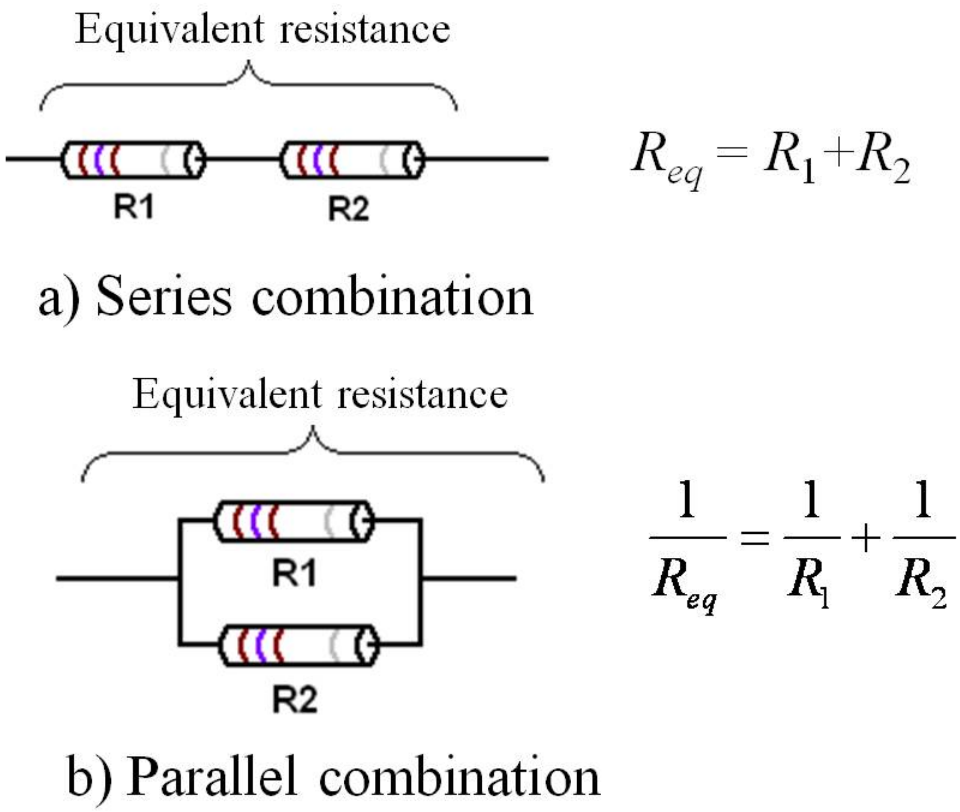

Figure 3.2: Series and parallel connections of resistors.

Resistors connected in this manner are said to be **connected in series**. We may replace a series connection of two resistors by a single equivalent resistance, $\mathbf{R_{eq}}$. From an electrical standpoint, the single equivalent resistance will behave exactly the same as the combination of the two resistors connected in series. The equivalent resistance of two resistors connected in series can be calculated by summing the resistance value of each of the two resistors.

$$R_{\text{eq}} = R_1 + R_2 \tag{3.13}$$

Let us now consider the case where the two resistors are placed side-by-side and then connected at both ends. This situation is depicted in Figure 2 (b) and is called a parallel combination of resistors. Whenever resistors are connected in this manner, they are said to be connected in parallel. The equivalent resistance of two resistors connected in parallel obeys the following relationship.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{3.14}$$

Once this quantity is calculated, one may easily take the reciprocal of the result to obtain the value of $\mathbf{R_{eq}}$.

The rule governing the determination of the equivalent resistance for the series connection of more than two resistors can be expanded to accommodate any number (**n**) of resistors. For **n** resistors connected in series, the equivalent resistance is equal to the sum of the **n** resistance values.

In addition, the rule governing the determination of the equivalent resistance for the parallel connection of more than two resistors can be expanded. For **n** resistors connected in parallel,

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n} \tag{3.15}$$

It is clear to see that one's ability to determine the equivalent resistance of parallel resistors depends upon one's ability to add fractions. The following exercises are included to reinforce this idea.

Example 1: Two resistors are connected in series. The value of resistance for the first resistor is 5 Ω, while that of the second is 9 Ω. Find the equivalent resistance of the series combination.

$$R_{\text{eq}} = 5\Omega + 9\Omega = 14\Omega \tag{3.16}$$

Example 2: Consider the two resistors presented in Example 1. Let the two resistors now be connected in parallel. Find the equivalent resistance of the parallel combination.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{5\Omega} + \frac{1}{9\Omega} \tag{3.17}$$

The lowest common denominator is 45 Ω. So we incorporate it into our analysis.

$$\frac{1}{R_{\text{eq}}} = \frac{9}{45\Omega} + \frac{5}{45\Omega} \tag{3.18}$$

$$\frac{1}{R_{\text{eq}}} = \frac{14}{45\Omega} \tag{3.19}$$

$$R_{\text{eq}} = \frac{45\Omega}{14} = 3.21\Omega \tag{3.20}$$

Example 3: Three resistors of values 2 kΩ, 3 kΩ, and 5 kΩ are connected in parallel. Find the equivalent resistance.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{2,000\Omega} + \frac{1}{3,000\Omega} + \frac{1}{5,000\Omega} \tag{3.21}$$

The lowest common denominator for the fractional terms is

$$\text{LCD} = 2 \times 3 \times 5 \times 1,000\Omega = 30,000\Omega \tag{3.22}$$

We rewrite the original equation to reflect the lowest common denominator

$$\frac{1}{R_{\text{eq}}} = \frac{15}{30,000\Omega} + \frac{10}{30,000\Omega} + \frac{6}{30,000\Omega} \tag{3.23}$$

$$\frac{1}{R_{\text{eq}}} = \frac{31}{30,000\Omega} \tag{3.24}$$

$R_{\text{eq}} = \frac{30,000\Omega}{31} = 968\Omega$

3.1.4 Summary

This module has presented how to add fractions using the lowest common denominator method. Also presented is the relationship among voltage, current and resistance that is known as Ohm's Law. Examples illustrating the use of the lowest common denominator method to solve for equivalent resistances of parallel combinations of resistors are also provided.

3.1.5 Exercises

1. Consider a 10 kΩ and a 20 kΩ resistor. (a) What is the equivalent resistance for their series connection? (b) What is the equivalent resistance for their parallel connection?
2. Consider a parallel connection of three resistors. The resistors have values of 25 Ω, 75 Ω, and 100 Ω. What is the equivalent resistance of the parallel connection?
3. Consider a parallel connection of four resistors. The resistors have values of 100 Ω, 100 Ω, 200 Ω and 200 Ω. What is the equivalent resistance of the parallel connection?
4. Conductance is defined as the reciprocal of resistance. Conductance which is typically denoted by the symbol, G, is measured in the units, Siemens. Suppose that you are presented with two resistors of value 500 Ω and 1 kΩ. What is the conductance of each resistor?
5. The equivalent conductance of a parallel connection of two resistors is equal to the sum of the conductance associated with each resistor. What is the equivalent conductance of the parallel connection of the resistors described in exercise 4?
6. What is the equivalent resistance of the parallel connection of resistors described in exercise 5?
7. Suppose that you are presented with 2 resistors. Each resistor has the same value of resistance (say, R). Derive an expression for the equivalent resistance of their parallel connection.
8. Three resistors with resistance values of 100 kΩ, 50 kΩ, and 100 kΩ are connected in parallel. What is the equivalent resistance? (Hint: You may use the result of Exercise 2 to simplify your work.)
9. Four resistors, each with a value of 10 Ω, are connected in parallel. What is the equivalent conductance of the parallel connection? What is the equivalent resistance of the parallel connection?
10. A 30 Ω resistance is connected in series with a parallel connection of two resistors, each with a value of 40 Ω. What is the equivalent resistance of this series/parallel connection?

Chapter 4

Exponents[1]

4.1 Exponents

4.1.1 Introduction

Exponentiation is a mathematical operation that is employed extensively in applications in fields that range from science and engineering to finance and economics. This module will begin with a brief discussion of the terminology and the mathematics involved with exponentiation. This will be followed with some applications of exponentiation in science and engineering.

Scientific notation and engineering notation are introduced in this module as two common means for expressing physical quantities. Each of these representational schemes involve the use of exponents. Applications are presented on topics including electrical power, gravitatitational force and electrostatic force are presented as a means for illustrating how exponentiation can be useful in problem solving.

4.1.2 Definition and Terminology

Exponentiation is an operation in mathematics that makes use of two numbers known as the base (x) and the exponent (n) and is expressed as

$$x^n \tag{4.1}$$

In this module, we will restrict our discussion to situations where the exponent (n) is an integer. Being an integer, the exponent may be positive, negative or equal to 0. We will consider each case below.

If the exponent n is a positive integer, then the operation of exponentiation is equivalent to the multiplication of the base x with itself a total of n times.

In the situation where the base (x) is 5 and the exponent (n) is 3, we obtain the result

$$5^3 = 5 \cdot 5 \cdot 5 = 125 \tag{4.2}$$

Now, let us the case where the exponent is a negative integer. In this case, the process of exponentiation is equivalent to dividing one by the base x by n times.

Let us consider an example where the base (x) is 5, but now the exponent (n) is -3. In this example, we obtain the result

$$5^{-3} = \frac{1}{5 \cdot 5 \cdot 5} = \frac{1}{125} = 0.008 \tag{4.3}$$

[1]This content is available online at <http://cnx.org/content/m38520/1.3/>.

Suppose that we form the product of 5^3 with 5^{-3}.

$$5^3 \times 5^{-3} = 125 \times 0.008 \tag{4.4}$$

$$5^{3-3} = 1 \tag{4.5}$$

$$5^0 = 1 \tag{4.6}$$

The result of the multiplication reminds us of the property of exponents that states whenever a base (x) is raised to an exponent (n) that is zero, the result is 1.

4.1.3 Scientific and Engineering Notation

One of the most important uses of exponents in the fields of science and engineering is that of scientific notation. As is often the case, in the fields of science and engineering one often deals with numbers that are extremely large or extremely small. Scientific notation is an effective means for writing such number that makes use of exponents. In many cases, scientific notation enables one to write very large or very small numbers in a manner that is more convenient, insightful and compact that writing numbers using decimal notation.

Numbers can be written in scientific notation in the following form

$$a \times 10^b \tag{4.7}$$

where a is the coefficient which can be any real number in the range 1 < |a| <10 and b is an integer that represents the exponent.

Example (Avogadro's Number)

In chemistry, the quantity of an element having a weight in grams numerically equal to that element's atomic weight is the *gram atomic weight* of that element. This quantity is often refererred to as a *gram atom*. A gram atom of any element contains the same number of atoms as the gram atom of any other element. The number of atoms in any gram atom is called *Avogadro's number* (N). Through meticulous experimental study, the value of Avogadro's number has been determined as

$$N = 602,300,000,000,000,000,000,000 \tag{4.8}$$

Question: *How can Avogadro's number be expressed in scientific notation?*

Solution: Avogadro's number can be written as the product of a real number (6.023) by the term (100,000,000,000,000,000,000,000)

$$N = 6.023 \times 100,000,000,000,000,000,000,000 \tag{4.9}$$

$$N = 6.023 \times 10^{23} \tag{4.10}$$

Rather than trying to perform calculations using the decimal form for Avogadro's number, chemists have learned to appreciate the more mathematically tractable scientific notation form of this important constant.

Engineering notation is quite similar to scientific notation. Numbers are written in engineering notation in the following form

$$a \times 10^c \tag{4.11}$$

where the coefficient a is a real number in the range 1 < |a |< 1,000 and the exponent c is restricted to be an integer multiple of 3.

An additional restriction on the coefficient (a) requires that a be expressed using no more than 3 significant digits. The restriction that the exponent be an integer multiple of 3 allows the numbers that result from the transformation to engineering notation to be expressed using the standard prefixes associated with the *Scientifique Internationale* (SI) system of units.

Example (Width of the Asteroid Belt)

Let's consider the following application of engineering notation. The width of the asteroid belt has been determined to be 280,000,000 m.

Question: *What is the width of the asteroid belt expressed in engineering notation?*

Solution: Let us begin by expressing this quantity using scientific notation

$$280,000,000m = 2.80 \times 10^8 m \tag{4.12}$$

We notice that the exponent is not an integer multiple of 3, so this quantity is not yet expressed in engineering notation. We do know that

$$10^8 = 10^{2+6} = 10^2 \times 10^6 \tag{4.13}$$

This quantity can be substituted into the previous equation to yield the expression for the width of the asteroid belt in engineering notation

$$280,000,000m = 2.80 \times 10^8 m \tag{4.14}$$

$$280,000,000m = 2.80 \times 10^2 \times 10^6 m \tag{4.15}$$

$$280,000,000m = (2.80 \times 100) \times 10^6 m \tag{4.16}$$

$$280,000,000m = 280 \times 10^6 m \tag{4.17}$$

$$280,000,000m = 280\text{Mm} \tag{4.18}$$

4.1.4 Application: Electrical Power

We will begin our investigation of applications of exponents with a discussion of electrical power. Consider the electrical circuit diagram that shows a source voltage (V) attached to a resistor (R) to produce a current (I).

The relationship between V, R and I is summarized by Ohm's Law

$$V = I \times R \tag{4.19}$$

where V is measured in volts (V) , I is measured in amps (A), and R is measured in ohms (Ω).

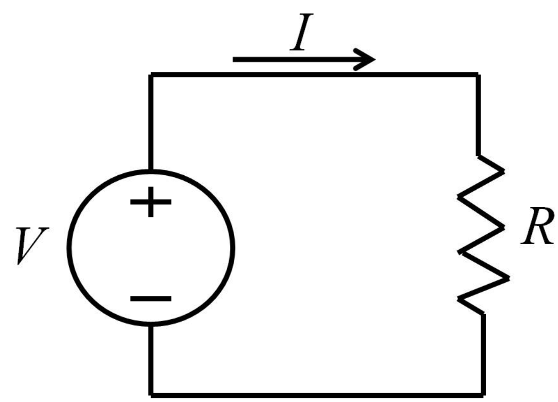

Figure 4.1: Simple electric circuit.

We may write an expression for the current as

$$I = \frac{V}{R} \tag{4.20}$$

The power that is absorbed by the resistor is known to be the product of the current flowing through the resistor time the potential difference (voltage) across the terminals of the resistor. If we denote the power absorbed by the resistor as PR, then it can be expressed mathematically as

$$P_R = I \times V \tag{4.21}$$

Substitution of the expression for I obtained from Ohm's Law yields an equivalent expression for the power absorbed by the resistor

$$P_R = \left(\frac{V}{R}\right) \cdot V = \frac{V^2}{R} \tag{4.22}$$

Paying attention to the exponent, we can say that the power absorbed by the resistor is the *square of the voltage* across the terminals of the resistor divided by the resistance. The units associated with power are Watts (W).

We can obtain an alternative form for the power absorbed by the resistor. We begin with the general expression for the power absorbed by a resistor

$$P_R = I \times V \tag{4.23}$$

If we now substitute the expression for V provided by Ohm's Law into this equation, we obtain an alternative expression for power

$$P_R = I \cdot (I \times R) \tag{4.24}$$

$$P_R = I^2 R \tag{4.25}$$

This relationship expresses the electrical power absorbed by the resistor as a product of the *square of the current* by the resistance.

Thus we have two equally useful formulas for the computation of the power absorbed by a resistor. One involves the square of the voltage, while the other incorporates the square of the current. Each formula involves a term that is raised to an exponent of 2.

Example (Electrical Power)

A 9 Volt battery is connected to a single 10 kΩ resistor.

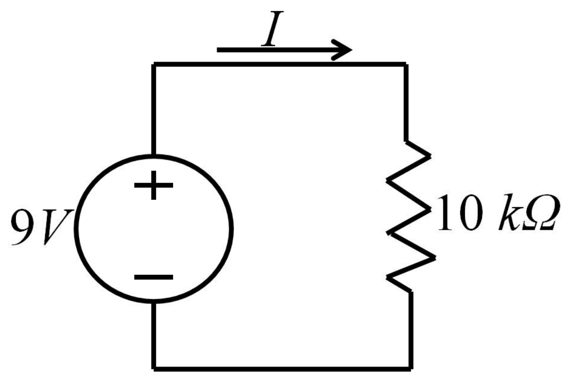

Figure 4.2: Electric circuit.

Question: *How much power is dissipated in the resistor? (Express the result in engineering notation.)*

$$P_R = \frac{(9V)^2}{10 \times 10^3 \Omega} \tag{4.26}$$

$$P_R = \frac{81V^2}{10^4 \Omega} \tag{4.27}$$

$$P_R = 81 \times 10^{-4} \left(\frac{V^2}{\Omega} \right) \tag{4.28}$$

$$P_R = 81 \times \left(10^{-1} \times 10^{-3} \right) W \tag{4.29}$$

$$P_R = \left(81 \times 10^{-1} \right) \times 10^{-3} W \tag{4.30}$$

$$P_R = 8.1 \times 10^{-3} W \tag{4.31}$$

$$P_R = 8.1 \mathrm{mW} \tag{4.32}$$

4.1.5 Gravitational Force

Every object of finite mass near the surface of the earth experiences a force that pulls it downward. This force is called weight. In the 17^{th} Century, Sir Isaac Newton postulated that this force which attracts objects to the earth might well be a particular example of a more general kind of interaction between objects. Newton was engaged in the study of the motion of planets around the sun and felt that the planets should obey the same laws of motion as objects falling toward the earth due to their weight. In one of humankind's greatest bits of insight, Newton discovered that the forces responsible for both planetary motion and the phenomenon of weight could be equally understood if one hypothesized that every object of finite mass attracts every other object of finite mass with a force that depends upon the individual masses of the objects. Such forces are known as gravitational forces and they serve as a basis for what is now known as *Newton's law of gravitation*.

Let us consider two objects of respective masses *m1* an *m2* that are separated by a distance that we denote as r. Newton's law of gravitation states that each object will exert on the other an attractive force, directed along a straight line connecting the two objects. The magnitude of the force that mass m_1 exerts on m_2 is equal to the magnitude of the force that m_2 exerts on m_1. The magnitude of the force that the first mass exerts on the second is provided by the equation

$$F = G\frac{m_1 m_2}{r^2}. \tag{4.33}$$

Here, G represents the universal gravitational constant and is given by

$$G = 6.67 \times 10^{-11} N - m^2/\mathrm{kg}^2. \tag{4.34}$$

Because the universal constant is so small, the gravitational forces that exists between objects of ordinary mass are extremely small.

We can incorporate the value for G along with the use of exponents in the equation for the magnitude of the force to yield the expression that follows.

$$F = \left(6.67 \times 10^{-11} \mathrm{Newton} - \mathrm{meter}^2/\mathrm{kg}^2\right)(m_1 m_2)\, r^{-2} \tag{4.35}$$

Thus we can see that exponents play an important role in the formulation of Newton's law of gravitation. Exponents are important to the expression of the universal gravitational constant and the attractive force between two objects depends upon the reciprocal of the square of the distance between the objects.

Example (Gravitational Force)

The mass of the sun is 2×10^{30}kg. The mass of the earth is 5.97×10^{24}kg. The approximate distance between the sun and the earth is 149.6×10^6km.

Question: What is the magnitude of the gravitational force that the sun exerts on the earth? (Express the result in scientific notation.)

Solution:

$$F = \left(6.67 \times 10^{-11} N - m^2/\mathrm{kg}^2\right)\left(2 \times 10^{30}\mathrm{kg}\right)\left(5.97 \times 10^{24}\mathrm{kg}\right) / \left(149.6 \times 10^6\mathrm{km}\right)^2 \tag{4.36}$$

$$F = \left(6.67 \times 10^{-11} N - m^2/\mathrm{kg}^2\right)\left(2 \times 10^{30}\mathrm{kg}\right)\left(5.97 \times 10^{24}\mathrm{kg}\right) / \left(149.6 \times 10^9 m\right)^2 \tag{4.37}$$

$$F = \left(6.67 \times 10^{-11} N - m^2\right)\left(2 \times 10^{30}\right)\left(5.97 \times 10^{24}\right) / \left(149.6 \times 10^9 m\right)^2 \tag{4.38}$$

$$F = \left(6.67 \times 10^{-11} N - m^2\right)\left(2 \times 10^{30}\right)\left(5.97 \times 10^{24}\right) / \left(22,380 \times 10^{18} m^2\right) \tag{4.39}$$

$$F = \left(6.67 \times 10^{-11} N\right)\left(2 \times 10^{30}\right)\left(5.97 \times 10^{24}\right) / \left(22,380 \times 10^{18}\right) \tag{4.40}$$

$$F = (6.67 \times 2 \times 5.97) \times \left(10^{-11} \times 10^{30} \times 10^{24}\right) N / \left(2.238 \times 10^{22}\right) \tag{4.41}$$

$$F = \left(79.64 \times 10^{43}\right) N / \left(2.238 \times 10^{22}\right) \quad (4.42)$$

$$F = (79.64/2.238) \times 10^{43-22} N \quad (4.43)$$

$$F = (35.6) \times 10^{21} N \quad (4.44)$$

$$F = 3.56 \times 10^{22} N \quad (4.45)$$

4.1.6 Coulomb's Law

Electrons and protons are examples of charged particles. In the case of the proton, charge is positive. The charge of a proton is thus said to have positive polarity. On the other hand, an electron has a negative charge. Thus the charge of an electron is said to have negative polarity.

An *electrostatic force* exists between two charged particles. If the charges associated with the two particles are of the same polarity, the electrostatic force will be *repulsive*. On the other hand if the charges for two particles have opposite polarity, the electrostatic force will be *attractive*.

Through extensive laboratory work, the physicist Charles Coulomb first established a mathematical expression that calculates the magnitude of the electrostatic force that results from the interaction of two charged particles. This expression which is known as Coulomb's law for electrostatic forces is given by

$$F = k\left(\frac{qQ}{r^2}\right) \quad (4.46)$$

where q and Q are the values of the charges measured in the units Coulombs, r is the distance measured in meters that separates the charged particles and k is a constant

$$k = 9 \times 10^9 \text{Newton} - \text{meters}^2/\text{Coulomb}^2 \quad (4.47)$$

It is interesting to note the similarity of the general form of Coulomb's law for electrostatic forces with Newton's law of gravitational force. Just as was the case with the gravitational force, the magnitude of the electrostatic force decreases with the *square of the distance* separating the two particles.

Example (Electrostatic Force)

The charge on an electron is given by the equation

$$q_e = -1.60 \times 10^{-19} \text{Coulombs}\,(C) \quad (4.48)$$

The charge associated with a proton is

$$q_p{=}{+}1.60 \times 10^{-19} \text{Coulombs}\,(C) \quad (4.49)$$

Suppose that an electron and a proton are separated by a distance of 10^{-6}nanometers.

Questions: What is the magnitude of the electrostatic force that the protonexerts on the electron? Is the force attractive or repulsive?

Solution:

$$F = k\frac{q_e q_p}{r^2} \quad (4.50)$$

$$F = \frac{\left(9 \times 10^9 N - m^2/C^2\right)\left(-1.60 \times 10^{-19} C\right)\left(1.60 \times 10^{-19} C\right)^2}{\left(10^{-6} \times \text{nanometers}\right)} \quad (4.51)$$

$$F = \frac{\left(9 \times 10^9\right)\left(-1.60 \times 10^{-19}\right)\left(1.60 \times 10^{-19}\right) N - m^{2^2}}{\left(10^{-6} \times 10^{-9} m\right)} \quad (4.52)$$

$$F = \frac{(9)(-1.60)(1.60)\left(10^9 \times 10^{-19} \times 10^{-19}\right) N - m^2}{\left(10^{-15} m\right)} \quad (4.53)$$

$$F = \frac{\left(23.0 \times 10^{-29}\right) N^2}{\left(10^{-15}\right)} \quad (4.54)$$

$$F = 23.0 \times 10^{-29} \times 10^{30} N \quad (4.55)$$

$$F = 23.0 \times 10^1 N \quad (4.56)$$

$$F = 230N \quad (4.57)$$

Because the charges on the electron and the proton differ in polarity, the electrostatic force is *attractive*.

4.1.7 Exercises

1. The weight of the Space Shuttle is 4,470,000 lb. Express this weight in scientific notation.
2. The weight of a honey bee is 0.000 385 05 lb. Express this weight in scientific notation.
3. Perform the following multiplications. (a) (37.5 x 10^7) x (2.87 x 10^5), (b) (37.5 x 10^7) x (2.87 x 10^{-5}).
4. Perform the following divisions. (a) (37.5 x 10^7) / (2.87 x 10^5), (b) (37.5 x 10^7) / (2.87 x 10^{-5}).
5. The area of the United States is 9.83 x 10^6 km^2. Its population is 3.10 x 10^6. What is the population density (people/km^2) of the United States?
6. The distance between the Sun and the Earth is 1.47 x 10^{11} m. The speed of light is 299,792,458 m/s. How long does it take for light to travel between the Sun and the Earth?
7. The planned Blythe Solar Power Plant in California is expected to produce 968 MW. The Aswan Dam Power Plant in Egypt produces 2.1 gigawatts of power. How may solar power plants with similar power production as that of the Blythe Solar Power Plant would be needed to match the power production of the Aswan Dam Power Plant?
8. An electric circuit consists of a 9.00 V battery in series with a load that has a resistance of 13.78 kΩ. (a) Find the power delivered by the battery. (b) Find the power absorbed by the load.
9. The mass of the Earth is 5.97 x 10^{24} kg, while that of the moon is 7.36 x 10^{22} kg. The average distance between the Earth and the Moon is 384,403 km. What is he force exerted on the Moon by the Earth?
10. Two charges are separated by 4.66 x 10^{-8} m. Each has a positive charge of 3.6 x 10^{-18} Coulombs. What is the repulsive force that they exert on one another?

Chapter 5

Linear Equations[1]

5.1 Linear Equations

5.1.1 Introduction

This module is intended to illustrate concepts related to the solution of engineering problems using straight lines. It has formed the basis of a Laboratory session associated with a MATH 1508 (Precalculus) course taught at the University of Texas at El Paso. The examples contained herein are drawn from the fields of fluid mechanics, mechanics, and electric circuits. Exercises are included at the end of this module.

