Sound, Physics and Music

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C O N N E X I O N S

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Chapter 1

Basic Concepts

1.1 Talking about Sound and Music[1]

Music is the art of sound, so let's start by talking about sound. Sound is invisible waves moving through the air around us. In the same way that ocean waves are made of ocean water, sound waves are made of the air (or water or whatever) they are moving through. When something vibrates, it disturbs the air molecules around it. The disturbance moves through the air in waves - each vibration making its own wave in the air - spreading out from the thing that made the sound, just as water waves spread out from a stone that's been dropped into a pond. You can see a short animation of a noise being created here[2].

Surf rolling down a beach, leaves rustling in the wind, a book thudding on a desk, or a plate crashing on the floor all make sounds, but these sounds are not music. **Music** is sound that's organized by people on purpose, to dance to, to tell a story, to make other people feel a certain way, or just to sound pretty or be entertaining.

Music is organized on many different levels. Sounds can be arranged into notes[3], rhythms[4], textures[5] and phrases[6]. Melodies[7] can be organized into anything from a simple song to a complex symphony. Beats[8], measures[9], cadences[10], and form[11] all help to keep the music organized and understandable. But the most basic way that music is organized is by arranging the actual sound waves themselves so that the sounds are interesting and pleasant and go well together.

A rhythmic, organized set of thuds and crashes is perfectly good music - think of your favorite drum solo - but many musical instruments are designed specifically to produce the regular, evenly spaced waves that we hear as particular pitches (musical notes). Crashes, thuds, and bangs are loud, short jumbles of lots of different wavelengths. The sound of surf, rustling leaves, or bubbles in a fish tank are also **white noise**, the term that scientists and engineers use for sounds that are mixtures of all the different wavelengths (just as white light is made of all the different wavelengths, or colors, of light).

[1] This content is available online at <http://cnx.org/content/m12373/1.10/>.
[2] See the file at <http://cnx.org/content/m12373/latest/blah.swf>
[3] "Duration: Note Lengths in Written Music" <http://cnx.org/content/m10945/latest/>
[4] "Rhythm" <http://cnx.org/content/m11646/latest/>
[5] "The Textures of Music" <http://cnx.org/content/m11645/latest/>
[6] "Melody": Section Melodic Phrases <http://cnx.org/content/m11647/latest/#s2>
[7] "Melody" <http://cnx.org/content/m11647/latest/>
[8] "Time Signature": Section Beats and Measures <http://cnx.org/content/m10956/latest/#s1>
[9] "The Staff": Section The Staff <http://cnx.org/content/m10880/latest/#s1>
[10] "Cadence in Music" <http://cnx.org/content/m12402/latest/>
[11] "Form in Music" <http://cnx.org/content/m10842/latest/>

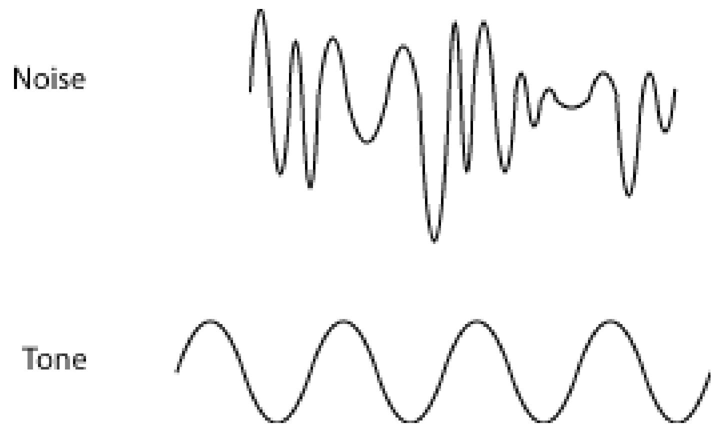

Figure 1.1

A tone (the kind of sound you might call a musical note) is a specific kind of sound. The vibrations that cause it are very regular - all the same size and same distance apart. Musicians have terms that they use to describe tones. But this kind of (very regular) wave is useful for things other than music, so scientists and engineers also have terms that describe tonal sound waves. It can be very useful to know both the scientific and the musical terms and how they are related to each other.

For example, the closer together the waves of a tonal sound are, the higher the note sounds. Musicians talk about the pitch[12] of the sound, or name specific notes[13], or talk about tuning (Section 2.4). Scientists and engineers, on the other hand, talk about the frequency (Figure 1.6: Wavelength, Frequency, and Pitch) and the wavelength (Figure 1.6: Wavelength, Frequency, and Pitch) of the sound. They are all essentially talking about the same thing. The scientific terms aren't necessary for the musician, but they can be very helpful in understanding and talking about what's happening when people make music.

The Concepts and Where to Find Them

- **Wavelength** - An introduction to wavelength, frequency, and pitch is presented in Frequency, Wavelength, and Pitch (Section 1.3). You can find out more about the (Western) musical concept of pitch in Pitch: Sharp, Flat, and Natural Notes[14].
- **Wave Size** - The other measurement you can make of regular, tonal waves is the size of each individual wave - its "height" or "intensity" rather than its wavelength. In sound waves, this is a measurement of the loudness of the sound. Amplitude (Section 1.4) is a short discussion of wave size. Musicians have many terms to discuss what they call Dynamics[15].
- **Types of Waves** - There are two basic types of waves. Most diagrams show **transverse** waves which "wave" up-and-down as they move left-and-right. These are easier to show in a diagram, and most of the familiar kinds of waves - light waves, radio waves, water waves - are transverse. But sound is made of **longitudinal** waves, which "wave" in the same direction that they move. These are harder to draw, and a little harder to imagine, than transverse waves, but you will find some helpful suggestions at Transverse and Longitudinal Waves (Section 1.2).
- **Standing Waves** - Most natural sounds are not tones. In order to produce the extremely regular vibrations that make tonal sound waves, musical instruments, se3e Standing Waves and Musical In-

[12]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[13]"Duration: Note Lengths in Written Music" <http://cnx.org/content/m10945/latest/>
[14]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[15]"Dynamics and Accents in Music" <http://cnx.org/content/m11649/latest/>

struments (Section 3.1) and Standing Waves and Wind Instruments (Section 3.2). To find out more about how the waves created in an instrument are related to each other musically, see Harmonic Series (Section 2.2) and Tuning Systems (Section 2.4).
- **Sound and Ears** - For a brief description of what happens when a sound reaches your ear, see Sound and Ears (Section 1.5.2)
- **The Math** - Students struggling with the math needed for these ideas can look at Musical Intervals, Frequency and Ratio (Section 2.1) and Powers, Roots, and Equal Temperament (Section 2.3).

Suggestions for Presenting These Concepts in the Classroom

- Decide which of the concepts you will be presenting to your class, and prepare your lectures/presentations accordingly. You will probably need about one class period for each related set of concepts. Sound and Ears (Section 1.5.2) is particularly geared towards younger students. The concepts in Frequency, Wavelength, and Pitch (Section 1.3), Transverse and Longitudinal Waves (Section 1.2), and Amplitude (Section 1.4) can be presented to just about any age. Standing Waves and Musical Instruments (Section 3.1), Standing Waves and Wind Instruments (Section 3.2), Harmonic Series (Section 2.2) and Tuning Systems (Section 2.4) are probably best presented to older students (middle school and up). Musical Intervals, Frequency and Ratio (Section 2.1) and Powers, Roots, and Equal Temperament (Section 2.3) can be used either to remind older students of the math that they have learned and its relevance to music, or as extra information for younger students working on these math concepts.
- Include suggested activities, worksheets, and demonstrations whenever possible, particularly for younger students.
- Younger students will benefit from the activities and worksheets in Sound and Music (Section 1.5.1).
- Worksheets that cover the basic concepts for older students are available here. Download and copy these PDF files as handouts for your class: Sound Waves handout[16] and Waves Worksheet[17]. There is also a Worksheet Answer Key[18]. In case you have any trouble with the PDF files, these handouts are also included as figures at the end of this module, but they will look better if you print out the PDF files.
- Use the exercises in the modules for class participation and discussion.

[16]See the file at <http://cnx.org/content/m12373/latest/waves1.pdf>
[17]See the file at <http://cnx.org/content/m12373/latest/waves3.pdf>
[18]See the file at <http://cnx.org/content/m12373/latest/waves4.pdf>

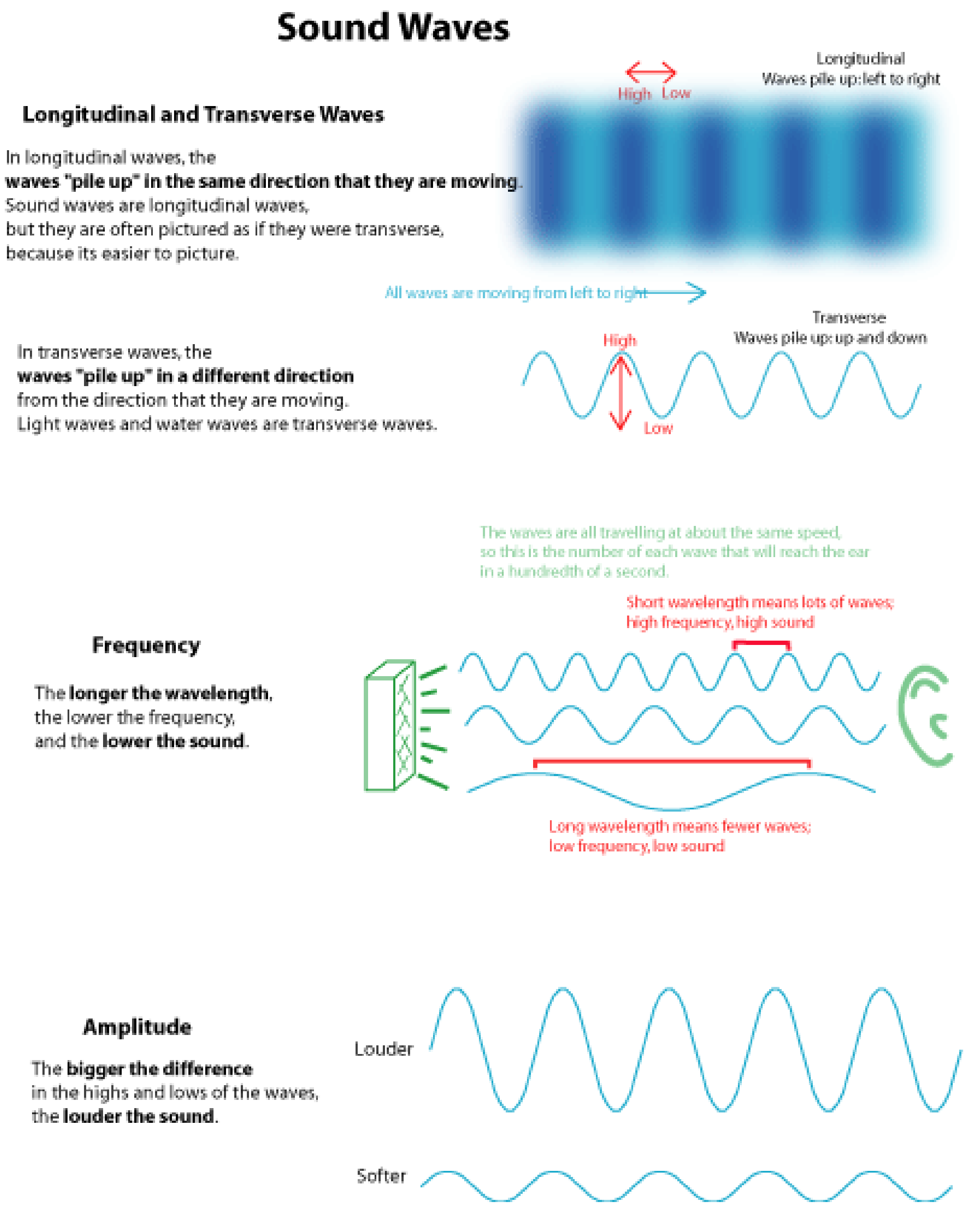
Sound Waves
Longitudinal and Transverse Waves
In longitudinal waves, the
waves "pile up" in the same direction that they are moving.
Sound waves are longitudinal waves,
but they are often pictured as if they were transverse,
because its easier to picture.
High Low
Longitudinal
Waves pile up: left to right
All waves are moving from left to right
In transverse waves, the
waves "pile up" in a different direction
from the direction that they are moving.
Light waves and water waves are transverse waves.
High
Low
Transverse
Waves pile up: up and down
The waves are all travelling at about the same speed,
so this is the number of each wave that will reach the ear
in a hundredth of a second.
Short wavelength means lots of waves;
high frequency, high sound
Frequency
The longer the wavelength,
the lower the frequency,
and the lower the sound.
Long wavelength means fewer waves;
low frequency, low sound
Amplitude
The bigger the difference
in the highs and lows of the waves,
the louder the sound.
Louder
Softer

Figure 1.2

Sound and Music Worksheet

Match both the science/engineering terms on the left and the music terms on the right with the definitions in the middle. You will use some of the definiitons twice.

____ Low Frequency

____ Longitudinal Waves

____ Frequency

____ High Amplitude

____ White Noise

____ Amplitude

____ Sound Waves

____ Standing Waves

____ Transverse Waves

____ Wavelength

____ High Frequency

____ Low Amplitude

A. Waves in the air caused by vibrations

B. Waves that move in one direction, but "wave" in another direction

C. Waves that move and "wave" in the same direction

D. The distance between one wave and the next wave

E. How often a single wave goes by

F. How big the difference is between the high points and the low points of the waves

G. Big difference between highs and lows

H. Small difference between highs and lows

I. Lots of short waves

J. Very few long waves

K. Waves that can keep vibrating in or on something for a long time. because they "fit"

L. A sound that is a mixture of all wavelengths

M. Sounds that are organized by people

____ Low note

____ Pitch

____ Dynamic level

____ Soft note

____ Music

____ High note

____ Sounds

____ Loud note

Give short answers:

1. Can sound travel through empty space? Why or why not?

2. How are sound waves like water waves? How are they not like water waves?

3. Name 2 ways a player of a musical instrument can change the sound of the instrument.

4. How can an instrument with only 4 strings get more than 4 different pitches?

5. When a trumpet player pushes down a valve, she opens an extra loop of tubing. What does this do to the trumpet? To the sound?

Figure 1.3

Sound and Music Worksheet

Match both the science/engineering terms on the left and the music terms on the right with the definitions in the middle. You will use some of the definiitons twice.

A. Waves in the air caused by vibrations

B. Waves that move in one direction, but "wave" in another direction

C. Waves that move and "wave" in the same direction

D. The distance between one wave and the next wave

E. How often a single wave goes by

F. How big the difference is between the high points and the low points of the waves

G. Big difference between highs and lows

H. Small difference between highs and lows

I. Lots of short waves

J. Very few long waves

K. Waves that can keep vibrating in or on something for a long time, because they "fit"

L. A sound that is a mixture of all wavelengths

M. Sounds that are organized by people

__J__ Low Frequency
__C__ Longitudinal Waves
__E__ Frequency
__G__ High Amplitude
__L__ White Noise
__F__ Amplitude
__A__ Sound Waves
__K__ Standing Waves
__B__ Transverse Waves
__D__ Wavelength
__I__ High Frequency
__H__ Low Amplitude

__J__ Low note
__E__ Pitch
__F__ Dynamic level
__H__ Soft note
__M__ Music
__I__ High note
__A__ Sounds
__G__ Loud note

Give short answers:

1. Can sound travel through empty space? Why or why not?
   No; there can be no sound vibrations where there is no air.

2. How are sound waves like water waves? How are they not like water waves?
   Both can have frequency and amplitude, but water waves are transverse and sound waves are longitudinal.

3. Name 2 ways a player of a musical instrument can change the sound of the instrument.
   They can make the pitch higher or lower or make the sound louder or softer.

4. How can an instrument with only 4 strings get more than 4 different pitches?
   You can make the vibrating part of the string shorter, and the pitch higher, by holding the string down with one finger.

5. When a trumpet player pushes down a valve, she opens an extra loop of tubing. What does this do to the trumpet? To the sound?
   This in effect makes the trumpet longer, so the sound is lower.

Figure 1.4

1.2 Transverse and Longitudinal Waves[19]

Waves are disturbances; they are changes in something - the surface of the ocean, the air, electromagnetic fields. Normally, these changes are travelling (except for Standing Waves); the disturbance is moving away from whatever created it.

Most kinds of waves are **transverse** waves. In a transverse wave, as the wave is moving in one direction, it is creating a disturbance in a different direction. The most familiar example of this is waves on the surface of water. As the wave travels in one direction - say south - it is creating an up-and-down (not north-and-south) motion on the water's surface. This kind of wave is very easy to draw; a line going from left-to-right has up-and-down wiggles. So most diagrams of waves - even of sound waves - are pictures of transverse waves.

But sound waves are not transverse. Sound waves are **longitudinal waves**. If sound waves are moving south, the disturbance that they are creating is making the air molecules vibrate north-and-south (not east-and-west, or up-and-down. This is very difficult to show clearly in a diagram, so most diagrams, even diagrams of sound waves, show transverse waves.

> NOTE: It's particularly hard to show amplitude (Figure 1.8) in longitudinal waves. Sound waves definitely have amplitude; the louder the sound, the greater the tendency of the air molecules to be in the "high" points of the waves, rather than in between the waves. But it's easier show exactly how intense or dense a particular wave is using transverse waves.

Longitudinal waves may also be a little difficult to imagine, because there aren't any examples that we can see in everyday life. A mathematical description might be that in longitudinal waves, the waves (the disturbances) are along the same axis as the direction of motion of the wave; transverse waves are at right angles to the direction of motion of the wave. If this doesn't help, try imagining yourself as one of the particles that the wave is disturbing (a water drop on the surface of the ocean, or an air molecule). As it comes from behind you, a transverse waves lifts you up and then drops you down; a longitudinal wave coming from behind pushes you forward and then pulls you back. You can view animations of longitudinal and transverse waves here[20], single particles being disturbed by a transverse wave or by a longitudinal wave[21], and particles being disturbed by transverse and longitudinal waves[22].

[19] This content is available online at <http://cnx.org/content/m12378/1.12/>.
[20] See the file at <http://cnx.org/content/m12378/latest/Waves.swf>
[21] See the file at <http://cnx.org/content/m12378/latest/Pulses.swf>
[22] See the file at <http://cnx.org/content/m12378/latest/Translong.swf>

Transverse and Longitudinal Waves

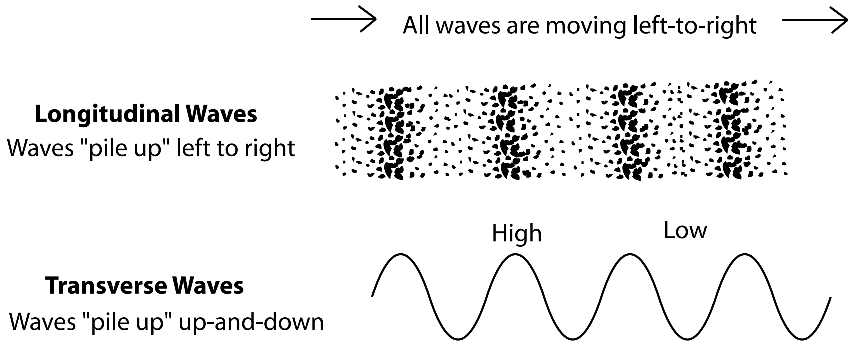

Figure 1.5: In water waves and other **transverse waves**, the ups and downs are in a different direction from their forward movement. The highs and lows of sound waves and other **longitudinal waves** are arranged in the "forward" direction.

1.2.1 Presenting These Concepts in a Classroom

Watching movies or animations of different types of waves can help younger students understand the difference between transverse and longitudinal waves. The handouts and worksheets at Talking about Sound and Music include transverse and longitudinal waves. Here are some classroom demonstrations you can also use.

1.2.1.1 Waves in Students

Procedure

1. You will not need any materials or preparation for this demonstration, except that you will need some room.
2. Have most of the students stand in a row at one side of the classroom, facing out into the classroom. Let some of the students stand across the room from the line so that they can see the "waves".
3. Starting at one end of the line, have the students do a traditional stadium "wave". If they don't know how, have them all start slightly bent forward with hands on knees. Explain that the student on the end will lift both arms all the way over their heads and then put both down again. Each student should do the same motion as soon as (but not before) they feel the student beside them do it.
4. If they do it well, the students watching should see a definite transverse wave travelling down the line of students.
5. Starting with the same end student, next have the line make a longitudinal wave. Have the students start with their arms out straight in front of them. As the wave goes by, each student will swing both arms first toward, and then away, from the next student in line.
6. Let the students take turns being the first in line, being in line, and watching the line from the other side of the room. Let them experiment with different motions: hopping in place, swaying to the left and right, taking a little step down the line and back, doing a kneebend, etc. Which kind of wave does each motion create?

1.2.1.2 Jumpropes and Slinkies

Materials and Preparation

- **Rope** - A jump-rope is ideal, or any rope of similar weight and suppleness
- **Coil** - A Slinky toy works, or any metal or plastic coil with enough length and elasticity to support a visible longitudinal wave
- **Pole** - A broomstick is fine, or a dowel, rod, pipe, or any long, thin, rigid, smooth cylinder.
- You may want to practice with these items before the demonstration, to make certain that you can produce visible traveling waves.

Procedure

1. Load the slinky onto the broomstick and stretch it out a bit. Have two people holding the broomstick horizontally at waist level, as steadily as possible, or secure the ends of the broomstick on desks or chairs.
2. Holding one end of the slinky still, have someone jerk the other end of the slinky forward and back along the broomstick as quickly as possible. This should create a longitudinal wave that travels down the slinky to the other end. (If the other end is being held very tightly, but without interfering with its coils, you may even be able to see the wave reflect and travel back up the slinky.)
3. Secure or have someone hold one end of the jumprope very still at waist height. Stretch the jumprope out taut, horizontally.
4. Have the person at the other end of the jumprope suddenly jerk the end of the rope up and down again. You should see a transverse wave travel to the other end of the rope. If the other end is secured very tightly, you may even be able to see a reflection of the wave travel back to the other end.
5. With both of these setups, you can experiment with sending single pulses, multiple waves, or even try to set up standing waves. In fact, a jumprope is usually used to make a sort of three-dimensional standing wave of the fundamental (p. 35) of the rope length. Try making the standing wave in two dimensions, going just up-and-down (without the forward and back part of the motion). With a good rope and some practice, you may be able to get a second harmonic (Section 2.2) standing wave, with one side of the rope going up while the other side goes down, and a node in the middle of the rope.