5.1.2 Fluid Mechanics - Continuity Equation

Figure 1 illustrates a piping system that consists of two pipes. Pipe1 has a radius of r_1 while the radius of Pipe 2 is r_2. These two pipes are joined so that water can pass from the left to the right.

Quite some time ago, engineers observed that the velocity of a fluid in Pipe 1 (v_1) for such a system would be quite a bit lower in value than the velocity of the fluid in Pipe 2 (v_2). As a consequence, engineers sought a means for determining the relationship between the velocity of fluids in the two pipes that comprise this sort of system.

Their solution to the problem of determining the relationship between the two velocities (v_1 and v_2) is provided by a very important principle in fluid mechanics that is called the *continuity equation*. The continuity equation states

$$A_1 v_1 = A_2 v_2 \tag{5.1}$$

where A_1 represents the cross-sectional area of Pipe 1 and A_2 represents the cross-sectional area of Pipe 2. For the continuity equation to be valid it is important to recognize that the flow of water be continuous as it passes from the first to the second pipe.

[1] This content is available online at <http://cnx.org/content/m38533/1.3/>.

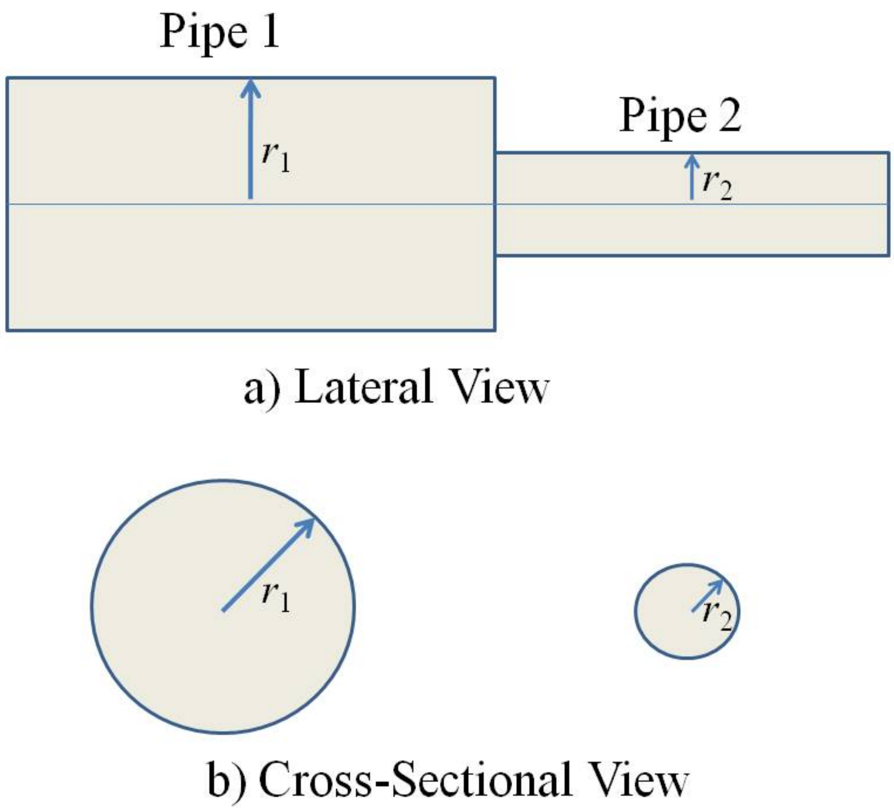

Figure 5.1: Connection of two pipes of different sizes.

By rearranging terms in equation (1), we can formulate an equivalent expression that may be a bit more insightful

$$\frac{A_1 v_1}{A_2} = \frac{A_2 v_2}{A_2} \tag{5.2}$$

$$\left(\frac{A_1}{A_2}\right) v_1 = v_2 \tag{5.3}$$

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 \tag{5.4}$$

Thus we can conclude that the ratio of the area of the two pipes provides a multiplicative constant relating the velocities of the fluid in the two pipes.

We recall that the equation for a straight line takes the form

$$y = \mathrm{mx} + b \tag{5.5}$$

Equation (4) indicates that there is a linear (straight line) relationship between the velocity ($\mathbf{v}_2$) of the fluid in the second pipe and the velocity ($\mathbf{v}_1$) of the fluid in the first pipe. If we establish v_1 as the independent variable, then the dependent variable v_2 is defined as a linear function of v_1. The slope (m) of the line is the ratio of the cross-sectional areas $\left(\frac{A_2}{A_1}\right)$, while the y-intercept (**b**) is zero.

Let us now apply our knowledge of the continuity equation to a problem.

Question: Suppose that the radius of Pipe 1 is 4.00 cm and the radius of Pipe 2 is 2.50 cm. The velocity of the water in Pipe 1 is measured to be 3.00 m/sec. Find the velocity of the water in Pipe 2.

Solution: We begin our solution by establishing the cross-sectional areas of each pipe. Because the cross-sectional profile of each pipe is a circle, we may compute the two cross-sectional areas using the formula for the area of a circle:

$$A_1 = \pi r_{1^2} = \pi \left(4.00\text{cm}\right)^2 \tag{5.6}$$

$$A_2 = \pi r_{2^2} = \pi \left(2.50\text{cm}\right)^2 \tag{5.7}$$

We can apply equation (4) to find the velocity of the fluid in Pipe 2. The steps are shown below

$$v_2 = \left(\frac{\pi \left(4.00\text{cm}\right)^{2^2}}{\pi \left(2.50\text{cm}\right)}\right)\left(v_1\right) \tag{5.8}$$

$$v_2 = \frac{16.00}{6.25}\left(v_1\right) \tag{5.9}$$

$$v_2 = 2.56 v_1 \tag{5.10}$$

$$v_2 = 2.56 \times 3.00 m/s \tag{5.11}$$

$$v_2 = 7.68 m/s \tag{5.12}$$

If we compare our result for the velocity of the fluid in Pipe 2 with the velocity of fluid in Pipe 1, we immediately see that the velocity of the fluid increases as it moves from a pipe with a larger cross-sectional area to another pipe with a smaller cross-sectional area. This result is intuitive with what we may have observed through personal experience.

Question: Make a plot that relates the dependent variable ($\mathbf{v}_2$) and the independent variable ($\mathbf{v}_1$).

Referring to equation (10), we note that a linear relationship exists between $\mathbf{v}_2$ and $\mathbf{v}_1$. For the straight line, the slope is 2.56 and the y-intercept is 0. We plot the line below

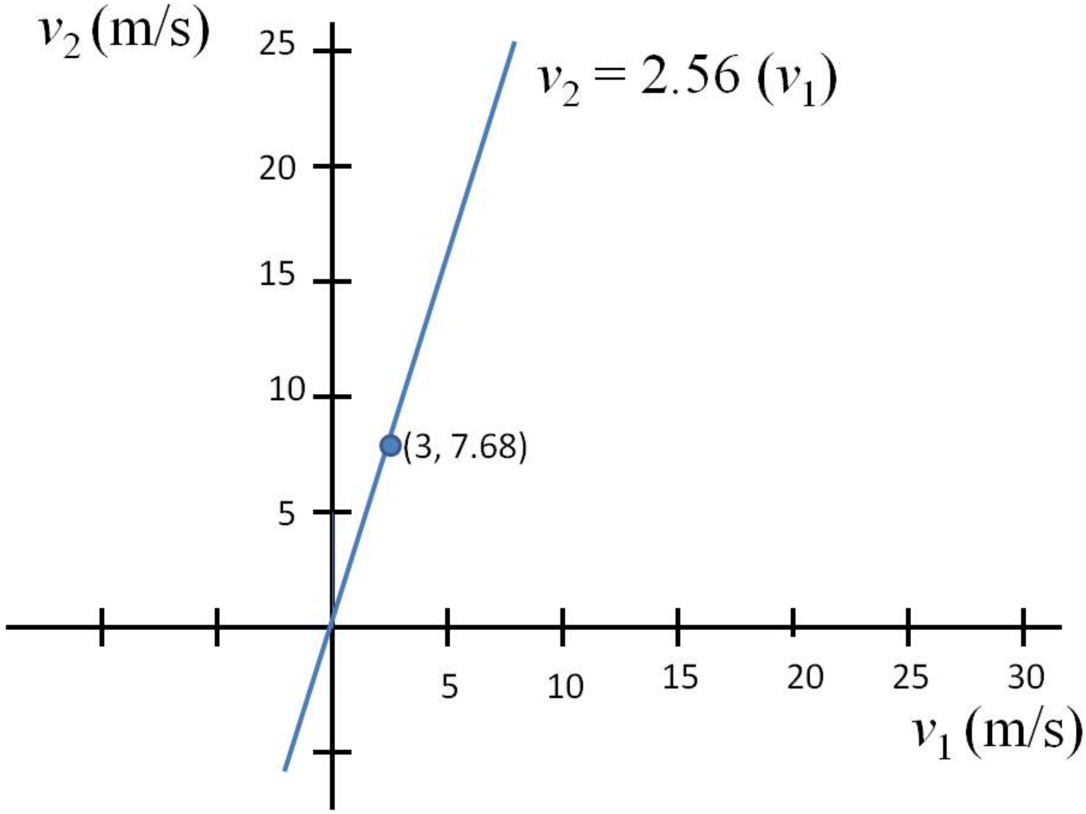

Figure 5.2: Graph of the linear relationship between velocities.

It is important to note that each axis is labeled and includes the units associated with each variable.

5.1.3 Mechanics – Velocity and Acceleration

Let us consider a car that is traveling at an unknown *initial velocity* (v_0). The driver of the car decides to enter the on-ramp of a freeway. The driver knows that he will need to increase his velocity in order blend in with the other traffic on the freeway. While situated in the on-ramp, the driver of the car applies pressure to the accelerator of the car. Let us consider that this action occurs at a specific instant of time (t_0). By applying constant pressure on the accelerator, the driver causes the car to accelerate at a *constant acceleration* (a).

This constant acceleration causes the velocity of the car to increase. During the time interval that the driver applies constant pressure to the accelerator, the *velocity* of the car can be expressed as a function of time

$$v(t) = at + v_0 \tag{5.13}$$

Inspection of this equation reveals that the velocity is a linear function of time. Here the dependent variable would be velocity and the independent variable would be time. The slope of the straight line that is associated with the equation would equal to the acceleration (**a**). The y-intercept of the equation would be the initial velocity ($\mathbf{v}_0$).

Let us apply what we have learned about the relationship between velocity and acceleration coupled with our knowledge of linear equations to work a problem.

Question: At an instant of time ($t_1 = 1.00$ s) after depressing the accerlerator, the driver observes that the car is traveling at a velocity ($v_1 = 17.00$ m/s). At an instant of time one second later (that is at $t_2 =$ 2.00 s), the driver observes that the velocity of the car has increased to a value ($v_2 = 22.0$ m/s). Determine the initial velocity of the vehicle and the value for the constant acceleration (a).

Solution: We know the values of the velocity at two instants of time, 1.00 seconds and 2.00 seconds. Because the acceleration is constant, we also know that the relationship between velocity and time is linear.

The slope of the line that relates velocity to time is equal to the acceleration (a) and the y-intercept corresponds to the intial velocity (v_0).

We begin by stating the two points on the line which are known. They are (t_1, v_1) and (t_2, v_2). The numerical values of these points are (1.00 s, 17.00 m/s) and (2.00 s, 22.00 m/s) respectively.

Our knowledge of straight lines tells us that we can calculate the slope through a differencing operation

$$m = \frac{v_2 - v_1}{t_2 - t_1} \tag{5.14}$$

Next we enter the numerical values of the problem

$$m = \frac{22.00m/s - 17.00m/s}{2.00s - 1.00s} \tag{5.15}$$

$$m = 5.00m/s^2 \tag{5.16}$$

We can therefore conclude that the acceleration is 5.00 m/s^2. We can incorporate this value into the linear equation that relates velocity to time

$$v(t) = \left(5.00m/s^2\right) t + v_0 \tag{5.17}$$

Either of the data points can be used to solve for the intital velocity. Let us substitute the values associated with the first data point into the equation. We obtain

$$17.00m/s = \left(5.00m/s^2\right) \times (1.00s) + v_0 \tag{5.18}$$

$$17.00m/s = 5.00m/s + v_0 \tag{5.19}$$

$$v_0 = 12.00m/s \tag{5.20}$$

So we conclude that the initial velocity of the vehicle is 12.00 m/s and the constant acceleration of the car while it is situated on the on-ramp is 5.00 m/s^2.

5.1.4 Electric Circuits – Variable Source Voltage

Suppose that we are presented with an electric circuit that contains a fixed voltage source (**v**), a variable source voltage ($\mathbf{v}_\mathrm{s}$) and a resistor (**R**). This situation is shown in Figure 3. In this figure, the variable source voltage is indicated by the circle and represented by the variable v_s. Its units are Volts. The current that flows through the circuit is indicated by the variable i and flows in the direction indicated by the arrow. The current has the units Amps. The fixed voltage source is represented by the constant v.

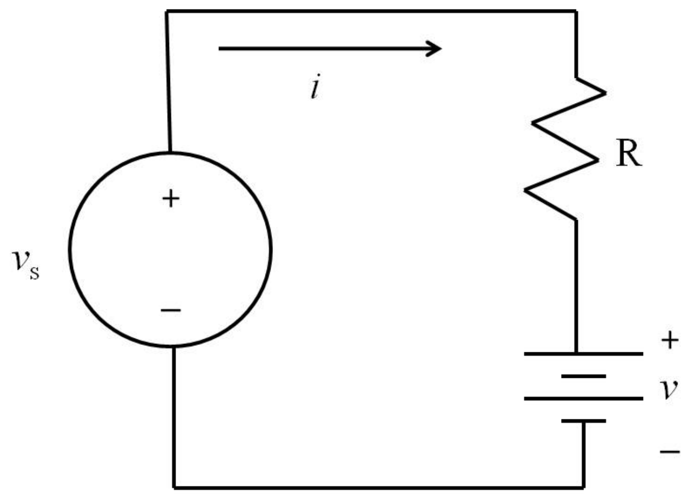

Figure 5.3: Electrical circuit with a variable source voltage.

One of the most important laws of Physics that govern the behavior of electric circuits is Kirchoff's Voltage Law. This law states the algebraic sum of the voltage drops experienced as one passes through a complete path through a circuit is equal to zero.

Application of Kirchoff's Voltage Law to this circuit yields the equation

$$-v_s + Ri + v = 0 \tag{5.21}$$

The terms of this equation may be arranged to produce the following equation

$$v_s = Ri + v \tag{5.22}$$

Let us consider the source voltage ($\mathbf{v}_\mathrm{s}$) as the dependent variable and the current ($\mathbf{i}$) as the independent variable. Examination of the equation reveals that there is a linear relationship between $\mathbf{v}_\mathrm{s}$ and $\mathbf{i}$. For this linear equation, the value of the resistor (**R**) is the slope and the fixed voltage (**v**) represents the **y**-intercept.

Let us apply our knowledge of linear equations to solve a problem associated with this circuit.

Question: It is observed through measurement that the when the variable source voltage is 6.00 Volts, the current takes on the value of 1.00 Amp. When the variable source voltage is raised to 12.00 Volts, the current rises to a value of 1.50 Amps. Find the values for the resistance and the fixed voltage.

Solution: We can draw some insight into the solution of this problem by applying our knowledge of straight lines.

Let us begin by finding the value of the slope or equivalently the value of the resistance. We have two ordered points to consider (1.00 A, 6.00 V) and (1.50 A, 12.00 V). The slope of the line that connects these two points is

$$m = \frac{v_2 - v_1}{i_2 - i_1} \tag{5.23}$$

$$m = \frac{12.00V - 6.00V}{1.50A - 1.00A} \tag{5.24}$$

$$m = 12.0\,(V/A) \tag{5.25}$$

We recognize that the ratio (volts/amps) is equivalent to the unit (Ω). We make the substitution to yield

$$m = 12.0\Omega \tag{5.26}$$

Earlier we stated that the slope of the line would be equal to the value of the resistance, so we have the following result

$$R = 12.0\Omega \tag{5.27}$$

The next step in the solution is to solve for the value of the fixed voltage. Incorporation of the slope that was just found into the equation of the line yields the equation

$$v_s = 12.0\,(V/A) \times i + v \tag{5.28}$$

Let us substitute the values 1.00 A and 6.00 V into this equation

$$6.00V = 12.00\,(V/A) \times 1.00A + v \tag{5.29}$$

$$6.00V = 12.00V + v \tag{5.30}$$

$$v = -6.00V \tag{5.31}$$

So we conclude that the value of the fixed voltage is -6.00 V and that the value for the resistance is 12.0 Ω.

5.1.5 Summary

Knowling how to apply the knowledge of linear equations and straight lines is critical for students in engineering. In this module, we have seen how knowledge of linear equations can be used to solve engineering problems. Applications from the fields of fluid mechanics, the mechanics of motion and electric circuits have been presented. Other applications in engineering abound.

5.1.6 Exercises

1. Water flows through a piping system that consists of two pipes that are joined together. The cross-sectional area of the first pipe is 10.00 cm^2, while that of the second pipe is 1.25 cm^2. If the velocity of the water in the first pipe is known to be 5 cm/s, then what is the velocity of the water in the second pipe. Assume that the continuity equation holds.
2. A vehicle is traveling through a neighborhood at an initial velocity (v_0). The driver of the vehicle notices a child who runs out into the street in front of her car. She applies her brakes to reduce the speed of her car and eventually stops. The velocity of her car obeys the linear relationship $v(t) = v_0 + at$ Determine the initial velocity and the acceleration (a) if the velocity is known to be 30 m/s at $t = 0.50$ s and the velocity is 3 m/s at the time $t = 1.25$ s. Also calculate the total time that it will take the vehicle to stop after the driver applies her brakes.
3. Consider the circuit depicted in Figure 3. The following two facts are known. When the variable voltage is set to 9 Volts, the current is 100 mA. When the variable voltage is sent to 18 Volts, the current is 1.20 A. What are the values for R and v?

Chapter 6

Quadratic Equations[1]

6.1 Quadratic Equations

6.1.1 Introduction

Quadratic equations play an important role in the modeling of many physical situations. Finding the roots of quadratic equations is a necessary skill. Being able to interpret these roots is an important ability that is important in understanding physical problems. In this module, we will present a number of applications of quadratic equations in several fields of engineering.

6.1.2 Determining the Roots of Quadratic Equations

A quadratic equation has the following form

$$\mathrm{ax}^2 + \mathrm{bx} + c = 0 \tag{6.1}$$

Because a quadratic equation involves a polynomial of order 2, it will have two roots. In general, a quadratic equation will either have two roots that are both real or have two roots that are both complex. For the present module, we will restrict our attention to quadratic equations that have two real roots.

There are three methods that are effective in solving for the roots of a quadratic equation. They are:

- Solution by factoring
- Solution by completing the square
- Solution by the quadratic formula

The applications that follow will include examples of each of these three methods of solution.

6.1.3 Motion of an Object under Uniform Acceleration

We will begin our study of quadratic equations by considering an application that you will likely encounter later in physics and mechanical engineering classes. Let us consider an object that is subject to a uniform acceleration. By uniform, we mean an acceleration that is constant. Such an object might be an automobile, an aircraft, a rocket, etc. The motion of an object subjected to uniform acceleration can be expressed mathematically by the following equation.

$$s\left(t\right) = \frac{1}{2}at^2 + v_0 t + s_0 \tag{6.2}$$

where $\mathbf{s}(\mathbf{t})$ represents the position of the object as function of time $\mathbf{t}$,

[1]This content is available online at <http://cnx.org/content/m38578/1.3/>.

a represents the constant acceleration of the object,
$\mathbf{v}_0$ represents the value of the object's velocity at time **t** = 0, and
$\mathbf{s}_0$ represents the position of the object at time **t** = 0.

An equation of this sort is called an **equation of motion**. We will illustrate its use in the following exercise.

Example 1: For our first example, let us consider a dragster on a drag strip of length one-quarter mile. For time t < 0, the dragster is at rest at the starting line. At time = 0, the driver depresses his gas pedal to produce a uniform acceleration of 50 m/s^2. Under these conditions, how far will the dragster travel in 1 second?

Because the dragster travels in a horizontal direction, we will represent its distance from the starting point as a fuction of time as **x(t)**. We also know that the value for the acceleration (**a**) is 30 $\mathbf{m/s^2}$. We can incorporate these changes in equation (1) to produce a new equation of motion for the dragster.

$$x(t) = \frac{1}{2}\left(50m/s^2\right)t^2 + v_0 t + x_0 \tag{6.3}$$

Because the dragster is initially motionless, its initial velocity ($\mathbf{v}_0$) is 0. Also since the dragster is situated at the starting point at time t = 0, its initial position ($\mathbf{x}_0$) is 0. We substitute these values into (2) to obtain

$$x(t) = \frac{1}{2}\left(50m/s^2\right)t^2 \tag{6.4}$$

We can determine the distance that the dragster travels in 1 second by substituting the value 1 for **t**.

$$x(1) = \frac{1}{2}\left(50m/s^2\right)(1s)^2 = 25m \tag{6.5}$$

Thus we conclude that the dragster travels 25 meters in its first second of travel.

Example 2: How far will the dragster travel in 2 seconds?

We may make use of the same equation (2) as before. In this case, we substitute 2 for **t** to obtain the distance traveled by the dragster in 2 seconds.

$$x(2) = \frac{1}{2}\left(50m/s^2\right)(2s)^2 = 100m \tag{6.6}$$

Example 3: Most drag strips are one-quarter mile in length. Assuming that the acceleration remains uniform, how long will it take the dragster to complete the one-quarter mile strip?

Because the equation of motion for the dragster is written is MKS units, we must convert one-quarter mile to meters. We can do so by applying the unit conversion factor 1 mile = 1,609.344 m. Thus,

$$\frac{1}{4}\text{mile}\frac{1,609.344m}{\text{mile}} = 402.3m \tag{6.7}$$

Once again, equation (2) can be used to model the motion of the dragster. We will use this equation to solve for the amount of time ($\mathbf{t}_1$) that it will take the dragster to travel 402.3 meters. The steps are shown below.

$$402.3m = \frac{1}{2}\left(50m/s^2\right)(t_1)^2 \tag{6.8}$$

$$\frac{2 \times 402.3m}{50m/s^2} = (t_1)^2 \tag{6.9}$$

$$(t_1)^2 = 16.092s^2 \tag{6.10}$$

$$t_1 = 4.01s \tag{6.11}$$

So we conclude that it will take the dragster slightly 4.01 seconds to complete the quarter mile track.

6.1.4 Projectile Motion

Just as we could model a dragster as an object under constant acceleration, we can also model a projectile using a similar approach. A **projectile** is an object that is hurled into the air. Rockets, missiles and artillery shells are examples of projectiles.

Projectiles experience uniform acceleration as they travel through the sky. The uniform acceleration is due to the gravitational pull exerted on the projectile by the Earth. This acceleration is denoted as **g**. It value in the MKS system is 9.8 $\mathbf{m/s}^2$. In the British system of units, the value of **g** is equal to 32 ft/s^2.

Let us represent the height of a projectile as a function of time as $\mathbf{y}(\mathrm{t})$. Let us represent the initial velocity in the vertical direction of the projectile and the initial height of the projectile to be $\mathbf{v}_0$ and $\mathbf{y}_0$, respectively. We can incorporate these constants into equation (1) to develop the following equation of motion for a projectile.

$$y(t) = -\frac{1}{2}gt^2 + v_0 t + y_0 \tag{6.12}$$

The presence of the minus sign in the equation of motion for a projectile is due to the fact that the gravitation force exerted on the projectile is directed toward the Earth which is opposite to the direction of increasing vertical position.

We can use this equation to model the height of a projectile as a function of time. The following exercises illustrate this principle. In the next three exercises you can ignore any affects due to the atmosphere or wind.

Example 4: Let us consider a bottle rocket that is launched at time ($\mathbf{t} = 0$) from a flat surface. The initial velocity of the bottle rocket is 124 ft/s. Determine the height of the bottle rocket 1.5 seconds after it is launched.

Clearly $\mathbf{v}_0$ is 124 ft/s. Since the bottle rocket is launched from the surface, the value for $\mathbf{y}_0$ is 0. We will substitute these values into the equation of motion for a projectile.

$$y(t) = -\frac{1}{2}gt^2 + (124)\,t + 0 \tag{6.13}$$

Next, we substitute 32 ft/s2 for g. We use this value because in this exercise, we make use of the British system of units.

$$y(t) = -\frac{1}{2}\left(32\mathrm{ft}/s^2\right)t^2 + (124\mathrm{ft}/s)\,t \tag{6.14}$$

To find the height of the bottle rocket at a time 1.5 seconds after its launch, we make the substitution $\mathbf{t} =$ 1.5 s.

$$y(1.5s) = -\frac{1}{2}\left(32\mathrm{ft}/s^2\right)(1.5s)^2 + (124\mathrm{ft}/s)(1.5s) \tag{6.15}$$

$$y(1.5s) = -36\mathrm{ft} + 186\mathrm{ft} = 150\mathrm{ft} \tag{6.16}$$

So we conclude that the bottle rocket attains a height of 150 ft at a time 1.5 seconds after launch.

Example 5: Let us enforce that same conditions on the bottle rocket as were presented in Example 4. Find the length of time it will take for the bottle rocket to strike the surface.

In order to determine the time at which the bottle rocket will strike the surface, we must find the value for t that leads to a height of 0. As before, we make use of the equation of motion

$$y(t) = -\frac{1}{2}\left(32\mathrm{ft}/s^2\right)t^2 + (124\mathrm{ft}/s)\,t \tag{6.17}$$

Because we are interested in determining when the bottle rocket hits the surface, we set the left hand side of the equation to 0

$$0 = -\frac{1}{2}\left(32\mathrm{ft}/s^2\right)t^2 + (124\mathrm{ft}/s)\,t \tag{6.18}$$

If we restrict the units of **t** to seconds, we can simplify this equation

$$-16t^2 + 124t = 0 \tag{6.19}$$

The left hand side of the equation can be factored as

$$t \times (-16t + 124) = 0 \tag{6.20}$$

The roots of this equation can be found by setting each factor to 0. That is

$$t = 0 \tag{6.21}$$

$$-16t + 124 = 0 \tag{6.22}$$

As a consequence, the root associated with the first factor which is 0 corresponds to the time of the launch. The second equation yields the root, 7.75 seconds. This root corresponds to the time at which the bottle rocket strikes the surface.

Example 6: Let us consider the same bottle rocket as before, but with one major exception. In this exercise, assume that the launch site of the bottle rocket is moved to a bluff whose height is 100 ft above the surface. Find the length of time it will take for the bottle rocket to strike the surface below the bluff.

The situation is depicted in Fig 1.

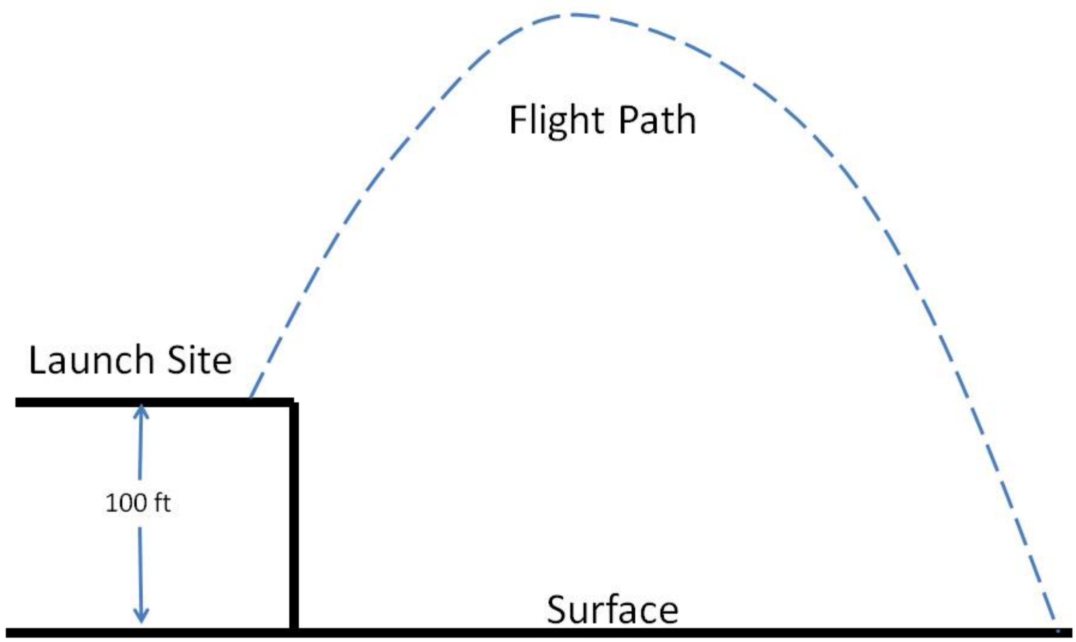

Figure 6.1: Flight path of a projectile.

Once again, we will make use of the equation of motion for a projectile. However in this case we establish the term $\mathbf{y}_0$ as 100 ft. This is due to the bottle rocket being launched from the bluff which is 100 ft above the surface. Thus we have

$$y(t) = -\frac{1}{2}\left(32\text{ft}/s^2\right)t^2 + (124\text{ft}/s)\,t + 100\text{ft} \tag{6.23}$$

To find the length of time in seconds at which the bottle rocket will strike the surface, we must solve for the roots of the following quadratic equation.

$$0 = -16t^2 + 124t + 100 \tag{6.24}$$

The quadratic formula provides us with a simple way to determine the roots of a quadratic equation of the form

$$0 = at^2 + bt + c \tag{6.25}$$

The quadratic formula tells us that the roots of this equation are given in terms of the coefficients (**a**, **b**, and **c**) as follows

$$\text{roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{6.26}$$

For our problem we express the roots as

In the problem at hand, the values for **a**, **b**, and **c** are -16, 124, and 100 respectively. Substitution of these values into the quadratic formula yield

$$\text{roots} = \frac{-124 \pm \sqrt{15,376 + 6,400}}{-32} \tag{6.27}$$

Further arithmetic manipulation on this expression leads us to determine the two roots to be -0.736 and 8.49 seconds. The first root is not physically possible, so it can be ignored. The second root (8.49 seconds) tells us the time at which the bottle rocket will strike the surface.

6.1.5 Quantitative Decision Making

Industrial engineers are often tasked by firms to make decisions that affect manufacturing operations. In making decisions, industrial engineers must weigh a variety of factors. These factors include cost, safety, regulatory constraints, ergonomics and others.

In determining which design alternatives for manufacturing are the best, industrial engineers often form mathematical models. With such models, decisions can be made that are optimal in terms of various criteria. The following exercise illustrates the sort of decision making that might be involved in a hypothetical manufacturing situation.

Example 7: A manufacturing firm produces commercial aircraft. The firm plans to expand its manufacturing operations to produce aircraft by opening a second plant at another site. An industrial engineer oversees the manufacturing operations for the firm and is charged with making decisions that impact the operations at the new site.

The industrial engineer has a deep understanding of two processes that could be used to govern manufacturing at the new site. For convenience let us refer to these two processes as process A and process B. The industrial engineer knows that manufacturing cost is a very important issue with the firm. In researching the two options, the industrial engineer has determined mathematical expressions that express the manufacturing cost for each of the two options.

In order to produce n aircraft during a 30-day production run, the cost in millions of dollars associated with process A is known to be

$$8n^2 + n + 2 \tag{6.28}$$

The cost in millions of dollars associated with manufacturing n aircraft over the same span of time for process B is known to be

$$7n^2 + 2n + 14 \tag{6.29}$$

Which process has the lowest associated cost?

We begin the process of determining which process is most cost effective by forming the difference between the cost associated with process A and the cost associated with process B. Let us designate this difference by the variable D

$$D = \left(8n^2 + n + 2\right) - \left(7n^2 + 2n + 14\right) = n^2 - n - 12 \tag{6.30}$$

Whenever **D** is greater than zero, one may conclude that the cost associated with process A is greater than that associated with process B. Whenever **D** is less than zero, one may conclude that the cost associated with process B is greater than that associated with process A. The cost is the same for each process whenever **D** equals zero.

D is a quadratic polynomial, so we begin by solving its roots using the equation below

$$n^2 + n - 12 = 0 \tag{6.31}$$

We can solve for the roots of this equation using the method commonly known as solution by completing the square. Let us rearrange the equation with the terms involving n2 and n on the left hand side and the constant on the right.

$$n^2 + n = 12 \tag{6.32}$$

If we add $\frac{1}{4}$ to the left and the right sides we obtain

$$n^2 + n + 1/4 = 12.25 \tag{6.33}$$

The left hand side can be expressed as the square $(\mathbf{n} + \frac{\mathbf{1}}{\mathbf{2}})^2$. We reflect this below

$$(n + 1/2)^2 = 12.25 \tag{6.34}$$

If we take the square root of each side, we are left with

$$n + 1/2 = \pm 3.5 \tag{6.35}$$

This yields the pair of roots 3 and -4.

Let us recall that the variable **n** represents the number of aircraft produced during a 30-day production run. The root (-4) is physically impossible for it is impossible to produce a negative number of aircraft. The remaining root (3) tells us that the cost associated with process A is equal to the cost associated with process B when 3 aircraft are produced during a 30-day production run.

The term **D** can be expressed as

$$D = (n + 1/2)^2 - 12.25 \tag{6.36}$$

We can tell much about the nature of the plot of D by examining this equation. **D** is a parabola with a vertex at (-1/2, -12.25). The parabola would be concave positive. The parabola would cross the **n**-axis at the values of the roots, namely -4 and 3. Figure 2 presents a sketch of **D** as a function of **n**.

This sketch presents us with invaluable information with regard to weighing the costs associated with the two manufacturing processes. To begin, we see that the value of **D** is positive for values of $\mathbf{n} > 3$. We interpret this to mean that whenever the number of aircraft produced in a 30-day production run exceeds the number 3, it is more cost efficient to implement manufacturing process B. Alternatively, whenever the number of aircraft produced in a 30-day production run is greater than or equal to 0 and less than 3, manufacturing process A is the most cost efficient. If 3 aircraft are manufactured in a 30-day production run, the costs associated with process A and process B are equal.

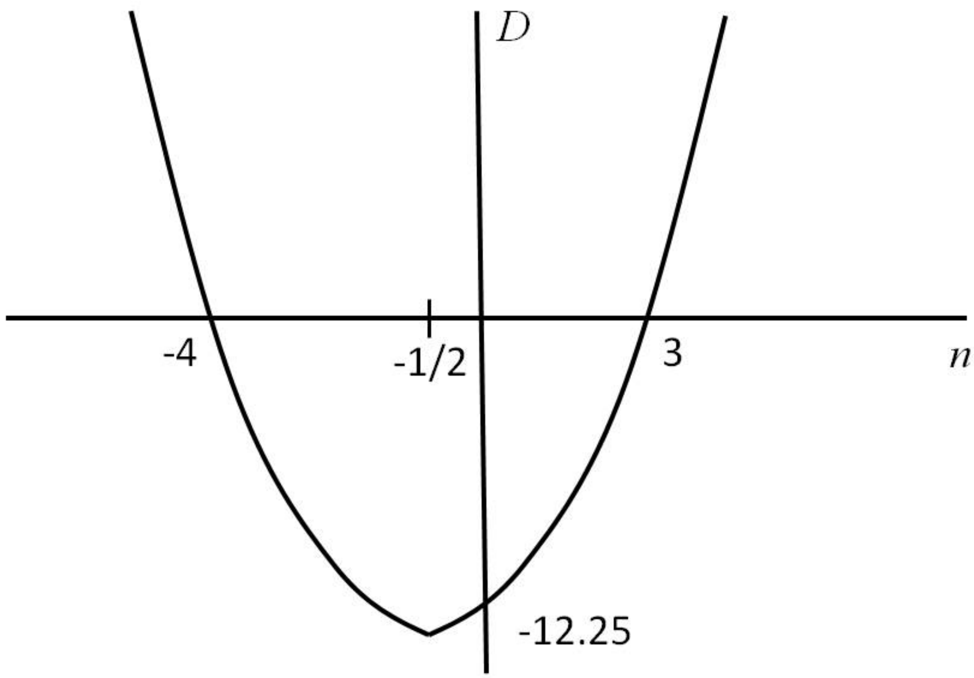

Figure 6.2: Plot of the difference in cost for competing plans as a function of production.

Here we recognize that it is physically impossible to manufacture a negative number of aircraft. Thus we may ignore the part of the curve to the left of the D axis.

6.1.6 Summary

A thorough knowledge of quadratic equations is crucial for an engineer. This module has presented several applications of quadratic equations in the context of problems that occur in the study of engineering. We see that the motion of dragsters, the flight of rockets and determining the most cost-effective approach to competing manufacturing processes involve quadratic polynomials and equations.

6.1.7 Exercises

1. Wind power is currently being considered as a primary source for generating electricity. The pressure due to the wind (P: measured in lbs/ft^2) is related to the velocity of the wind (v: measured in miles/hr) by the

equation

$$P = \frac{3v^2}{1,000} \tag{6.37}$$

Determine the approximate wind velocity (v) if the pressure is measured to be 11.75 lb/ft^2.

2. The distance (d: measured in miles) to the apparent visible horizon of an observer at a height (h: measured in miles) above sea level is given by the formula

$$d^2 = h\,(h + 8,000) \tag{6.38}$$

On a clear day, how far away is the visible horizon for a passenger flying aboard an aircraft at a height of 6 miles?

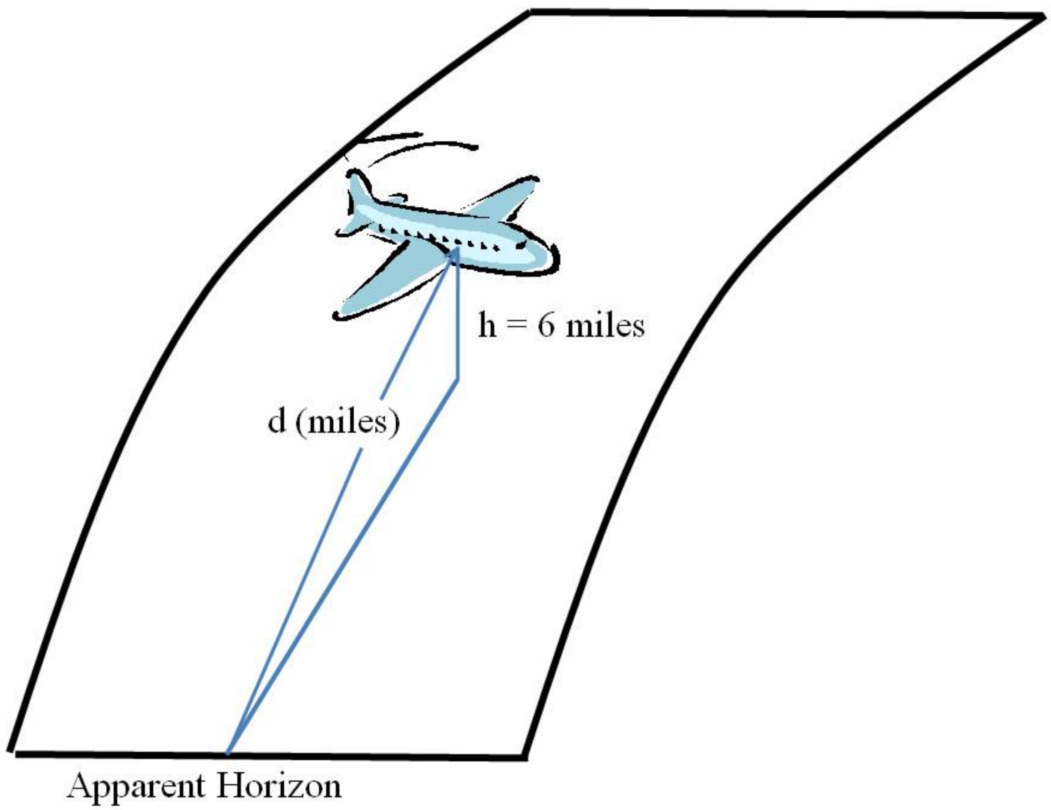

Figure 6.3: Depiction of apparent horizon from an aircraft.

3. The braking distance (d: measured in feet) for a car travelling at a velocity = v miles/hour with good tires and well maintained brakes, on dry pavement can be calculated by the formula, $d = \frac{v(v+20)}{20}$. If a child runs into the street 100 feet in front of your car and you react immediately, what is the maximum speed that you could be driving and still stop without hitting the child?

4. An industrial engineer using supply chain management techniques, estimates that the hourly cost (C measured in dollars/hr) of producing n sub-assemblies for a robotic application by the formula, $C = 2n^2 + 50n - 30$

How many subassemblies can be produced when $C = \$270/\mathrm{hr}$?

5. The **Little Old Lady from Pasadena** is a very cautious driver who is on her way to the grocery store. The speedometer on her car registers an initial velocity of of 25 miles/hr. She presses on her accelerator to produce a constant acceleration of 3,600 miles/hr^2. Assuming that she is 10 miles from her final destination when she begins to accelerate, how long will it for her to reach the grocery store?

6. A mortar shell is launched at time, $t = 0$, with an initial velocity of 10,000 m/s. When will the shell hit the Earth?

7. Sketch the height of the mortar shell presented in exercise 6 as a function of time.

8. A mortar shell is launched from a bluff overlooking a battlefield. The height of the bluff is 250 m above the target of the shell. Assuming that the initial velocity of the shell is 5,000 m/s, how long will it take for the shell to travel to its target?

9. The expected profit from the manufacturing of a particular item is governed by a quadratic equation $P(n) = -n^2 + 120n$. In the equation, n represents the number of items manufactured. Sketch the curve of $P(n)$.

10. Using the information given in exercise 9, what is the value of the number of items that should be manufactured to produce the maximum amount of profit?

Chapter 7

Rational Expressions and Equations[1]

7.1 Rational Expressions and Equations

7.1.1 Introduction

We have seen that much of the analysis that it takes to simplify rational expressions and to solve rational equations is an outgrowth of the mathematics associated with fractions. In most cases, one can begin to solve a problem involving a rational equation by factoring the polynomials that are constituents of the rational equation. Then, one may seek to identify if there are any common factors in the numerator and denominator that cancel with one another. If there are, cancelling these terms will often make our job of solving the equation simpler.

The fields of science and engineering are filled with rational equations that describe many different application areas. In this section of notes, we will focus on how rational expressions and equations are solved in several different applications. Before we do so, let us examine an example that involves the manipulation of rational expressions to solve a numerical problem involving positive integers.

7.1.2 Numerical Problem Involving Rational Expressions

Question: One positive integer is 3 more than another positive integer. When the reciprocal of the smaller integer is added to the reciprocal of the larger integer, the resulting sum is $\frac{1}{2}$. Find the positive integer.

As a reminder the reciprocal of an integer is 1 divided by the integer. For example, the reciprocal of 10 is 1/10.

Solution: We begin the solution by defining the variable x.

Let x = smaller positive integer

With this definition of x, we know that the larger positive integer can be expressed algebraically as (x + 3).

Using the definition of the reciprocal, we can translate the problem statement into a rational equation

$$\left(\frac{1}{x}\right)+\left(\frac{1}{x+3}\right)=\frac{1}{2} \tag{7.1}$$

The rational equation consists of three fractions. The lowest common denominator for these three fractions is $(x)(x+3)(2)$. We can multiply each side of the rational equation by the lowest common denominator to obtain the equation

$$(2)(x)(x+3)\{\left(\frac{1}{x}\right)+\left(\frac{1}{x+3}\right)\}=(2)(x)(x+3)\frac{1}{2} \tag{7.2}$$

[1]This content is available online at <http://cnx.org/content/m38602/1.4/>.

We can simplify things on a term by term basis

$$2\,(x+3) + 2\,(x) = x\,(x+3) \tag{7.3}$$

$$2x + 6 + 2x = x^2 + 3x \tag{7.4}$$

$$4x + 6 = x^2 + 3x \tag{7.5}$$

$$x^2 - x - 6 = 0 \tag{7.6}$$

This quadratic equation can be solved a variety of ways. One simple way to do so is to employ factoring.

$$(x-3)\,(x+2) = 0 \tag{7.7}$$

The roots are therefore x = 3 and x = -2.

From the problem statement, we know that the solution for x must be positive. We therefore can ignore the root of -2.

Our answer is therefore 3.

7.1.3 Rate Applications

Engineering and science abound with problems that make use of rates. Examples of rate include concepts such as speed which is a measure of the rate of change of distance per unit of time. Flow rates are common in fluid mechanics problems. The flow rate of liquid in a pipe can be expressed as a number of liters of liquid per unit of time.

An important consideration in working rate problems is to recognize that a rate can be written as a ratio of the quantity of a particular entity over time. The following two examples illustrate the use of rates to solve engineering problems.

7.1.4 Computing the Time It Takes to Finish a Construction Project

Civil engineers perform important duties in the management of construction projects. Suppose that a civil engineer is charged with the the completion of a construction project. The civil engineer knows that if he (she) hires Subcontractor A to complete the construction project, it will take 10 months. On the other hand, if Subcontractor B is hired, the construction project can be completed in 15 months

Instead of hiring just one of the subcontractor, the civil engineer decides to hire both Subcontractor A and Subcontractor B. By doing so, the civil engineer expects the project to be completed more quickly.

Question: How long will it take both subcontractors working together to complete the construction project?

Let us begin by considering Subcontractor A. Working alone, it would take Subcontractor A 10 months to complete the project. If we consider the completion of the construction project as a job, then Subcontractor A can complete (1/10) of the job in one month. Another way to look at this is that the rate at which Subcontractor A can complete the job is as (1/10) job/month.

Now, let us look at Subcontractor B. Working alone, it would take Subcontractor B 15 months to complete the construction project. Thus Subcontractor B can complete (1/15) of job in one month. The rate of Subcontractor B can be stated as (1/15) job/month

Let us define the variable t

t = the time that it takes to complete the entire job using both of the subcontractors,

where t is measured in months.

In a length of time equal to t, the fraction of the job that Subcontractor A can complete is equal to the product

$$\left(\frac{1}{10}\right)t = \frac{t}{10} \quad (7.8)$$

Similarly the fraction of the job that Subcontractor B can complete in the same length of time is

$$\left(\frac{1}{15}\right)t = \frac{t}{15} \quad (7.9)$$

We may obtain the fraction of the job that can be completed in an interval of time equal to t by adding the fraction associated with the Subcontractor A to that of Subcontractor B, which is $\frac{t}{10} + \frac{t}{15}$

If we set this sum to one, we obtain a rational equation that can be solved to produce the amount of time that it will take both subcontractors working together to complete the entire job

$$\frac{t}{10} + \frac{t}{15} = 1 \quad (7.10)$$

The solution follows

$$\frac{15t}{150} + \frac{10t}{150} = 1 \quad (7.11)$$

$$150\left(\frac{15t + 10t}{150}\right) = 150\,(1) \quad (7.12)$$

Simplification of this equation yields

$$25t = 150 \quad (7.13)$$

From this, we conclude that the two subcontractors working together can complete the job in 6 months.

7.1.5 Motion of an Automobile Using Two Different Speeds

Question: Suppose that you are interested in taking an automobile trip to visit a friend in Van Horn, Texas. On your journey, you will travel 40 miles inside the City of El Paso. Once you leave the city limits of El Paso, you will have to travel an additional 90 miles to reach Van Horn. Because of traffic, you anticipate that your average speed within the city limits of El Paso will be only half of the average speed once you leave the city limits. You also project that the total time of your trip from El Paso to Van Horn will be 2.5 hours,

What is your average speed within the city limits of El Paso? What would be your average velocity outside the city limits?

Solution: We begin by defining the following variable

x = average speed (miles/hr) within in the city limits of El Paso.

The speed at which you would drive outside the city limits would be twice that of the velocity within the city limits. Thus

2 x = average velocity (miles/hr) outside the city limits of El Paso.

The amount of time that you would spend driving in El Paso would be

$$\frac{40\text{miles}}{x} \quad (7.14)$$

The amount of time that you would spend driving outside the city limits would be

$$\frac{90\text{miles}}{2x} \quad (7.15)$$

If we sum these together we obtain the total time of the trip

$$\frac{40\text{miles}}{x}+\frac{90\text{miles}}{2x}=2.5\text{hr} \tag{7.16}$$

This rational equation can be solved as follows

$$\frac{2\,(40)\,\text{miles}}{2\,(x)}+\frac{90\text{miles}}{2x}=2.5\text{hr} \tag{7.17}$$

which can be simplified as

$$170\text{miles}=5x\text{hr} \tag{7.18}$$

We solve this for x to obtain the result of 34 miles/hr. This represents the average speed within the city limits of El Paso. Outside the city limits would be twice that within the city limits or 68 miles/hr.

7.1.6 Exercises

1. The difference between the reciprocals of two consecutive positive odd integers is 2/15. Find the two positive odd integers.

2. Two pipes (A and B) attach to the top of a holding tank that is part of a waste water processing system. The pipes are of different size. If liquid is supplied to the tank by Pipe A only, it takes 12 minutes to fill the tank. Pipe B has a smaller cross-sectional area than that of pipe A. If liquid is supplied to the tank using Pipe B only, it takes 15 minutes to fill the tank.

A civil engineer is tasked with managing this process. In order to minimize the time needed to fill the tank, the civil engineer decides to supply liquid to the tank via both Pipe A and Pipe B. If this is done, how long will it take to fill the tank?

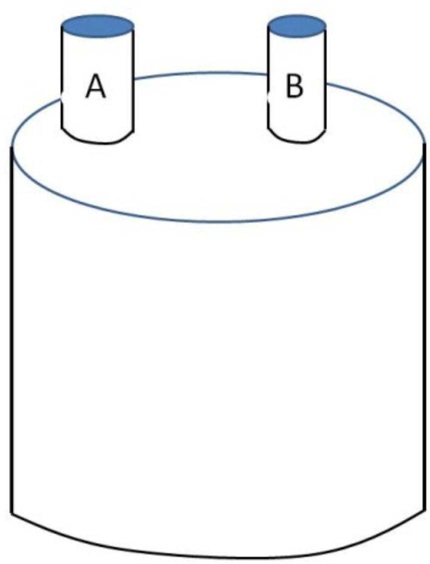

Figure 7.1: Storage tank with two input pipes.

3. The first leg of Mary's Spring Break trip consisted of 120 miles of traffic. When the traffic cleared she was able to drive twice as fast for 300 miles. If the total trip took 9 hours, how long was Mary stuck in traffic?

4. Three resistors are connected in parallel. There individual values are 10 Ω, 20 Ω and 50 Ω. What is the equivalent resistance of the parallel combination?

Chapter 8

Radicals[1]

8.1 Radicals

8.1.1 Introduction

Equations involving radicals abound in the various fields of engineering. Students of engineering must therefore gain confidence and competence in solving equations that include radical expressions. In this module, several different applications that involve the use of radicals to solve engineering problems are presented along with several exercises.

8.1.2 Centripetal Force

Centripetal force is the inward directed force that is exerted on one body as it moves in a circular path about another body.

Figure 1 illustrates a body that is in circular motion about a center point.

[1] This content is available online at <http://cnx.org/content/m38603/1.3/>.