1.3 Frequency, Wavelength, and Pitch[23]

Any sound that you hear as a **tone** is made of regular, evenly spaced waves of air molecules. The most noticeable difference between various tonal sounds is that some sound higher or lower than others. These differences in the pitch[24] of the sound are caused by different spacing in the waves; the closer together the waves are, the higher the tone sounds. The spacing of the waves - the distance from the high point of one wave to the next one - is the **wavelength**.

All sound waves are travelling at about the same speed - the speed of sound. So waves with a longer wavelength don't arrive (at your ear, for example) as often (frequently) as the shorter waves. This aspect of a sound - how often a wave peak goes by, is called **frequency** by scientists and engineers. They measure it in **hertz**, which is how many wave peaks go by in one second. People can hear sounds that range from about 20 to about 17,000 hertz.

The word that musicians use for frequency is **pitch**. The shorter the wavelength, the higher the frequency, and the higher the pitch[25], of the sound. In other words, short waves sound high; long waves sound low. Instead of measuring frequencies, musicians name the pitches[26] that they use most often. They might call

[23]This content is available online at <http://cnx.org/content/m11060/2.14/>.
[24]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[25]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[26]"Clef" <http://cnx.org/content/m10941/latest/>

a note "middle C" or "2 line G" or "the F sharp in the bass clef". (See Octaves and Diatonic Music[27] and Tuning Systems (Section 2.4) for more on naming specific frequencies.) These notes do have definite frequencies (Have you heard of the "A 440" that is used as a tuning note?), but musicians usually find it easier just to use the note names.

Wavelength, Frequency, and Pitch

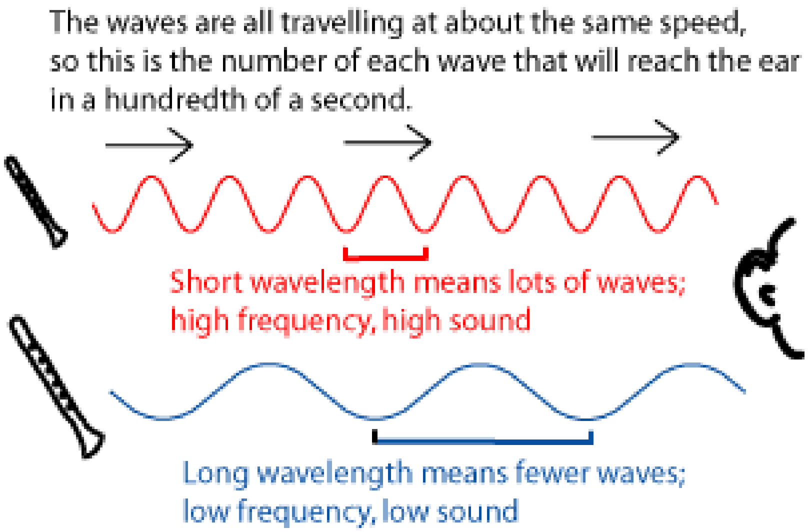

Figure 1.6: Since the sounds are travelling at about the same speed, the one with the shorter wavelength will go by more frequently; it has a higher frequency, or pitch. In other words, it sounds higher.

1.3.1 Ideas for Introducing These Concepts in the Classroom

- For younger students, the "Strings Instruments" and "Wind Instruments" activities in Sound and Music (Section 1.5.1) give children a chance to create higher and lower pitched sounds. There are also handouts and worksheets for younger students covering basic acoustics terms, including frequency and wavelength.
- For older students, there are more advanced handouts and worksheets in Talking about Sound and Music (Section 1.1) that cover acoustics concepts, including frequency, wavelength, and pitch.
- If it can be arranged, a demonstration with a real musical instrument (or two) should be popular. A live show-and-tell-style demonstration would be most memorable, although a video or a recording with pictures will do. Include a discussion on why and how instruments produce higher and lower sounds. Have the musician demonstrate low and high notes, and explain and demonstrate how the sounding part of the instrument is being made shorter or longer to get different notes. Point out that smaller, shorter instruments make shorter waves and higher sounds, and larger, longer instruments make longer waves and lower sounds. Ask the students if they are listening to a small, high-sounding instrument, or a large, low-sounding one.

[27] "Octaves and the Major-Minor Tonal System" <http://cnx.org/content/m10862/latest/>

1.4 Sound Amplitude and Musical Dynamics[28]

When sound waves come as a very regular, pitched tone, there are two useful measurements you can make that tell you something about both the sound waves and about the tone they are making. One measurement is the distance between one wave and the next. This is the wavelength (Figure 1.6: Wavelength, Frequency, and Pitch), which is also related to the frequency (Figure 1.6: Wavelength, Frequency, and Pitch) and the pitch[29] of the sound. The other measurement you can make is the size of each individual wave - its "height" or "intensity" rather than its length. This is the **amplitude** of the wave, and it determines the loudness of the sound.

Wavelength and Amplitude

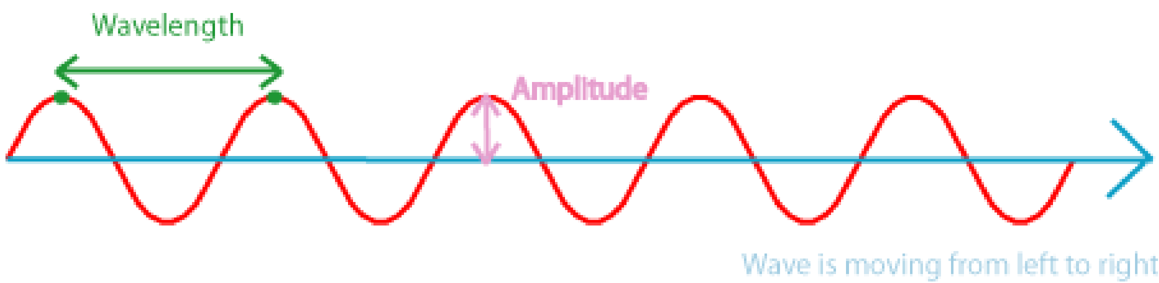

Figure 1.7: The **wavelength** is the distance between the "crests" of two waves that are next to each other. The **amplitude** is how high the crests are.

You may want to note that sound waves are not the type of waves shown in the figure above. (Please see Transverse and Longitudinal Waves (Section 1.2) for more on this.) Rather than piling up high in the crests of the waves, as water on the surface of the ocean does, the air molecules in sound waves pile into the waves. So the bigger the amplitude of the wave, the more air molecules are in the "crest" of each wave, and the fewer air molecules are left in the "low" spots. The amplitude of the wave is still measuring the same thing - how much change there is during one wave - but this is more difficult to show clearly in a diagram with sound-type longitudinal Waves (Section 1.2) waves.

[28]This content is available online at <http://cnx.org/content/m12372/1.8/>.
[29]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>

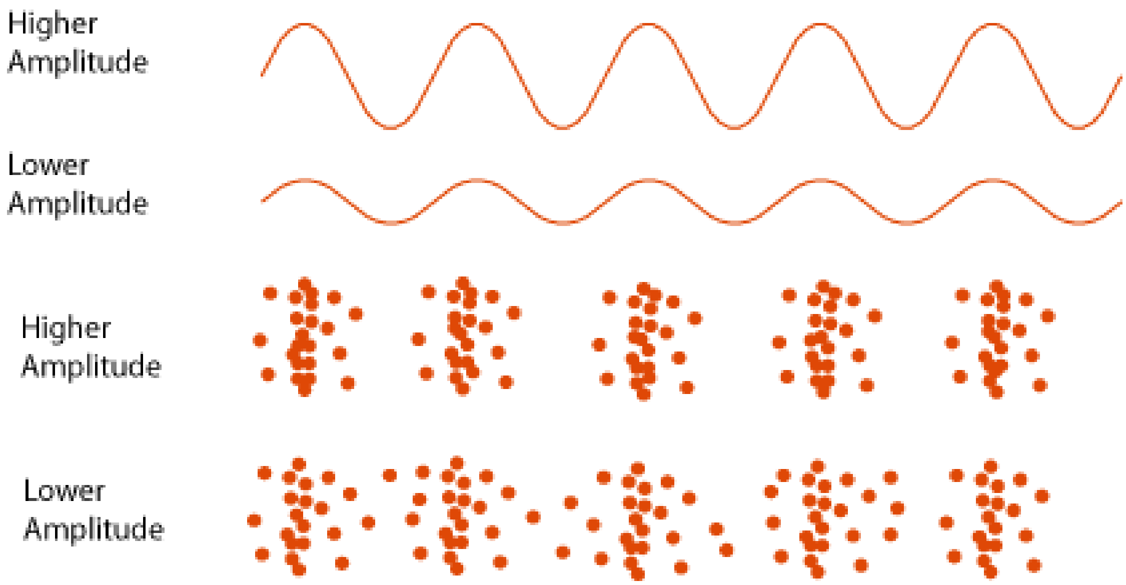

Figure 1.8: It's easier to spot differences in amplitude at a glance when figures use transverse (Section 1.2) waves.

Engineers and scientists call how big a wave is its **amplitude**. They measure the amplitude of sound waves in **decibels**. Leaves rustling in the wind are about 10 decibels; a jet engine is about 120 decibels.

Musicians call the loudness of a note its **dynamic level**. **Forte** (pronounced "FOR-tay") is a dynamic level meaning "loud"; **piano** is a dynamic level meaning "soft". Dynamic levels don't correspond to a measured decibel level. For example, an orchestra playing "fortissimo" (which basically means "even louder than forte") sounds much louder than a string quartet playing "fortissimo". (See Dynamics[30] for more of the terms that musicians use to talk about loudness.)

[30]"Dynamics and Accents in Music" <http://cnx.org/content/m11649/latest/>

Amplitude is Loudness

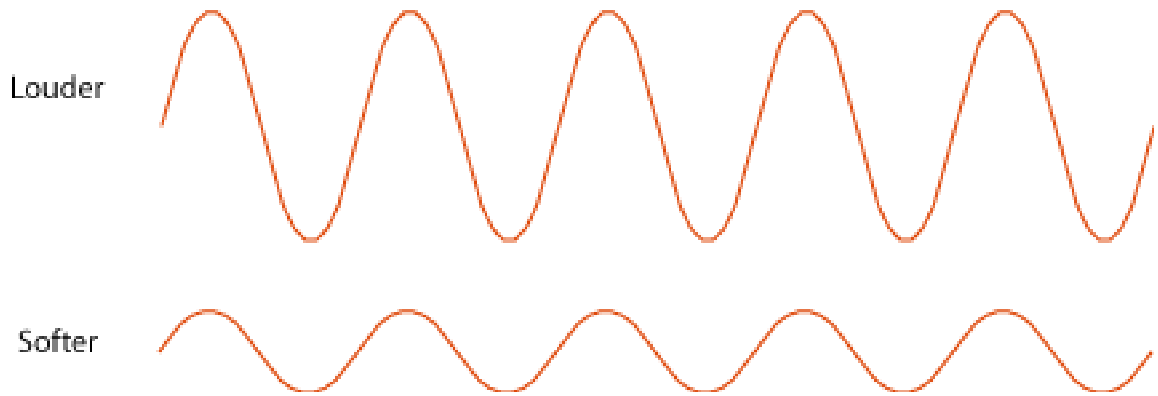

Figure 1.9: The size of a wave (how much it is "piled up" at the high points) is its **amplitude**. For sound waves, the bigger the amplitude, the louder the sound.

1.5 Presenting Concepts to Younger Students

1.5.1 Sound and Music Activities[31]

1.5.1.1 Introduction

Different musical instruments produce sounds in very different ways, but all of them take advantage of some of the fundamental properties of sound - the physics of sound - to make a variety of interesting and pleasant sounds. You will find here a Strings Activity (Section 1.5.1.3: Strings Activity), Wind Instrument Activity (Section 1.5.1.4: Wind Instruments Activity), Percussion Activity (Section 1.5.1.5: Percussion Activity), and Resonance Activity (Section 1.5.1.6: Instrument Body Activities), as well as worksheets (Figure 1.10) appropriate for younger students. All of these explore some basic concepts (Section 1.5.1.2: Terms and Concepts) of sound wave physics (**acoustics**) while demonstrating how various musical instruments produce sounds.

Goals and Standards

- **Goals** - The student will develop an understanding of the physical (scientific) causes of musical sounds, and be able to use appropriate scientific and/or musical terminology to discuss the variety of possible musical sounds.
- **Music Standards Addressed** - National Standards for Music Education[32] standard 8 (understanding relationships between music, the other arts, and disciplines outside the arts)
- **Other Subjects Addressed** - In encouraging active exploration of the effects of physics on music and musical instruments, these activities also address National Science Education Standards[33] in **physical science** and in **science and technology**.
- **Grade Level** - 3-8

[31] This content is available online at <http://cnx.org/content/m11063/2.15/>.
[32] http://musiced.nafme.org/resources/national-standards-for-music-education/
[33] http://www.nap.edu/readingroom/books/nses/overview.html#content

- **Student Prerequisites** - If younger students are not ready to conduct their own lightly-supervised investigations, these activities should be done as full-classroom demonstrations.
- **Teacher Expertise** - Teacher expertise in music is not necessary to present this activity. The teacher should be familiar and comfortable with basic acoustics terms and concepts (see Acoustics for Music Theory[34]).
- **Time Requirements** - Reserve one (approximately 45-minute) class period for each activity/discussion, and one class period to finish discussions, draw conclusions, do worksheets, and reinforce terms and concepts. If you have a longer period of time and a large area to work in, you may want to set up each experiment as a "work station" and have student groups move from one station to another.

You can do any one or any combination of the activities. While doing them, introduce whichever of the terms and concepts you think will benefit your students. You can either use only the scientific terms, or only the musical terms, or both. To reinforce the concepts and terms with younger students, follow the activities with the worksheets in the Terms and Concepts (Section 1.5.1.2: Terms and Concepts) section below. For older students, present the relevant information from Frequency, Wavelength, and Pitch (Section 1.3), Amplitude and Dynamics (Section 1.4), and Transverse and Longitudinal Waves (Section 1.2), and include the worksheet and handout from Talking About Sound and Music (Section 1.1).

1.5.1.2 Terms and Concepts

During or after your activities, introduce the following terms and concepts to the students. Worksheets to help you do this with younger students are available here as PDF files: Terms Worksheet[35], Matching Worksheet[36], Answer sheet[37]. (Or you may copy the figures (Figure 1.10).) With younger students, you may also want to study Sound and Ears (Section 1.5.2). For older students, use the worksheet and handout in Talking About Sound and Music (Section 1.1). For more detailed information on this subject, you may also see Talking about Sound and Music (Section 1.1), Frequency, Wavelength, and Pitch (Section 1.3), Amplitude and Dynamics (Section 1.4), Transverse and Longitudinal Waves (Section 1.2), Standing Waves and Musical Instruments (Section 3.1), Standing Waves and Wind Instruments (Section 3.2), or Acoustics for Music Theory[38]. Use the discussion questions during and after the activities to help the students reach conclusions about their investigations.

Terms and Concepts

- **Sound Waves** - When something vibrates, it makes a sound. The vibrations travel out in all directions from the "something" in the same way that ripples travel out from a pebble that has been dropped in water. But instead of being waves of water, these are waves of vibrations of air: **sound waves**. Because it is the air itself that is vibrating, sound waves, unlike water waves, are invisible.
- **Frequency** - or **Pitch** - Think of water waves again. They can be close together or far apart. If they are close together, there are more of them; they are more frequent. **Frequency** is the term that scientists and engineers use to describe how many pulses of a sound wave arrive at your ear in one second. Musicians use the term **pitch**. A sound with a higher frequency (more waves) has a higher pitch, and sounds higher.
- **Amplitude** - or **Dynamic Level** - Water waves can also be great, big, tall waves, or small ripples. The size of a wave is called its **amplitude**. In sound waves, the bigger the wave, the louder the sound is. Musicians call the loudness of a sound its **dynamic level**.

[34]"Acoustics for Music Theory" <http://cnx.org/content/m13246/latest/>
[35]See the file at <http://cnx.org/content/m11063/latest/soundhandout.pdf>
[36]See the file at <http://cnx.org/content/m11063/latest/soundhandout2.pdf>
[37]See the file at <http://cnx.org/content/m11063/latest/soundhandout3.pdf>
[38]"Acoustics for Music Theory" <http://cnx.org/content/m13246/latest/>

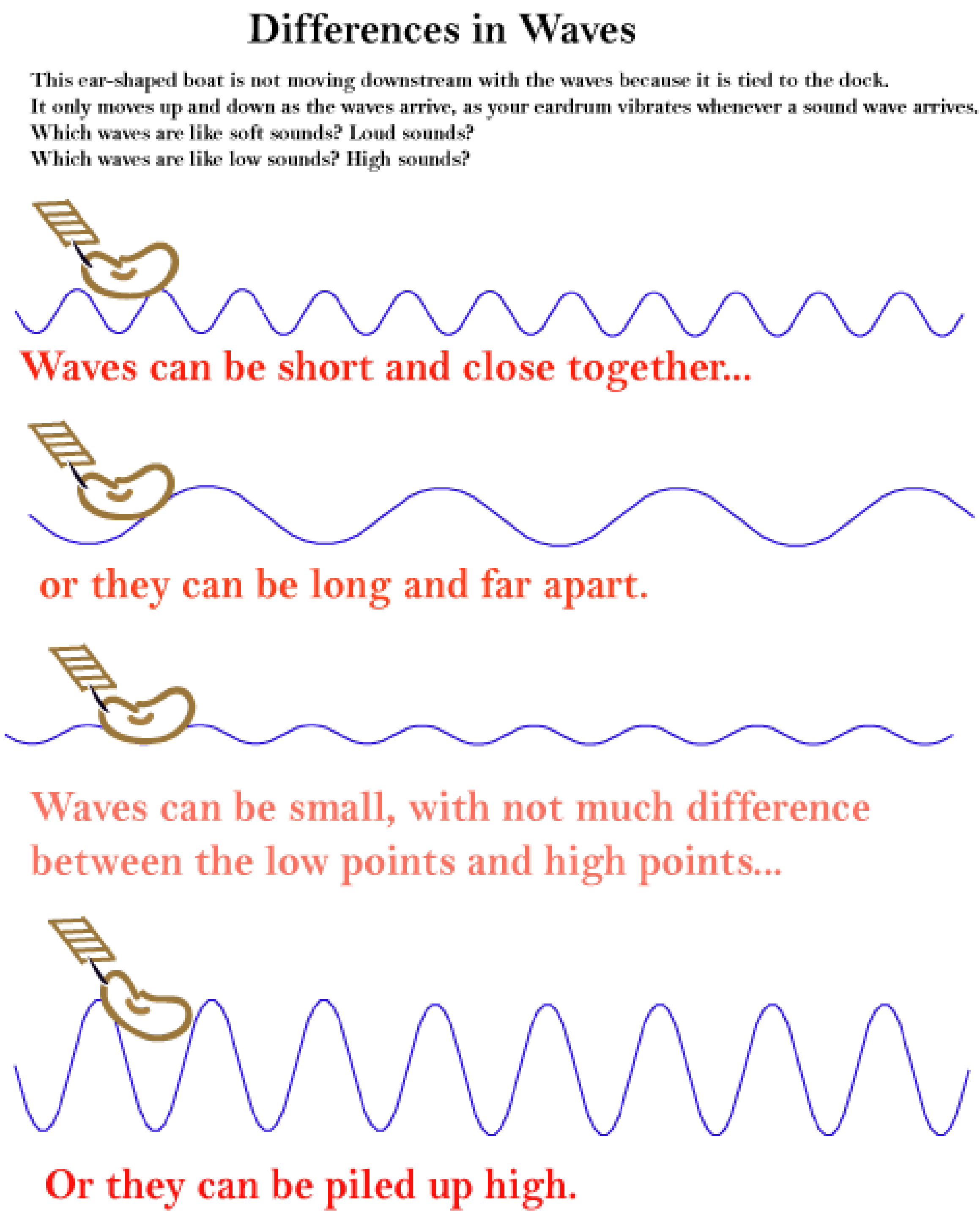
Differences in Waves
This ear-shaped boat is not moving downstream with the waves because it is tied to the dock.
It only moves up and down as the waves arrive, as your eardrum vibrates whenever a sound wave arrives.
Which waves are like soft sounds? Loud sounds?
Which waves are like low sounds? High sounds?
Waves can be short and close together...
or they can be long and far apart.
Waves can be small, with not much difference
between the low points and high points...
Or they can be piled up high.

Figure 1.10

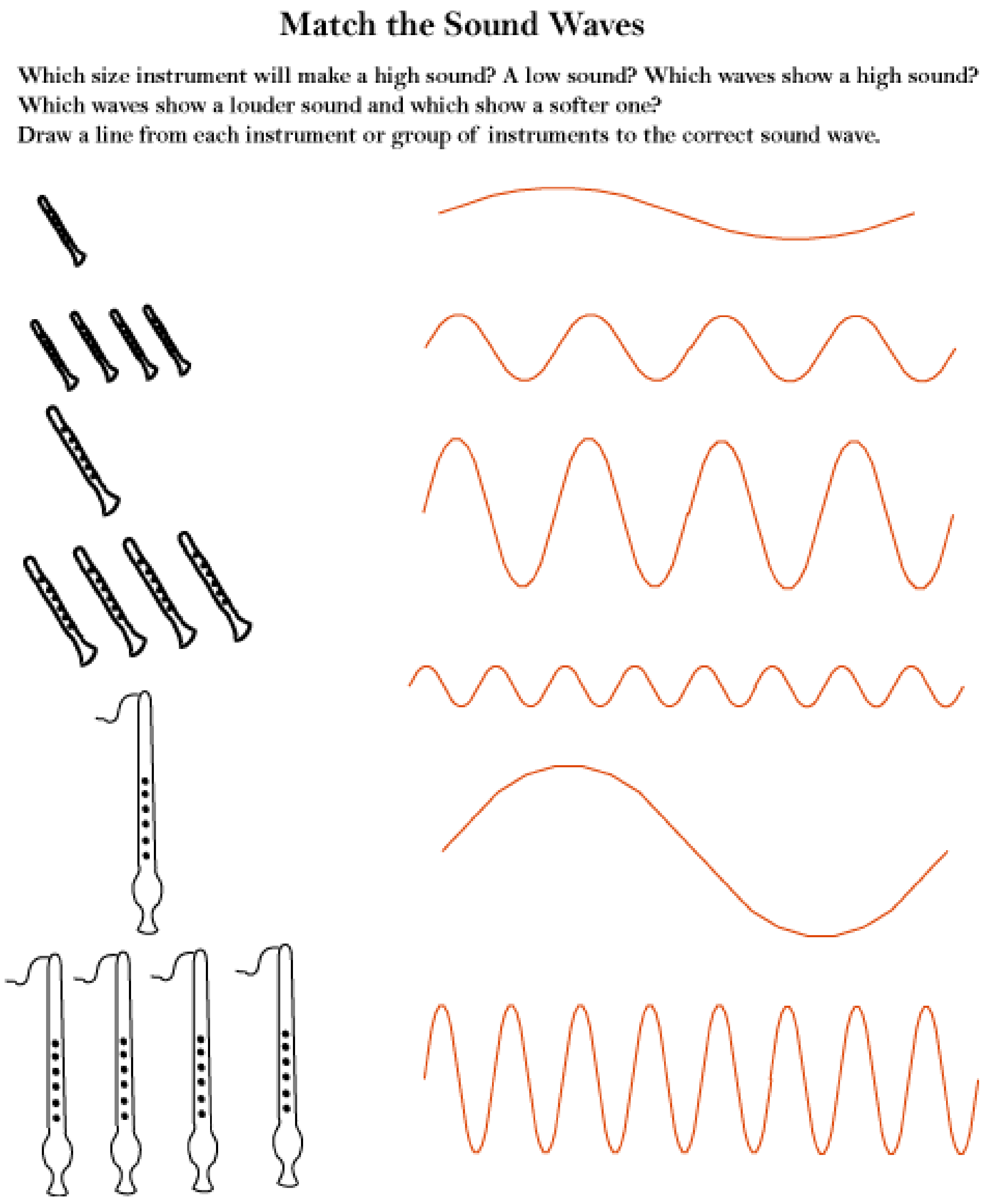
Match the Sound Waves
Which size instrument will make a high sound? A low sound? Which waves show a high sound?
Which waves show a louder sound and which show a softer one?
Draw a line from each instrument or group of instruments to the correct sound wave.

Figure 1.11

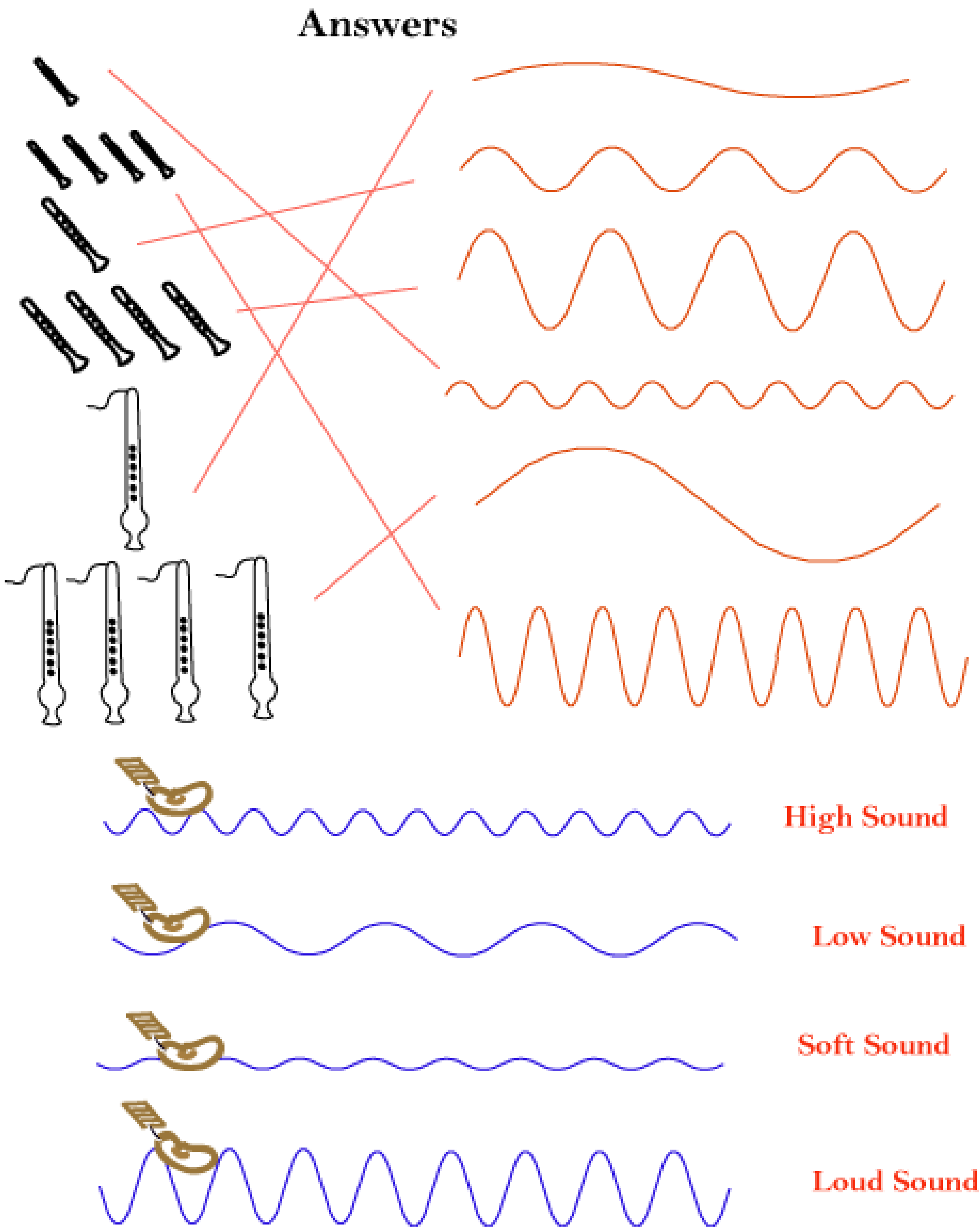

Figure 1.12

1.5.1.3 Strings Activity

Objectives and Assessment

- **Objectives** - The student will construct a simplified version of a stringed instrument, using rubber bands as strings, and will use the instrument to explore the effects of various string characteristics on frequency and amplitude.
- **Evaluation** - Assess student learning using worksheets or answers to discussion questions.

Materials and Preparation

- Most students will be able to do this experiment alone or in small groups. If you do not want students working with thumbtacks, plan to use boxes or pans as instrument bodies.
- You will need lots of rubber bands, as many different lengths and thicknesses and tightnesses as you can find. If you are using boxes, the rubber bands must be long enough to stretch around a box.
- You will also need either small, sturdy cardboard or plastic boxes or containers, with or without lids, OR pieces (about 8" X 10" or so) of thick, flat cardboard, OR square or rectangular baking pans, one for each student or group.
- If you are using flat cardboard, you will also need thumbtacks or push pins.
- If you are using a lidded box, pencils, pens, or other objects approximately the size and shape of a pencil (a couple for each instrument) will be useful.
- You may want scissors that are strong enough to cut the cardboard or plastic.
- If a stringed-instrument player is available for a show-and-tell, you may want to include this after the activity, to demonstrate and reinforce some of the main points. Any stringed instrument (guitar, violin, harp, etc.) will do.
- For older or more independent students, you may want to make copies of the discussion questions.[39]

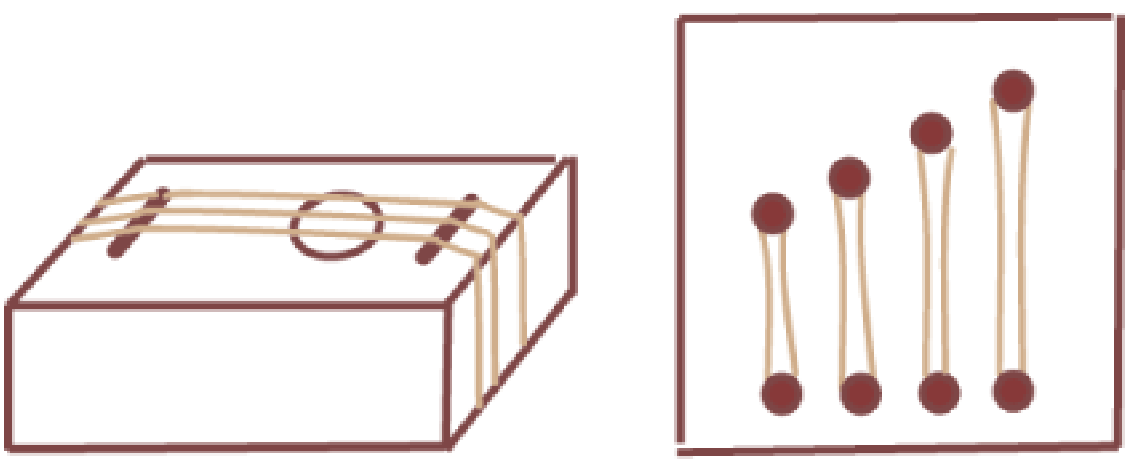

Figure 1.13

Procedure

[39]See the file at <http://cnx.org/content/m11063/latest/StringQuestions.pdf>

1. Each student or group should choose a variety of rubber bands (3-6, depending on the size of their "instruments") to start with.
2. If you are using flat cardboard, stretch each rubber band between two thumbtacks so that it is tight enough to give a particular pitch.
3. If using a box or baking pan, stretch the rubber bands around the box or pan.
4. Have the students pluck each rubber band separately and listen carefully to the "twang". They are listening for which ones sound higher and which sound lower.
5. To try many different thicknesses and tightnesses, students can trade rubber bands with each other or trade off from the central pile if there are enough.
6. Students with the thumbtack instruments can vary length and tightness by changing the distance between the thumbtacks.
7. Students with box or pan instruments can vary tightness by pulling on the rubber band at the side of the box while plucking it at the top. Students with lidded box instruments can vary length by slipping a pencil under each end of the rubber bands on the top of the box and then varying the distance between the pencils, or even holding the rubber band down tightly with a finger between the pencils, in the same way as a real string player.
8. Students with box instruments can also see if the body of the instrument makes any difference to the sound. Can they play the instrument with the lid off and with it on? Does cutting a hole in the lid change the sound? Does it make it easier to play? Does adding the pencils change the sound or make it easier to play? Do different boxes make a different sound with the same rubber bands? Do cardboard boxes sound different from plastic ones?
9. Ask younger students the discussion questions while they are experimenting. Allow them to check and answer immediately. Summarize the answers for them on the board, or remind them and let them write them down when they are done experimenting. Give older students a list of the discussion questions before they begin.

Discussion Questions

- Do thicker rubber band "strings" sound higher or lower than thinner ones? (Answer: thicker should sound lower.)
- Do tighter strings sound higher or lower than looser ones? (Tighter should sound higher.)
- Do shorter strings sound higher or lower than longer ones? (Shorter should sound higher.)
- Do there seem to be differences in how loud and soft or how dull or clear a string sounds? If so, what seems to cause those differences?
- What determines whether the sound of a string is loud or soft?
- What happens to the sound if they pluck with one finger while touching the string lightly with another finger? (No "twang"; the touch stops the vibrations.) If their instrument design allows it, what happens when they hold the string tightly down against the instrument and then pluck it? (The shorter vibrating length should give a higher pitch.)
- After their experiments, can they explain what happens when a player holds a string down with a finger? What if the same string is held down in a different spot?
- Based on their observations, do the students feel they could tell which strings of an instrument are the low strings just by looking at them closely? (For an extra activity, arrange for them to try this with a real instrument.)
- Can the students come up with possible reasons why the thickness, length, and tightness of a string affect its frequency/pitch in the way that they do? (For example, why does a shorter string have a higher frequency/pitch?) (It may help on length to remind them that the longer the waves are, the less frequent they will be.)

1.5.1.4 Wind Instruments Activity

Objectives and Assessment

- **Objectives** - The student will explore the effects of air column size (and shape) on the frequency and amplitude of standing waves in the air column, using empty glass bottles, and water if necessary to vary air column size.
- **Evaluation** - Assess student learning using worksheets or answers to discussion questions.

Materials and Preparation

- If you do not want your students working with glass jugs and water, plan to do this as a demonstration.
- You will need several narrow-necked bottles, all the same size and shape OR several narrow-necked bottles of varying sizes and shapes. Bottles should be empty and clean. Make sure before the class begins that your bottles give a clear, reasonably loud sound when you blow across the top of them. If necessary, practice getting a sound. Large glass jugs with an inner lip diameter of approximately one inch work well.
- If using bottles of the same size, you will also need water to fill them to varying depths. If you are using this approach, food coloring is very useful to clearly show the depth of the water.
- If plastic recorders are available to your students, or a player of a woodwind[40] or brass[41] instrument is available for a show-and-tell, they can be used for an extra demonstration.
- For older or more independent students, you may want to make copies of the discussion questions.[42]

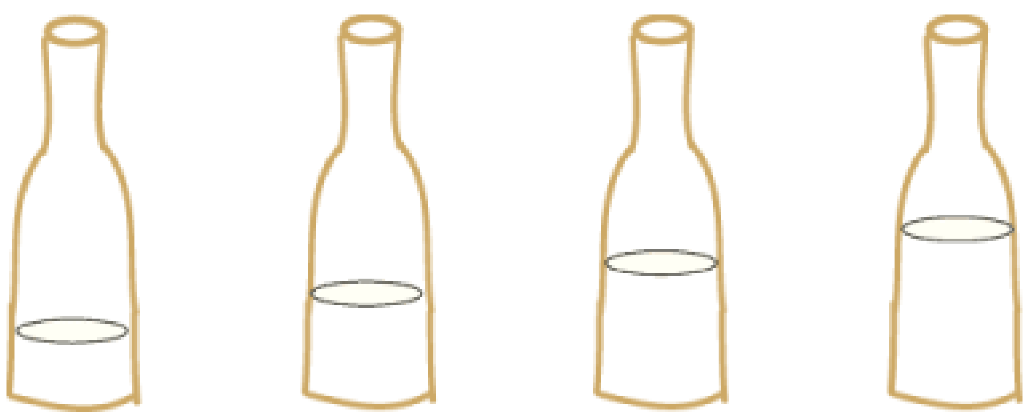

Figure 1.14

Procedure

1. If using same-size bottles and water, fill each bottle to a different depth (for example, an inch in one bottle, two inches in another, three inches in a third and so on). If you have food coloring, add a few drops to the water in each bottle so it is easy to see the depths.
2. Make the air in a bottle vibrate by blowing steadily across the top of the bottle.
3. "Play" each bottle in turn, and arrange them in order from the highest sound to the lowest.
4. If you have the time and inclination, you can even try to "tune" the bottles by adding or pouring out water.

[40]"Orchestral Instruments": Section Woodwinds <http://cnx.org/content/m11897/latest/#s12>
[41]"Orchestral Instruments": Section Brass <http://cnx.org/content/m11897/latest/#s13>
[42]See the file at <http://cnx.org/content/m11063/latest/WindQuestions.pdf>

5. If recorders or a wind instrument are available, demonstrate how covering and uncovering the holes on the instrument changes the pitch. Explain that the main vibration in the instrument is happening in the air inside the instrument (just like the air in the bottles), in between the mouthpiece and the first hole that the air can escape from..

Discussion Questions

- If using bottles of different shapes and sizes, how does the size of the bottle affect the pitch/frequency? Does the shape of the bottle seem to affect it?
- Does the size and shape of the bottle seem to affect anything else, like the loudness of the sound or the tone quality?
- What do you think explains these effects?
- If using water in bottles, how does the amount of water affect the pitch/frequency? Why? (You may need to remind the students that it is the air in the bottle that is vibrating; more water means a smaller space for the air; smaller space means shorter waves and higher frequency/pitch).
- How is a bottle "instrument" the same as a wind[43] instrument, and how is it different?
- If demonstrating with instruments: How does opening and closing the holes of the instruments change the pitch? Why? (Answer: the shorter the distance between the mouthpiece and the first open hole, the shorter the waves and the higher the pitch/frequency. Opening and closing other holes further down the instrument from the first open hole may have no discernible effect - they are not changing the length of the vibrating column of air - or if they are affecting the vibrating air a little, they may change the sound enough to make it more or less in tune.) If a brass instrument is used, what is the effect of opening a valve or extending the slide? (Opening valves actually lengthens the instrument, by opening up extra tubing, lowering the pitch.)

1.5.1.5 Percussion Activity

Objectives and Assessment

- **Objectives** - The student will assist in constructing a "found objects" chime, and will use the instrument to explore the effects of various object characteristics on frequency and amplitude.
- **Evaluation** - Assess student learning using worksheets or answers to discussion questions.

Materials and Preparation

- Each working group will need a dowel, rod, or small beam, around 4-6 feet long, held at both ends about five feet off the ground.
- Each group will need a variety of objects of different sizes and materials. Forks, spoons, spatulas, rulers, wind chimes, lengths of chain, lengths of pipe or bamboo or tubing, are all easy to line up below the dowel because they are long and thin. Objects that have holes or handles (slotted spoons, pan lids) making it easier to keep them tied on, are also a good idea. Objects that are metal, hardwood, hard plastic, hollow, and/or made in a single piece are most likely to make easy-to-hear, interesting sounds.
- It may be easier to answer some of the discussion question if some of the objects are similar objects in a variety of sizes, for example small medium and large metal spoons.
- You will need enough string to hang the objects from the dowels, and may need tape to keep the objects on the string. Keep in mind, though, that tape will probably dampen the vibrations of the object so that it won't "ring" as long.
- You will need something the students can use to strike the objects; a wooden spoon, short stick, pen or pencil, or ruler. Or they can experiment with using different objects as "drumsticks". Which do the students prefer and why?

[43] "Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/>

- For older or more independent students, you may want to make copies of the discussion questions.[44]

Figure 1.15

Procedure

1. Have the students hang the objects securely from the dowel.
2. The students should then strike the objects one at a time, listening carefully to the sound each object typically makes.

Discussion Questions

- Does the size of the object seem to affect its pitch/frequency? Its loudness?
- Does the shape of the object seem to affect its pitch/frequency? Its loudness?
- Does the object's material seem to affect its pitch/frequency? Its loudness?
- Can you tell what effects the thickness of an object has on its sound?
- What seems to affect how long a sound lasts?
- What objects make the sounds that you like best? Which do you think would make good percussion instruments? Why?
- Which of these effects do you think you can explain in terms of waves and the vibrations the objects must be making?

1.5.1.6 Instrument Body Activities

Objectives and Assessment

- **Objectives** - The student will construct a simple megaphone, and will use the megaphone and a music box in several simple investigations to explore the effects that the body of an instrument has on its sound.
- **Evaluation** - Assess student learning using worksheets or answers to discussion questions.

Materials and Preparation

[44]See the file at <http://cnx.org/content/m11063/latest/PercussionQuestions.pdf>

- Decide whether each step of this investigation will be a teacher demonstration or an individual or small-group activity.
- You will need a music box.
- You will need several large, flat surfaces of different types of materials - different types of wood and metal as well as plastic and softer surfaces will be particularly instructive. A box or drawer made of hardwood is optional.
- You will also need large sheets of paper, construction paper, newspaper, soft, pliable plastic or foam or poster board, and some tape, OR a megaphone. If you have a variety of megaphone materials, have different students use different materials to see if material choice affects the sound.
- For older or more independent students, you may want to make copies of the discussion questions.[45]

Procedure

1. Wind the music box and let everyone listen to it while holding it in your hand.
2. Place the box on different surfaces and listen to the difference it makes in the sound. Continue to wind it as necessary to hear a long example of each surface. If you can, place the music box inside a wooden box or drawer.
3. If you do not have a real megaphone to demonstrate, let the students make their own megaphones by rolling the paper into a cone shape, open at both ends. Tape it if necessary to hold the shape.
4. Let them talk or sing into their megaphones and otherwise experiment with how the megaphone changes sounds. Experiment with different megaphone sizes and shapes (narrow or widely flaring).

Discussion Questions

- What effect does each surface have on the sound from the music box? What is causing these effects? (Answer: some surfaces will vibrate with the music box if they are touching. See Resonance[46].)
- Why do instruments have bodies; why aren't they just a bunch of strings or a reed or a membrane to beat on?
- Why would an instrument maker choose to make an instrument body out of wood (like a violin or piano)? Why might metal be chosen (as in brass and many percussion instruments)? Of the other materials you experimented with, would you make instruments out of them? What kind of instrument with each material? Why?
- How does a megaphone shape change a sound? Does it matter whether the megaphone is narrow or flaring?
- How do you think the megaphones would have changed if they had been made of wood or of metal?
- Would a violin sound louder if you were sitting right in front of it or next to it? What about a trumpet? What's the difference?
- Based on your observations, what do you think the shape of the instrument does to the sound of a tuba, trumpet, trombone, clarinet, or saxophone? What about flutes and bassoons (which do not flare)?

1.5.2 Sound and Ears[47]

1.5.2.1 Introduction

The ear is the sense organ that picks up sound waves (Section 1.3) from the surrounding air and turns them into nerve impulses that can be sent to the brain. The sound waves carry lots of information - language, music, and noises - all mixed up together. The task of the ear is to turn the signals in these waves of bouncing air molecules into electrical nerve signals, while keeping as much of the information in the signal as possible. (Then it's the brain's job to sort the signals and make sense out of them.) It's not easy to turn one kind of signal into another kind without losing information, but the ear is well designed for the task.