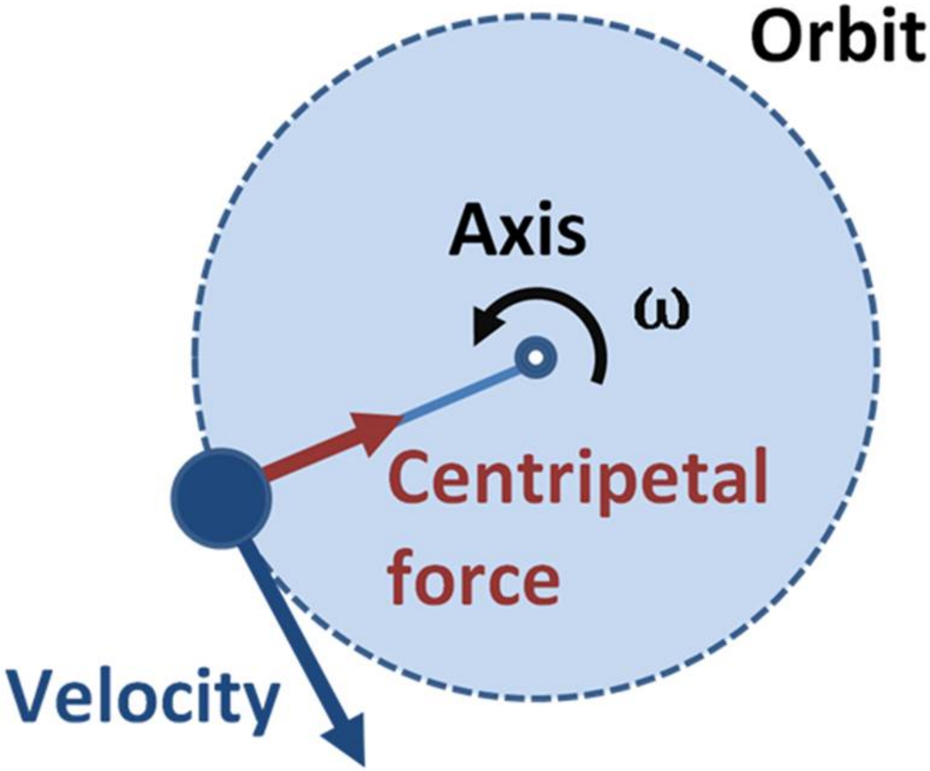

Figure 8.1: Centripetal force for an object under rotation.

As the object moves about the circle, its angle changes. This time rate of change of the angle is called the *angular velocity* and is denoted by the symbol ω. The angular velocity has units of radians/sec. As an example, if the object makes 2 revolutions in a second, it would have an angular velocity

$$\omega = \frac{2\text{revolutions}}{s} = \frac{2\,(2\pi\text{rad})}{s} = 4\pi\text{rad}/s \tag{8.1}$$

Examination of Figure 1 shows the centripetal force being directed inward toward the center of the circular path of the object. The velocity of the object is illustrated as being in the direction of the tangent at the point on the circle occupied by the object. If for any reason the body were released from its orbit about the center point, it would travel in a straight line path indicated in the direction of the velocity.

Quite often, one may measure the amount of time that it takes for the object to complete a complete revolution and denote it as the variable (T). This value which is usually expressed in seconds is called the *period* of revolution. For the example given previously where the object makes 2 revolutions per second, the period of revolution (T) is $\frac{1}{2}$ second.

The period of revolution (T) measured in seconds can be calculated by means of a relationship that involves the magnitude of the centripetal force (F) measured in Newtons, the mass of the object (m) measured in kilograms, and the radius (R) of the circle measured in meters.

$$T = \sqrt{\frac{4mR\pi^2}{F}} \tag{8.2}$$

Question: A mass of 2 kg revolves about an axis. The radius of the object about the axis is 0.5 m. It takes 0.25 seconds for the mass to make a single revolution. What is the value of the centripetal force?

Solution: We begin by replacing the variables of equation (2) by their numeric values

$$0.25 = \sqrt{\frac{4\,(2)\,(0.5)\,\pi^2}{F}} \tag{8.3}$$

Next we take the square of each side of the equation

$$(0.25)^2 = \frac{4\pi^2}{F} \tag{8.4}$$

We can isolate F on the left hand side of the equation as

$$F = \frac{4\pi^2}{0.625} \tag{8.5}$$

Which leads to the result $F = 632N$.

8.1.3 Nozzle Characteristics for Aircraft De-Icing

The presence of ice on the wings and fuselage on an aircraft can lead to severe problems during stormy winter weather. Equipment is used to spray aircraft with a de-icing agent prior to take-off in order to remove the ice from the wing surfaces and fuselage of planes.

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Figure 8.2: Photograph of a high-pressure nozzle.

There are several important parameters that relate to the performance of a nozzle. These include the diameter of the nozzle (d), the nozzle pressure (P) and the flow rate (r). The nozzle diameter is measured in inches; the flow rate is measured in gallons/minute; and the nozzle pressure is measured in pounds/square inch. The relationship between these parameters can be expressed via the radical equation

$$r = 30d^2\sqrt{P} \tag{8.6}$$

Question: Water flows at a rate of 2.5 pounds/s through a nozzle whose diameter is 0.25 inches. What is the value of the nozzle pressure at the exit?

Solution: We can begin by substituting values into equation (5).

$$2.5 = 30\,(0.25)^2\,\sqrt{P} \tag{8.7}$$

This can be written as

$$\sqrt{P} = \frac{2.5}{30\,(0.0625)} = 1.33 \tag{8.8}$$

Squaring each side of the equation yields the result $P = 1.778 \mathrm{lb}/\mathrm{in}^2$

8.1.4 Motion of a Pendulum

A *pendulum* is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced from its resting or equilibrium point, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position. When released, the restoring force combined with the pendulum's mass causes it to swing back and forth. The time for one complete cycle, a left swing and a right swing, is called the period. A pendulum swings with a specific period which depends on factors such as its length. From its discovery around 1602 by Galileo, the regular motion of pendulums was used for timekeeping, and was the world's most accurate timekeeping technology until the 1930s.

Figure 3 shows a picture of a pendulum.

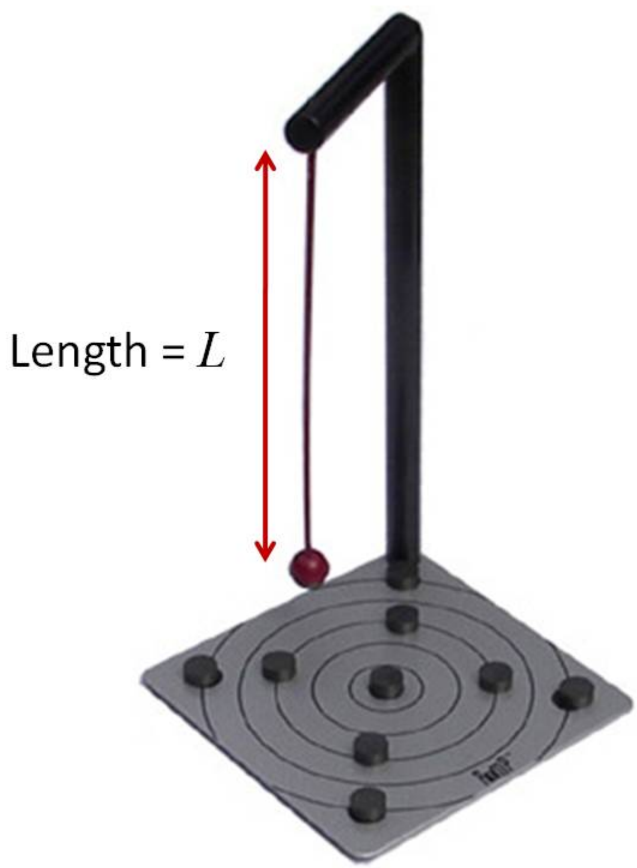

Figure 8.3: Simple pendulum.

The period of the pendulum can be represented by the variable T. The period is typically measured in seconds. The length of the pendulum can be modeled by the variable L and is measured in feet. Under such conditions, the relationship between the period and the length of the pendulum is summarized by the equation

$$T = 2\pi\sqrt{\frac{L}{32}} \tag{8.9}$$

Question: The arm of a pendulum makes a complete cycle every two seconds. What is the length of the pendulum?

Solution: We insert the appropriate value for the period into equation (9)

$$2 = 2\pi\sqrt{L/32} \tag{8.10}$$

Next, we square each side of the equation

$$4 = 4\pi^2 \left(L/32\right) \tag{8.11}$$

which can be re-arranged as

$$L = \frac{32}{\pi^2} \tag{8.12}$$

So our solution is $L = 3.24\text{ft}$

8.1.5 Exercises

1. Use algebra to derive a formula that expresses F as a function of T, m and R.
2. A passenger rides around in a ferris wheel of radius 20 m which makes 1 revolution every 10 seconds. If the passenger has a mass of 75 kg, what is the centripetal force exerted on the passenger? (Use the formula you derived in Exercise 1 to solve for the centripetal force.)
3. Find the centripetal force exerted on the passenger described in Exercise 2 if the ferris wheel takes 8 seconds to complete one revolution.
4. What can you say qualitatively about the relationship between the centripetal force and the amount of time it takes to complete one revolution?
5. Apply algebra to equation (3) to produce a formula for P as a function of r and d.
6. Find the nozzle pressure P for a nozzle whose diameter is 1.25 inches for a flow rate of 250 gallons/minute.
7. Find the nozzle pressure P for a nozzle whose diameter is 1.50 inches for a flow rate of 250 gallons/minute.
8. What can you say qualitatively about the relationship between the pressure P and the diameter of the nozzle d?
9. A grandfather clock has a pendulum of length 3.5 feet. How long will it take for the pendulum to swing back and forth one time?
10. To achieve a period of 2 seconds, how long must a pendulum be?

Chapter 9

Complex Numbers[1]

9.1 Complex Numbers

9.1.1 Introduction

It is essential that engineers master the concept of complex numbers because the important role that complex numbers play in a variety of application areas. In this module applications in the field of electric circuits are provided.

9.1.2 Alternating Current (AC) Electric Circuits

Earlier we introduced a number of components that are typically found in common electric circuits. These included voltage sources, current sources and resistors. We also observed that the behavior of an electric circuit could be predicted by using several laws from Physics, including Ohm's Law and Kirchoff's Laws.

In this laboratory exercise, we will introduce two additional components of electric circuits: the inductor and the capacitor. These elements are typically found in electric circuits which involve sinusoidally varying voltage or current sources. These circuits are called *alternating current* or *AC* circuits. AC circuits abound in the physical world. The voltage and current that power household appliances comes from AC sources.

Figure 1 shows the plot for a sinusoidally varying waveform that represents the output of an AC voltage source. Such a waveform could also be used to represent the current that is supplied by an AC current source. It is important to note that the waveform has a repetitive or *periodic* nature.

[1]This content is available online at <http://cnx.org/content/m38605/1.3/>.

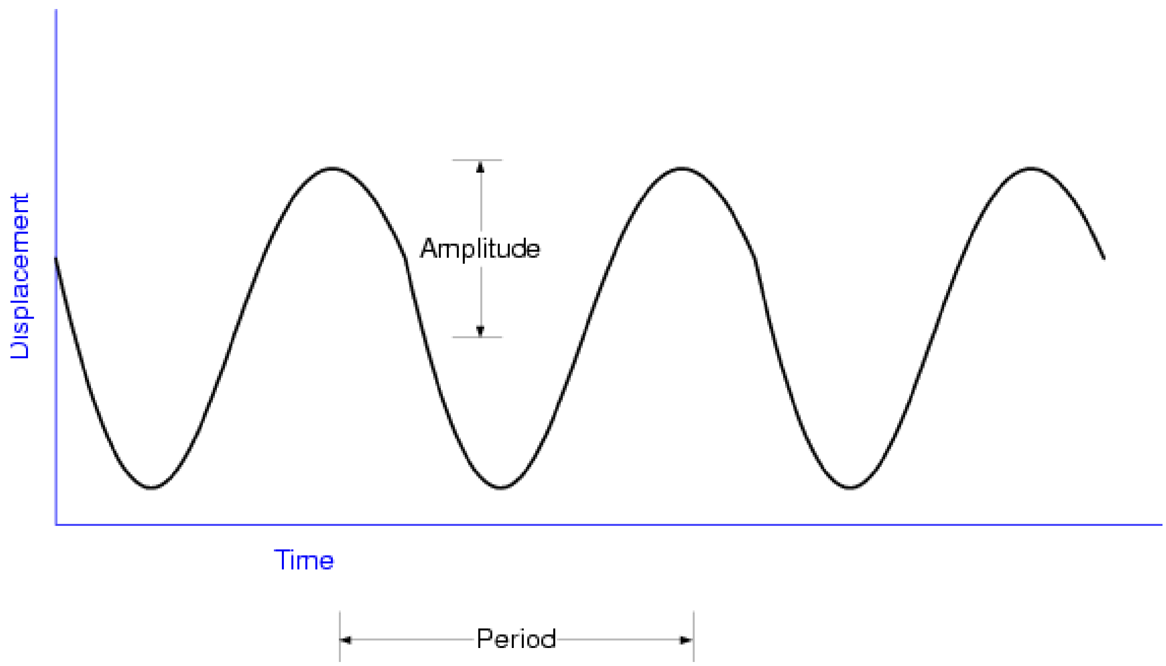

Figure 9.1: Sketch of a sinusoidal waveform.

In the figure, we note that the amount of time that occurs between successive maxima of the sinusoidal waveform is equal to the *period*. The *angular frequency* of the waveform is denoted by the symbol ω and is defined in terms of the period by the equation

$$\omega = \frac{2\pi}{T}\text{rad}/s \tag{9.1}$$

If we denote the *amplitude* as $Vmax$, then we can express the sinusoidal waveform for the voltage mathematically as

$$v\left(t\right) = V_{\max}\cos\left(\omega t + \theta_v\right) \tag{9.2}$$

Here the instantaneous value of the voltage is measured in the units volts. The term θv is called the *phase angle* of the sinusoidal waveform. It is measured in degrees. Its usage and importance in the analysis of AC circuits will be discussed later in the course during the study of trigonometry.

Inductors and capacitors are found in circuits of all types and designs, so their understanding is critical to the education of an engineer or scientist. One important distinction between resistors and these two new components (inductors and capacitors) is that they are analyzed using different mathematic techniques. In the case of a resistor, it was quite easy to determine the relationship between the current, voltage and resistance present in a circuit by means of simple algebra. In the case of the inductor and the capacitor, we will see that we must expand our knowledge of mathematics particulary in the are of complex numbers to analyze circuits that contain inductors and capacitors.

9.1.3 Inductors

An *inductor* is an electrical component that stores energy in the form of a magnetic field. In its simplest form, an inductor consistsof a wire loop or coil. Figure 2 depicts an inductor next to a coin to show its relative size and structure.

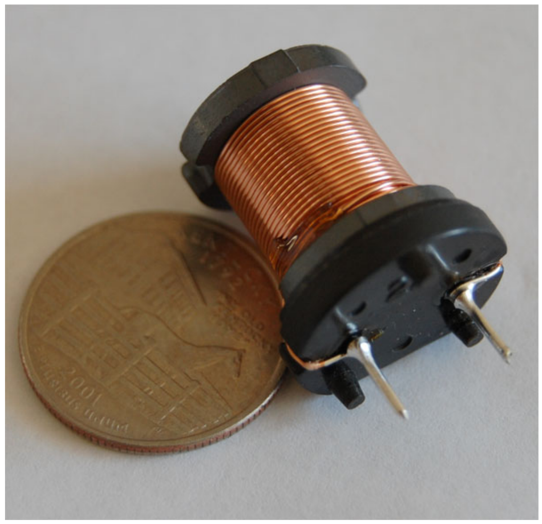

Figure 9.2: Photograph of an inductor beside a coin.

The *inductance* of the component is directly proportional to the number of turns present in the wire that makes up the coil. Inductance also depends on the radius of the coil and on the type of material around which the coil is wound. The standard unit of inductance is the Henry (H).

The schematic symbol for an inductor is shown in Figure 3.

Figure 9.3: Schematic symbol for an inductor.

9.1.4 Capacitor

A *capacitor* is an electrical device consisting of two conducting plates separated by an electrical insulator (the **dielectric**), designed to hold an electric charge. Charge builds up when a voltage is applied across the plates, creating an electric field between them. Current can flow through a capacitor only as the voltage across it is changing, not when it is constant. Capacitors are used in power supplies, amplifiers, signal processors, oscillators, and logic gates.

The standard unit of capacitance is the farad (F). Typical capacitance values are small. Common capacitors have values of capacitance that are expressed in units of microfarads (μF). Figure 4 shows a photograph of a several different capacitors.

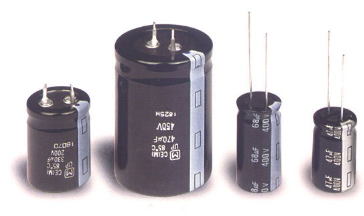

Figure 9.4: Photograph of capacitors of various values.

The standard symbol for a capacitor is shown in Figure 5.

Figure 9.5: Schematic symbol for a capacitor.

9.1.5 Impedance

In the case of electric circuits that are driven by a sinusoidally varying voltage source, the **impedance** serves to restrict the flow of current. Like the resistance, impedance is measured in ohms (Ω). However, the impedance differs from resistance in that the impedance is a complex quantity. Because the impedance is a complex quantity, we will represent the impedance as a complex number

$$Z = R + jX \tag{9.3}$$

The real part of Z as stated in equation (3) is R and the imaginary part is X.

Resistors, inductors and capacitors serve to contribute to the impedance present in a sinusoidally varying electric circuit. The impedance of a resistor is merely the value of its resistance.

The *impedance of an inductor* (Z_{L}) can be easily computed via the relationship

$$Z_L = j\omega L \tag{9.4}$$

The impedance of an inductor is measured in the units Ω. The term ω is equal to the angular frequency of the sinusoidally varying source voltage. Examination of equation (4) indicates that as the angular frequency increase, so too does the impedance of the inductor. At very high frequencies an inductor will essentially inhibit all flow of current through itself.

The *impedance of a capacitor* (Z_c) is given by the equation

$$Z_C = -j\left(\frac{1}{\omega C}\right) \tag{9.5}$$

The impedance of a capacitance is measured in the units Ω. Once again, the term ω is equal to the angular frequency of the sinusoidally varying source voltage. Examination of equation (5) indicates that as the angular frequency increases, the impedance of the capacitor decreases. At very high frequencies a capacitor will behave as a short circuit. That is, its effect at very high frequencies is to allow current to flow through it in an unimpeded manner.

9.1.6 Series RL Circuit

Just as resistors can be combined using series and parallel connections, so too can impedances. In the case of series connections, impedances are merely added. One distinction is that the addition is performed using complex arithmetic. Let us consider the RL circuit shown in Figure 6.

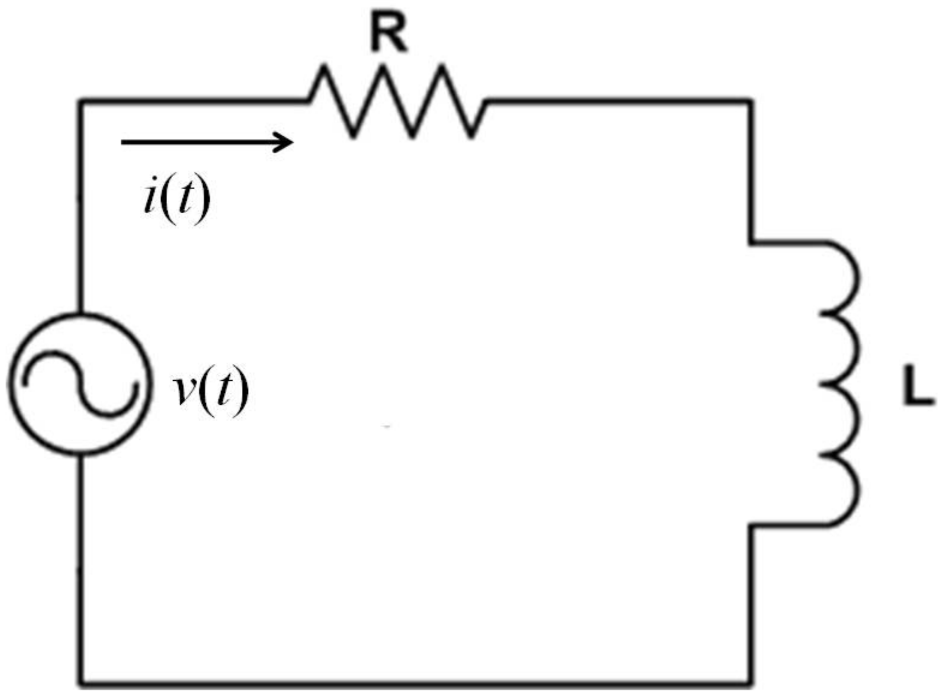

Figure 9.6: Series RL circuit.

The series impedance is equal to the sum of the resistance with the impedance of the inductor. Suppose that for this circuit the value of R is 10 Ω and that the value for the inductance is 100 mH. Suppose that the frequency (ω) of the source voltage is 100 rad/sec. For this specification of values, we can compute the impedance of the series connection

$$Z = 10 + j\,(100)\left(100 \times 10^{-3}\right)\Omega = 10 + j10\Omega \tag{9.6}$$

The square of the magnitude of the impedance can be obtained by use of the complex conjugate.

$$\mid Z \mid^2 = Z \times Z * (10 + j10)\,(10 - j10) = 100 + 100 = 200 \tag{9.7}$$

So we calculate the magnitude of the impedance to be

$$\mid Z \mid = 14.17\Omega \tag{9.8}$$

An important property of AC circuits that contain an AC source voltage along with resistors, inductors and capacitors is that if the current i(t) will take the form of a sinusoid

$$i\,(t) = I_{\max}\cos\left(\omega t + \theta_i\right) \tag{9.9}$$

The instantaneous value of the current is measured in Amps. The angular frequency of the current sinusoid (ω) will be the same as that of the sinusoid that represents the supply voltage. In addition, electric circuits involving resistors, capacitors and inductors will contribute to a change in the phase angle (θi) of the current sinuosoid. In general the phase angle of the voltage sinusoid (θv) will differ from that of the current sinusoid (θi). Once again, the discussion of how the phase angle of the current can be computed will be deferred until our later discussion of trigonometry.

Once we know the magnitude of the impedance, we can use it to calculate the amplitude of the sinusoidally varying current, $Imax$. This is accomplished by the following formula.

$$I_{\max} = \frac{V_{\max}}{\mid Z \mid} \tag{9.10}$$

So the amplitude of the current is $Vmax/141.7$. It is interesting to note that this formula is similar to Ohm's Law. The differences lie in the fact that the magnitude of the impedance appears instead of the resistance. Also, the amplitudes of the sinusoidally varying current and voltage appear.

The formula expressed above is useful in determining the amplitude of the current for a wide range of sinusoidally varying AC circuits. These circuits can combine resistors with inductors and capacitors to create a wide range of design options for electrical engineers. The following exercises illustrate the application of complex numbers to the analysis of AC circuits.

9.1.7 Exercises

Consider the series RC circuit shown below

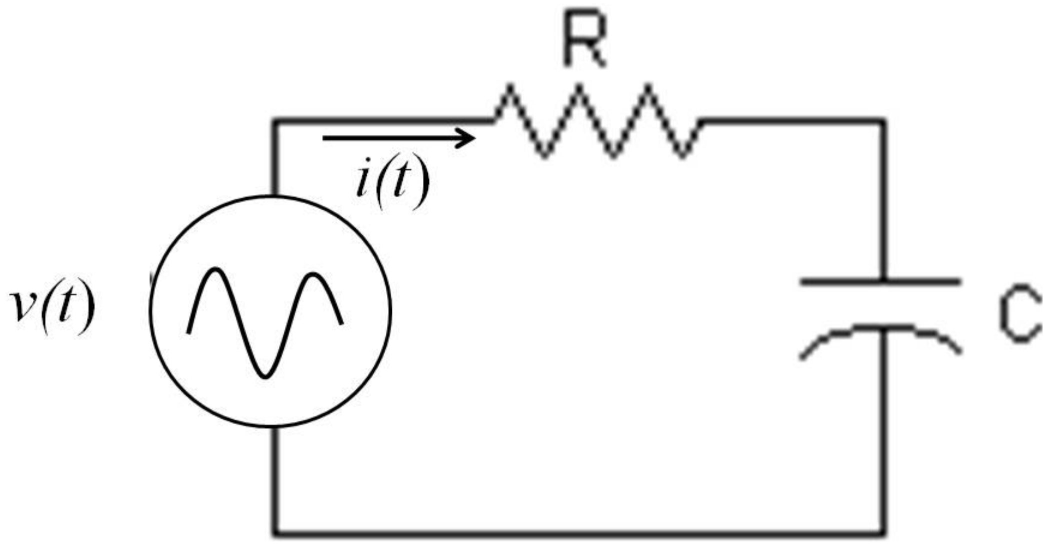

Figure 9.7: Series RC circuit.

Suppose that the sinusoidally varying source voltage is given as $v\left(t\right)=20\cos\left(10t\right)V$.

1. What are the amplitude and the radian frequency of the source voltage?
2. If the value of the capacitance is C = 100 μF, what is the impedance of the capacitor?
3. If the value for the resistor is R = 5 KΩ, what is the series impedance of the circuit?
4. Find the magnitude of the series impedance?
5. What is the amplitude of the sinusoidally varying current, $i(t)$?

Chapter 10

Logarithms[1]

10.1 Logarithms

10.1.1 Introduction

This module is intended to present some areas of engineering in which logarithms are used. By reading the material and solving the associated problems, you will learn about some important applications of logarithms in engineering.

10.1.2 Decibels

The **decibel (dB)** is a logarithmic unit that indicates the ratio of a physical quantity relative to a specified or implied reference level. The decibel is used for a wide variety of measurements in science and engineering, most prominently in acoustics, electronics, communications, radar, sonar and control systems.

Decibels are frequently used as a means to express the power ratio for physical systems. It is computed by multiplying the factor 10 by the base 10 logarithm of the ratio of the quantities under consideration. Equation (1) shows the computation that is used to express the ratio of two powers using decibels

$$L_{\mathrm{DB}} = 10\log_{10}\left(\frac{P_2}{P_1}\right) \tag{10.1}$$

Gain of an Amplifier: We will begin our discussion of decibels with an application in the field of electronics. An amplifier is an electronic device that is capable of boosting the power present in an input signal to produce an output signal with more power. It can be thought of as a black box as shown in Figure 1.

[1] This content is available online at <http://cnx.org/content/m38608/1.5/>.

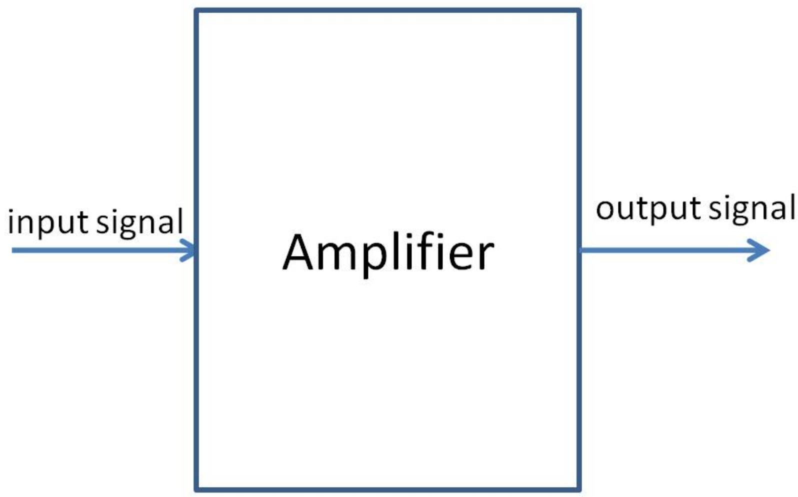

Figure 10.1: Block diagram of an amplifier.

In practical cases, the ratio of the power in the output signal to the power in the input signal is a positive quantity whose value is greater than unity. The decibel measurement of this ratio of power is often called the *gain* of the amplifier and is given as

$$\text{Gain} = 10\log_{10}\left(\frac{P_{\text{output}}}{P_{\text{input}}}\right)\text{dB} \tag{10.2}$$

Question: An electronic signal is passed through an amplifier. Suppose that the power present in the signal at the input to the amplifier is 10 W. The power present in the signal at the output of the amplifier is 20 W. Express the gain of the amplifier in decibels.

We can use equation (2) to easily express the gain of the amplifier in terms of decibels

$$\text{Gain} = 10\log_{10}\left(\frac{20W}{10W}\right) = 10\log_{10}(2) = 3.01\text{dB} \approx 3\text{dB} \tag{10.3}$$

10.1.3 Signal to Noise Ratio

Electrical signals are often corrupted by a random phenomenon known as *noise* when they are transmitted from one point to another . Because it is impossible to know the exact value of the noise at any point in time, it is often becomes difficult to extract the orignal signal at the receiver without the application of some form of *signal processing algorithm* such as a *filter*. The situation is depicted in Figure 2.

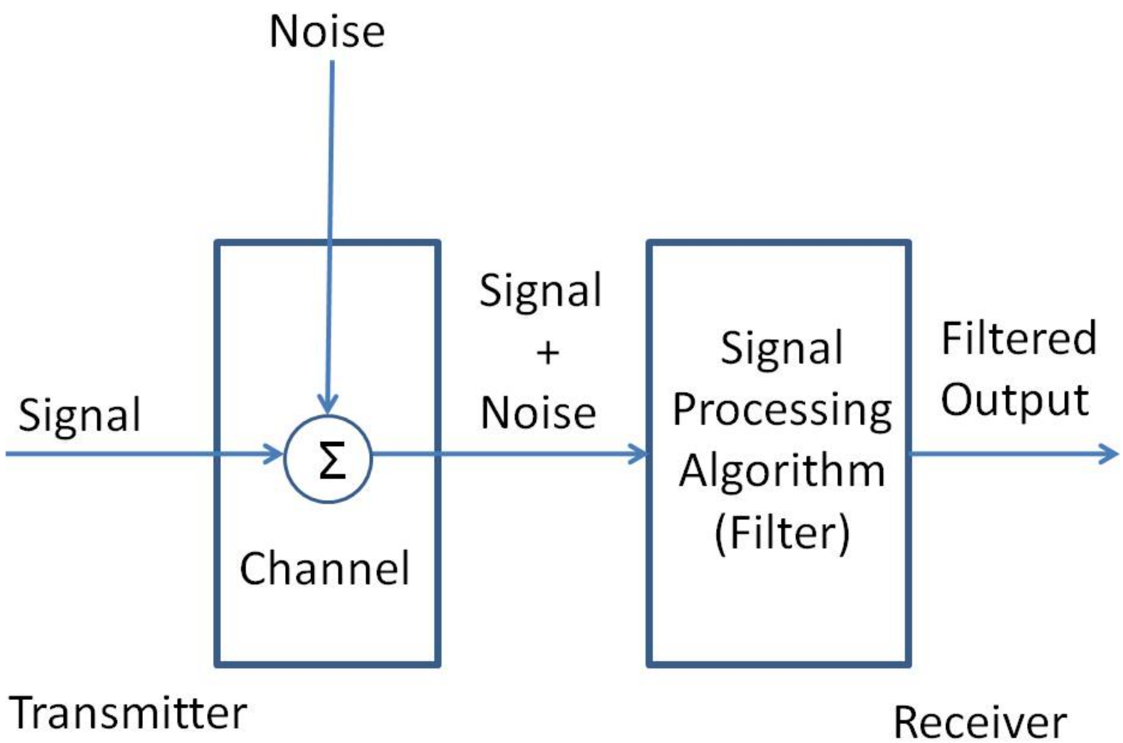

Figure 10.2: Communication system with signal processing.

A common figure of merit of communication systems is the *signal-to-noise ratio*. Communication systems that are characterized by high signal-to-noise ratios are in general superior to those that are characterized by low signal-to-noise ratios.

By definition the signal-to-noise ratio or *SNR* is given as the ratio of the power in a signal divided by the power in the noise that is responsible for corrupting the signal. The signal-to-noise ratio can be expressed in decibels as follows

$$\mathrm{SNR} = 10\log_{10}\left(\frac{P_{\text{signal}}}{P_{\text{noise}}}\right)\mathrm{dB} \tag{10.4}$$

where $Psignal$ and $Pnoise$ represent the values for the signal power and noise power respectively.

At the receiver, the original signal arrives with an added noise component. Communication engineers use signal processing algorithms in the form of filters to minimize the power of the noise that is present in the received noisy signal while seeking to retain as much of the power of the original signal as possible. Such algorithms are based on the premise that although random, noise may well fall into different regions of angular frequency in the electromagnetic spectrum than the signal itself. Thus, frequency selective filters can be utilized to increase the signal-to-noise ratio.

Suppose that we represent the signal-to-noise ratio at the input to the signal processing algorithm as

SNRin and that at its output as *SNRout*. For a good designs, SNR_{out} will be significantly higher that that of *SNRin*. This increase in signal-to-noise ratio results in superior performance and more reliable communication of information.

Question: A signal with power 30 W is sent through a noisy communication channel. The noise introduced by the communication channel has an associated power of 5 W. What is the signal-to-noise ratio of the noisy signal that is received from the communication channel in decibels?

Solution: We can find the SNR by substituting the appropriate values for the parameters into equation (4)

$$\text{SNR} = 10\text{log}_{10}\left(\frac{P_{\text{signal}}}{P_{\text{noise}}}\right) = 10\text{log}_{10}\left(\frac{30W}{5W}\right) = 10\text{log}_{10}\left(6\right) = 7.78\text{dB} \tag{10.5}$$

Question: Consider the noisy signal described in Example 2. Suppose that a signal processing filter is applied to it. The filter reduces the noise power by 95% while it reduces the signal power by only 10%. What is the signal-to-noise ratio of the filtered output?

Solution: The amount of signal power present in the filtered output is 30 W – 0.1 (30 W) or 27 W. The amount of noise power present in the filtered output is 5 W – 0.95 (5 W) or 0.25 W.

The signal-to-noise ratio of the filtered output is

$$\text{SNR} = 10\text{log}_{10}\left(\frac{27W}{0.25W}\right) = 10\text{log}_{10}\left(108\right) = 10 \cdot 2.03 = 20.33\text{dB} \tag{10.6}$$

We see that the primary result obtained by the application of the signal processing filter is to raise the signal to noise ratio from 7.78 dB to 20.33 *dB*. This represents an increase in SNR of 20.33 – 7.78 *dB* = 12.55 *dB*. Along with this increase in SNR, one would observe an increase in reliable communication of information.

10.1.4 Transient Response (Exponential Decay)

The transient response of a circuit is response of a circuit that is expressed as a function of time. Determining the transient response of a circuits is important in that knowledge of the transient response reveals how a circuit will behave whenever perturbations arise, such as the opening or closing of a switch.

Let us examine the RC circuit present in the figure below

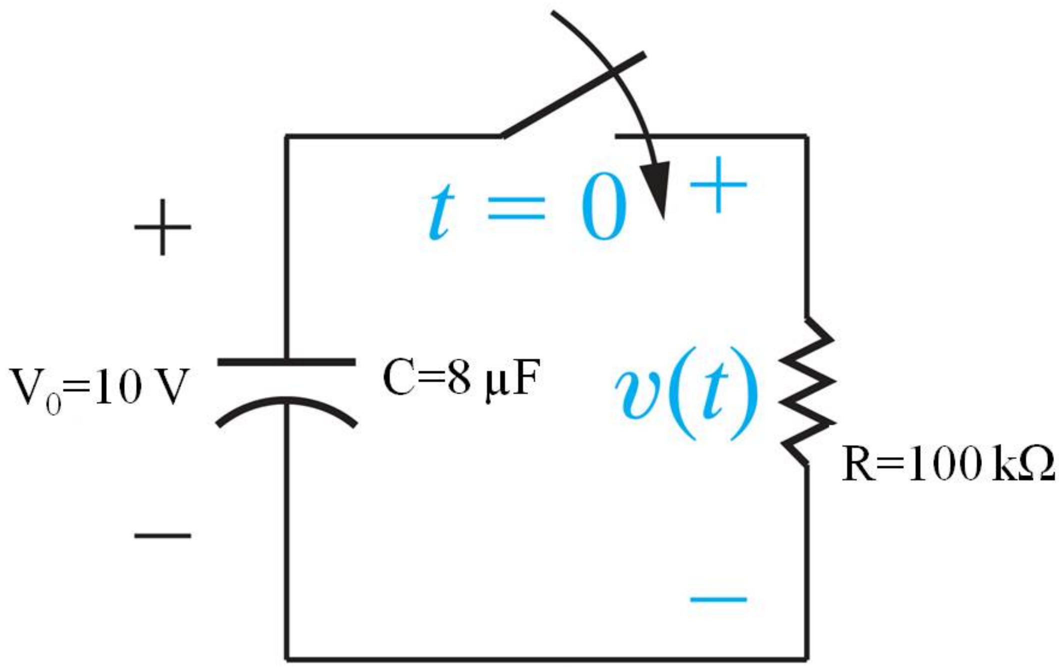

Figure 10.3: Switched RC circuit.

This circuit is called an RC circuit in that it contains a resistor and a capacitor.

The switch is open for values of time (t) less than 0. Thus for negative values of time there is no current flow. Because there is no current flow, the voltage across the resistor ($v(t)$) is zero. The capacitor is initially charged. The initial voltage across the capacitor (V_0) is 10 V.

At t =0, the switch is closed. Because the voltage across the capacitor cannot change instantaneously, the value of the voltage across the resistor immediately after the closing of the swith ($v(0^+)$ will be 10 V. After the switch has been closed, current will flow throughout the circuit and the voltage across the resistor will diminish exponentially. Because the decay can be described by the use of an exponential function, this is an example of **exponential decay**.

We can write an expression for the transient response, that is, the voltage across the resistor for positive time

$$v\left(t\right) = 10e^{-t/\mathrm{RC}} \tag{10.7}$$

Here, the units of $v(t)$ are Volts.

The product of the resistance and the capacitance is called the time constant for the circuit and is often denoted as (τ). The time constant is measured in seconds. For this example, $\tau = \left(100 \times 10^3\right)\left(8 \times 10^{-6}\right) = 800 \times 10^{-3} = 0.800s$

So the transient response of the circuit becomes

Let us now work a problem about this circuit that involves logarithms.

Question: At what instant of time will the transent response be equal to 5 Volts?

Solution: We have been asked to find the value of t that satisfies the following equation.

$$5 = 10e^{-t/0.8} \tag{10.8}$$

Let us divide each side of the equation by (10) and interchange the sides

$$e^{-t/0.8} = 0.5 \tag{10.9}$$

We now take the natural logarithm of each side

$$\frac{-t}{0.8} = -0.693 \tag{10.10}$$

$$t = (0.8)\,(0.693) = 0.555 \tag{10.11}$$

So we conclude that at the time 0.555 seconds after the switch closes, the value of the voltage across the resistor will be 5.0 volts.

10.1.5 Compound Interest

It is always essential to consider the financial aspects of an engineering project. The field of industrial engineering provides us with the analytical tools necessary to weigh the merits of competing designs.

Most situations surrounding the financial aspects of an engineering project involve the determination of what is most economical in the long run. That is, engineers must be aware of the costs and benefits of a project over a considerable period of time. In situations as these, it is important to consider the **time value** of money. Because of the existence of interest, the current value of a dollar is worth more than the value of a dollar some time in the future.

Interest can be defined as money that is paid for the use of money that has been borrowed. The rate of interest is the ratio between the interest chargeable or payable at the end of a period of time. This period of time is typically yearly, quarterly or monthly. In this module, we will restrict our attention to interest that is paid yearly or **per annum**.

As an example, a sum of money is invested at an annual rate of interest of 4%. One year later, the interest that would be paid on the investment would be $40 (4% of $1,000). So after one year, the initial sum would grow to a value of $1,040 one year later.

Suppose that the $1,040 were invested for a second year at the end of the first year. At the end of the second year, the amount of interest that would be payable would be 4% of $1,040 or $41.60. The amount of interest earned in the second year exceeds the amount earned in the first year because of a phenomenon known as **compound interest**.

Interest calculations can be quantified by mathematical formulas. Suppose that we are concerned with making an investment of P dollars at an annual rate of interest of i for n years. Here P denotes the present value of the investment, n represents the number of years the money is to be invested, and i is the interest rate per annum. Let us denote by F the future value of the investment at the end of n years. The value of F can be calculated via the formula

$$F = P\,(1 + i)^{n} \tag{10.12}$$

Let us illustrate the use of this formula by means of a problem.

Question: A couple has just had a baby. The couple wishes to make an investment in an account that will be used to fund college expenses. The couple visits an investment advisor and establishes a tax-free college savings account. By tax-free, we mean an account where income tax is not paid yearly. The couple is advised that this feature will enable the account to grow faster.

The couple deposits $25,000 in the account. The couple is told that the value of the account will appreciate at an annual rate of 8% . What will be the value of the account when the child turns 18 years of age?

Solution: We are asked to find the value of F. From the problem statement, P is $25,000, i is 0.08 and n is 18. We substitute into the interest formula to obtain the result

$$F = (25,000)\left(1 + 0.08\right)^{18} = (25,000)\left(1.08\right)^{18} = (25,000)\left(3.996\right) = 99,900 \tag{10.13}$$

We conclude that the account will be worth $99,900 at the end of 18 years.

Let us now a related problem that involves the use of logarithms.

Question: An individual inherits $10,000 from a relative. The individual wishes to invest the sum in a tax-free account. He/she plans to use the proceeds of this account as a down payment on a future purchase of a home. The annual interest rate of the account is 6%,

The individual anticipates that he/she will need at least $20,000 for the down payment. How long will it take for the value of the account to grow to $20,000?

Solution: Once again, we begin with the identification of the parameters of the problem. Here, P is 10,000, i is 0.06, and F is 20,000. We incorporate these values into the interest formula

$$20,000 = (10,000)\left(1 + 0.06\right)^{n} \tag{10.14}$$

Let us divide each side by (10,000) and re-arrange terms

$$\left(1.06\right)^{n} = 2 \tag{10.15}$$

Now let us take the logarithm of each side of the equation. We will use 1.06 as the base of the logarithm

$$n = \log_{1.06}\left(2\right) \tag{10.16}$$

The base 1.06 logarithm is related to the base 10 logarithm as follows

$$\log_{1.06}\left(x\right) = \frac{\log_{10}\left(x\right)}{\log_{10}\left(1.06\right)} \tag{10.17}$$

We will use this relationship to help us find the value for n

$$n = \frac{\log_{10}\left(2\right)}{\log_{10}\left(1.06\right)} = \frac{0.301}{0.0253} = 11.90 \tag{10.18}$$

So the individual should plan on waiting 11.90 years for the account to grow to a value of $20,000.

Problems such as the previous one often make use of the relationship

$$\log_{a}\left(x\right) = \frac{\log_{b}\left(x\right)}{\log_{b}\left(a\right)} \tag{10.19}$$

This relationship allows one to convert from the logarithm of any base to a logarithm of another base. Typically, scientific calculators are only able to compute base 10 and natural (base e) logarithms. One should become acquainted with the conversion of logarithms of other bases to base 10 or base e logarithms.

10.1.6 Exercises

1. The power of the signal entering an amplifier is 15 mW. The power of the signal that leaves the amplifier is 25 W. Express the gain of the amplifier in decibels.
2. Unlike an amplifier, some devices reduce the power of input signals. This process is called attenuation. Suppose that the power that enters an attenuator is 60 W and the power at the output is 0.9 W. Express the gain of the attenuator in decibels.

3. Consider a signal processing scheme such as that shown in Figure 2. The power of the input signal before it passes through the noisy channel is 40 W. After passing through the noisy channel, the original signal is corrupted by noise. The noise component has a power of 10 W. What is the Signal to Noise ratio of the signal that emerges from the noisy channel?
4. Consider the situation described in exercise 3. The noisy signal enters a signal processor. The signal processor diminishes the noise power of the signal by 5%, while diminishing the power in the noise component by 95%, What is the SNR of the output of the signal processor?
5. Consider the circuit shown in Figure 3. Let us replace the resistor with another whose value is 200 kΩ. (a) What is the new value of the time constant of the circuit. (b) Find the value of the transient response when $t = 0.8$ seconds. (c) Determine the value of time at which the transient response decays to a value of 1 Volt.

Chapter 11

Simultaneous Equations[1]

11.1 Simultaneous Equations

11.1.1 Introduction

The applications of simultaneous equations in the modeling, analysis and design of engineering systems are numerous. This module presents several applications drawn from engineering that illustrate uses of simultaneous equations.

11.1.2 Unmanned Air Vehicle (Robotics)

Many applications in engineering involve the quantities speed, distance and time. In some applications, one is presented with two situations. In the two situations, the speed of an object differs. The following is an example involving an Unmanned Air Vehicle (UAV) whose speed varies depending upon whether the UAV travels in the same direction of the wind as opposed to the situation where the UAV travels in the opposite direction in opposition to the wind. Simultaneous equations can be used to solve problems relating to situations such as this.

Example 1: In a closed course surveillance flight, the downwind leg of length18.6 miles is completed in 2.0 hours. The upwind leg which is also 18.6 miles in length is completed in 3.5 hours. Find both the speed of the UAV and the speed of the wind.

To address such a problem it is critical to begin with the definition of variables. We will let **V** be the airspeed of the UAV measured in miles/hr. The wind speed expressed in miles/hr will be represented by the variable **W**.

We know that the distance that an object travels is equal the product of its speed and time. When the UAV travels downwind, its speed will be equal to the sum of its airspeed and the windspeed. When the UAV travels upwind, its total speed will equal to the difference of its airspeed minus the windspeed. The distance that the UAV travels is the same (18.6 miles) whether it travels downwind or upwind.

We can use this information to establish two equations

$$18.6 = 2.0 \times (V + W) \tag{11.1}$$

$$18.6 = 3.5 \times (V - W) \tag{11.2}$$

From equation (1) we obtain

$$18.6 = 2.0 \times (V + W) \tag{11.3}$$

[1] This content is available online at <http://cnx.org/content/m38616/1.4/>.

which can can yield an expression for the variable V

$$V = 9.3 - W \tag{11.4}$$

Next, we can substitute this expression for V into equation (2). By doing so, we will be able to solve for W.

$$18.6 = 3.5 \times ((9.3 - W) - W) \tag{11.5}$$

Dividing each side of the equation by (3.5) yields

$$5.31 = (9.3 - W) - W \tag{11.6}$$

This equation can be readily solved for the variable W as follows

$$\begin{gathered} -3.99 = -2W \\ W = 1.995 \text{ miles / hr} \end{gathered} \tag{11.7}$$

Next, we substitute this value for W into equation (4) to solve for V.

$$V = 9.3 - 1.995 = 7.305\text{miles/hr} \tag{11.8}$$

Thus we conclude that the UAV airspeed is 7.305 miles/hr and the windspeed is 1.995 miles/hr.

11.1.3 Analysis of a Torsion Beam

Torque is a term that is used to describe the tendency of a force to rotate an object about an axis, a fulcrum or a pivot. Just as a force is a push or a pull, a torque can be thought of as a twist. Loosely speaking, torque is a measure of the turning force on an object such as a bolt. For example, pushing or pulling the handle of a wrench connected to a nut or bolt produces a torque (turning force) that loosens or tightens the nut or bolt.

Consider the torsion beam shown in Figure 1.

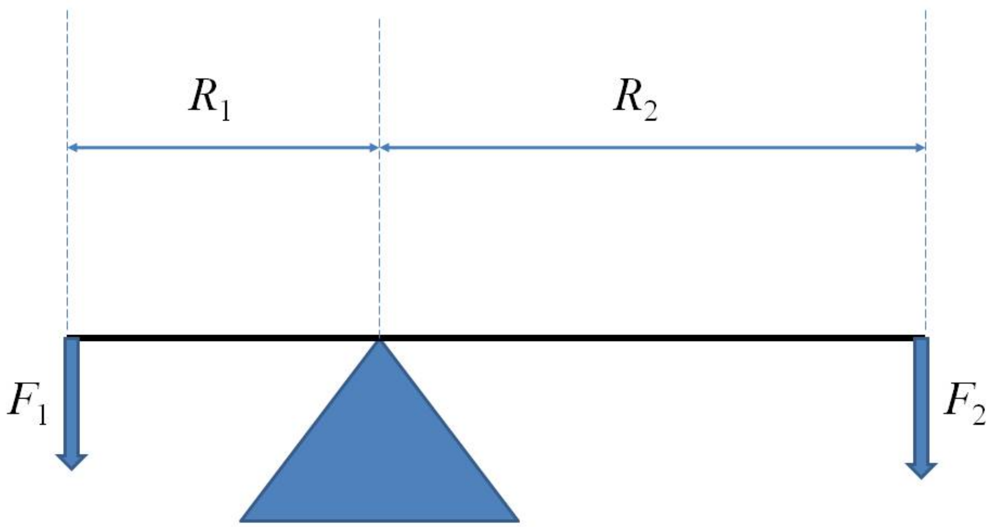

Figure 11.1: Diagram of a torsion beam.

Figure 1. Torsion beam.

In this figure, we observe a situation in which forces are applied at each end of the beam. We also note that each force is applied at a right angle to the beam. Under such conditions, torque can be calculated as the product of the force and the length of the beam from its end to its support point atop the pivot point of the beam.

The force (F_1) contributes a counter-clockwise torque ($TCCW$) on the beam. The value of this torque can be calculated

$$T_{\text{CCW}} = F_1 \times R_1 \tag{11.9}$$

The force (F_2) contributes a clockwise torque (TCW) on the beam. The value of this torque can be calculated as

$$T_{\text{CW}} = F_2 \times R_2 \tag{11.10}$$

Suppose that the beam is not in motion. Engineers would say that the beam is static or that the beam system is in equillibrium. A necessary condition for equillibrium of the system is that the two torques be equal, that is

$$T_{\text{CW}} = T_{\text{CCW}} \tag{11.11}$$

We now will apply what we know about simultaneous equations to solve a problem involving a torsion beam.

Example 2: Assume that a a beam of length 56 inches is supported by a fulcrum. Force is applied at each end of the beam. Assume that the force (F_1) on the left hand side of the beam is 24 lbs while the force on the right hand side of the beam (F_2) is 32 lbs. If the beam is static, then what are the values of R_1 and R_2?

We know from that the clockwise and counter-clockwise torques must balance as a condition for equilibrium. This leads to the first equation that we will use to solve this problem

$$24R_1 = 32R_2 \tag{11.12}$$

which can be written as

$$24R_1 - 32R_2 = 0 \tag{11.13}$$

We also know that the overall length of the beam is 56 inches. Therefore,

$$R_1 + R_2 = 56 \tag{11.14}$$

which can be solved for R_1 as

$$R_1 = 56 - R_2 \tag{11.15}$$

We may substitute the expression for R_1 in equation () to obtain

$$24\left(56 - R_2\right) - 32R_2 = 0 \tag{11.16}$$

Multiplication and rearrangement yield the equation

$$1,344 = 56R_2 \tag{11.17}$$

which can be solved for R_2.

$$R_2 = 24\text{in} \tag{11.18}$$

The value for R1 can be easily found as

$$R_1 = 56 - R_2 = 56 - 24 = 32\text{inches} \tag{11.19}$$

11.1.4 Parallel Processing

Parallel processing is a term drawn from Compute Science that involves the simultaneous use of more than one central processing unit (CPU) to execute a program. Ideally, parallel processing makes programs run faster because there are more than one processors in use to execute the program. Though most computers have just one CPU, newer computer architectures that feature several CPU's are becoming the norm more prevalent.

Example 3: Suppose that we are interested in implementing a computer algorithm on two processors. Let us call these processors A and B. The steps involved in the accomplishment of the algorithm are divided among the two processors. In all there are 17,000,000 computations which will need to be performed on the two processors. Working in parallel, the two processors accomplish the algorithm. In doing so, the processor A is in service for 3 seconds while the processor B is in services for 2 seconds.

Suppose that a second algorithm is to be accomplished by processors A and B working in parallel. In all, the second algorithm requires the execution of 15,500,000 computations. In executing the second algorithm, 2 seconds of computing time are required of processor A while 3 seconds of computing time are required of processor B.

What are the processor rates for processor A and processor B?