[45] See the file at <http://cnx.org/content/m11063/latest/InstrumentQuestions.pdf>
[46] "Resonance and Musical Instruments" <http://cnx.org/content/m13537/latest/>
[47] This content is available online at <http://cnx.org/content/m12365/1.4/>.

NOTE: The human ear also has some other functions not related to hearing; those won't be discussed here.

When something vibrates, the vibrations can travel as waves through solids, liquids, and gases. Even animals that have no ears can often feel these vibrations. But in order to understand language and hear music, the brain has to be given more information than just "there's a vibration". It needs to know the frequency (Figure 1.6: Wavelength, Frequency, and Pitch) and amplitude of all the waves that the ear is collecting. Interestingly, the ear sends this information to the brain very accurately by turning the sound waves in the air (vibrations in a gas) into vibrations in bones (solid), and then into waves in a fluid in the inner ear (a liquid), before they become (electrical) nerve signals. This might seem like a lot of unnecessary translation, but it allows the sense of hearing to be both sturdy and very sensitive, as explained below.

1.5.2.2 Parts of the Ear

The ear has three main sections. In the **outer ear**, the sound waves are still moving in air. In the **middle ear**, the sound waves are being conducted by three small bones. In the **inner ear**, the waves are moving through the fluid-filled **cochlea**.

The Ear

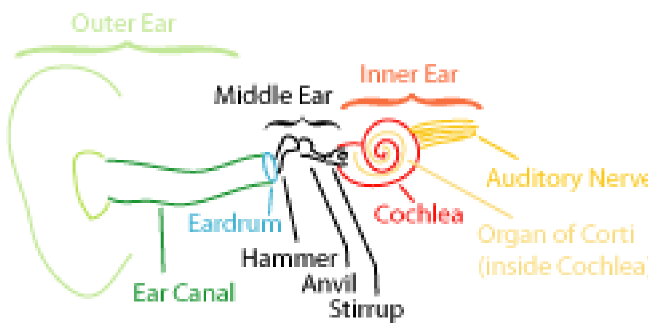

Figure 1.16: The parts of the ear that aren't involved in hearing have been left out.

1.5.2.2.1 The Outer Ear

The part of the human ear that you can see is simply a sound wave collector. Its shape helps to funnel the sound waves into the **auditory canal** (or **ear canal**) so that you get plenty of signals from even soft sounds, particularly ones from the direction that you are looking at. At the other end of the ear canal is the **eardrum** (or **tympanic membrane**). This is a membrane that is stretched tight, like the membranes on the tops of drums (including tympani). And thin, taut membranes are very good at vibrating, which is why they can be found both on drums and inside ears. The eardrum picks up the vibrations in the ear canal and vibrates with them.

1.5.2.2.2 The Middle Ear

On the other side of the eardrum are the three tiny bones of the middle ear, the **hammer**, the **anvil**, and the **stirrup**. They are named for their shapes. Vibrations in the eardrum are passed to the hammer, which transmits them to the anvil, which makes the stirrup vibrate against the **oval window** of the cochlea. Bone is a very good conductor of vibrations, and the bones of the middle ear are specially arranged so that they can amplify (make louder) very quiet sounds. On the other hand, if things get too loud, tiny muscles in your middle ear can relax the eardrum a bit. A relaxed eardrum doesn't vibrate as much (think of a relaxed rubber band as opposed to a taut one), and this helps to keep things from getting damaged.

1.5.2.2.3 The Inner Ear

The **cochlea** is a fluid-filled spiral (shaped something like a snail shell) about the size of a pea. Vibrations in the stirrup make waves in the fluid that travel down the spiral. In the fluid, in a long strip following the spiral, is the **organ of Corti**. This organ is covered with (about 20,000) tiny, incredibly sensitive hairs that are waving around inside the Cochlear fluid. Each of these hairs is a nerve ending that is picking up specific information about the vibrations in the fluid. At the end of the organ of Corti, the nerves are bundled together as the **auditory nerve**, which brings the information to the brain.

> NOTE: The fragile, sensitive hairs on the organ of Corti would never stand up to the rough conditions in the ear canal. Even protected in the Coclear fluid, they don't last forever, especially the ones that can sense the highest-frequency vibrations. That is why most people begin to lose their sense of hearing as they grow older.

1.5.2.3 Presenting this Module to Children

You can present the information above in the form of a classroom lecture/presentation to elementary or middle school classes. Here are some suggestions for making the presentation more interactive and engaging.

- Locate a poster or large diagram of the ear to use as a visual aid.
- If you don't have a poster, or if the printing on it is small, write the names of the parts of the ear on the board as you discuss them.
- Make copies of this PDF file worksheet[48] for a class handout. Have the students label the parts of the ear during or after your presentation.
- When you discuss the outer ear, have the students make their own simple funnels out of paper and tape. Have each student hold the small end of the funnel up to an ear to see if it helps the ear collect sounds even better, especially in the direction that the funnel is pointing. A very simple version of this is to simply cup the hands behind the ears.
- You may want to have a classroom discussion on why it might be useful to have ears that "focus" on the sounds that are directly ahead of you. If the class is also studying animals, you can bring in pictures of various ears. Which are large and which small? Which are pointed straight ahead? How would that be useful? Which can swivel in different directions? How would that be useful? If they cannot come up with any ideas, give them a hint by asking which animals are hunters and which are hunted. You may also want to discuss animals that pick up vibrations with parts of their bodies that are very unlike human ears. You can even turn this into a class project by asking students to research and report on different animals (reptiles, elephants, and insects are particularly interesting).
- For the eardrum, you can simply use rubber bands to demonstrate that things vibrate more clearly when they are taut. Or if you want to be more adventurous (and messy), stretch a sheet of thick cellophane or thin rubber, leather, hide, or close-woven fabric across the opening of a bowl, can, or small tub, and sprinkle some rice over it. Try hitting your stretched membrane with a stick when it is relaxed, fairly taut, and very taut. When is it best at transmitting the vibrations and making the rice jump? Can you get it taut enough to act like an eardrum - so taut that even a loud sound nearby (say, hitting a different can) will make it vibrate and the rice jump?
- When discussing vibrations in bone, let them talk while pressing their fingers gently on the back of their jawbones (below the ears). They should be able to feel the vibrations from their own speech in the bone almost as well as when they press against their throats, where the sounds are being produced.

[48] See the file at <http://cnx.org/content/m12365/latest/earworksheet.pdf>

But they probably won't feel any vibrations from their noses, cheeks, outer ear, or hair. You can point out that: a lot of what you hear when you hear your own voice is coming to your ear through your jawbone. That's why your voice sounds so different to you when you hear a recording of it.

- When discussing the Cochlea and organ of Corti, ask if the students have seen underwater plants moving back and forth in the waves. If you really want to be hands-on, you can get a tank of water, hang some long thin ribbons or plant fronds in it, and let them make waves and watch the "hairs" move.

Chapter 2

The Physics and Math of Intervals and Tuning

2.1 Musical Intervals, Frequency, and Ratio[1]

In order to really understand tuning, the harmonic series, intervals, and harmonic relationships, it is very useful to understand a little bit about the physics of sound and to be comfortable discussing ratios, fractions, and decimals. This lesson is a short review of some basic math concepts for students who want to understand some of the math and physics principles that underlie music theory.

Ratios, fractions, and decimals are basically three different ways of saying the same thing. (So are percents, but they don't have anything to do with music.)

Example 2.1

If you have two apples and three oranges, that's five pieces of fruit altogether. You can say:

- The ratio of apples to oranges is 2:3, or the ratio of oranges to apples is 3:2.
- The ratio of apples to total fruit is 2:5, or the ratio of oranges to total fruit is 3:5.
- 2/5 of the fruit are apples, and 3/5 of the fruit are oranges.
- There are 2/3 as many apples as oranges, and 1 and 1/2 times (or 3/2) as many oranges as apples.
- There are 1.5 times as many oranges as apples, or there are only .67 times as many apples as oranges.
- 0.4 (Four tenths) of the fruit is apples, and 0.6 (six tenths) of the fruit is oranges.

NOTE: You should be able to see where the numbers for the ratios and fractions are coming from. If you don't understand where the decimal numbers are coming from, remember that a fraction can be understood as a quick way of writing a division problem. To get the decimal that equals a fraction, divide the numerator by the denominator.

Example 2.2

An adult is walking with a child. For every step the adult takes, the child has to take two steps to keep up. This can be expressed as:

- The ratio of adult to child steps is 1:2, or the ratio of child to adult steps is 2:1.

[1] This content is available online at <http://cnx.org/content/m11808/1.8/>.

- The adult takes half as many (1/2) steps as the child, or the child takes twice as many (2/1) steps as the adult.
- The adult takes 0.5 as many steps as the child, or the child takes 2.0 times as many steps as the adult.

Exercise 2.1.1 ***(Solution on p. 55.)***

The factory sends shirts to the store in packages of 10. Each package has 3 small, 3 medium, and 4 large shirts. How many different ratios, fractions, and decimals can you write to describe this situation?

What has all this got to do with music? Quite a bit, as a matter of fact. For example, every note in standard music notation is a fraction of a beat, and every beat is a fraction of a measure. You can explore the relationship between fractions and rhythm in Fractions, Multiples, Beats, and Measures[2], Duration[3] and Time Signature[4].

The discussion here will focus on the relationship between ratio, frequency, and musical intervals. The interval[5] between two pitches depends on the ratio of their frequencies (Figure 1.6: Wavelength, Frequency, and Pitch). There are simple, ideal ratios as expressed in a harmonic series (Section 2.2), and then there is the more complex reality of equal temperament (Section 2.4.3: Temperament), in which the frequency ratios are not so simple and are best written as roots or decimals. Here is one more exercise before we go on to discussions of music.

Exercise 2.1.2 ***(Solution on p. 55.)***

The kind of sound waves that music is made of are a lot like the adult and child walking along steadily in the example above (Example 2.2). Low notes have long wavelengths, like the long stride of an adult. Their frequencies, like the frequency of the adult's steps, are low. High notes have shorter wavelengths, like the small stride of the child. Their frequencies, like the frequency of a child's steps, are higher. (See Sound, Physics and Music (Section 1.3) for more on this.)

You have three notes, with frequencies 220, 440, and 660. (These frequencies are in hertz, or waves per second, but that doesn't really matter much; the ratios will be the same no matter what units are used.)

1. Which note sounds highest, and which sounds lowest?
2. Which has the longest wavelength, and which the shortest?
3. What is the ratio of the frequencies? What is it in lowest terms?
4. How many waves of the 660 frequency are there for every wave of the 220 frequency?
5. Use a fraction to compare the number of waves in the 440 frequency to the number of waves in the 660 frequency.

It is easy to spot simple frequency relationships, like 2:1, but what about more complicated ratios? Remember that you are saying **the ratio of one frequency to another IS (equals) another ratio(or fraction or decimal)**. This idea can be written as a simple mathematical expression. With enough information and a little bit of algebra, you can solve this equation for any number that you don't have.

If you remember enough algebra, you'll notice that the units for frequency in this equation must be the same: if frequency #1 is in hertz, frequency #2 must be in hertz also. In all the examples and problems below, I am going to assume all frequencies are in hertz (waves per second), but you can use any frequency unit **as long as they are both the same**. Most musicians don't talk about frequency much, and when they do, they rarely mention units, but just say, for example, "A 440".

[2] "Fractions, Multiples, Beats, and Measures" <http://cnx.org/content/m11807/latest/>
[3] "Duration: Note Lengths in Written Music" <http://cnx.org/content/m10945/latest/>
[4] "Time Signature" <http://cnx.org/content/m10956/latest/>
[5] "Interval" <http://cnx.org/content/m10867/latest/>

The idea

"The ratio of frequency #1 to frequency # 2 is Y"

can be written:

$$\frac{\text{frequency \#1}}{\text{frequency \#2}} = Y$$

Y can be another ratio written as a fraction, or it can be a decimal, a root, or a whole number. You can use this equation, or an algebraic rearrangement of it, to find a frequency or a ratio that you do not know. Since musical intervals depend on frequency ratios, you can also use it to find an interval if you know the frequencies involved.

Figure 2.1: Remember that ratios, fractions and decimals are all just different ways of writing the same idea. If you write the ratio as a fraction it becomes easy to use in simple algebra equations.

Example 2.3

Say you would like to compare the frequencies of two sounds. Sound #1 is 630 and sound #2 is 840. If you use the expression given above and do the division on a calculator, the answer will be a decimal. If you simply reduce the fraction to lowest terms, or if you know the fraction that these decimals represent, you can see that you have a simple ratio of 3:4. Notice that if you switch the frequencies in the expression, the ratio also switches from 3:4 to 4:3. So it doesn't really matter which frequency you put on top; you will get the right answer as long as you keep track of which frequency is which.

$$\frac{\text{frequency \#1}}{\text{frequency \#2}} = Y \qquad \frac{\text{frequency \#2}}{\text{frequency \#1}} = Z$$

$$\frac{630}{840} = .75 = \frac{3}{4} \qquad \frac{840}{630} = 1.\overline{333} = \frac{4}{3}$$

Figure 2.2

Sound waves in the real world of musical instruments often do have simple ratios like these. (See Standing Waves and Musical Instruments (Section 3.1) for more about this.) In fact, a vibrating string or a tube of vibrating air will generate a whole series of waves, called a harmonic series (Section 2.2), that have fairly simple ratios. Musicians describe sounds in terms of pitch[6] rather than frequency and call the distance

[6]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>

between two pitches (how far apart their frequencies are) the interval[7] between the pitches. The simple-ratio intervals between the harmonic-series notes are called pure intervals (Section 2.4.2.1: Pythagorean Intonation). (The specific names of the intervals, such as "perfect fifth" are based on music notation and traditions rather than physics. If you need to understand interval names, please see Interval[8].)

Example 2.4

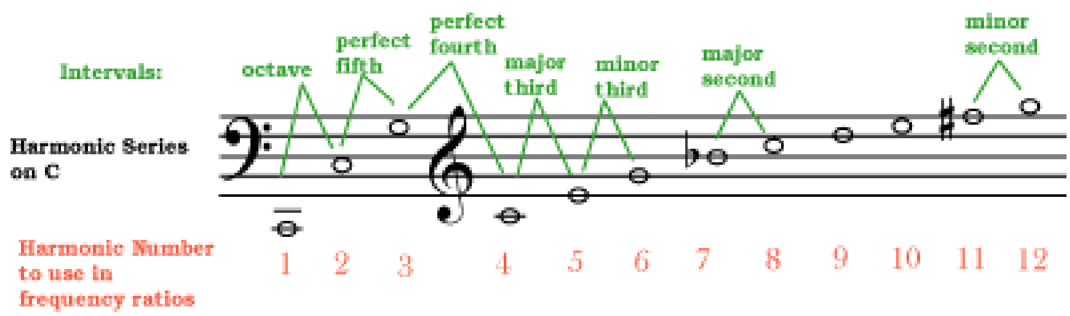

Figure 2.3: You can use a harmonic series (Section 2.2) to find frequency ratios for pure intervals (Section 2.4.2.1: Pythagorean Intonation). For example, harmonics 2 and 3 are a perfect fifth[9] apart, so the frequency ratio of a perfect fifth is 2:3. Harmonics 4 and 5 are a major third apart, so the frequency ratio for major thirds is 4:5. Harmonics 4 and 1 are two octaves apart, so the frequency ratio of notes two octaves apart is 4:1.

Perhaps you would like to find the frequency of a note that is a perfect fifth higher or lower than another note. A quick look at the harmonic series here shows you that the ratio of frequencies of a perfect fifth is 3:2.

NOTE: It does not matter what the actual notes are! If the ratio of the frequencies is 3:2, the interval between the notes will be a perfect fifth.

The higher number in the ratio will be the higher-sounding note. So if you want the frequency of the note that is a perfect fifth higher than A 440, you use the ratio 3:2 (that is, the fraction 3/2). If you want the note that is a perfect fifth lower than A 440, you use the ratio 2:3 (the fraction 2/3).

[7]"Interval" <http://cnx.org/content/m10867/latest/>
[8]"Interval" <http://cnx.org/content/m10867/latest/>
[9]"Interval": Section Perfect Intervals <http://cnx.org/content/m10867/latest/#s21>

Perfect Fifth Higher

$$\frac{\text{frequency \#1}}{\text{frequency \#2}} = \frac{2}{3}$$

$$\frac{440}{\text{frequency \#2}} = \frac{2}{3}$$

$$440 \times \frac{3}{2} = \text{frequency \#2}$$

$$660 = \text{frequency \#2}$$

Perfect Fifth Lower

$$\frac{\text{frequency \#1}}{\text{frequency \#2}} = \frac{3}{2}$$

$$\frac{440}{\text{frequency \#2}} = \frac{3}{2}$$

$$440 \times \frac{2}{3} = \text{frequency \#2}$$

$$293.\overline{33} = \text{frequency \#2}$$

$$\text{frequency \#2} = \text{frequency \#1} \times \text{ratio of \#2 over \#1}$$

Figure 2.4: Remember that it is important to put the ratio numbers in the right place; if #2 is the higher frequency, then #2 must be the higher number in the ratio, too. If you want #2 to be the lower frequency, then #2 should be the lower ratio number, too. Always check your answer to make sure it makes sense; a higher note should have a higher frequency.

In this example, I have done the algebra for you to show that you are really using the same equation as in example 1, just rearranged a bit. If you are uncomfortable using algebra, use the red expression if you know the interval but don't know one of the frequencies.

Pure intervals (Section 2.4.2.1: Pythagorean Intonation) that are found in the physical world (such as on strings or in brass tubes) are nice simple ratios like 2:3. But musicians in Western musical genres typically do not use pure intervals; instead they use a tuning system called equal temperament (Section 2.4.3: Temperament). (If you would like to know more about how and why this choice was made, please read Tuning Systems (Section 2.4).) In equal temperament, the ratios for notes in equal temperament (Section 2.4.3: Temperament) are based on the twelfth root of two. (For more discussion and practice with roots and equal temperament, please see Powers, Roots, and Equal Temperament (Section 2.3).) This evens out the intervals between the notes so that scales are more uniform, but it makes the math less simple.

Example 2.5

Frequency Ratios in Equal Temperament

Interval	Frequency ratio as a power of the twelfth root of 2		Decimal Equivalent (to the nearest ten thousandth)
Unison	$(\sqrt[12]{2})^0$	$\simeq$	1.0000
Minor Second	$(\sqrt[12]{2})^1$	$\simeq$	1.0595
Major Second	$(\sqrt[12]{2})^2$	$\simeq$	1.1225
Minor Third	$(\sqrt[12]{2})^3$	$\simeq$	1.1892
Major Third	$(\sqrt[12]{2})^4$	$\simeq$	1.2599
Perfect Fourth	$(\sqrt[12]{2})^5$	$\simeq$	1.3348
Tritone	$(\sqrt[12]{2})^6$	$\simeq$	1.4142
Perfect Fifth	$(\sqrt[12]{2})^7$	$\simeq$	1.4983
Minor Sixth	$(\sqrt[12]{2})^8$	$\simeq$	1.5874
Major Sixth	$(\sqrt[12]{2})^9$	$\simeq$	1.6818
Minor Seventh	$(\sqrt[12]{2})^{10}$	$\simeq$	1.7818
Major Seventh	$(\sqrt[12]{2})^{11}$	$\simeq$	1.8897
Octave	$(\sqrt[12]{2})^{12}$	$\simeq$	2.0000

Figure 2.5

Say you would like to compare a pure major third from the harmonic series to a equal temperament major third.

Pure Major Third

$$\frac{\text{frequency \#1}}{\text{frequency \#2}} = Y$$

$$\frac{5}{4} = 1.2500$$

Equal Temperament Major Third

$$(\sqrt[12]{2})^4 = 1.2599$$

Figure 2.6

By comparing the ratios as decimal numbers, you can see that a pure major third is quite a bit smaller than an equal temperament major third.

Exercise 2.1.3 *(Solution on p. 55.)*

A note has frequency 220. Using the pure intervals of the harmonic series, what is the frequency of the note that is a perfect fourth higher? What is the frequency of the note that is a major third lower?

Exercise 2.1.4 ***(Solution on p. 56.)***

The frequency of one note is 1333. The frequency of another note is 1121. What equal temperament interval will these two notes sound like? (Hint: compare the frequencies, and then compare your answer to the frequencies in the equal temperament figure above. (Figure 2.5))

2.2 Harmonic Series[10]

2.2.1 Introduction

Have you ever wondered how a trumpet[11] plays so many different notes with only three valves[12], or how a bugle plays different notes with no valves at all? Have you ever wondered why an oboe[13] and a flute[14] sound so different, even when they're playing the same note? What is a string player doing when she plays "harmonics"? Why do some notes sound good together while other notes seem to clash with each other? The answers to all of these questions will become clear with an understanding of the harmonic series.

2.2.2 Physics, Harmonics and Color

Most musical notes are sounds that have a particular pitch[15]. The pitch depends on the main frequency (Figure 1.6: Wavelength, Frequency, and Pitch) of the sound; the higher the frequency, and shorter the wavelength, of the sound waves, the higher the pitch is. But musical sounds don't have just one frequency. Sounds that have only one frequency are not very interesting or pretty. They have no more musical color[16] than the beeping of a watch alarm. On the other hand, sounds that have too many frequencies, like the sound of glass breaking or of ocean waves crashing on a beach, may be interesting and even pleasant. But they don't have a particular pitch, so they usually aren't considered musical notes.

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[10] This content is available online at <http://cnx.org/content/m11118/2.19/>.
[11] "Trumpets and Cornets" <http://cnx.org/content/m12606/latest/>
[12] "Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/#p2f>
[13] "The Oboe and its Relatives" <http://cnx.org/content/m12615/latest/>
[14] "Flutes" <http://cnx.org/content/m12603/latest/>
[15] "Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[16] "Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>

Frequency and Pitch

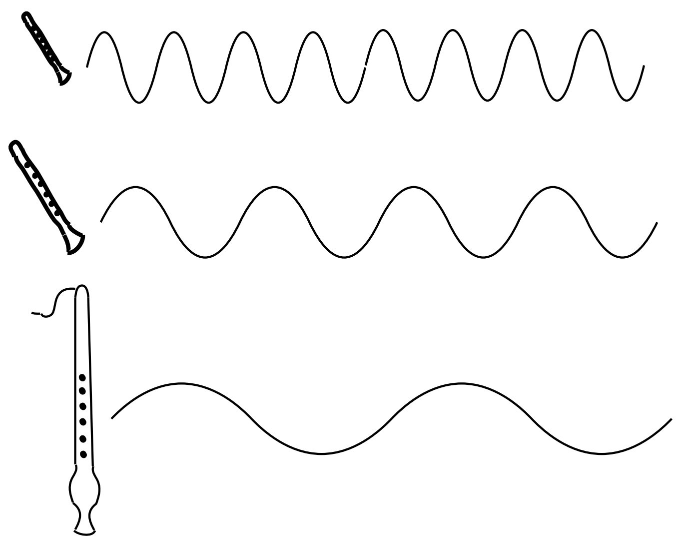

Figure 2.7: The higher the frequency, the higher the note sounds.

When someone plays or sings a note, only a very particular set of frequencies is heard. Imagine that each note that comes out of the instrument is a smooth mixture of many different pitches. These different pitches are called **harmonics**, and they are blended together so well that you do not hear them as separate notes at all. Instead, the harmonics give the note its color.

What is the color[17] of a sound? Say an oboe plays a middle C. Then a flute plays the same note at the same loudness as the oboe. It is still easy to tell the two notes apart, because an oboe sounds different from a flute. This difference in the sounds is the **color**, or **timbre** (pronounced "TAM-ber") of the notes. Like a color you see, the color of a sound can be bright and bold or deep and rich. It can be heavy, light, murky, thin, smooth, or transparently clear. Some other words that musicians use to describe the timbre of a sound are: reedy, brassy, piercing, mellow, thin, hollow, focussed, breathy (pronounced BRETH-ee) or full. Listen to recordings of a violin[18] and a viola[19]. Although these instruments are quite similar, the viola has a noticeably "deeper" and the violin a noticeably "brighter" sound that is not simply a matter of the violin playing higher notes. Now listen to the same phrase played by an electric guitar[20], an acoustic guitar with twelve steel strings[21] and an acoustic guitar with six nylon strings[22]. The words musicians use to describe timbre are somewhat subjective, but most musicians would agree with the statement that, compared with each other, the first sound is mellow, the second bright, and the third rich.

Exercise 2.2.1 ***(Solution on p. 56.)***

Listen to recordings of different instruments playing alone or playing very prominently above a group. Some suggestions: an unaccompanied violin or cello sonata, a flute, oboe, trumpet, or horn concerto, native American flute music, classical guitar, bagpipes, steel pan drums, panpipes,

[17] "Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>
[18] See the file at <http://cnx.org/content/m11118/latest/timvl.mp3>
[19] See the file at <http://cnx.org/content/m11118/latest/timvla.mp3>
[20] See the file at <http://cnx.org/content/m11118/latest/electricGUITARS.wav>
[21] See the file at <http://cnx.org/content/m11118/latest/12stringGUITARS.wav>
[22] See the file at <http://cnx.org/content/m11118/latest/nylonGUITARS.wav>

or organ. For each instrument, what "color" words would you use to describe the timbre of each instrument? Use as many words as you can that seem appropriate, and try to think of some that aren't listed above. Do any of the instruments actually make you think of specific shades of color, like fire-engine red or sky blue?

Where do the harmonics, and the timbre, come from? When a string vibrates, the main pitch you hear is from the vibration of the whole string back and forth. That is the **fundamental**, or first harmonic. But the string also vibrates in halves, in thirds, fourths, and so on. Each of these fractions also produces a harmonic. The string vibrating in halves produces the second harmonic; vibrating in thirds produces the third harmonic, and so on.

NOTE: This method of naming and numbering harmonics is the most straightforward and least confusing, but there are other ways of naming and numbering harmonics, and this can cause confusion. Some musicians do not consider the fundamental to be a harmonic; it is just the fundamental. In that case, the string halves will give the first harmonic, the string thirds will give the second harmonic and so on. When the fundamental is included in calculations, it is called the first **partial**, and the rest of the harmonics are the second, third, fourth partials and so on. Also, some musicians use the term **overtones** as a synonym for harmonics. For others, however, an overtone is any frequency (not necessarily a harmonic) that can be heard resonating with the fundamental. The sound of a gong or cymbals will include overtones that aren't harmonics; that's why the gong's sound doesn't seem to have as definite a pitch as the vibrating string does. If you are uncertain what someone means by the second harmonic or by the term overtones, ask for clarification.

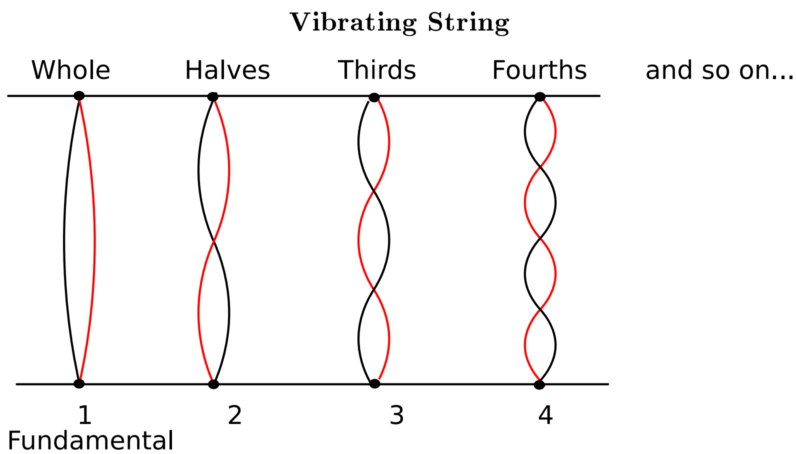

Figure 2.8: The fundamental pitch is produced by the whole string vibrating back and forth. But the string is also vibrating in halves, thirds, quarters, fifths, and so on, producing **harmonics**. All of these vibrations happen at the same time, producing a rich, complex, interesting sound.

A column of air vibrating inside a tube is different from a vibrating string, but the column of air can also vibrate in halves, thirds, fourths, and so on, of the fundamental, so the harmonic series will be the same. So why do different instruments have different timbres? The difference is the relative loudness of all the different harmonics compared to each other. When a clarinet[23] plays a note, perhaps the odd-numbered

[23]"Clarinets" <http://cnx.org/content/m12604/latest/>

harmonics are strongest; when a French horn[24] plays the same notes, perhaps the fifth and tenth harmonics are the strongest. This is what you hear that allows you to recognize that it is a clarinet or horn that is playing.

> NOTE: You will find some more extensive information on instruments and harmonics in Standing Waves and Musical Instruments (Section 3.1) and Standing Waves and Wind Instruments (Section 3.2).

2.2.3 The Harmonic Series

A harmonic series can have any note as its fundamental, so there are many different harmonic series. But the relationship between the frequencies (Figure 1.6: Wavelength, Frequency, and Pitch) of a harmonic series is always the same. The second harmonic always has exactly half the wavelength (and twice the frequency) of the fundamental; the third harmonic always has exactly a third of the wavelength (and so three times the frequency) of the fundamental, and so on. For more discussion of wavelengths and frequencies, see Frequency, Wavelength, and Pitch (Section 1.3).

Harmonic Series Wavelengths and Frequencies

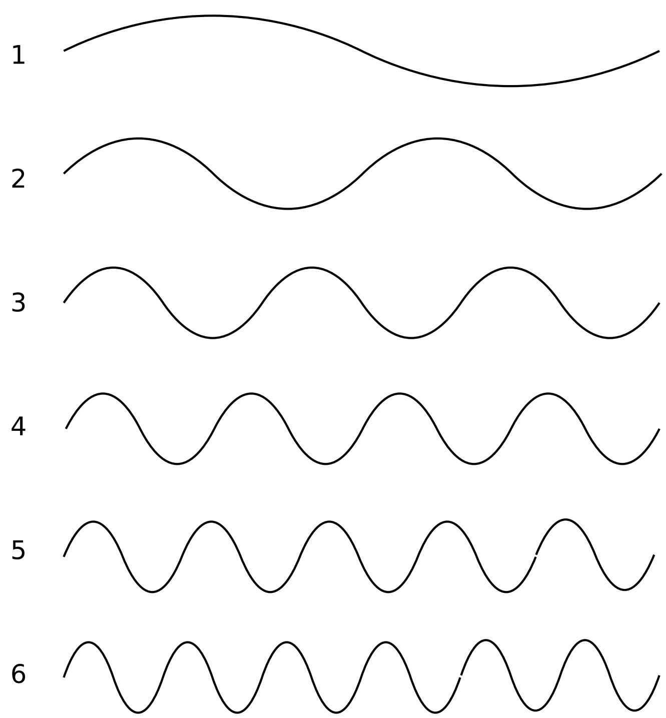

Figure 2.9: The second harmonic has half the wavelength and twice the frequency of the first. The third harmonic has a third the wavelength and three times the frequency of the first. The fourth harmonic has a quarter the wavelength and four times the frequency of the first, and so on. Notice that the fourth harmonic is also twice the frequency of the second harmonic, and the sixth harmonic is also twice the frequency of the third harmonic.

[24] "The French Horn" <http://cnx.org/content/m11617/latest/>

Say someone plays a note, a middle C. Now someone else plays the note that is twice the frequency of the middle C. Since this second note was already a harmonic of the first note, the sound waves of the two notes reinforce each other and sound good together. If the second person played instead the note that was just a litle bit more than twice the frequency of the first note, the harmonic series of the two notes would not fit together at all, and the two notes would not sound as good together. There are many combinations of notes that share some harmonics and make a pleasant sound together. They are considered consonant[25]. Other combinations share fewer or no harmonics and are considered dissonant[26] or, when they really clash, simply "out of tune" with each other. The scales and chords of most of the world's musics are based on these physical facts.

> NOTE: In real music, consonance and dissonance also depend on the standard practices of a musical tradition, especially its harmony practices, but these are also often related to the harmonic series.

For example, a note that is twice the frequency of another note is one octave[27] higher than the first note. So in the figure above, the second harmonic is one octave higher than the first; the fourth harmonic is one octave higher than the second; and the sixth harmonic is one octave higher than the third.

Exercise 2.2.2 ***(Solution on p. 56.)***

1. Which harmonic will be one octave higher than the fourth harmonic?
2. Predict the next four sets of octaves in a harmonic series.
3. What is the pattern that predicts which notes of a harmonic series will be one octave apart?
4. Notes one octave apart are given the same name. So if the first harmonic is a "A", the second and fourth will also be A's. Name three other harmonics that will also be A's.

A mathematical way to say this is "if two notes are an octave apart, the ratio (Section 2.1) of their frequencies is two to one (2:1)". Although the notes themselves can be any frequency, the 2:1 ratio is the same for all octaves. And all the other intervals[28] that musicians talk about can also be described as being particular ratios of frequencies.

A Harmonic Series Written as Notes

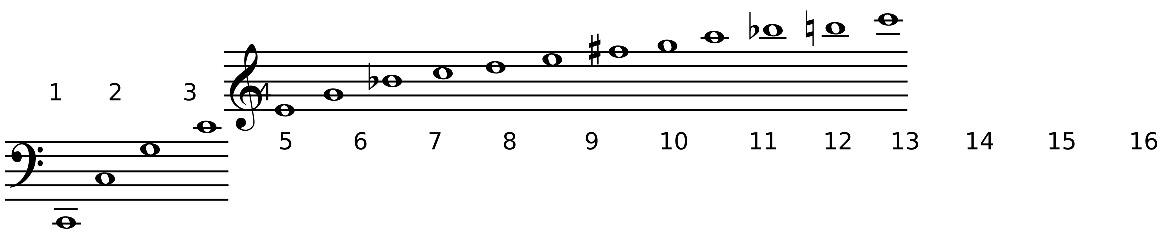

Figure 2.10

Take the third harmonic, for example. Its frequency is three times the first harmonic (ratio 3:1). Remember, the frequency of the second harmonic is two times that of the first harmonic. So the ratio (Section 2.1) of the frequencies of the second to the third harmonics is 2:3. From the harmonic series shown above, you can see that the interval[29] between these two notes is a perfect fifth[30]. The ratio of the frequencies of all

[25] "Consonance and Dissonance" <http://cnx.org/content/m11953/latest/>
[26] "Consonance and Dissonance" <http://cnx.org/content/m11953/latest/>
[27] "Octaves and the Major-Minor Tonal System" <http://cnx.org/content/m10862/latest/>
[28] "Interval" <http://cnx.org/content/m10867/latest/>
[29] "Interval" <http://cnx.org/content/m10867/latest/>
[30] "Interval": Section Perfect Intervals <http://cnx.org/content/m10867/latest/#s21>

perfect fifths is 2:3.

Exercise 2.2.3 *(Solution on p. 56.)*

1. The interval between the fourth and sixth harmonics (frequency ratio 4:6) is also a fifth. Can you explain this?
2. What other harmonics have an interval of a fifth?
3. Which harmonics have an interval of a fourth?
4. What is the frequency ratio for the interval of a fourth?

NOTE: If you have been looking at the harmonic series above closely, you may have noticed that some notes that are written to give the same interval have different frequency ratios. For example, the interval between the seventh and eighth harmonics is a major second, but so are the intervals between 8 and 9, between 9 and 10, and between 10 and 11. But 7:8, 8:9, 9:10, and 10:11, although they are pretty close, are not exactly the same. In fact, modern Western[31] music uses the equal temperament (Section 2.4.3.2: Equal Temperament) tuning system, which divides the octave into twelve notes that are spaced equally far apart. The positive aspect of equal temperament (and the reason it is used) is that an instrument will be equally in tune in all keys. The negative aspect is that it means that all intervals except for octaves are slightly out of tune with regard to the actual harmonic series. For more about equal temperament, see Tuning Systems (Section 2.4.3: Temperament). Interestingly, musicians have a tendency to revert to true harmonics when they can (in other words, when it is easy to fine-tune each note). For example, an a capella choral group or a brass ensemble, may find themselves singing or playing perfect fourths and fifths, "contracted" major thirds and "expanded" minor thirds.

2.2.4 Brass Instruments

The harmonic series is particularly important for brass instruments. A pianist or xylophone player only gets one note from each key. A string player who wants a different note from a string holds the string tightly in a different place. This basically makes a vibrating string of a new length, with a new fundamental.

But a brass player, without changing the length of the instrument, gets different notes by actually playing the harmonics of the instrument. Woodwinds also do this, although not as much. Most woodwinds can get two different octaves with essentially the same fingering; the lower octave is the fundamental of the column of air inside the instrument at that fingering. The upper octave is the first harmonic.

But it is the brass instruments that excel in getting different notes from the same length of tubing. The sound of a brass instruments starts with vibrations of the player's lips. By vibrating the lips at different speeds, the player can cause a harmonic of the air column to sound instead of the fundamental.

So a bugle player can play any note in the harmonic series of the instrument that falls within the player's range. Compare these well-known bugle calls to the harmonic series above (Figure 2.10: A Harmonic Series Written as Notes).

[31]"What Kind of Music is That?" <http://cnx.org/content/m11421/latest/>

Bugle Calls

Figure 2.11: Although limited by the fact that it can only play one harmonic series, the bugle can still play many well-known tunes.

For centuries, all brass instruments were valveless. A brass instrument could play only the notes of one harmonic series. The upper octaves of the series, where the notes are close together, could be difficult or impossible to play, and some of the harmonics sound quite out of tune to ears that expect equal temperament. The solution to these problems, once brass valves were perfected, was to add a few valves to the instrument. Three is usually enough. Each valve opens an extra length of tube, making the instrument a little longer, and making available a whole new harmonic series. Usually one valve gives the harmonic series one half step lower than the valveless intrument, another one whole step lower, and another one and a half steps lower. The valves can be used at the same time, too, making even more harmonic series. So a valved brass instrument can find, in the comfortable middle of its range (its **middle register**), a valve combination that will give a reasonably in-tune version for every note of the chromatic scale[32]. (For more on the history of valved brass, see History of the French Horn[33]. For more on how and why harmonics are produced in wind instruments, please see Standing Waves and Wind Instruments (Section 3.2))

> NOTE: Trombones use a slide instead of valves to make their instrument longer. But the basic principle is still the same. At each slide "position", the instrument gets a new harmonic series. The notes in between the positions aren't part of the chromatic scale, so they are usually only used for special effects like **glissandos** (sliding notes).

[32]"Half Steps and Whole Steps" <http://cnx.org/content/m10866/latest/#p0bb>
[33]"The French Horn": Section History <http://cnx.org/content/m11617/latest/#s2>

Overlapping Harmonic Series in Brass Instruments

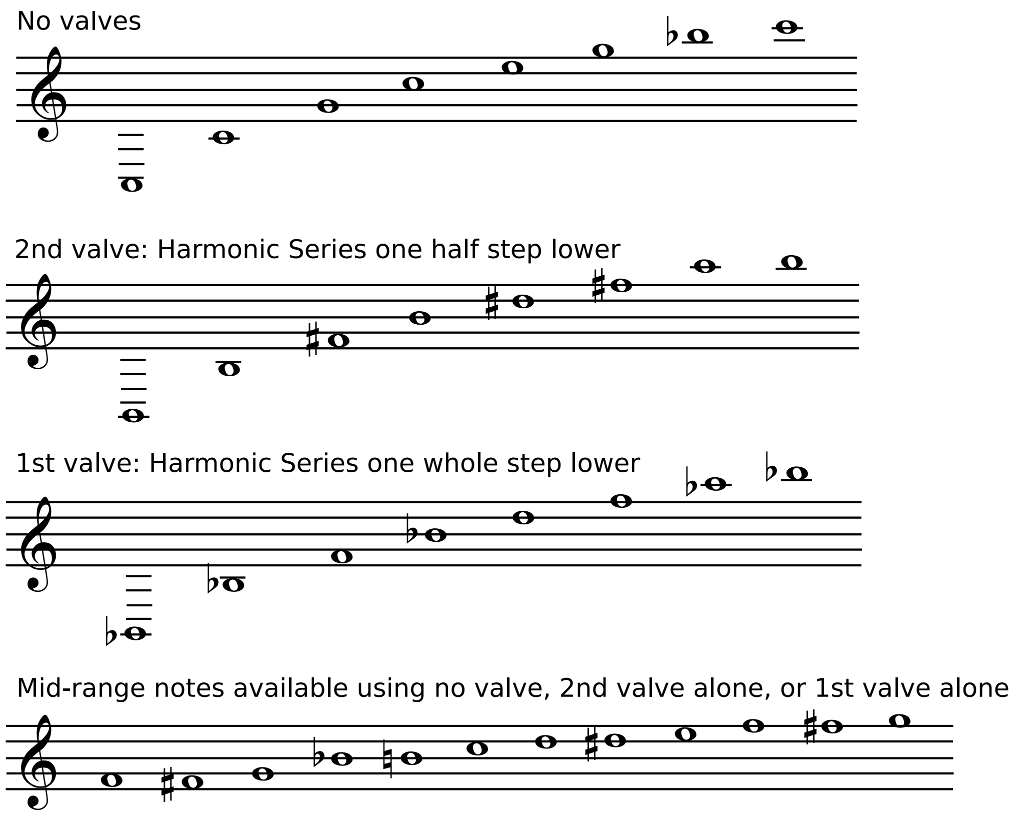

Figure 2.12: These harmonic series are for a brass instrument that has a "C" fundamental when no valves are being used - for example, a C trumpet. Remember, there is an entire harmonic series for every fundamental, and any note can be a fundamental. You just have to find the brass tube with the right length. So a trumpet or tuba can get one harmonic series using no valves, another one a half step lower using one valve, another one a whole step lower using another valve, and so on. By the time all the combinations of valves are used, there is some way to get an in-tune version of every note they need.

Exercise 2.2.4 *(Solution on p. 57.)*

Write the harmonic series for the instrument above when both the first and second valves are open. (You can use this PDF file[34] if you need staff paper.) What new notes are added in the instrument's middle range? Are any notes still missing?

NOTE: The French horn[35] has a reputation for being a "difficult" instrument to play. This is also because of the harmonic series. Most brass instruments play in the first few octaves of the harmonic series, where the notes are farther apart and it takes a pretty big difference in the mouth and lips (the embouchure[36], pronounced AHM-buh-sher) to get a different note. The range of the French

[34] See the file at <http://cnx.org/content/m11118/latest/staffpaper1.pdf>
[35] "The French Horn" <http://cnx.org/content/m11617/latest/>
[36] "Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/#p2a>

horn is higher in the harmonic series, where the notes are closer together. So very small differences in the mouth and lips can mean the wrong harmonic comes out.

2.2.5 Playing Harmonics on Strings

String players also use harmonics, although not as much as brass players. Harmonics on strings have a very different timbre[37] from ordinary string sounds. They give a quieter, thinner, more bell-like tone, and are usually used as a kind of ear-catching-special-effect.

Normally when a string player puts a finger on a string, he holds it down tight. This basically forms a (temporarily) shorter vibrating string, which then produces an entire harmonic series, with a shorter (higher) fundamental.

In order to play a harmonic, he touches the string very, very lightly instead. So the length of the string does not change. Instead, the light touch interferes with all of the vibrations that don't have a node at that spot. (A **node** is a place in the wave where the string does not move back-and-forth. For example, the ends of the string are both nodes, since they are held in place.)