Suppose that we define the processing rates (measured in computations/second) for processors A and B to be represented by the variables x and y respectively. With this definition of variables, the number of computations that processor A can perform in 3 seconds is the product ($3\ x$). Likewise the number of computations that processor B can perform in 2 seconds is the product ($2\ y$). The total number of computations that must be performed to complete the first algorithm is 17,000,000. Therefore, we obtain the first of two simultaneous equations

$$3x + 2y = 17 \times 10^6 \tag{11.20}$$

Let us now consider the implementation of the second algorithm. The number of computations that processor A can make in 2 seconds is equal to the product ($2\ x$). Similarly, the number of computations that processor B can make in 3 seconds is the product ($3\ y$). We obtain the second simultaneous equation as

$$2x + 3y = 15.5 \times 10^6 \tag{11.21}$$

Let us use the first equation to solve for x in terms of y

$$3x = -2y + 17 \times 10^6 \tag{11.22}$$

which results in the expression

$$x = \frac{17 \times 10^6 - 2y}{3} \tag{11.23}$$

Next, we substitute this expression for x into equation ()

$$2\left(\frac{17 \times 10^6 - 2y}{3}\right) + 3y = 15.5 \times 10^6 \tag{11.24}$$

This equation can be simplified as follows

$$17 \times 10^6 - 2y + \frac{3}{2}(3y) = \frac{3}{2}\left(15.5 \times 10^6\right) \tag{11.25}$$

$$17 \times 10^6 - 2y + 4.5y = 23.25 \times 10^6 \tag{11.26}$$

$$2.5y = 6.25 \times 10^6 \tag{11.27}$$

This leads to the solution

$$y = 2.5 \times 10^6 \text{computations}/s \tag{11.28}$$

We now may solve for x through the use of equation ()

$$x = \frac{17 \times 10^6 - 2\left(2.5 \times 10^6\right)}{3} \tag{11.29}$$

which lead to the result

$$x = \frac{12 \times 10^6}{3} = 4 \times 10^6 \tag{11.30}$$

Thus we conclude that processor A can perform 4 million computations per second while processor B can perform 2.5 million computations per second.

11.1.5 Alloy Compositions

Metallurgical engineering involves the study of the physical and chemical behavior of metallic elements and their mixtures, which are called alloys. It is also involves the technology of metals. That is, metallurgical engineering encompasses the way in which the science of metals is applied to produce compounds for practical use.

The following example presents how simultaneous equations can be applied in the study of alloys.

Example 4: Let us consider two alloys comprised of different percentages of copper and zinco. Suppose that the first alloy is made up of a mixture of 70% copper and 30% zinc. The second alloy is comprised of 40% copper and 60% zinc.

By melting the two alloys and re-combining their constituents into one 300 gram casting, the resulting alloy is 60% copper and 40% zinc.

Find the amount of copper and zinc in each of the original alloys.

We begin our solution by establishing the variables A and B as being equal to the mass (in grams) of the first alloy and the second alloy respectively. We know that the final casting has a mass of 300 grams, so our first simultaneous equation is

$$A + B = 300 \tag{11.31}$$

Because 60% of the final casting is copper, the amount of copper in the final casting is the product 0.60 (300 grams) or 180 grams.

We know that 70% of the first alloy is copper and that 40% of the second alloy is copper. Thus we may write the second simultaneous equation as

$$0.7A + 0.4B = 180 \tag{11.32}$$

The variable A can be expressed as (300 – B) and substituted into the equation

$$0.7A + 0.4B = 180 \tag{11.33}$$

to yield

$$0.7\,(300 - B) + 0.4B = 180 \tag{11.34}$$

This equation can be simplified as

$$210 - 0.7B + B = 180 \tag{11.35}$$

Combining terms yields

$$-0.3B = -30 \tag{11.36}$$

The solution for the variable B is 100. So we find that the mass of the second alloy is 100 grams. Because the sum of the masses of the two alloys is 300 grams, the mass of the first allow is 200 grams.

11.1.6 Photovoltaic Arrays

A **photovoltaic system** is a system which uses one or more solar panels to convert sunlight into electricity. It consists of multiple components, including the photovoltaic modules, mechanical and electrical connections and mountings and means of regulating and/or modifying the electrical output.

Due to the low voltage of an individual solar cell (typically on the order of 0.5V), several cells are wired in series in the manufacture of a "laminate". The laminate is assembled into a protective weatherproof enclosure, thus making a photovoltaic module or solar panel. Modules may then be strung together into a photovoltaic array. The electricity generated can be either stored, used directly (island/standalone plant)or fed into a large electricity grid powered by central generation plants (grid-connected/grid-tied plant) or combined with one or many domestic electricity generators to feed into a small grid (hybrid plant)

A *photovoltaic array* (or *solar array*) is a linked collection of solar panels. The power that one module can produce is seldom enough to meet requirements of a home or a business, so the modules are linked together to form an *array*. Most arrays use a device known as an inverter to convert the DC power produced by the modules into alternating current that can be used to power lights, household appliances, motors, and other loads. The modules in a photovoltaic array are usually first connected in series to obtain the desired voltage. Once this is accomplished, the individual strings are then connected in parallel to allow the system to produce more current.

Example 6: A photvoltaic array is to be installed. It is a requirement that the length of the array be 8 inches longer than its width. A second requirement states that the total area of the array be 180 in^2.

Find the length (L) and width (W) of the array.

We may use the first requirement to construct the first simultaneous equation

$$L = W + 8 \tag{11.37}$$

The second requirement contributes a second simultaneous equation

$$LW = 180 \tag{11.38}$$

We may substitute the expression for L given by equation (18) into equation (19)

$$W^2 + 8W = 180 \tag{11.39}$$

or equivalently

$$W^2 + 8W - 180 = 0 \tag{11.40}$$

This quadratic equation can be factored as

$$(W + 18)(W - 10) = 0 \tag{11.41}$$

The two roots for W are 10 *in* and -18 *in*. Because it is physically impossible to have a negative value for width, we ignore the second root and conclude that the width of the photovoltaic array is 10 in. We use this value to solve for the length (L) of the array

$$L = W + 8 = 10 + 8\text{in} = 18\text{in} \tag{11.42}$$

Therefore, the dimensions of the photovoltaic array are 18 x 10 in.

11.1.7 Exercises

1. A rowing team competes in a tournament that has two legs. The first leg covers 15 miles in the direction of the current. The second leg covers the same 15 miles, but against the current. It takes 25 minutes to complete the first leg and 30 minutes to complete the second leg. Assuming that there is no current present, what would be the speed of the boat? What is the speed of the current?
2. A beam of length 3 meters is supported by a fulcrum. Force is applied at each end of the beam. Assume that the force (F_1) on the left hand side of the beam is 6 N while the force on the right hand side of the beam (F_2) is 7 N. If the beam is static, then what are the lengths between the each end of the beam and its fulcrum?
3. Suppose that we are interested in implementing a computer algorithm on three processors. Let us call these processors A and. The steps involved in the accomplishment of the algorithm are divided among the three processors. In all there are 36,000,000 computations which will need to be performed on the three processors. In performing the overall task, processor A is in service for 2 seconds while the processor B is in services for 4 seconds. A second algorithm is implemented in parallel on processors

A and B. In it, a total of 23,000,000 computations are required. In performing the task, processor A is in service for 1 second and processor B is in service for 1.5 seconds. Find the processing rate (computations/s) for each processor.

4. One alloy is made up of a concentration of 25% copper and 75% manganese. A second alloy consists of a concentration of 50% copper and 50% manganese. The two alloys are melted and re-combed. The resulting mixture weighs 450 gram and consists of 35% copper and 65% zinc. Find the weights of the original alloys.

Chapter 12

Cramer's Rule[1]

12.1 Cramer's Rule

12.1.1 Introduction

Cramer's Rule is a technique that can be used to solve simultaneous linear equations. It is most often utilized when one is required to solve the system by hand rather than by computer. This is due to the fact that there are quicker, more efficient procedures such as Gauss elimination that can be implemented on the computer. The approach is based upon the use of determinants.

12.1.2 Mathematical Preliminaries

Before we describe the procedure known as Cramer's Rule, we begin with some mathematical preliminaries. Let us consider a pair of two simultaneous linear equations in two unknowns (x_1 and x_2). We can write these equations as

$$a_{11}x_1 + a_{12}x_2 = b_1 \tag{12.1}$$

and

$$a_{21}x_1 + a_{22}x_x = b_2 \tag{12.2}$$

Here the coefficients a_{11}, a_{12}, a_{21}, and a_{22} are known constants.

Now, let us solve this system of equations via Gauss elimination. We should recall that the basic idea behind Gauss elimination is to reduce the original set of equations into an equivalent form which is triangular and to use back-substitution once the first unknown is discovered.

We begin by multiplying each side of equation (1) by the value ($-a_{21}/a_{11}$). This yields an equivalent equation of the form

$$-a_{21}x_1 - \frac{a_{21}a_{12}}{a_{11}}x_2 = -\frac{a_{21}}{a_{11}}b_1 \tag{12.3}$$

Next, we add equation (3) to equation (2). In doing so, we note that the term involving x_1 is removed. The result of the addition of the two equations is

$$\left(a_{22} - \frac{a_{21}a_{12}}{a_{11}}\right)x_2 = b_2 - \left(\frac{a_{21}}{a_{11}}\right)b_1 \tag{12.4}$$

[1] This content is available online at <http://cnx.org/content/m38619/1.3/>.

The value for the unknown x_2 can be easily found using equation (4). The solution is

$$x_2 = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}} \tag{12.5}$$

This result can be substituted back into equation (1) to produce an equation that can be solved for the unknown x_1.

$$a_{11}x_1 + a_{12}\left(\frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}}\right) = b_1 \tag{12.6}$$

The solution for the unknown, x_1, proceeds as follows. The equation ( ) tells us

$$a_{11}x_1 = b_1 - a_{12}\left(\frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}}\right) \tag{12.7}$$

which can be expressed as

$$a_{11}x_1 = \frac{a_{11}a_{22}b_1 - a_{12}a_{21}b_1 - a_{11}a_{12}b_2 + a_{12}a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}} \tag{12.8}$$

This can be reduced to the following equation

$$a_{11}x_1 = \frac{a_{11}a_{22}b_1 - a_{11}a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}} \tag{12.9}$$

Dividing through by the constant a_{11} yields an expression for x_1.

$$x_1 = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}} \tag{12.10}$$

Equations (5) and (10) provide us the solution for the variables in terms of the set of constants associated with the original equations. Examination of equations (5) and (10) reveals that the solution for each variable includes the common term

$$\Delta = a_{11}a_{22} - a_{12}a_{21} \tag{12.11}$$

Suppose we write our original equations in matrix-vector form as follows

$$A\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{12.12}$$

where we define the coefficient matrix as

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \tag{12.13}$$

Clearly, we see that the term Δ is equal to the determinant of the coefficient matrix

$$\Delta = \det(A) = | A | \tag{12.14}$$

Next let us consider the numerator for the solution of the unknown x_1 as expressed in equation (10). We recognize that it, too, can be expressed by means of a determinant as is shown below

$$x_1 = a_{2,2}b_1 - a_{1,2}b_2 = \det\begin{bmatrix} b_1 & a_{1,2} \\ b_2 & a_{2,2} \end{bmatrix} \tag{12.15}$$

We note that the matrix in the equation can be obtained by merely replacing the first column of the original coefficient matrix with the vector

$$B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{12.16}$$

So the solution for the unknown x_1 can be written as a ratio of determinants

$$x_1 = \frac{\det \begin{bmatrix} b_1 & a_{1,2} \\ b_2 & a_{2,2} \end{bmatrix}}{\Delta} \tag{12.17}$$

Before we solve for the variable x2, we replace the second column of the original coefficient matrix with the vector B. With this replacement accomplished, we may write the solution for the unknown x_2 as a ratio of determinants

$$x_2 = \frac{\det \begin{bmatrix} a_{1,1} & b_1 \\ a_{2,1} & b_2 \end{bmatrix}}{\Delta} \tag{12.18}$$

In the following section, we will outline a procedure that can be used to solve simultaneous linear equations based upon determinants which is called solution via Cramer's Rule.

12.1.3 Solution via Cramer's Rule

It is important that one begin by writing the set of simultaneous equations in normal form. That is, the equations should be written as

$$a_{11}x_1 + a_{12}x_2 = b_1 \tag{12.19}$$

$$a_{21}x_1 + a_{22}x_x = b_2 \tag{12.20}$$

Next, we form the coefficient matrix

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,s} \end{bmatrix} \tag{12.21}$$

At this point, we can solve for the value of Δ by taking the determinant of A.

In anticipation of solving for the unknown x_1, we replace the first column of A with the elements contained in the column vector

$$B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{12.22}$$

Once this is accomplished we can express the solution for x_1 as the ratio

$$x_1 = \frac{\det \begin{bmatrix} b_1 & a_{1,2} \\ b_2 & a_{2,2} \end{bmatrix}}{\Delta} \tag{12.23}$$

To obtain the solution for the unknown x_2, we return to the original coefficient matrix A. This time, we replace the second column of A with the column vector B. Now, we can solve for x_2 as a ratio

$$x_2 = \frac{\det \begin{bmatrix} a_{1,1} & b_1 \\ a_{2,1} & b_2 \end{bmatrix}}{\Delta} \tag{12.24}$$

This constitutes the procedure for solving a system of two linear equations in two unknowns via Cramer's Rule.

12.1.4 Example: Mesh Current Analysis

Mesh current analysis is one of the techniques that are often employed to analyze an electric circuit that contains more than one mesh or loop. Figure 1 provides an example of an electric circuit containing two meshs.

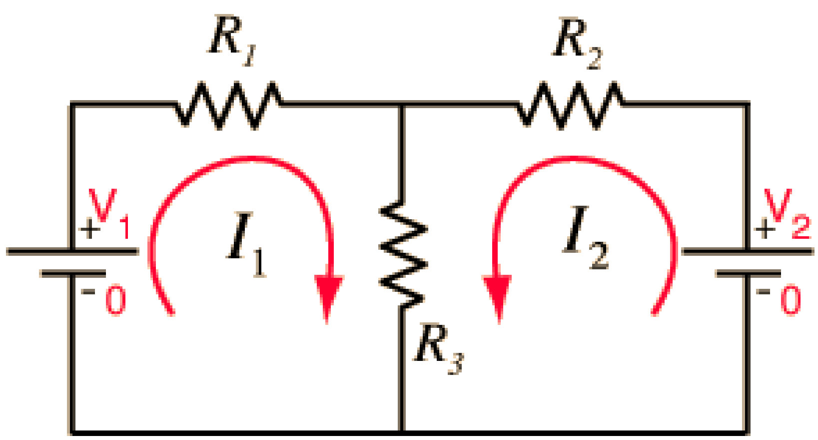

Figure 12.1: Electric circuit with two independent mesh currents.

The mesh currents are identified as I_1 and I_2. The set of equations that govern the behavior of the circuit in terms of the mesh currents are

$$\left(R_1 + R_3\right) I_1 + R_3 I_1 = V_1 \tag{12.25}$$

$$R_3 I_1 + \left(R_2 + R_3\right) = V_2 \tag{12.26}$$

Suppose that the values for R_1, R_2 and R_3 are 2 Ω, 3Ω, and 1 Ω respectively. Also, suppose that values for V_1 and V_2 are 6 V and 9 V. With these values defined, the set of equations can be written as

$$3I_1 + I_2 = 6 \tag{12.27}$$

and

$$I_1 + 4I_2 = 9 \tag{12.28}$$

We can use Cramer's Rule to find the mesh currents. We begin by finding the value for Δ

$$\Delta = \det \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix} = 12 - 1 = 11 \tag{12.29}$$

Next, we find the value for I_1.

$$I_1 = \frac{\det \begin{bmatrix} 6 & 1 \\ 9 & 4 \end{bmatrix}}{\Delta} = \frac{24 - 9}{11} = 1.364A \tag{12.30}$$

By a similar approach, we solve for I_2.

$$I_2 = \frac{\det \begin{bmatrix} 3 & 6 \\ 1 & 9 \end{bmatrix}}{\Delta} = \frac{27 - 6}{11} = 1.909A \tag{12.31}$$

12.1.5 Summary

This module has presented Cramer's Rule as a technique for solving simultaneous linear equations. The discussion in this module was limited to systems involving two simultaneous equations. This limitation was deliberate in that Cramer's Rule is typically not applied for linear systems comprised of large numbers of equations. An application involving the mesh analysis of an electric circuit was provided.

12.1.6 Exercises

1. Consider the two mesh circuit depicted in Figure 1. Assume the following values for the resistors in the circuit: $R_1 = 10\Omega, R_2 = 20\Omega, R_3 = 50\Omega$. Let $V_1 = 24V$and $V_2 = 6V$. Find the two mesh currents through the use of Cramer's Rule.
2. A civil engineering firm plans to sign a contract with a customer. The contract calls for the construction of two office buildings which are denoted as Building X and Building Y. According to estimates derived in the preliminary design phase, the firm knows that the total cost of the project will be \$50.000,000. It is also known that Building X will cost \$5,000,000 more to construct than Building Y. Use Cramer's Rule to find the cost of each building.
3. Two types of pumps provide input into a municipal reservoir. Let us refer to the two types of pumps as A and B. If 4 type A and 2 type B pumps operate at maximum flow, the input to the reservoir is 1,200 gallons/min. If 3 type A and 5 type B pumps operate at maximum flow, the input to the reservoir is 1,600 gallons/min. Find the flow rates for type A and type B pumps using Cramer's Rule.
4. The combined cost of 12 microprocessors and 36 random access memory chips is \$7,200. The combined cost of 8 microprocessors and 42 random access memory chips is \$6,600. Find the cost of each microprocessor chip and each random access memory chip using Cramer's Rule.
5. The design of an electronic thermometer is based in part upon the incorporation of a component known as a thermistor. A thermistor has the property that its resistance varies linearly as a function of temperature. This linear relationship is $R = R_0 + mT$. The term (R^0) represents the value of the resistance at 0^0 C. At a temperature $T = 25^0$ C, the resistance of the thermistor (R) is 100 Ω. A a temperature of 55^0 C, the resistance of the thermistor is 104 Ω. Use Cramer's Rule to find the values for R_0 and m.

Chapter 13

Matrices[1]

13.1 Matrices

13.1.1 Solution of an Electric Circuit with 2 Unknowns by Matrix Inversion

Let us apply our knowledge of matrices to assist us in the analysis of an electric circuit. We consider the circuit shown below.

[1] This content is available online at <http://cnx.org/content/m38632/1.3/>.

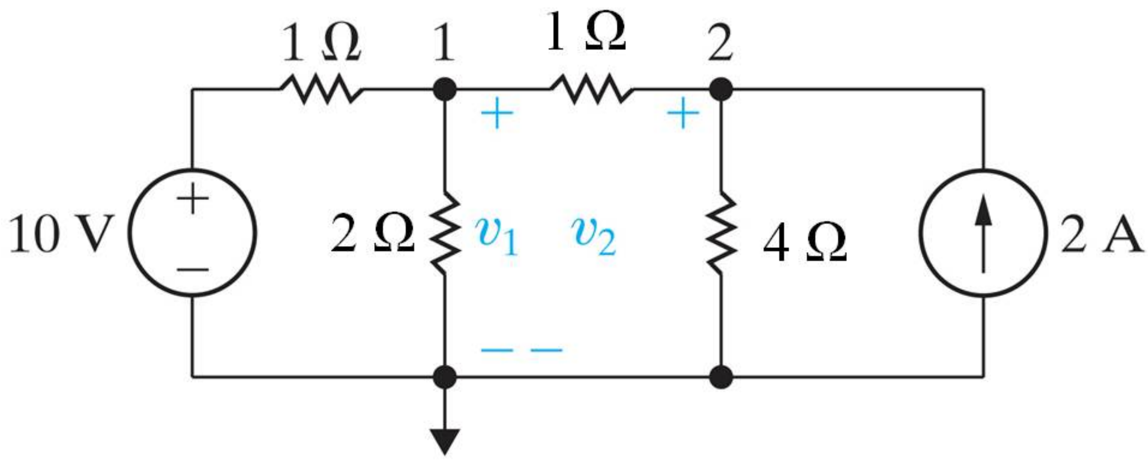

Figure 13.1: Electric circuit with two node voltages specified.

In this example, we wish to solve for the two node voltages v_1 and v_2. Since there are two unknowns in this problem, we must first establish two independent equations that reflect the operation of the circuit.

Kirchoff's Current Law tells us that the sum of the currents that enter a node must equal the sum of the currents that leave a node. Let us focus first on node 1. The current that enters node 1 from the left can be stated mathematically as

$$\frac{10V - v_1}{1\Omega} \tag{13.1}$$

The current that enters node 1 from the right can be stated as

$$\frac{v_2 - v_1}{1\Omega} \tag{13.2}$$

The current that travels downward from node 1 is

$$\frac{v_1}{2\Omega} \tag{13.3}$$

We can arrange the expressions for each of the currents in terms of an equation via Kirchoff's Current Law

$$\frac{10V - v_1}{1\Omega} + \frac{v_2 - v_1}{1\Omega} = \frac{v_1}{2\Omega} \tag{13.4}$$

We can combine and rearrange these terms into the equation

$$5v_1 - 2v_2 = 20V \tag{13.5}$$

Now let us turn our attention to node 2. The current entering node 2 from the left is given by the expression

$$\frac{v_1 - v_2}{1\Omega} \tag{13.6}$$

The current entering node 2 from the right is 2 A. The current leaving node 2 in a downward direction is

$$\frac{v_2}{4\Omega} \tag{13.7}$$

We proceed to combine these currents via Kirchoff's Current Law

$$\frac{v_1 - v_2}{1\Omega} + 2A = \frac{v_2}{4\Omega} \tag{13.8}$$

This equation can be rearranged as

$$-4v_1 + 5v_2 = -8V \tag{13.9}$$

So the pair of equations that we will use to solve for the two unknowns are

$$5v_1 - 2v_2 = 20V \tag{13.10}$$

and

$$-4v_1 + 5v_2 = -8V \tag{13.11}$$

These equations may be expressed in matrix-vector form as

$$\begin{bmatrix} 5 & -2 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ -8 \end{bmatrix} \tag{13.12}$$

or

$$A \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ -8 \end{bmatrix} \tag{13.13}$$

$$A = \begin{bmatrix} 5 & -2 \\ -4 & 5 \end{bmatrix} \tag{13.14}$$

Let us find the inverse of the matrix A. The coefficients of this matrix are given by

$$a_{1,1} = 5 \tag{13.15}$$

$$a_{1,2} = -2 \tag{13.16}$$

$$a_{2,1} = -4 \tag{13.17}$$

$$a_{2,2} = 5 \tag{13.18}$$

The inverse can be found making use of the following formula

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} a_{2,2} & -a_{1,2} \\ -a_{2,1} & a_{1,1} \end{bmatrix} \tag{13.19}$$

It should be noted that this formula works only with (2 x 2) matrices. For matrices of higher rank, other methods need to be applied.

For this example, the determinant of A is found as

$$\det A = (5)(5) - (-4)(-2) = 25 - 8 = 17 \tag{13.20}$$

We can incorporate this information to express the inverse matrix as

$$A^{-1} = \frac{1}{17} \begin{bmatrix} 5 & 2 \\ 4 & 5 \end{bmatrix} \tag{13.21}$$

which can be written as

$$A^{-1} = \begin{bmatrix} \frac{5}{17} & \frac{2}{17} \\ \frac{4}{17} & \frac{5}{17} \end{bmatrix} \tag{13.22}$$

We can apply A^{-1} to solve for the unknowns

$$A^{-1}A \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = A^{-1} \begin{bmatrix} 20 \\ -8 \end{bmatrix} \tag{13.23}$$

Recognizing that $A^{-1}A = I$, we find that

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = A^{-1} \begin{bmatrix} 20 \\ -8 \end{bmatrix} = \frac{1}{17} \begin{bmatrix} 5 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 20 \\ -8 \end{bmatrix} = \frac{1}{17} \begin{bmatrix} 84 \\ 40 \end{bmatrix} = \begin{bmatrix} 4.94 \\ 2.35 \end{bmatrix} \tag{13.24}$$

So the node voltages are given as

$$v_1 = 4.94V \tag{13.25}$$

and

$$v_2 = 2.35V \tag{13.26}$$

13.1.2 Solution of an Electric Circuit with 3 Unknowns by Gaussian Elimination

Let us consider the electric circuit that is shown below.

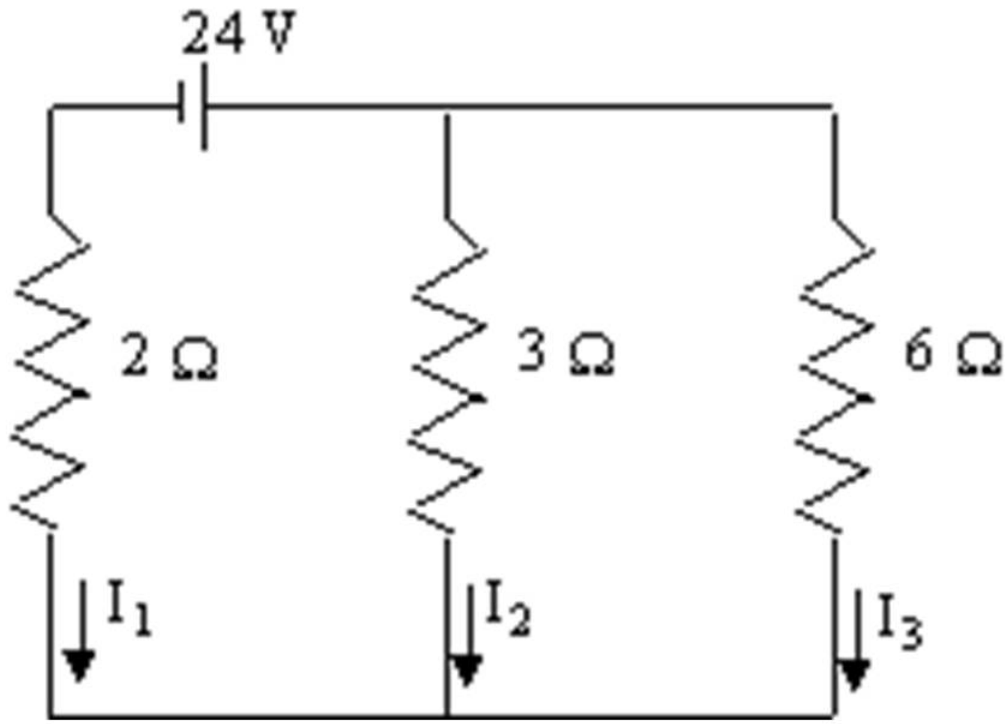

Figure 13.2: Electric circuit with three unknown currents.