String Harmonics

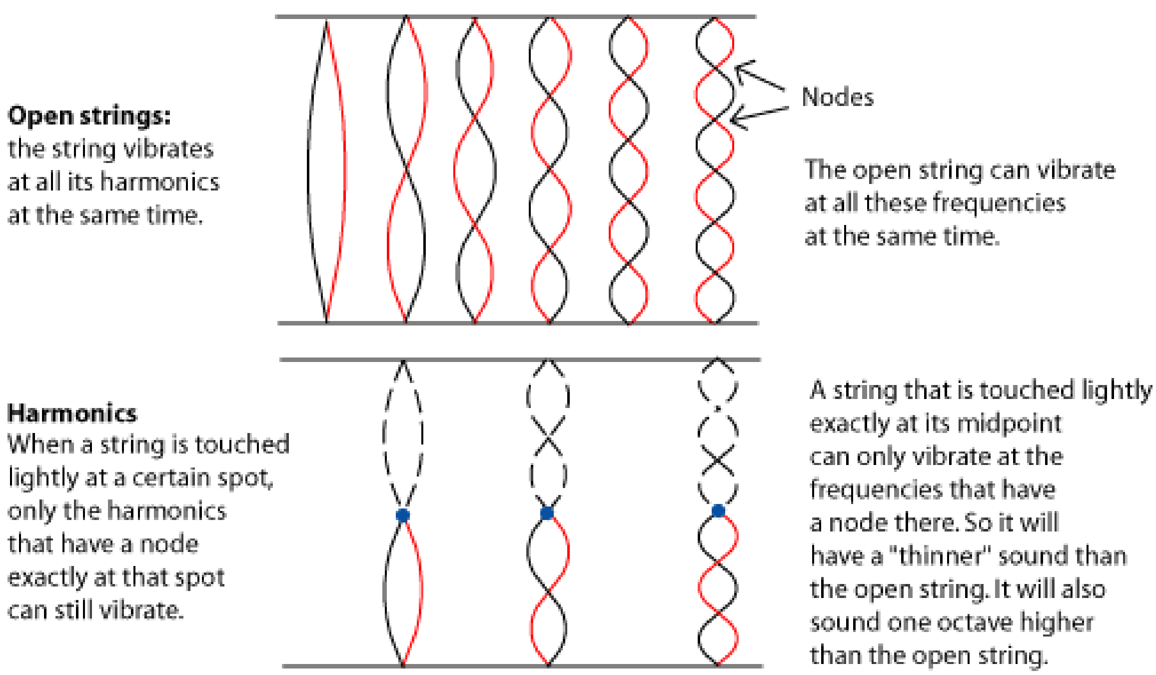

Figure 2.13

The thinner, quieter sound of "playing harmonics" is caused by the fact that much of the harmonic series is missing from the sound, which will of course be heard in the timbre (p. 34). Lightly touching the string in most spots will result in no sound at all. It only works at the precise spots that will leave some of the main harmonics (the longer, louder, lower-numbered ones) free to vibrate.

[37]"Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>

2.3 Powers, Roots, and Equal Temperament[38]

You do not need to use powers and roots to discuss music unless you want to talk about frequency relationships. They are particularly useful when discussing equal temperament. (See Tuning Systems (Section 2.4.4: A Comparison of Equal Temperament with the Harmonic Series).)

Powers are simply a shorthand way to write "a certain number times itself so many times".

Example 2.6

$$2^4 = 2 \times 2 \times 2 \times 2$$
$$4^2 = 4 \times 4$$
$$5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$$
$$10^3 = 10 \times 10 \times 10$$

Figure 2.14

Roots are the opposite of powers. They are a quick way to write the idea "the number that, multiplied by itself so many times, will give this number".

Example 2.7

$\sqrt[4]{16} = 2$, because $2 \times 2 \times 2 \times 2 = 16$

$\sqrt[3]{125} = 5$, because $5 \times 5 \times 5 = 125$

$\sqrt{16} = \sqrt[2]{16}$ ("the square root of 16") $= 4$, because $4 \times 4 = 16$

Often one number does not go into another number evenly, so
most roots end up being irrational numbers
(decimals that go on and on with no stopping point)
For example:

$\sqrt[2]{3} = 1.7320508...$

Figure 2.15

Roots and powers are relevant to music because equal temperament divides the octave into twelve equal half steps. A note one octave higher than another note has a frequency that is two times higher. So if you divide the octave into twelve equal parts (half steps), the size of each half step is "the twelfth root of two".

[38] This content is available online at <http://cnx.org/content/m11809/1.5/>.

(Notice that it is **not** "2 divided by twelve" or "one twelfth". For more on this, see Equal Temperament (Section 2.4.4: A Comparison of Equal Temperament with the Harmonic Series).)

Example 2.8

Of course, you can mix roots and powers in the same math problem. For example,

$(\sqrt[4]{16})^3$ = the fourth root of 16, to the third power

= 2, to the third power (because $2 \times 2 \times 2 \times 2 = 16$)

= 8 (because $2 \times 2 \times 2 = 8$)

Following the same steps,

$(\sqrt{9})^4 = 81$

Do you see why?

Figure 2.16

Exercise 2.3.1 *(Solution on p. 57.)*

Using a scientific calculator, find

1. The frequency ratio of a half step (the twelfth root of 2), to the nearest ten thousandth (four decimal places).
2. The frequency ratio of a perfect fourth (five half steps, or the twelfth root of 2 raised to the fifth power), to the nearest ten thousandth.
3. The frequency ratio of a major third (four half steps), to the nearest ten thousandth.
4. The frequency ratio of an octave.

2.4 Tuning Systems[39]

2.4.1 Introduction

The first thing musicians must do before they can play together is "tune". For musicians in the standard Western music[40] tradition, this means agreeing on exactly what pitch[41] (what frequency[42]) is an "A", what is a "B flat" and so on. Other cultures not only have different note names and different scales, they may even

[39]This content is available online at <http://cnx.org/content/m11639/1.27/>.
[40]"What Kind of Music is That?" <http://cnx.org/content/m11421/latest/>
[41]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[42]"Acoustics for Music Theory": Section Wavelength, Frequency, and Pitch <http://cnx.org/content/m13246/latest/#s2>

have different notes - different pitches - based on a different tuning system. In fact, the modern Western tuning system, which is called **equal temperament**, replaced (relatively recently) other tuning systems that were once popular in Europe. All tuning systems are based on the physics of sound[43]. But they all are also affected by the history of their music traditions, as well as by the tuning peculiarities of the instruments used in those traditions. Pythagorean (Section 2.4.2.1: Pythagorean Intonation), mean-tone (Section 2.4.2.2: Mean-tone System), just intonation (Section 2.4.2.3: Just Intonation), well temperaments (Section 2.4.3.1: Well Temperaments), equal temperament (Section 2.4.3.2: Equal Temperament), and wide tuning (Section 2.4.5: Beats and Wide Tuning).

To understand all of the discussion below, you must be comfortable with both the musical concept of interval and the physics concept of frequency. If you wish to follow the whole thing but are a little hazy on the relationship between pitch and frequency, the following may be helpful: Pitch[44]; Acoustics for Music Theory[45]; Harmonic Series I: Timbre and Octaves[46]; and Octaves and the Major-Minor Tonal System[47]. If you do not know what intervals are (for example, major thirds and perfect fourths), please see Interval[48] and Harmonic Series II: Harmonics, Intervals and Instruments[49]. If you need to review the mathematical concepts, please see Musical Intervals, Frequency, and Ratio (Section 2.1) and Powers, Roots, and Equal Temperament. Meanwhile, here is a reasonably nontechnical summary of the information below: Modern Western music uses the equal temperament (Section 2.4.3.2: Equal Temperament) tuning system. In this system, an octave[50] (say, from C to C) is divided into twelve equally-spaced notes. "Equally-spaced" to a musician basically means that each of these notes is one half step[51] from the next, and that all half steps sound like the same size pitch change. (To a scientist or engineer, "equally-spaced" means that the ratio of the frequencies of the two notes in any half step is always the same.) This tuning system is very convenient for some instruments, such as the piano, and also makes it very easy to change key[52] without retuning instruments. But a careful hearing of the music, or a look at the physics of the sound waves involved, reveals that equal-temperament pitches are not based on the harmonics[53] physically produced by any musical sound. The "equal" ratios of its half steps are the twelfth root of two, rather than reflecting the simpler ratios produced by the sounds themselves, and the important intervals that build harmonies can sound slightly out of tune. This often leads to some "tweaking" of the tuning in real performances, away from equal temperament. It also leads many other music traditions to prefer tunings other than equal temperament, particularly tunings in which some of the important intervals are based on the pure, simple-ratio intervals of physics. In order to feature these favored intervals, a tuning tradition may do one or more of the following: use scales in which the notes are not equally spaced; avoid any notes or intervals which don't work with a particular tuning; change the tuning of some notes when the key[54] or mode[55] changes.

2.4.2 Tuning based on the Harmonic Series

Almost all music traditions recognize the octave[56]. When note Y has a frequency[57] that is twice the frequency of note Z, then note Y is one octave higher than note Z. A simple mathematical way to say this is that the ratio (Section 2.1) of the frequencies is 2:1. Two notes that are exactly one octave apart sound good together because their frequencies are related in such a simple way. If a note had a frequency, for example, that was

[43]"Acoustics for Music Theory" <http://cnx.org/content/m13246/latest/>
[44]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[45]"Acoustics for Music Theory" <http://cnx.org/content/m13246/latest/>
[46]"Harmonic Series I: Timbre and Octaves" <http://cnx.org/content/m13682/latest/>
[47]"Octaves and the Major-Minor Tonal System" <http://cnx.org/content/m10862/latest/>
[48]"Interval" <http://cnx.org/content/m10867/latest/>
[49]"Harmonic Series II: Harmonics, Intervals, and Instruments" <http://cnx.org/content/m13686/latest/>
[50]"Octaves and the Major-Minor Tonal System" <http://cnx.org/content/m10862/latest/>
[51]"Half Steps and Whole Steps" <http://cnx.org/content/m10866/latest/>
[52]"Major Keys and Scales" <http://cnx.org/content/m10851/latest/>
[53]"Harmonic Series I: Timbre and Octaves" <http://cnx.org/content/m13682/latest/>
[54]"Major Keys and Scales" <http://cnx.org/content/m10851/latest/>
[55]"Modes and Ragas: More Than just a Scale" <http://cnx.org/content/m11633/latest/>
[56]"Octaves and the Major-Minor Tonal System" <http://cnx.org/content/m10862/latest/>
[57]"Acoustics for Music Theory": Section Wavelength, Frequency, and Pitch <http://cnx.org/content/m13246/latest/#s2>

2.11 times the frequency of another note (instead of exactly 2 times), the two notes would not sound so good together. In fact, most people would find the effect very unpleasant and would say that the notes are not "in tune" with each other.

To find other notes that sound "in tune" with each other, we look for other sets of pitches that have a "simple" frequency relationship. These sets of pitches with closely related frequencies are often written in common notation[58] as a harmonic series[59]. The harmonic series is not just a useful idea constructed by music theory; it is often found in "real life", in the real-world physics of musical sounds. For example, a bugle can play only the notes of a specific harmonic series. And every musical note you hear is not a single pure frequency, but is actually a blend of the pitches of a particular harmonic series. The relative strengths of the harmonics are what gives the note its timbre[60]. (See Harmonic Series II: Harmonics, Intervals and Instruments[61]; Standing Waves and Musical Instruments (Section 3.1); and Standing Waves and Wind Instruments (Section 3.2) for more about how and why musical sounds are built from harmonic series.)

Harmonic Series on C

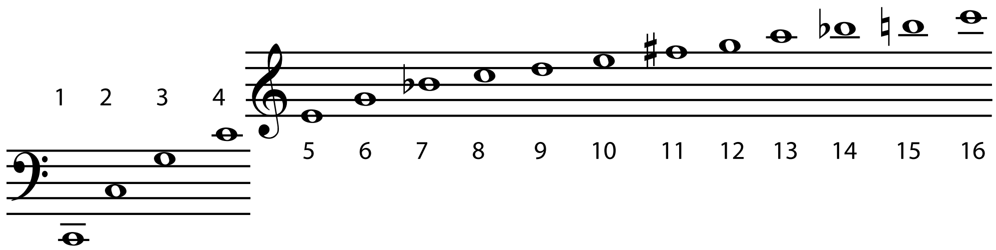

Figure 2.17: Here are the first sixteen pitches in a harmonic series that starts on a C natural. The series goes on indefinitely, with the pitches getting closer and closer together. A harmonic series can start on any note, so there are many harmonic series, but **every harmonic series has the same set of intervals and the same frequency ratios**.

What does it mean to say that two pitches have a "simple frequency relationship"? It doesn't mean that their frequencies are almost the same. Two notes whose frequencies are almost the same - say, the frequency of one is 1.005 times the other - sound bad together. Again, anyone who is accustomed to precise tuning would say they are "out of tune". Notes with a close relationship have frequencies that can be written as a ratio (Section 2.1) of two small whole numbers; the smaller the numbers, the more closely related the notes are. Two notes that are exactly the same pitch, for example, have a frequency ratio of 1:1, and octaves, as we have already seen, are 2:1. Notice that when two pitches are related in this simple-ratio way, it means that they can be considered part of the same harmonic series, and in fact the actual harmonic series of the two notes may also overlap and reinforce each other. The fact that the two notes are complementing and reinforcing each other in this way, rather than presenting the human ear with two completely different harmonic series, may be a major reason why they sound consonant[62] and "in tune".

NOTE: Nobody has yet proven a physical basis for why simple-ratio combinations sound pleasant to us. For a readable introduction to the subject, I suggest Robert Jourdain's *Music, the Brain, and Ecstasy*

[58] "The Staff" <http://cnx.org/content/m10880/latest/>
[59] "Harmonic Series I: Timbre and Octaves" <http://cnx.org/content/m13682/latest/>
[60] "Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>
[61] "Harmonic Series II: Harmonics, Intervals, and Instruments" <http://cnx.org/content/m13686/latest/>
[62] "Consonance and Dissonance" <http://cnx.org/content/m11953/latest/>

Notice that the actual frequencies of the notes do not matter. What matters is how they compare to each other - basically, how many waves of one note go by for each wave of the other note. Although the actual frequencies of the notes will change for every harmonic series, the comparative distance between the notes, their interval[63], will be the same.

For more examples, look at the harmonic series in Figure 2.17 (Harmonic Series on C). The number beneath a note tells you the relationship of that note's frequency to the frequency of the first note in the series - the **fundamental**. For example, the frequency of the note numbered 3 in Figure 2.17 (Harmonic Series on C) is three times the frequency of the fundamental, and the frequency of the note numbered fifteen is fifteen times the frequency of the fundamental. In the example, the fundamental is a C. That note's frequency times 2 gives you another C; times 2 again (4) gives another C; times 2 again gives another C (8), and so on. Now look at the G's in this series. The first one is number 3 in the series. 3 times 2 is 6, and number 6 in the series is also a G. So is number 12 (6 times 2). Check for yourself the other notes in the series that are an octave apart. You will find that the ratio for one octave[64] is always 2:1, just as the ratio for a unison is always 1:1. Notes with this small-number ratio of 2:1 are so closely related that we give them the same name, and most tuning systems are based on this octave relationship.

The next closest relationship is the one based on the 3:2 ratio, the interval[65] of the perfect fifth[66] (for example, the C and G in the example harmonic series). The next lowest ratio, 4:3, gives the interval of a perfect fourth[67]. Again, these pitches are so closely related and sound so good together that their intervals have been named "perfect". The perfect fifth figures prominently in many tuning systems. In Western[68] music, all major and minor chords contain, or at least strongly imply, a perfect fifth. (See Triads[69] and Naming Triads[70] for more about the intervals in major and minor chords.)

2.4.2.1 Pythagorean Intonation

The Pythagorean system is so named because it was actually discussed by Pythagoras, the famous Greek mathematician and philosopher, who in the sixth century B.C. already recognized the simple arithmetical relationship involved in intervals of octaves, fifths, and fourths. He and his followers believed that numbers were the ruling principle of the universe, and that musical harmonies were a basic expression of the mathematical laws of the universe. Their model of the universe involved the "celestial spheres" creating a kind of harmony as they moved in circles dictated by the same arithmetical relationships as musical harmonies.

In the Pythagorean system, all tuning is based on the interval of the pure fifth. **Pure intervals** are the ones found in the harmonic series, with very simple frequency ratios. So a pure fifth will have a frequency ratio of exactly 3:2. Using a series of perfect fifths (and assuming perfect octaves, too, so that you are filling in every octave as you go), you can eventually fill in an entire chromatic scale[71].

[63]"Interval" <http://cnx.org/content/m10867/latest/>
[64]"Octaves and the Major-Minor Tonal System" <http://cnx.org/content/m10862/latest/>
[65]"Interval" <http://cnx.org/content/m10867/latest/>
[66]"Interval" <http://cnx.org/content/m10867/latest/#p21b>
[67]"Interval" <http://cnx.org/content/m10867/latest/#p21b>
[68]"What Kind of Music is That?" <http://cnx.org/content/m11421/latest/>
[69]"Triads" <http://cnx.org/content/m10877/latest/>
[70]"Naming Triads" <http://cnx.org/content/m10890/latest/>
[71]"Half Steps and Whole Steps" <http://cnx.org/content/m10866/latest/#p0bb>

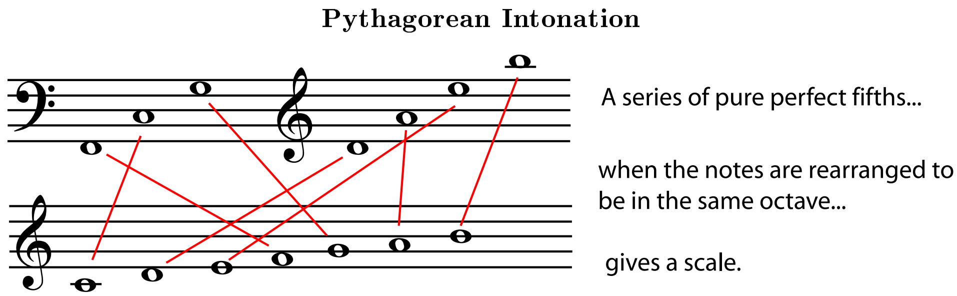

Figure 2.18: You can continue this series of perfect fifths to get the rest of the notes of a chromatic scale; the series would continue F sharp, C sharp, and so on.

The main weakness of the Pythagorean system is that a series of pure perfect fifths will never take you to a note that is a pure octave above the note you started on. To see why this is a problem, imagine beginning on a C. A series of perfect fifths would give: C, G, D, A, E, B, F sharp, C sharp, G sharp, D sharp, A sharp, E sharp, and B sharp. In equal temperament (which doesn't use pure fifths), that B sharp would be exactly the same pitch as the C seven octaves above where you started (so that the series can, in essence, be turned into a closed loop, the Circle of Fifths[72]). Unfortunately, the B sharp that you arrive at after a series of pure fifths is a little higher than that C.

So in order to keep pure octaves, instruments that use Pythagorean tuning have to use eleven pure fifths and one smaller fifth. The smaller fifth has traditionally been called a **wolf** fifth because of its unpleasant sound. Keys that avoid the wolf fifth sound just fine on instruments that are tuned this way, but keys in which the wolf fifth is often heard become a problem. To avoid some of the harshness of the wolf intervals, some harpsichords and other keyboard instruments were built with split keys for D sharp/E flat and for G sharp/A flat. The front half of the key would play one note, and the back half the other (differently tuned) note.

Pythagorean tuning was widely used in medieval and Renaissance times. Major seconds and thirds are larger in Pythagorean intonation than in equal temperament, and minor seconds and thirds are smaller. Some people feel that using such intervals in medieval music is not only more authentic, but sounds better too, since the music was composed for this tuning system.

More modern Western music, on the other hand, does not sound pleasant using Pythagorean intonation. Although the fifths sound great, the thirds[73] are simply too far away from the pure major and minor thirds of the harmonic series. In medieval music, the third was considered a dissonance and was used sparingly - and actually, when you're using Pythagorean tuning, it really is a dissonance - but most modern harmonies are built from thirds (see Triads[74]). In fact, the common harmonic tradition that includes everything from Baroque[75] counterpoint to modern rock is often called **triadic harmony**.

Some modern Non-Western music traditions, which have a very different approach to melody and harmony, still base their tuning on the perfect fifth. Wolf fifths and ugly thirds are not a problem in these traditions, which build each mode[76] within the framework of the perfect fifth, retuning for different modes as necessary. To read a little about one such tradition, please see Indian Classical Music: Tuning and

[72]"The Circle of Fifths" <http://cnx.org/content/m10865/latest/>
[73]"Interval": Major and Minor Intervals <http://cnx.org/content/m10867/latest/#list22a>
[74]"Triads" <http://cnx.org/content/m10877/latest/>
[75]"Music of the Baroque Period" <http://cnx.org/content/m14737/latest/>
[76]"Modes and Ragas: More Than just a Scale" <http://cnx.org/content/m11633/latest/>

Ragas[77].

2.4.2.2 Mean-tone System

The mean-tone system, in order to have pleasant-sounding thirds, takes rather the opposite approach from the Pythagorean. It uses the pure major third[78]. In this system, the whole tone (or whole step[79]) is considered to be exactly half of the pure major third. This is the **mean**, or average, of the two tones, that gives the system its name. A semitone (or half step[80]) is exactly half (another mean) of a whole tone.

These smaller intervals all work out well in mean-tone tuning, but the result is a fifth that is noticeably smaller than a pure fifth. And a series of pure thirds will also eventually not line up with pure octaves, so an instrument tuned this way will also have a problem with wolf (p. 47) intervals.

As mentioned above, Pythagorean tuning made sense in medieval times, when music was dominated by fifths. Once the concept of harmony in thirds took hold, thirds became the most important interval[81]; simple perfect fifths were now heard as "austere" and, well, medieval-sounding. So mean-tone tuning was very popular in Europe in the 16th through 18th centuries.

But fifths can't be avoided entirely. A basic major or minor chord, for example, is built of two thirds, but it also has a perfect fifth between its outer two notes (see Triads[82]). So even while mean-tone tuning was enjoying great popularity, some composers and musicians were searching for other solutions.

2.4.2.3 Just Intonation

In just intonation, the fifth and the third are both based on the pure, harmonic series interval. Because chords are constructed of thirds and fifths (see Triads[83]), this tuning makes typical Western harmonies particularly resonant and pleasing to the ear; so this tuning is often used (sometimes unconsciously) by musicians who can make small tuning adjustments quickly. This includes vocalists, most wind instruments, and many string instruments.