Suppose that we are interested in determining the value of the three unknown currents I_1, I_2 and I_3. In order to do so, we rely upon Ohm's Law and Kirchoff's Laws to develop a system of three independent, linear equations. We should note that because we have three unknowns (I_1, I_2 and I_3), we must have three independent, linear equations.

$$I_1 + I_2 + I_3 = 0 \tag{13.27}$$

$$-2I_1 + 3I_2 = 24 \tag{13.28}$$

$$-3I_2 + 6I_3 = 0 \tag{13.29}$$

Let us define the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 3 & 0 \\ 0 & -3 & 6 \end{bmatrix}$

In order to find the unknowns, we must first find the inverse of the matrix A. This can be accomplished using elimination. To start the process, we adjoin the vector $[0\ 24\ 0]^{\mathrm{T}}$ to the matrix A.

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ -2 & 3 & 0 & 24 \\ 0 & -3 & 6 & 0 \end{pmatrix} \tag{13.30}$$

Next, we wish to force the left-most constant of row 2 to take on a value of 0. We can do so by multiplying each value in the first row by (-2) and subtracting the result from the corresponding value in row 2. This process yields

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 5 & 2 & 24 \\ 0 & -3 & 6 & 0 \end{pmatrix} \tag{13.31}$$

Now, we divide each term in row 2 by (5) to yield

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2/5 & 24/5 \\ 0 & -3 & 6 & 0 \end{pmatrix} \tag{13.32}$$

Next, we turn our attention to eliminating the (-3) term in row 3. We can do so by multiplying each term of row 2 by (-3) and subtracting the results from the corresponding terms in row 3. This produces the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2/5 & 24/5 \\ 0 & 0 & 36/5 & 72/5 \end{pmatrix} \tag{13.33}$$

We can then divide the terms of row 3 by (36/5) to produce

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2/5 & 24/5 \\ 0 & 0 & 1 & 2 \end{pmatrix} \tag{13.34}$$

Interpretation of the third row tells us that the value for the third unknown (I_3) is 2 A. We can use the coefficients from the second row along with the value for I_3 to solve for I_2.

$$I_2 + \frac{2}{5}(I_3) = \frac{24}{5} \tag{13.35}$$

which yields the result

$$I_2 = 4A. \tag{13.36}$$

Lastly, we may use the coefficients of the first row along with the previously determined values for I_2 and I_3 to produce the result for I_1.

$I_1 + I_2 + I_3 = 0$

Insertion of the previously found unknowns yields

$$I_1 + 4 + 2 = 0 \tag{13.37}$$

So we find the value for I_1 to be -6 A.

13.1.3 Exercises

1. Company A has more cash than Company B. If Company A lends \$20 million to Company B, then the two companies would have the same amount of cash. If instead Company B gave Company A \$22 million, then Company A would have twice as much cash as Company B. Use the matrix inversion method to find how much cash each company has.
2. A computer manufacturer sells two types of units. One unit is primarily marketed to the professional community and sells for \$1,700. Another unit is marketed to students and sells for \$900. In a typical month, the manufacturer sells 2,000 units. This accounts for \$1,380,000 in sales. Use the matrix inversion method to find how many units of each type are sold.
3. A ship can travel 300 miles upstream in 80 hours. Under the same conditions, the same ship can travel 275 miles downstream in 65 hours. Use the matrix inversion method to find the speed of the current along with the speed of the ship.
4. The matrix $A = \begin{bmatrix} 2 & 1 & 3 & 8 \\ 0 & 6 & 2 & 4 \\ 1 & 0 & 1 & 2 \end{bmatrix}$ represents a linear system with three unknowns. Use Gaussian elimination to solve for the three unknowns.
5. A system of 3 independent linear equations that govern the operation of the circuit below are $i_1 + i_2 + i_2 = 0$, $-i_1 - 24 + 2i_2 = 0$, and $-2i_2 + 4i_3 = 0$. Use Gaussian elimination to solve for the three currents.
6. Suppose that the value of each resistor in the figure below is 1 Ω. The mesh equations that govern the circuit are $6V = 2i_a - i_b$ and $2i_b - i_a + 9V = 0$. Use the matrix inversion method to find the two mesh current.

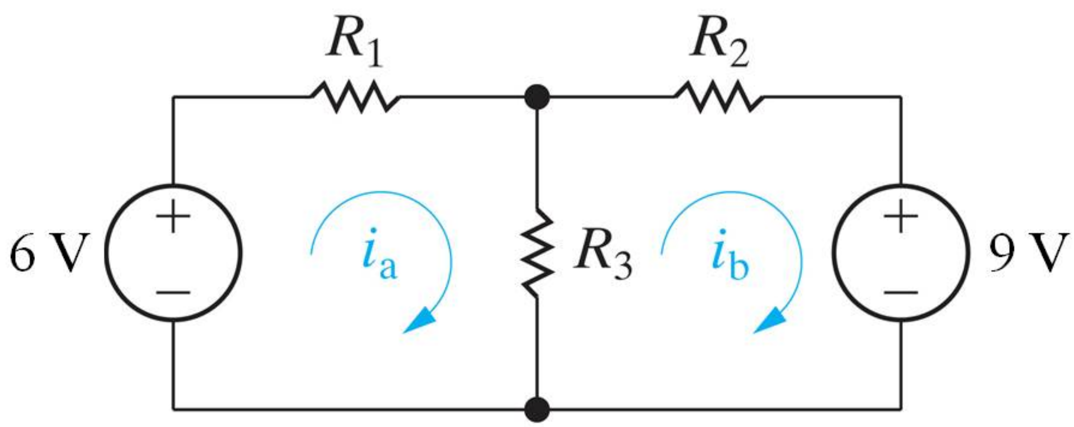

Figure 13.3: Electric circuit with two mesh currents.

Chapter 14

Trigonometry[1]

14.1 Trigonometry

14.1.1 Introduction

Trigonometry is a very important tool for engineers. This reading mentions just a few of the many applications that involve trigonometry to solve engineering problems.

Trigonometry means the study of the triangle. Most often, it refers to finding angles of a triangle when the lengths of the sides are known, or finding the lengths of two sides when the angles and one of the side lengths are known.

As you complete this reading, be sure to pay special attention to the variety of areas in which engineers utilize trigonometry to develop the solutions to problems. Rest assured that there are an untold number of applications of trigonometry in engineering. This reading only introduces you to a few. You will learn many more as you progress through your engineering studies.

Virtually all engineers use trigonometry in their work on a regular basis. Things like the generation of electrical current or a computer use angles in ways that are difficult to see directly, but that rely on the fundamental rules of trigonometry to work properly. Any time angles appear in a problem, the use of trigonometry usually will not be far behind.

An excellent working knowledge of trigonometry is essential for practicing engineers. Many problems can be easily solved by applying the fundamental definitions of trigonometric functions. Figure 1 depicts a right triangle for which we will express the various trigonometric functions and relationships that are important to engineers.

[1]This content is available online at <http://cnx.org/content/m38633/1.5/>.

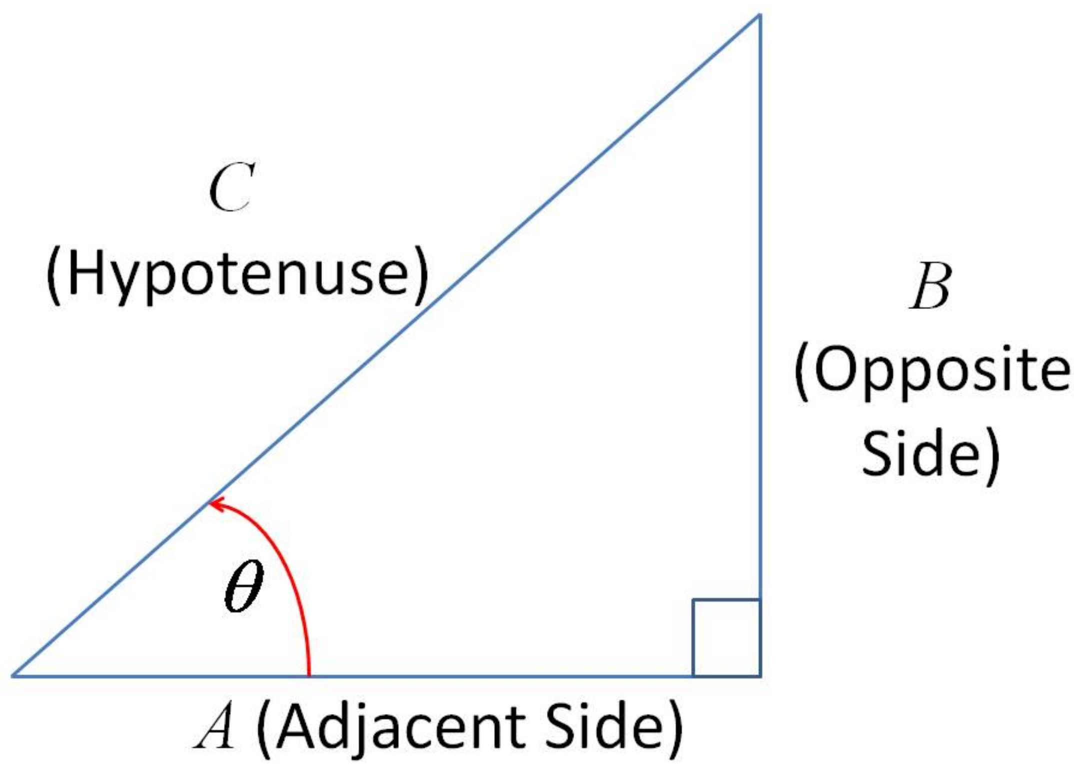

Figure 14.1: Triangle for definition of trigonometric functions.

Some of the most widely used trigonometric functions follow

$$\sin(\theta) = \frac{\text{oppositeside}}{\text{hypotenuse}} = \frac{B}{C} \tag{14.1}$$

$$\cos(\theta) = \frac{\text{adjacentside}}{\text{hypotenuse}} = \frac{A}{C} \tag{14.2}$$

$$\tan(\theta) = \frac{\text{oppositeside}}{\text{adjacentside}} = \frac{B}{A} \tag{14.3}$$

The Pythagorean Theorem often plays a key role in applications involving trigonometry in engineering. It states that the square of the hypotenuse is equal to the sum of the squares of the adjacent side and of the opposite side. This theorem can be stated mathematically by means of the equation that follows. Here we make use of the symbols (A, B, and C) that designate associated lengths.

$$A^2 + B^2 = C^2 \tag{14.4}$$

In the remainder of this module, we will make use of these formulas in addressing various applications.

14.1.2 Flight Path of an Aircraft

The following is representative of the type of problem one is likely to encounter in the introductory Physics course as well as in courses in Mechanical Engineering. This represents an example of how trigonometry can be applied to determine the motion of an aircraft.

Assume that an airplane climbs at a constant angle of 3^0 from its departure point situated at sea level. It continues to climb at this angle until it reaches its cruise altitude. Suppose that its cruise altitude is 31,680 ft above sea level.

With the information stated above, we may sketch an illustration of the situation

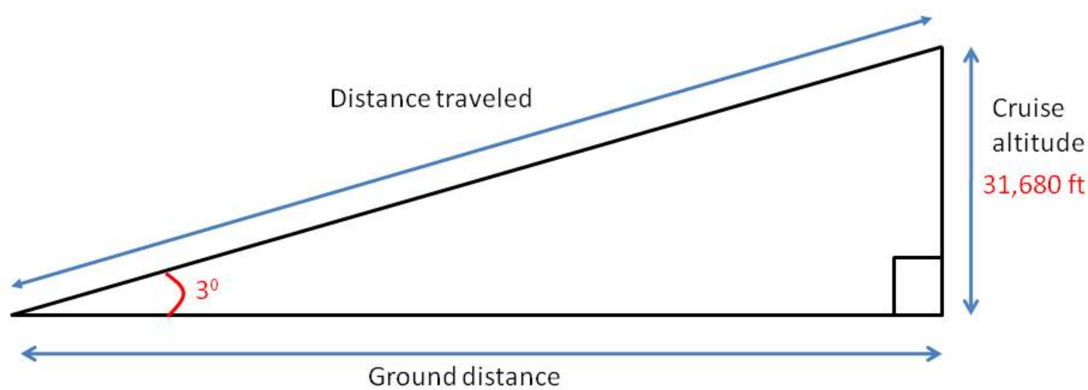

Figure 14.2: Depiction of aircraft motion.

Question 1: What is the distance traveled by the plane from its departure point to its cruise altitude?

The distance traveled by the plane is equal to the length of the hypotenuse of the right triangle depicted in Figure 2. Let us denote the distance measured in feet that the plane travels by the symbol C. We may apply the definition of the sine function to enable us to solve for the symbol C as follows.

$$\sin\left(3^0\right) = \frac{31,680}{C} \tag{14.5}$$

Rearranging terms we find

$$C = \frac{31,680}{\sin(3^o)} \tag{14.6}$$

Calculation and rounding to 3 significant digits yields the result

$$C = 605,320\text{ft} \tag{14.7}$$

Let us ponder a second question based upon the data presented in the original problem.

Question 2: What is the ground distance traveled by the airplane as it moves from its departure point to its cruise altitude?

Solution: Referring to Figure 2, we observe that we must find the length of the adjacent side in order to answer the question. We can use the definition of the tangent to guide our solution.

$$\tan\left(3^0\right) = \frac{\text{Oppositeside}}{\text{Adjacentside}} \tag{14.8}$$

Denoting the adjacent side by the symbol A, we obtain

$$A = \frac{\text{Oppositeside}}{\tan(3^0)} \tag{14.9}$$

$$A = \frac{31,680}{0.0524} \tag{14.10}$$

After rounding to 3 significant digits, we obtain the solution

$$A = 604,580\text{ft} \tag{14.11}$$

14.1.3 Inclined Plane

Work is an important concept in virtually every field of science and engineering. It takes work to move an object; it takes work to move an electron through an electric field; it takes work to overcome the force of gravity; etc.

Let's consider the case where we use an inclined plane to assist in the raising of a 300 pound weight. The inclined plane situated such that one end rests on the ground and the other end rests upon a surface 4 feet aove the ground. This situation is depicted in Figure 3.

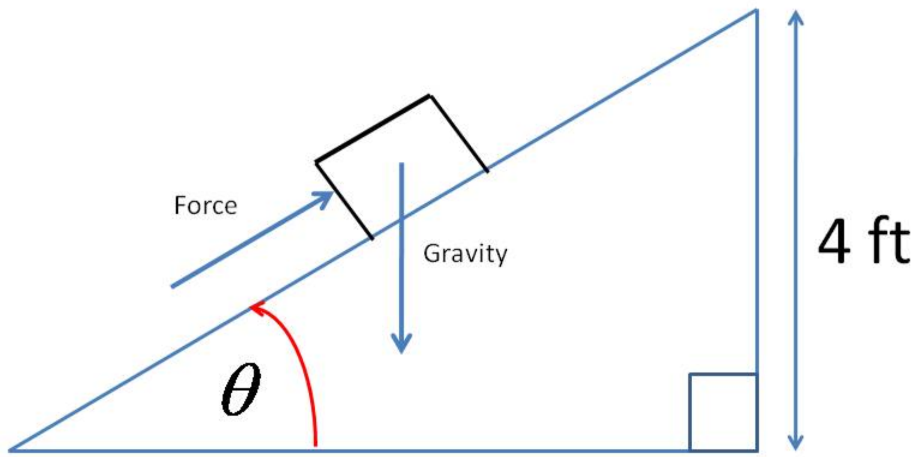

Figure 14.3: Object on an inclined plane.

Question 3: Suppose that the length of the inclined plane is 12 feet. What is the angle that the plane makes with the ground?

Clearly, the length of the inclined plane is same as that of the hypotenuse shown in the figure. Thus, we may use the sine function to solve for the angle

$$\sin(\theta) = \frac{4}{12} = 0.333 \tag{14.12}$$

In order to solve for the angle, we must make use of the inverse sine function as shown below

$$\sin^{-1}(\sin(\theta)) = \sin^{-1}(0.333) \tag{14.13}$$

$$\theta = 19.45^0 \tag{14.14}$$

So we conclude that the inclined plane makes a 19.85^0 angle with the ground.

Neglecting any effects of friction, we wish to determine the amount of work that is expended in moving the block a distance (L) along the surface of the inclined plane.

14.1.4 Surveying

Let us now turn our attention to an example in the field of surveying. In particular, we will investigate how trigonometry can be used to help forest rangers combat fires. Let us suppose that a fire guard observes a fire due south of her Hilltop Lookout location. A second fire guard is on duty at a Watch Tower that is located 11 miles due east of the Hilltop Lookout location. This second guard spots the same fire and measures the bearing (angle) at 215^0 from North. The figure below illustrates the geometry of the situation.

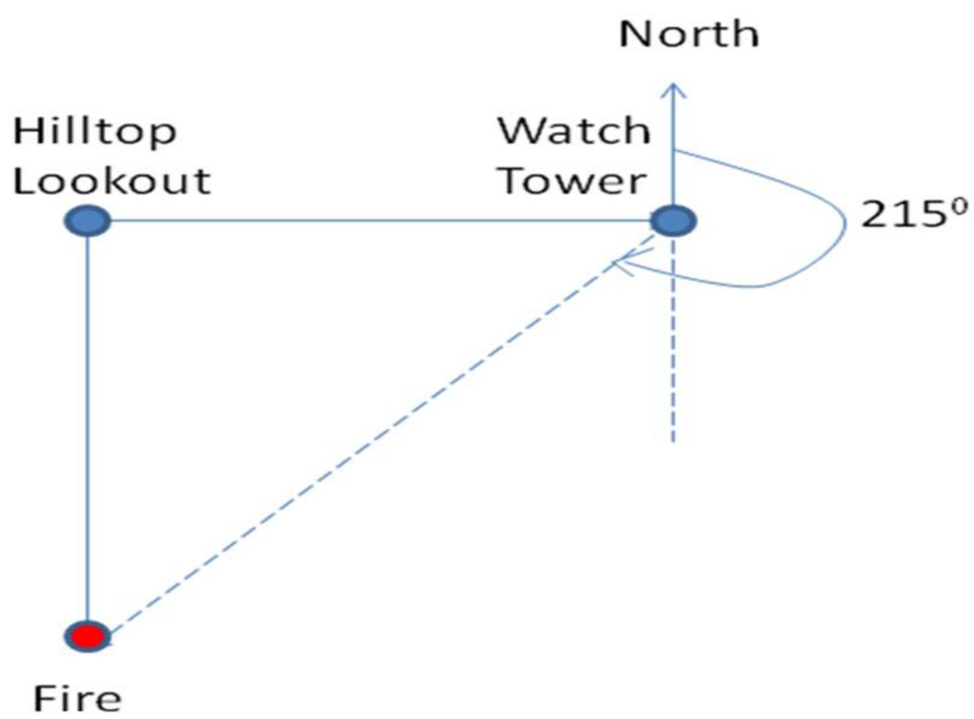

Figure 14.4: Depiction of a scenario associated with a forest fire.

Question: How far away is the fire from the Hilltop Lookout location?
We begin by identifying the angle θ in the figure below.

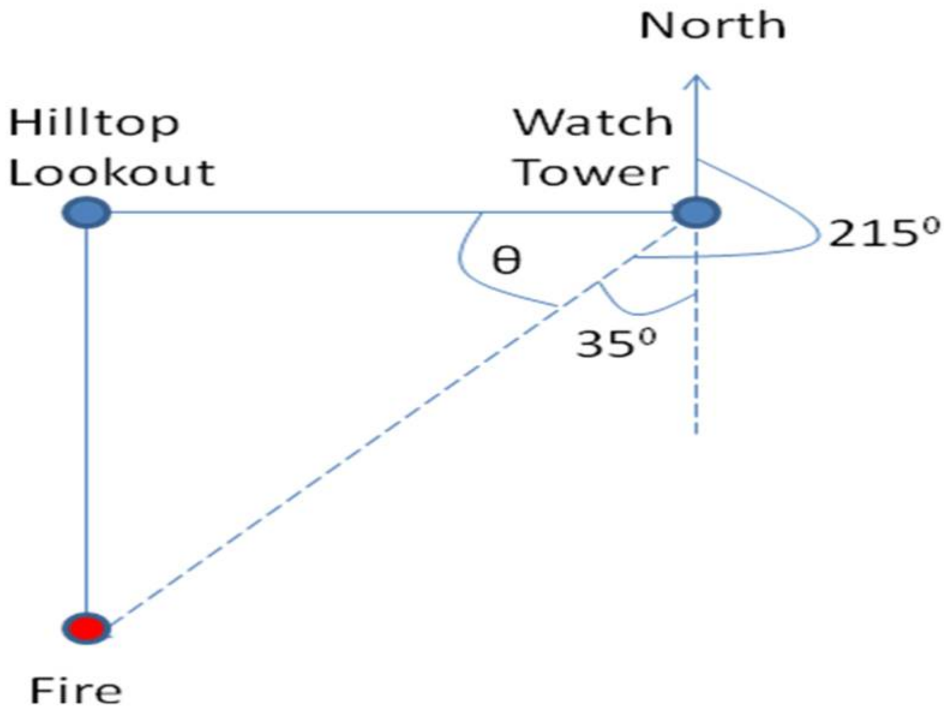

Figure 14.5: Refined depiction of scenario.

The value of θ can be found via the equation

$$\theta = 90^0 - 35^0 \tag{14.15}$$

$$\theta = 55^0 \tag{14.16}$$

So we can simplify the drawing as shown below.

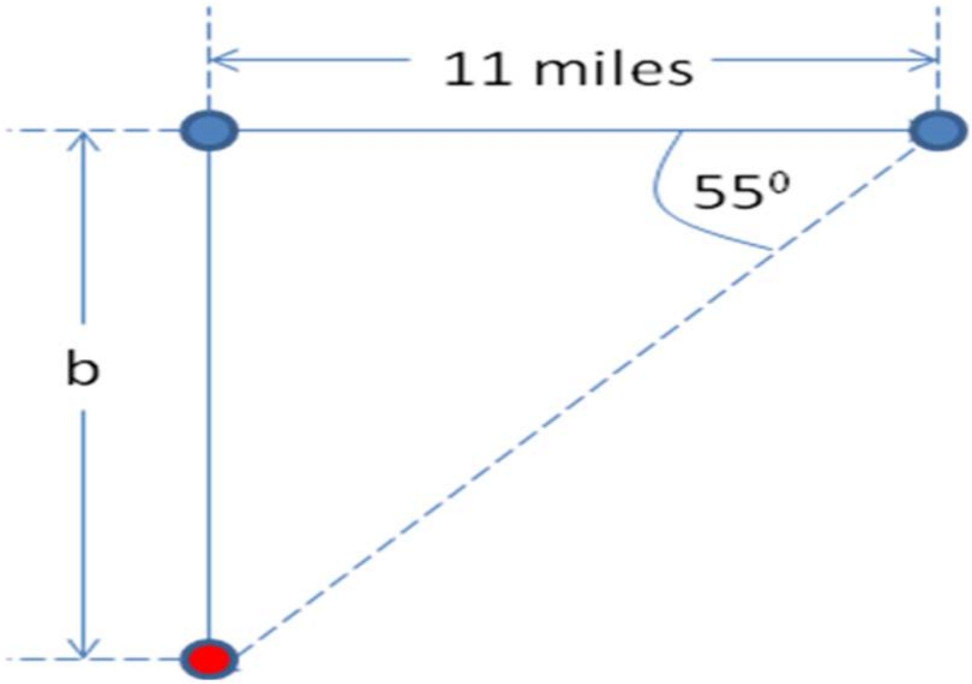

Figure 14.6: Trigonometric representation of scenario.

Our problem reduces to solving for the value of b.

$$\tan\left(55^0\right) = \frac{b}{11\text{miles}} \tag{14.17}$$

$$b = (1.43)\,(11\text{miles}) = 15.7\text{miles} \tag{14.18}$$

We conclude that the fire is located **15.7 miles** south of the Hilltop Lookout location.

Comment: Triangulation is a process that can be applied to solve problems in a number of areas of engineering including surveying, construction management, radar, sonar, lidar, etc.

14.1.5 Refraction

Refraction is a physical phenomenon that occurs when light passes from one transparent medium (such as air) through another (for example, glass.) It is known that light travels at different speeds through different transparent media. The index of refraction of a medium is a measure of how much the speed of light is reduced as it passes through the medium. In the case of glass, the index of refraction is approximately 1.5. This means that light travels as a speed of $\frac{1}{1.5} = \frac{2}{3}$times the speed of light in a vacuum.

Two common properties of transparent materials can be attributed to the index of refraction. One is that light rays change direction as they pass from one medium through another. Secondly, light is partially reflected when it passes from one medium to another medium with a different index of refraction. We will focus on the first of these properties in this reading.

In optics, which is a field of physics, you will learn about **Snell's law,** which is also known as Descartes' law after the scientist, Rene Descartes**.** Snell's law takes the form of an equation that states the relationship between the angle of incidence and the angle of refraction for light passing from one medium to another. Stated mathematically, Snell's law is

$$\frac{\sin\left(\theta_{\text{incidence}}\right)}{\sin\left(\theta_{\text{refraction}}\right)} = \frac{c_{\text{incidence}}}{c_{\text{refraction}}} \tag{14.19}$$

It follows that

$$\frac{\sin\left(\theta_{\text{incidence}}\right)}{\sin\left(\theta_{\text{refraction}}\right)} = \frac{I_2}{I_1} \tag{14.20}$$

where $I_1$and$I_2$are the Index of Refraction of medium 1 and medium 2 respectively.

Consider a situation where light rays pass are shined from air through a tank of water. This situation is illustrated below.

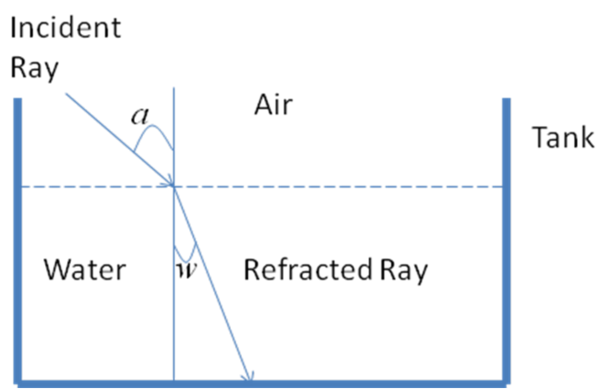

Figure 14.7: Depiction of light refraction.

The Index of refraction for air is 1.0003 and that of water is 1.3000. Let us assume that the angle that light enters the water is 21^0 40', what is the angle of refraction, w?