As explained above (p. 47), using pure fifths and thirds will require some sort of adjustment somewhere. Just intonation makes two accommodations to allow its pure intervals. One is to allow inequality in the other intervals. Look again at the harmonic series (Figure 2.17: Harmonic Series on C).

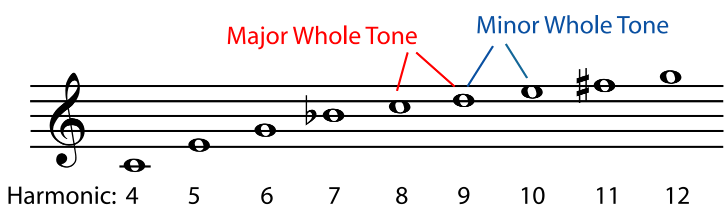

Figure 2.19: Both the 9:8 ratio and the 10:9 ratio in the harmonic series are written as whole notes. 9:8 is considered a **major whole tone** and 10:9 a **minor whole tone**. The difference between them is less than a quarter of a semitone.

[77]"Indian Classical Music: Tuning and Ragas" <http://cnx.org/content/m12459/latest/>
[78]"Interval": Major and Minor Intervals <http://cnx.org/content/m10867/latest/#list22a>
[79]"Half Steps and Whole Steps" <http://cnx.org/content/m10866/latest/>
[80]"Half Steps and Whole Steps" <http://cnx.org/content/m10866/latest/>
[81]"Interval" <http://cnx.org/content/m10867/latest/>
[82]"Triads" <http://cnx.org/content/m10877/latest/>
[83]"Triads" <http://cnx.org/content/m10877/latest/>

As the series goes on, the ratios get smaller and the notes closer together. Common notation[84] writes all of these "close together" intervals as whole steps (whole tones) or half steps (semitones), but they are of course all slightly different from each other. For example, the notes with frequency ratios of 9:8 and 10:9 and 11:10 are all written as whole steps. To compare how close (or far) they actually are, turn the ratios into decimals.

Whole Step Ratios Written as Decimals

- $9/8 = 1.125$
- $10/9 = 1.111$
- $11/10 = 1.1$

These are fairly small differences, but they can still be heard easily by the human ear. Just intonation uses both the 9:8 whole tone, which is called a **major whole tone** and the 10:9 whole tone, which is called a **minor whole tone**, in order to construct both pure thirds and pure fifths.

> NOTE: In case you are curious, the size of the whole tone of the "mean tone" system is also the mean, or average, of the major and minor whole tones.

The other accommodation with reality that just intonation must make is the fact that a single just-intonation tuning cannot be used to play in multiple keys. In constructing a just-intonation tuning, it matters which steps of the scale are major whole tones and which are minor whole tones, so an instrument tuned exactly to play with just intonation in the key of C major will have to retune to play in C sharp major or D major. For instruments that can tune almost instantly, like voices, violins, and trombones, this is not a problem; but it is unworkable for pianos, harps, and other other instruments that cannot make small tuning adjustments quickly.

As of this writing, there was useful information about various tuning systems at several different websites, including The Development of Musical Tuning Systems[85] , where one could hear what some intervals sound like in the different tuning systems, and Kyle Gann's Just Intonation Explained[86] , which included some audio samples of works played using just intonation.

2.4.3 Temperament

There are times when tuning is not much of an issue. When a good choir sings in harmony without instruments, they will tune without even thinking about it. All chords will tend towards pure fifths and thirds, as well as seconds, fourths, sixths, and sevenths that reflect the harmonic series. Instruments that can bend most pitches enough to fine-tune them during a performance - and this includes most orchestral instruments - also tend to play the "pure" intervals. This can happen unconsciously, or it can be deliberate, as when a conductor asks for an interval to be "expanded" or "contracted".

But for many instruments, such as piano, organ, harp, bells, harpsichord, xylophone - any instrument that cannot be fine-tuned quickly - tuning is a big issue. A harpsichord that has been tuned using the Pythagorean system or just intonation may sound perfectly in tune in one key - C major, for example - and fairly well in tune in a related key[87] - G major - but badly out of tune in a "distant" key like D flat major. Adding split keys or extra keys can help (this was a common solution for a time), but also makes the instrument more difficult to play. In Western music[88], the tuning systems that have been invented and widely used that directly address this problem are the various temperaments, in which the tuning of notes is "tempered" slightly from pure intervals. (Non-Western music traditions have their own tuning systems, which is too big a subject to address here. See Listening to Balinese Gamelan[89] and Indian Classical Music: Tuning and Ragas[90] for a taste of what's out there.)

[84]"The Staff" <http://cnx.org/content/m10880/latest/>
[85]http://www.midicode.com/tunings/index.shtml
[86]http://www.kylegann.com/tuning.html
[87]"The Circle of Fifths" <http://cnx.org/content/m10865/latest/>
[88]"What Kind of Music is That?" <http://cnx.org/content/m11421/latest/>
[89]"Listening to Balinese Gamelan: A Beginners' Guide" <http://cnx.org/content/m15795/latest/>
[90]"Indian Classical Music: Tuning and Ragas" <http://cnx.org/content/m12459/latest/>

2.4.3.1 Well Temperaments

As mentioned above (p. 47), the various tuning systems based on pure intervals eventually have to include "wolf" intervals that make some keys unpleasant or even unusable. The various **well temperament** tunings that were very popular in the 18th and 19th centuries tried to strike a balance between staying close to pure intervals and avoiding wolf intervals. A well temperament might have several pure fifths, for example, and several fifths that are smaller than a pure fifth, but not so small that they are "wolf" fifths. In such systems, tuning would be noticeably different in each key[91], but every key would still be pleasant-sounding and usable. This made well temperaments particularly welcome for players of difficult-to-tune instruments like the harpsichord and piano.

> NOTE: Historically, there has been some confusion as to whether or not well temperament and equal temperament are the same thing, possibly because well temperaments were sometimes referred to at the time as "equal temperament". But these well temperaments made all keys equally useful, not equal-sounding as modern equal temperament does.

As mentioned above (Section 2.4.2.2: Mean-tone System), mean-tone tuning was still very popular in the eighteenth century. J. S. Bach wrote his famous "Well-Tempered Klavier" in part as a plea and advertisement to switch to a well temperament system. Various well temperaments did become very popular in the eighteenth and nineteenth centuries, and much of the keyboard-instrument music of those centuries may have been written to take advantage of the tuning characteristics of particular keys in particular well temperaments. Some modern musicians advocate performing such pieces using well temperaments, in order to better understand and appreciate them. It is interesting to note that the different keys in a well temperament tuning were sometimes considered to be aligned with specific colors and emotions. In this way they may have had more in common with various modes and ragas[92] than do keys in equal temperament.

2.4.3.2 Equal Temperament

In modern times, well temperaments have been replaced by equal temperament, so much so in Western music[93] that equal temperament is considered standard tuning even for voice and for instruments that are more likely to play using just intonation when they can (see above (Section 2.4.2.3: Just Intonation)). In equal temperament, only octaves[94] are pure (Section 2.4.2.1: Pythagorean Intonation) intervals. The octave is divided into twelve equally spaced half steps[95], and all other intervals[96] are measured in half steps. This gives, for example, a fifth[97] that is a bit smaller than a pure fifth, and a major third[98] that is larger than the pure major third. The differences are smaller than the wolf tones (p. 47) found in other tuning systems, but they are still there.

Equal temperament is well suited to music that changes key[99] often, is very chromatic[100], or is harmonically complex[101]. It is also the obvious choice for atonal[102] music that steers away from identification with any key or tonality at all. Equal temperament has a clear scientific/mathematical basis, is very straightforward, does not require retuning for key changes, and is unquestioningly accepted by most people. However, because of the lack of pure intervals, some musicians do not find it satisfying. As mentioned above, just intonation is sometimes substituted for equal temperament when practical, and some musicians would also

91"Major Keys and Scales" <http://cnx.org/content/m10851/latest/>
92"Modes and Ragas: More Than just a Scale" <http://cnx.org/content/m11633/latest/>
93"What Kind of Music is That?" <http://cnx.org/content/m11421/latest/>
94"Octaves and the Major-Minor Tonal System" <http://cnx.org/content/m10862/latest/>
95"Half Steps and Whole Steps" <http://cnx.org/content/m10866/latest/>
96"Interval" <http://cnx.org/content/m10867/latest/>
97"Interval" <http://cnx.org/content/m10867/latest/#p21b>
98"Interval": Major and Minor Intervals <http://cnx.org/content/m10867/latest/#list22a>
99"Major Keys and Scales" <http://cnx.org/content/m10851/latest/>
100"What Kind of Music is That?" <http://cnx.org/content/m11421/latest/#p7f>
101"Beginning Harmonic Analysis" <http://cnx.org/content/m11643/latest/>
102"What Kind of Music is That?" <http://cnx.org/content/m11421/latest/#p7e>

like to reintroduce well temperaments, at least for performances of music which was composed with well temperament in mind.

2.4.4 A Comparison of Equal Temperament with the Harmonic Series

In a way, equal temperament is also a compromise between the Pythagorean approach and the mean-tone approach. Neither the third nor the fifth is pure, but neither of them is terribly far off, either. Because equal temperament divides the octave into twelve equal semi-tones (half steps), the frequency ratio of each semi-tone is the twelfth root of 2. If you do not understand why it is the twelfth root of 2 rather than, say, one twelfth, please see the explanation below (p. 52). (There is a review of powers and roots in Powers, Roots, and Equal Temperament if you need it.)

$$\sqrt[12]{2} = \text{a semitone (half step)}$$

$$\left(\sqrt[12]{2}\right)^2 = \text{a whole tone (whole step)}$$

$$\left(\sqrt[12]{2}\right)^4 = \text{a major third (four semitones)}$$

$$\left(\sqrt[12]{2}\right)^7 = \text{a perfect fifth (seven semitones)}$$

$$\left(\sqrt[12]{2}\right)^{12} = 2 = \text{an octave (twelve semitones)}$$

Figure 2.20: In equal temperament, the ratio of frequencies in a semitone (half step) is the twelfth root of two. Every interval is then simply a certain number of semitones. Only the octave (the twelfth power of the twelfth root) is a pure interval.

In equal temperament, the only pure interval is the octave. (The twelfth power of the twelfth root of two is simply two.) All other intervals are given by irrational numbers based on the twelfth root of two, not nice numbers that can be written as a ratio of two small whole numbers. In spite of this, equal temperament works fairly well, because most of the intervals it gives actually fall quite close to the pure intervals. To see that this is so, look at Figure 2.21 (Comparing the Frequency Ratios for Equal Temperament and Pure Harmonic Series). Equal temperament and pure intervals are calculated as decimals and compared to each other. (You can find these decimals for yourself using a calculator.)

Comparing the Frequency Ratios for Equal Temperament and Pure Harmonic Series

Interval	Equal Temperament Frequency Ratio			Approximate Difference	Harmonic Series Frequency Ratio		
Unison	$(\sqrt[12]{2})^{0}$	≃	1.0000	0.0	1.0000	≃	1/1
Minor Second	$(\sqrt[12]{2})^{1}$	≃	1.0595	0.0314	1.0909	≃	12/11
Major Second	$(\sqrt[12]{2})^{2}$	≃	1.1225	0.0025	1.1250	≃	9/8
Minor Third	$(\sqrt[12]{2})^{3}$	≃	1.1892	0.0108	1.2000	≃	6/5
Major Third	$(\sqrt[12]{2})^{4}$	≃	1.2599	0.0099	1.2500	≃	5/4
Perfect Fourth	$(\sqrt[12]{2})^{5}$	≃	1.3348	0.0015	1.3333	≃	4/3
Tritone	$(\sqrt[12]{2})^{6}$	≃	1.4142	0.0142	1.4000	≃	7/5
Perfect Fifth	$(\sqrt[12]{2})^{7}$	≃	1.4983	0.0017	1.5000	≃	3/2
Minor Sixth	$(\sqrt[12]{2})^{8}$	≃	1.5874	0.0126	1.6000	≃	8/5
Major Sixth	$(\sqrt[12]{2})^{9}$	≃	1.6818	0.0151	1.6667	≃	5/3
Minor Seventh	$(\sqrt[12]{2})^{10}$	≃	1.7818	0.0318	1.7500	≃	7/4
Major Seventh	$(\sqrt[12]{2})^{11}$	≃	1.8897	0.0564	1.8333	≃	11/6
Octave	$(\sqrt[12]{2})^{12}$	≃	2.0000	0.0	2.0000	≃	2/1

Figure 2.21: Look again at Figure 2.17 (Harmonic Series on C) to see where pure interval ratios come from. The ratios for equal temperament are all multiples of the twelfth root of two. Both sets of ratios are converted to decimals (to the nearest ten thousandth), so you can easily compare them.

Except for the unison and the octave, none of the ratios for equal temperament are exactly the same as for the pure interval. Many of them are reasonably close, though. In particular, perfect fourths and fifths and major thirds are not too far from the pure intervals. The intervals that are the furthest from the pure intervals are the major seventh, minor seventh, and minor second (intervals that are considered dissonant[103] anyway).

Because equal temperament is now so widely accepted as standard tuning, musicians do not usually even speak of intervals in terms of ratios. Instead, tuning itself is now defined in terms of equal-temperament, with tunings and intervals measured in cents. A **cent** is 1/100 (the hundredth root) of an equal-temperament semitone. In this system, for example, the major whole tone discussed above measures 204 cents, the minor whole tone 182 cents, and a pure fifth is 702 cents.

Why is a cent the hundredth root of a semitone, and why is a semitone the twelfth root of an octave? If it bothers you that the ratios in equal temperament are roots, remember the pure octaves and fifths of the harmonic series.

[103]"Consonance and Dissonance" <http://cnx.org/content/m11953/latest/>

Frequency Relationships

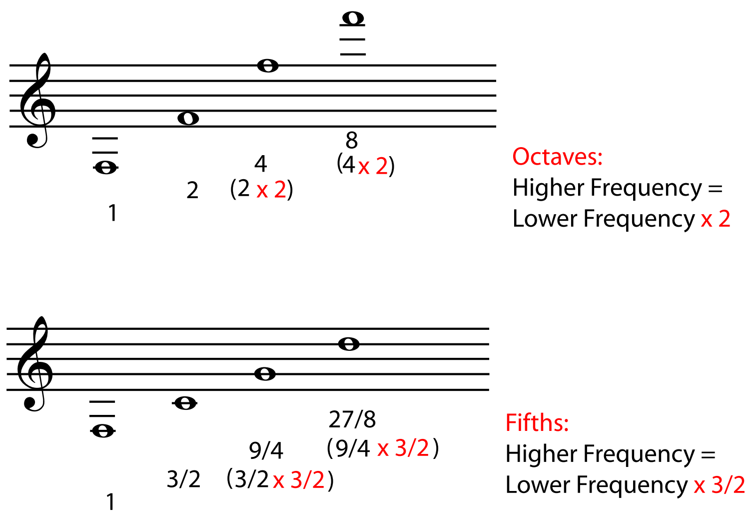

Figure 2.22: Remember that, no matter what note you start on, the note one octave higher has 2 times its frequency. Also, no matter what note you start on, the note that is a perfect fifth higher has exactly one and a half times its frequency. Since each of these intervals is so many "times" in terms of frequencies, when you **add** intervals, you **multiply** their frequencies. For example, a series of two perfect fifths will give a frequency that is 3/2 x 3/2 (or 9/4) the beginning frequency.

Every octave has the same frequency ratio; the higher note will have 2 **times** the frequency of the lower note. So if you go up another octave from there (another 2 times), that note must have 2 x 2, or 4 times the frequency of the lowest note. The next octave takes you up 2 times higher than that, or 8 times the frequency of the first note, and so on.

In just the same way, in every perfect fifth, the higher note will have a frequency one and a half (3/2) times the lower note. So to find out how much higher the frequency is after a series of perfect fifths, you would have to multiply (not add) by one and a half (3/2) every time you went up another perfect fifth.

All intervals work in this same way. So, in order for twelve semitones (half steps) to equal one octave, the size of a half step has to be a number that gives the answer "2" (the size of an octave) when you multiply it twelve times: in other words, the twelfth root of two. And in order for a hundred cents to equal one semitone, the size of a cent must be the number that, when you multiply it 100 times, ends up being the same size as a semitone; in other words, the hundredth root of the twelfth root of two. This is one reason why most musicians prefer to talk in terms of cents and intervals instead of frequencies.

2.4.5 Beats and Wide Tuning

One well-known result of tempered tunings is the aural phenomenon known as **beats**. As mentioned above (p. 44), in a pure interval (Section 2.4.2.1: Pythagorean Intonation) the sound waves have frequencies that are related to each other by very simple ratios. Physically speaking, this means that the two smooth waves line up together so well that the combined wave - the wave you hear when the two are played at the same time - is also a smooth and very steady wave. Tunings that are slightly off from the pure interval, however, will result in a combined wave that has an extra bumpiness in it. Because the two waves are each very even, the bump itself is very even and regular, and can be heard as a "beat" - a very regular change in the intensity of the sound. The beats are so regular, in fact, that they can be timed; for equal temperament they are on the order of a beat per second in the mid range of a piano. A piano tuner works by listening to and timing these beats, rather than by being able to "hear" equal temperament intervals precisely.

It should also be noted that some music traditions around the world do not use the type of precision tunings described above, not because they can't, but because of an aesthetic preference for **wide tuning**. In these traditions, the sound of many people playing precisely the same pitch is considered a thin, uninteresting sound; the sound of many people playing near the same pitch is heard as full, lively, and more interesting.

Some music traditions even use an extremely precise version of wide tuning. The gamelan[104] orchestras of southeast Asia, for example, have an aesthetic preference for the "lively and full" sounds that come from instruments playing near, not on, the same pitch. In some types of gamelans, pairs of instruments are tuned very precisely so that each pair produces beats, and the rate of the beats is the same throughout the entire range[105] of that gamelan. Long-standing traditions allow *gamelan* craftsmen to reliably produce such impressive feats of tuning.

2.4.6 Further Study

As of this writing:

- Kyle Gann's An Introduction to Historical Tunings[106] is a good source about both the historical background and more technical information about various tunings. It also includes some audio examples.
- The Huygens-Fokker Foundation has a very large on-line bibliography[107] of tuning and temperament.
- Alfredo Capurso, a researcher in Italy, has developed the Circular Harmonic System (c.ha.s), a tempered tuning system that solves the wolf fifth problem by adjusting the size of the octave as well as the fifth. It also provides an algorithm for generating microtonal scales. You can read about it at the Circular Harmonic System website[108] or download a paper[109] on the subject. You can also listen to piano performances using this tuning by searching for "CHAS tuning" at YouTube.
- A number of YouTube videos provide comparisons that you can listen to, for example comparisons of just intonation and equal temperament, or comparisons of various temperaments.

[104] "Balinese Gamelan" <http://cnx.org/content/m15796/latest/>
[105] "Range" <http://cnx.org/content/m12381/latest/>
[106] http://www.kylegann.com/histune.html
[107] http://www.huygens-fokker.org/docs/bibliography.html
[108] http://www.chas.it/
[109] http://math.unipa.it/~grim/Quaderno19_Capurso_09_engl.pdf

Solutions to Exercises in Chapter 2

Solution to Exercise 2.1.1 (p. 28)

- Ratio of small to medium is 3:3. Like fractions, ratios can be reduced to lowest terms, so ratio of 1:1 is also correct.
- Ratio of small to large, or medium to large, is 3:4; ratio of large to either of the others is 4:3.
- Ratio of small or medium to total is 3:10; ratio of large to total is 4:10.
- 3/10, or 0.3, of the shirts, are small; 3/10, or 0.3 of the shirts are medium, and 4/10, or 0.4 of the shirts, are large.
- There are 3/4 as many small or medium shirts as there are large shirts, and there are 4/3 as many large shirts as small or medium shirts.
- If you made more ratios, fractions, and decimals by combining various groups (say ratio of small and medium to large is 6:4, and so on), give yourself extra credit.

Solution to Exercise 2.1.2 (p. 28)

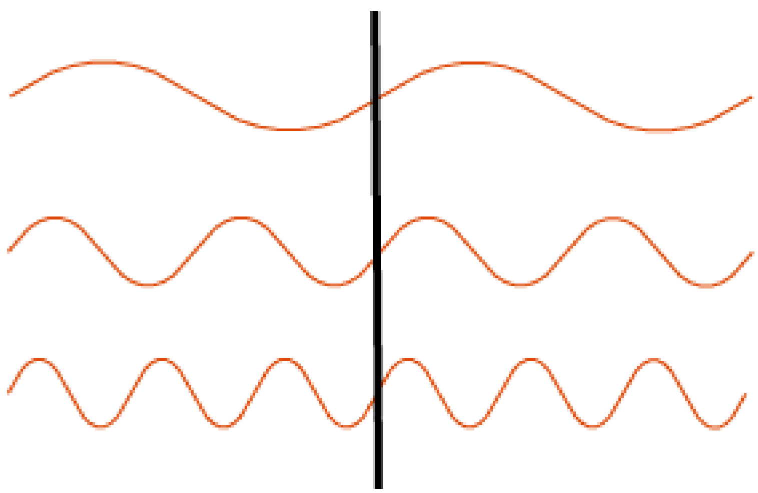

Figure 2.23: For every one wave of frequency 220, there are two of 440, and 3 of 660.

1. 660 sounds the highest; 220 lowest. (440 is a "tuning A" or A 440", by the way. 220 is the A one octave lower, and 660 is the E above A 440.)
2. 220 has the longest wavelength, and 660 the shortest.
3. 220:440:660 in lowest terms is 1:2:3
4. 3
5. There are only 2/3 as many waves in the 440 frequency as in the 660 frequency.

Solution to Exercise 2.1.3 (p. 32)

frequency #2 = frequency #1 x ratio of #2 over #1

Perfect Fourth Higher:
Ratio 4:3

frequency #2 =

$$220 \times \frac{4}{3} = 293.\overline{33}$$

Major Third Lower:
Ratio 4:5

frequency #2 =

$$220 \times \frac{4}{5} = 176$$

Figure 2.24

Solution to Exercise 2.1.4 (p. 33)

$$\frac{\text{frequency \#1}}{\text{frequency \#2}} = Y$$

$$\frac{1333}{1121} = 1.1891$$

This is very close to 1.1892,
so the interval will sound like a minor third.