From Snell's law, we know

$$\frac{I_W}{I_A} = \frac{\sin(a)}{\sin(w)} \tag{14.21}$$

$$\sin(w) = \frac{I_A}{I_W}\sin(a) \tag{14.22}$$

$\sin(w) = \frac{I_A}{I_W}\sin(a)$

Substituting in the numerical values for IA, IW and a yield

$$\sin(w) = 0.2841 \tag{14.23}$$

We now make use of the inverse sine function

$$w = \sin^{-1}(0.2841) \tag{14.24}$$

This leads to the result

$$w = 16^0 30^{'} \quad (14.25)$$

We conclude that the refracted ray will travel through the water at an angle of refraction of **160 30'**.

14.1.6 Exercises

1. A 50 ft ladder leans against the top of a building which is 30 ft tall. Determine the angle the ladder makes with the horizontal. Also determine the distance from the base of the ladder to the building.
2. A straight trail leads from the Alpine Hotel at elevation 8,000 feet to a scenic overlook at elevation 11,100 feet. The length of the trail is 14,100 feet. What is the inclination angle β in degrees? What is the value of β in radians?
3. A ray of light moves from a media whose index of refraction is 1.200 to another whose index of refraction is 1.450. The angle of incidence of the ray as it intersects the interface of the two media is 15^0. Sketch the geometry of the situation and determine the value of the angle of refraction.
4. One-link planar robots can be used to place pick up and place parts on work table. A one-link planar robot consists of an arm that is attached to a work table at one end. The other end is left free to rotate about the work space. If $l = 5$ cm, sketch the position of the robot and determine the (x, y) coordinates of point $p(x,y)$ for the following values for θ: (50 $^\circ$, $2\pi/3$ rad, -20 $^\circ$, and -5 $\pi/4$ rad).

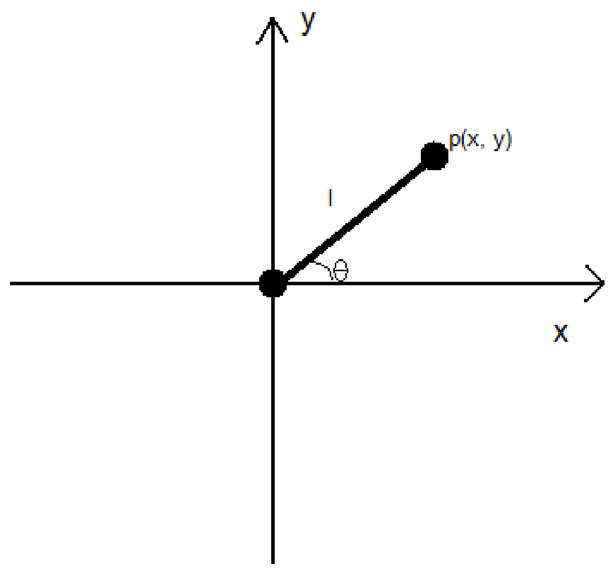

Figure 14.8: One-link planar robot.

Chapter 15

Two-Dimensional Vectors[1]

15.1 Two-Dimensional Vectors

15.1.1 Determining the Speed of an Airplane Relative to a Stationary Observer

On occasion objects move within a medium that is itself in motion with respect to an observer. For example, an airplane usually encounters a wind as it flies. The speed of the airplane with respect to a stationary observer situated on the ground will not be the same as the speedometer reading for the plane. In order to calculate the true speed of the plane, it is essential to include the effect due to the wind. In some cases, planes encounter headwinds which will diminish the speed of the plane relative to a stationary observer. I other cases, planes may encounter tail winds which will increase the speed relative to a stationary observer.

Question 1: Let us consider the example where an airplane is traveling with a speed of 880 km/hr with respect to the air. Suppose the airplane encounters a tailwind of velocity 30 km/hr. What is the resultant velocity of the airplane relative to an observer on the ground?

This problem can be addressed using the concept of vector addition. We will represent the velocity of the plane relative to the air as the vector V_1 and that of the tail wind as the vector V_2. The velocity of the plane relative to a stationary observer on the ground can be found by performing the vector addition of V_1 and V_2. This addition is accomplished by placing the tail of vector V2 to the head of vector V1. The result, 910 km/m, is shown in Figure 1 (a).

[1]This content is available online at <http://cnx.org/content/m38634/1.3/>.

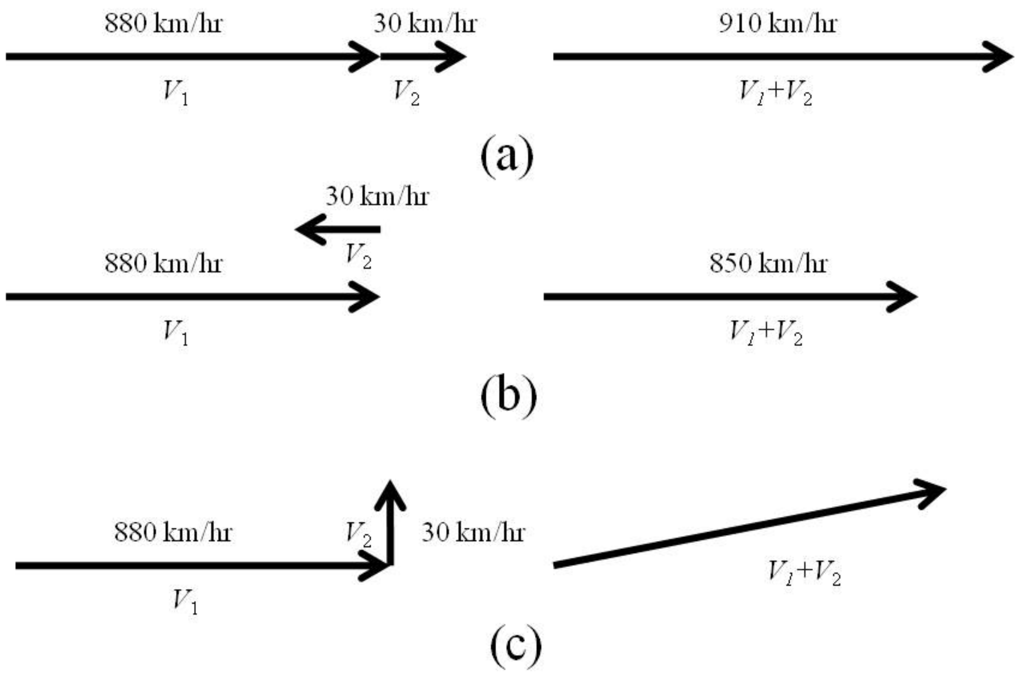

Figure 15.1: Combining velocities.

Let us now consider a different situation.

Question 2: Consider the same airplane as in Question 1. However, assume that the airplane is encountering a head wind of 30 km/hr. What is the speed of the plane relative to a stationary observer on the ground?

In this case, the head wind serves to slow the airplane. The situation is reflected in Figure 1 (b). We note that the vector V2 is directed opposite to that of vector V1 due to the fact that the head wind opposes the motion of the airplane. The resulting speed relative to an observer on the ground is computed as 850 km/hr.

Now, we will consider a situation in which we must use our knowledge of vectors and trigonometry to find the correct result.

Question 3: Consider the same airplane as in Question 1. However, assume that the airplane is encountering a cross wind of 30 km/hr. What is the speed of the plane relative to a stationary observer on the ground?

This situation is depicted in Figure 2.

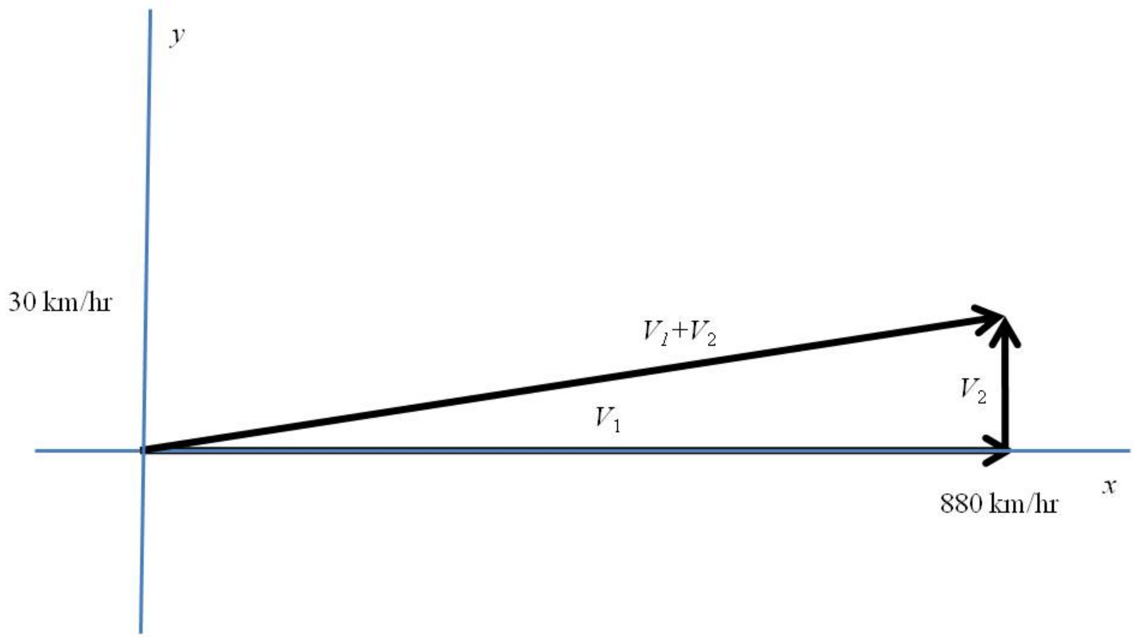

Figure 15.2: Adding orthogonal velocities.

In this case, the speed of the airplane is represented by the length of the sum of vectors, $V_1 + V_2$. Using the x-y coordinate system shown in the figure, we note that

$$V_1 = 880 \overset{\Theta}{x} (15.1)$$

and

$$V_2 = 30 \overset{\Theta}{y} (15.2)$$

we may write an expression for this sum in vector form as

$$V_1 + V_2 = 880 \overset{\Theta}{x} + 30 \overset{\Theta}{y} (15.3)$$

To determine the speed of the airplane relative to a stationary observer we must find the magnitude of the vector sum. We can do so by applying the Pythagorean Theorem

$$| V_1 + V_2 | = \sqrt{880^2 + 30^2} = 881\text{km/hr} \tag{15.4}$$

Thus, we determine speed of the airplane relative to a stationary observer on the ground to be 881 km/hr.

We note that the heading of the plane has been redirected in light of the cross wind. Let us find the new heading of the plane. We use the tangent function as follows

$$\tan(\theta) = \frac{30}{880} \tag{15.5}$$

Applying the inverse tangent leaves us with

$$\theta = \tan^{-1}(0.341) = 1.953^0 \tag{15.6}$$

So we conclude the heading of the airplane will be directed 1.953^0 from its initial heading.

15.1.2 Boat and Trailer on an Inclined Ramp

Let us now consider another example that will allow us to apply 2-D vectors. Suppose that a force of 750 pounds is required to pull a boat and trailer up a ramp that is inclined at an angle of 20^0 from the horizon. This situation is depicted in Figure 3 (a). Under the assumption of no friction, what is the combined weight of the boat and trailer?

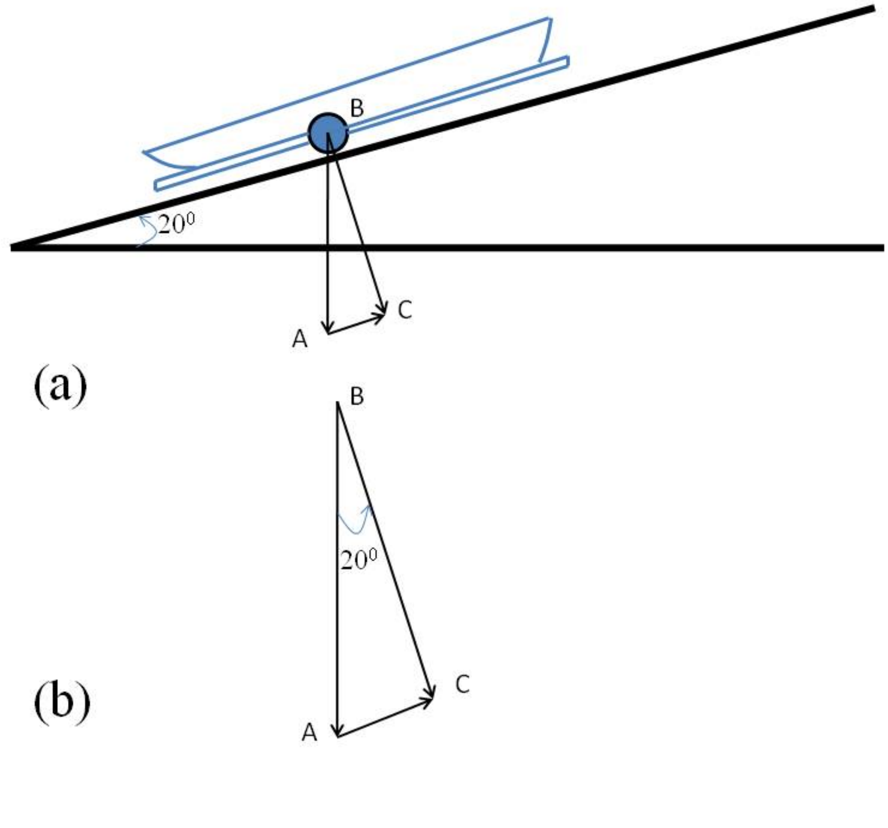

Figure 15.3: Depiction of a boat on a landing ramp along with a trigonometric description.

Let us now use the figure to interpret the vectors shown in Figure 3. The vector $\vec{BA}$ represents the combined weight of the boat and the trailer. This is the quantity that we need to find. The vector $\vec{BC}$ represents the force against the ramp. The vector $\vec{AC}$ which is parallel to the ramp represents the force applied to the boat and trailer. From the problem description we know that it has a magnitude of 750 pounds. We use the fact that this is a right triangle to simplify our efforts.

We can apply the definition of the sine function to obtain

$$\sin\left(20^0\right) = \frac{|\vec{AC}|}{|\vec{BA}|} \tag{15.7}$$

Substitution leads to the equation

$$\sin\left(20^0\right) = \frac{750}{|\vec{BA}|} \tag{15.8}$$

We can now solve for the weight of the boat and trailer

$$|\vec{BA}| = 750/0.342 = 2,192\text{lbs} \tag{15.9}$$

15.1.3 Cable Tension for a Hanging Weight

A 200-pound weight is suspended from a ceiling. The weight is supported by two cables. One cable makes a 20^0 angle away from the vertical and the other a 30^0 angle as shown in Figure 4. Find the tension in each of the support cables.

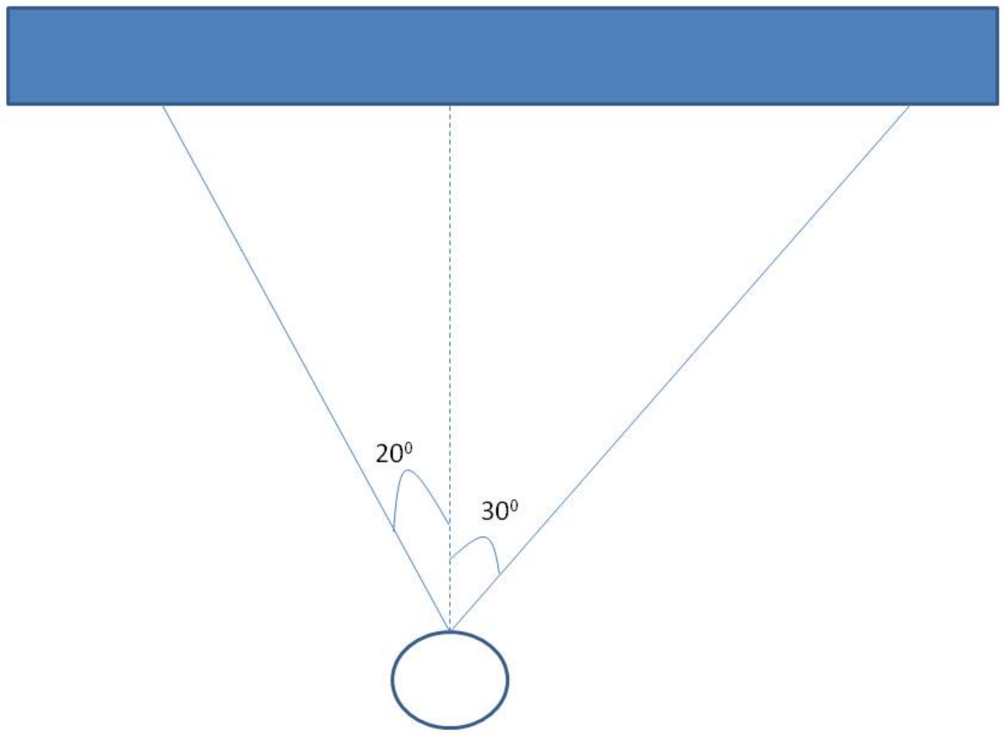

Figure 15.4: Mass suspended from a beam with two cables.

Statics is the field of engineering that is used to solve problems of this sort. Because the object does not move, it is said to be static. Another way to look at this, is that the object is at equilibrium. At equilibrium, the forces acting on an object must balance. Otherwise, the object would indeed move. To better analyze the situation, engineers often make use of what is known as a free body diagram. Figure 5 shows the free body diagram related to our problem.

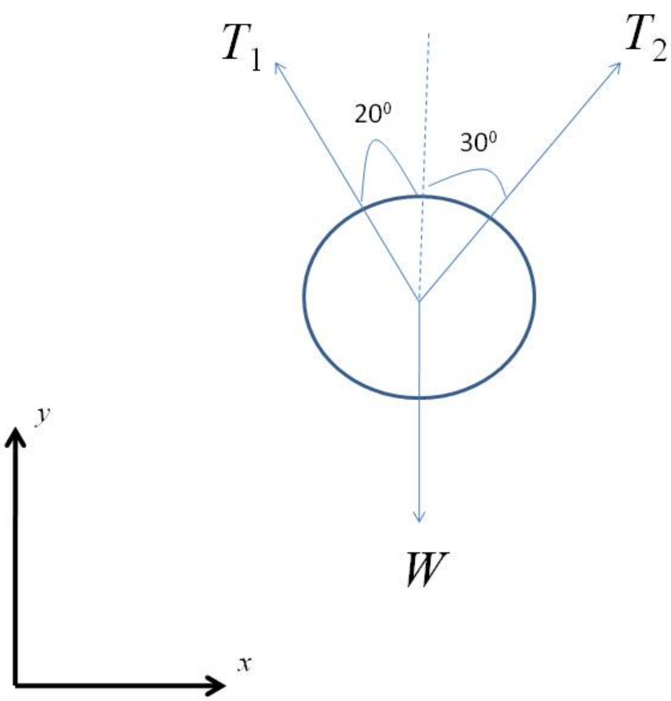

Figure 15.5: Free body diagram of mass suspended from a beam with two cables.

The free body diagram shows the three forces that act on the object. The Figure also shows the x-y coordinate system employed in this solution. The tension in cable 1 can be resolved into its x and y components and written in vector notation as

$$T_1 = - \mid T_1 \mid \sin\left(20^0\right) \overset{\ominus}{x} + \mid T_1 \mid \cos\left(20^0\right) \overset{\ominus}{y} (15.10)$$

Similarly, T_2 can be written as

$$T_2 = \mid T_2 \mid \sin\left(30^0\right) \overset{\ominus}{x} + \mid T_2 \mid \cos\left(30^0\right) \overset{\ominus}{y} (15.11)$$

The weight (W) is expressed as

$$W = -200 \overset{\ominus}{y} (15.12)$$

The sum of the forces acting on the object must be 0. Thus

$$T_1 + T_2 = W \qquad (15.13)$$

or

$$- \mid T_1 \mid \sin\left(20^0\right) \overset{\ominus}{x} + \mid T_1 \mid \cos\left(20^0\right) \overset{\ominus}{y} + \mid T_2 \mid \sin\left(30^0\right) \overset{\ominus}{x} + \mid T_2 \mid \cos\left(30^0\right) \overset{\ominus}{y} - 200 \overset{\ominus}{y} = 0 (15.14)$$

This problem can be simplified by writing an equation for solely the x-component

$$-\mid T_1 \mid \sin\left(20^0\right) + \mid T_2 \mid \sin\left(30^0\right) = 0 \tag{15.15}$$

We can do the same for the y-component

$$\mid T_1 \mid \cos\left(20^0\right) + \mid T_2 \mid \cos\left(30^0\right) - 200 = 0 \tag{15.16}$$

We begin by substituting in for the trigonometric function values to yield the following set of equations

$$-0.342 \mid T_1 \mid +0.5 \mid T_2 \mid = 0 \tag{15.17}$$

and

$$0.940 \mid T_1 \mid +0.866 \mid T_2 \mid = 200 \tag{15.18}$$

First, we find

$$\mid T_2 \mid = 0.684 \mid T_1 \mid \tag{15.19}$$

This expression can then be substituted into the other equation

$$0.940 \mid T_1 \mid +0.866\,(0.684) \mid T_1 \mid = 200 \tag{15.20}$$

This leads to the solution

$$\mid T_1 \mid = 130.5\text{lbs} \tag{15.21}$$

Next, we solve for the other variable

$$\mid T_2 \mid = 0.684 \mid T_1 \mid = 0.684\,(130.5) = 89.3\text{lbs} \tag{15.22}$$

So we conclude that the tension in the cables are 130.5 lbs and 89.3 lbs respectively.

15.1.4 Exercises

1. A load is of mass 150 kg is situated atop a moving dolly. A force with a magnitude of 15 Newtons is applied at an angle of 30^0 with respect to the horizontal. Resolve the force into its x- and y-components.
2. A ship travels 300 miles due East, then 700 miles North of due East. Sketch the geometry of the situation. Be sure to include all angles in your sketch. In all, how far does the ship travel on its voyage?
3. A small airplane travels at a velocity of 320 km/hr at an angle that is 40^0 South of East. The airplane ecounters a wind whose speed is 25 km/hr. (a) If the wind travels in a direction from West to East, what is the resulting speed and direction of the airplane? (b) Repeat for a 25 km/hr wind directed from East to West. (c) Repeat for a 25 km/hr wind directed from North to South. (d) Repeat for a 25 km/hr wind directed from South to North.
4. A 50 kg mass is suspended by two cables of equal length from a beam. Each cable makes a 450 angle with the horizontal beam. Sketch a free body diagram that represents the situation. Determine the tension in each cable.
5. A complex number has two components. One component is real while the other is imaginary. A complex number can be represented as a two-dimensional vector using what is known as the complex plane. The complex plane is a special plane whose abscissa is use to specify the real part of the complex number and whose ordinate is used to specify the imaginary part of the complex number. Consider the complex number $z = 4 + i6$. This complex number is shown as a vector in the complex plane in the figure below. The symbol M represents the magnitude of the complex number and θ represents the

angle or argument of the complex number. (a) Find the magnitude and angle of the complex number $z = 4 + i6$. (b) Repeat for the complex number $z = 4 - i6$. (c) Repeat for the complex number $z = -4 + i6$. (d) Repeat for the complex number $z = -4 - i6$.

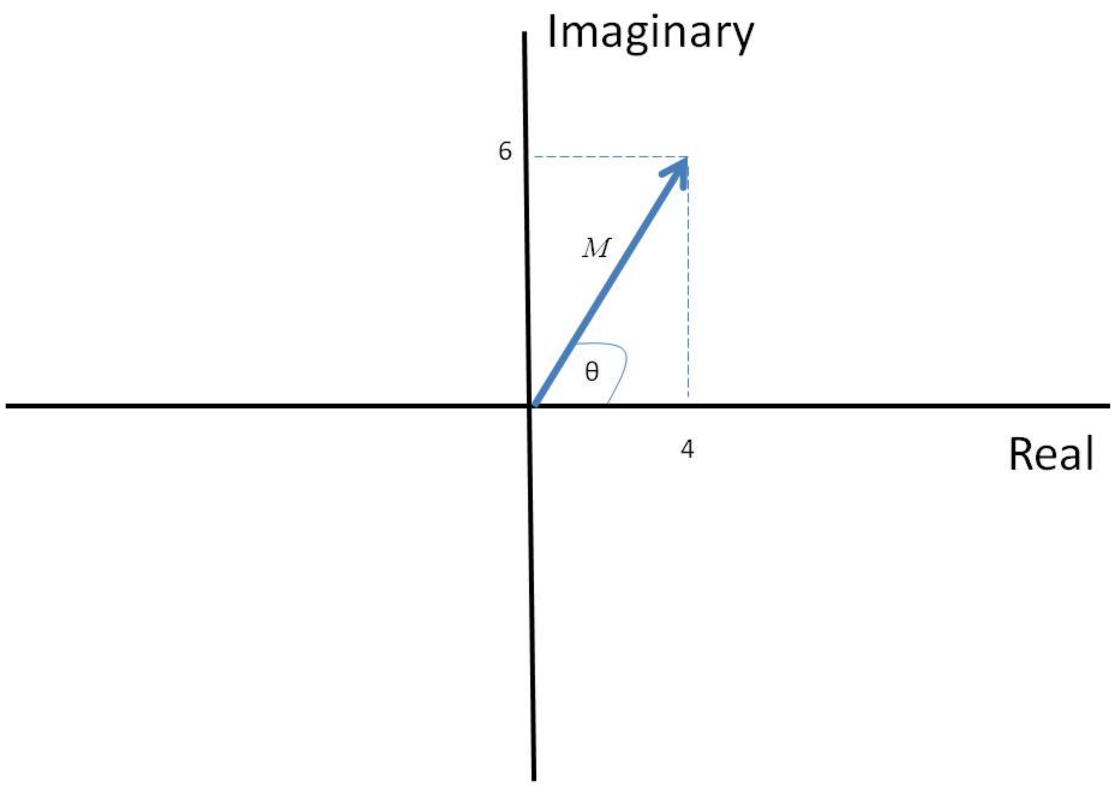

Figure 15.6: Complex number representation in the complex plane.

Index of Keywords and Terms

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