Figure 2.25

Solution to Exercise 2.2.1 (p. 34)
Although trained musicians will generally agree that a particular sound is reedy, thin, or full, there are no hard-and-fast right-and-wrong answers to this exercise.
Solution to Exercise 2.2.2 (p. 37)

1. The eighth harmonic
2. The fifth and tenth harmonics; the sixth and twelfth harmonics; the seventh and fourteenth harmonics; and the eighth and sixteenth harmonics
3. The note that is one octave higher than a harmonic is also a harmonic, and its number in the harmonic series is twice (2 X) the number of the first note.
4. The eighth, sixteenth, and thirty-second harmonics will also be A's.

Solution to Exercise 2.2.3 (p. 38)

1. The ratio 4:6 reduced to lowest terms is 2:3. (If you are more comfortable with fractions than with ratios, think of all the ratios as fractions instead. 2:3 is just two-thirds, and 4:6 is four-sixths. Four-sixths reduces to two-thirds.)
2. Six and nine (6:9 also reduces to 2:3); eight and twelve; ten and fifteen; and any other combination that can be reduced to 2:3 (12:18, 14:21 and so on).

3. Harmonics three and four; six and eight; nine and twelve; twelve and sixteen; and so on.
4. 3:4

Solution to Exercise 2.2.4 (p. 40)

Opening both first and second valves gives the harmonic series one-and-a-half steps lower than "no valves".

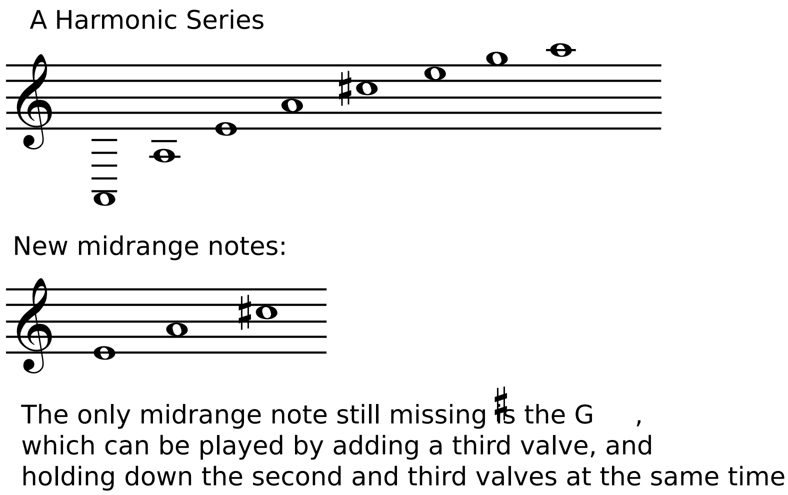

Figure 2.26

Solution to Exercise 2.3.1 (p. 43)

1. 1.0595
2. The twelfth root of 2, to the fifth power, is approximately 1.3348
3. The twelfth root of 2, to the fourth power, is approximately 1.2599
4. The twelfth root of 2, to the twelfth power, is 2

Chapter 3

Standing Waves and Instruments

3.1 Standing Waves and Musical Instruments[1]

3.1.1 What is a Standing Wave?

Musical tones (p. 59) are produced by musical instruments, or by the voice, which, from a physics perspective, is a very complex wind[2] instrument. So the physics of music is the physics of the kinds of sounds these instruments can make. What kinds of sounds are these? They are tones caused by standing waves produced in or on the instrument. So the properties of these standing waves, which are always produced in very specific groups, or series, have far-reaching effects on music theory.

Most sound waves, including the musical sounds that actually reach our ears, are not standing waves. Normally, when something makes a wave, the wave travels outward, gradually spreading out and losing strength, like the waves moving away from a pebble dropped into a pond.

But when the wave encounters something, it can bounce (reflection) or be bent (refraction). In fact, you can "trap" waves by making them bounce back and forth between two or more surfaces. Musical instruments take advantage of this; they produce pitches[3] by trapping sound waves.

Why are trapped waves useful for music? Any bunch of sound waves will produce some sort of noise. But to be a **tone** - a sound with a particular pitch[4] - a group of sound waves has to be very regular, all exactly the same distance apart. That's why we can talk about the frequency[5] and wavelength[6] of tones.

[1] This content is available online at <http://cnx.org/content/m12413/1.15/>.
[2] "Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/>
[3] "Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[4] "Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[5] "Acoustics for Music Theory" <http://cnx.org/content/m13246/latest/#p2b>
[6] "Acoustics for Music Theory" <http://cnx.org/content/m13246/latest/#p2a>

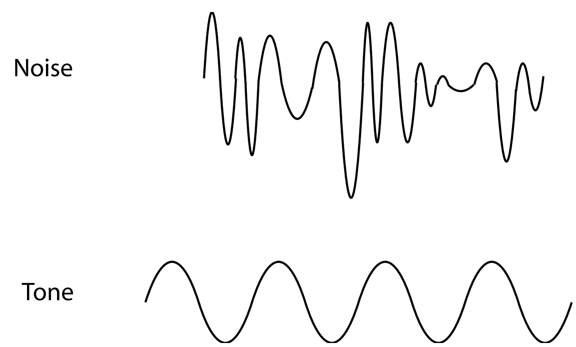

Figure 3.1: A noise is a jumble of sound waves. A tone is a very regular set of waves, all the same size and same distance apart.

So how can you produce a tone? Let's say you have a sound wave trap (for now, don't worry about what it looks like), and you keep sending more sound waves into it. Picture a lot of pebbles being dropped into a very small pool. As the waves start reflecting off the edges of the pond, they interfere with the new waves, making a jumble of waves that partly cancel each other out and mostly just roils the pond - noise.

But what if you could arrange the waves so that reflecting waves, instead of cancelling out the new waves, would reinforce them? The high parts of the reflected waves would meet the high parts of the oncoming waves and make them even higher. The low parts of the reflected waves would meet the low parts of the oncoming waves and make them even lower. Instead of a roiled mess of waves cancelling each other out, you would have a pond of perfectly ordered waves, with high points and low points appearing regularly at the same spots again and again. To help you imagine this, here are animations of a single wave reflecting back and forth[7] and standing waves[8].

This sort of orderliness is actually hard to get from water waves, but relatively easy to get in sound waves, so that several completely different types of sound wave "containers" have been developed into musical instruments. The two most common - strings and hollow tubes - will be discussed below, but first let's finish discussing what makes a good standing wave container, and how this affects music theory.

In order to get the necessary constant reinforcement, the container has to be the perfect size (length) for a certain wavelength, so that waves bouncing back or being produced at each end reinforce each other, instead of interfering with each other and cancelling each other out. And it really helps to keep the container very narrow, so that you don't have to worry about waves bouncing off the sides and complicating things. So you have a bunch of regularly-spaced waves that are trapped, bouncing back and forth in a container that fits their wavelength perfectly. If you could watch these waves, it would not even look as if they are traveling back and forth. Instead, waves would seem to be appearing and disappearing regularly at exactly the same spots, so these trapped waves are called **standing waves**.

> NOTE: Although standing waves are harder to get in water, the phenomenon does apparently happen very rarely in lakes, resulting in freak disasters. You can sometimes get the same effect by pushing a tub of water back and forth, but this is a messy experiment; you'll know you are getting a standing wave when the water suddenly starts sloshing much higher - right out of the tub!

For any narrow "container" of a particular length, there are plenty of possible standing waves that don't fit. But there are also many standing waves that do fit. The longest wave that fits it is called the **fundamental**. It is also called the **first harmonic**. The next longest wave that fits is the **second**

[7] See the file at <http://cnx.org/content/m12413/latest/ReflectingWave.swf>
[8] See the file at <http://cnx.org/content/m12413/latest/WaterWaves.swf>

harmonic, or the **first overtone**. The next longest wave is the **third harmonic**, or **second overtone**, and so on.

Standing Wave Harmonics

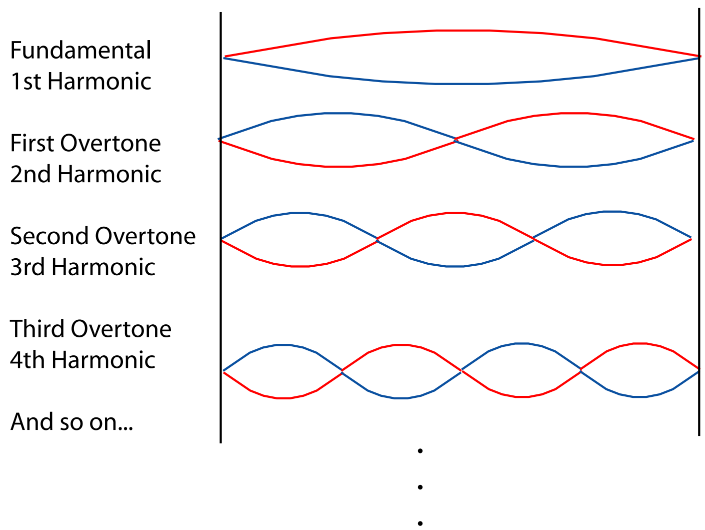

Figure 3.2: There is a whole set of standing waves, called **harmonics**, that will fit into any "container" of a specific length. This set of waves is called a **harmonic series**.

Notice that it doesn't matter what the length of the fundamental is; the waves in the second harmonic must be half the length of the first harmonic; that's the only way they'll both "fit". The waves of the third harmonic must be a third the length of the first harmonic, and so on. This has a direct effect on the frequency and pitch of harmonics, and so it affects the basics of music tremendously. To find out more about these subjects, please see Frequency, Wavelength, and Pitch (Section 1.3), Harmonic Series (Section 2.2), or Musical Intervals, Frequency, and Ratio (Section 2.1).

3.1.2 Standing Waves on Strings

You may have noticed an interesting thing in the animation (p. 60) of standing waves: there are spots where the "water" goes up and down a great deal, and other spots where the "water level" doesn't seem to move at all. All standing waves have places, called **nodes**, where there is no wave motion, and **antinodes**, where the wave is largest. It is the placement of the nodes that determines which wavelengths "fit" into a musical instrument "container".

Nodes and Antinodes

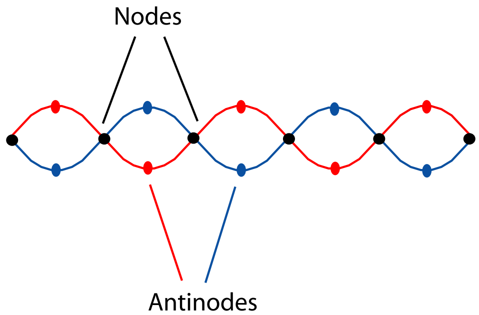

Figure 3.3: As a standing wave waves back and forth (from the red to the blue position), there are some spots called **nodes** that do not move at all; basically there is no change, no waving up-and-down (or back-and-forth), at these spots. The spots at the biggest part of the wave - where there is the most change during each wave - are called **antinodes**.

One "container" that works very well to produce standing waves is a thin, very taut string that is held tightly in place at both ends. Since the string is taut, it vibrates quickly, producing sound waves, if you pluck it, or rub it with a bow. Since it is held tightly at both ends, that means there has to be a node (p. 61) at each end of the string. Instruments that produce sound using strings are called chordophones[9], or simply strings[10].

[9]"Classifying Musical Instruments": Section Chordophones <http://cnx.org/content/m11896/latest/#s21>
[10]"Orchestral Instruments": Section Strings <http://cnx.org/content/m11897/latest/#s11>

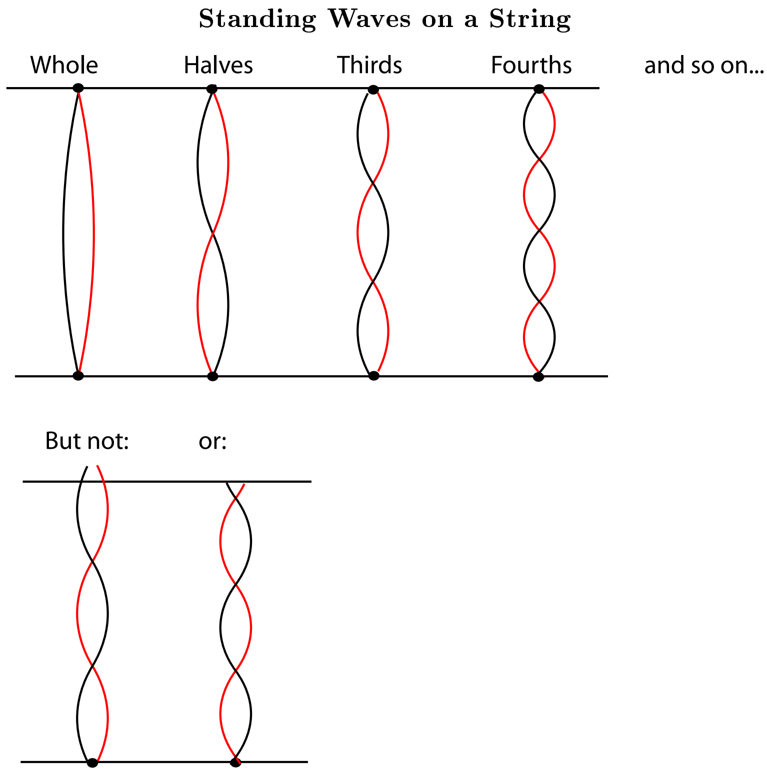

Figure 3.4: A string that's held very tightly at both ends can only vibrate at very particular wavelengths. The whole string can vibrate back and forth. It can vibrate in halves, with a node at the middle of the string as well as each end, or in thirds, fourths, and so on. But any wavelength that doesn't have a node at each end of the string, can't make a standing wave on the string. To get any of those other wavelengths, you need to change the length of the vibrating string. That is what happens when the player holds the string down with a finger, changing the vibrating length of the string and changing where the nodes are.

The fundamental (p. 60) wave is the one that gives a string its pitch[11]. But the string is making all those other possible vibrations, too, all at the same time, so that the actual vibration of the string is pretty complex. The other vibrations (the ones that basically divide the string into halves, thirds and so on) produce a whole series of **harmonics**. We don't hear the harmonics as separate notes, but we do hear them. They are what gives the string its rich, musical, string-like sound - its timbre[12]. (The sound of a single frequency alone is a much more mechanical, uninteresting, and unmusical sound.) To find out more about harmonics and how they affect a musical sound, see Harmonic Series (Section 2.2).

Exercise 3.1.1 ***(Solution on p. 77.)***

When the string player puts a finger down tightly on the string,

1. How has the part of the string that vibrates changed?
2. How does this change the sound waves that the string makes?

[11]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[12]"Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>

3. How does this change the sound that is heard?

3.1.3 Standing Waves in Wind Instruments

The string disturbs the air molecules around it as it vibrates, producing sound waves in the air. But another great container for standing waves actually holds standing waves of air inside a long, narrow tube. This type of instrument is called an aerophone[13], and the most well-known of this type of instrument are often called wind instruments[14] because, although the instrument itself does vibrate a little, most of the sound is produced by standing waves in the column of air inside the instrument.

If it is possible, have a reed player and a brass player demonstrate to you the sounds that their mouthpieces make without the instrument. This will be a much "noisier" sound, with lots of extra frequencies in it that don't sound very musical. But, when you put the mouthpiece on an instrument shaped like a tube, only some of the sounds the mouthpiece makes are the right length for the tube. Because of feedback from the instrument, the only sound waves that the mouthpiece can produce now are the ones that are just the right length to become **standing waves** in the instrument, and the "noise" is refined into a musical tone.

[13]"Classifying Musical Instruments": Section Aerophones <http://cnx.org/content/m11896/latest/#s22>
[14]"Orchestral Instruments": Section The Sections of the Orchestra <http://cnx.org/content/m11897/latest/#s1>

Standing Waves in Wind Instruments

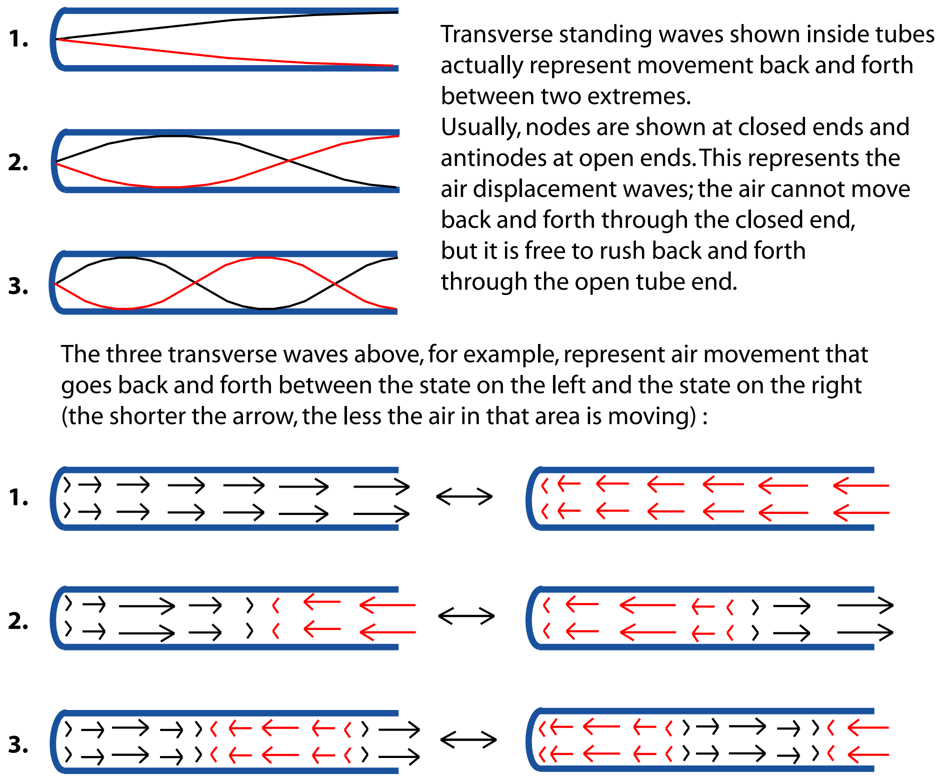

Figure 3.5: Standing Waves in a wind instrument are usually shown as displacement waves, with nodes at closed ends where the air cannot move back-and-forth.

The standing waves in a wind instrument are a little different from a vibrating string. The wave on a string is a **transverse wave**, moving the string back and forth, rather than moving up and down along the string. But the wave inside a tube, since it is a sound wave already, is a **longitudinal wave**; the waves do not go from side to side in the tube. Instead, they form along the length of the tube.

Longitudinal Waves in Pipes

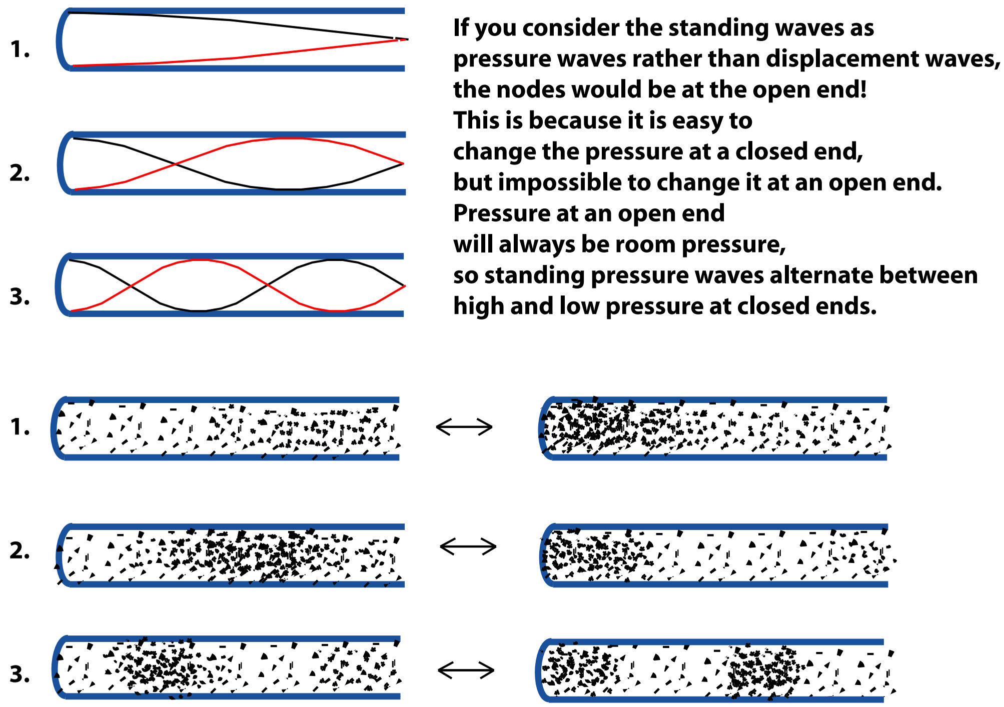

Figure 3.6: The standing waves in the tubes are actually longitudinal sound waves. Here the displacement standing waves in Figure 3.5 (Standing Waves in Wind Instruments) are shown instead as longitudinal air pressure waves. Each wave would be oscillating back and forth between the state on the right and the one on the left. See Standing Waves in Wind Instruments (Section 3.2) for more explanation.

The harmonics of wind instruments are also a little more complicated, since there are two basic shapes (cylindrical[15] and conical[16]) that are useful for wind instruments, and they have different properties. The standing-wave tube of a wind instrument also may be open at both ends, or it may be closed at one end (for a mouthpiece, for example), and this also affects the instrument. Please see Standing Waves in Wind Instruments (Section 3.2) if you want more information on that subject. For the purposes of understanding music theory, however, the important thing about standing waves in winds is this: the harmonic series they produce is essentially the same as the harmonic series on a string. In other words, the second harmonic is still half the length of the fundamental, the third harmonic is one third the length, and so on. (Actually, for reasons explained in Standing Waves in Wind Instruments (Section 3.2), some harmonics are "missing" in some wind instruments, but this mainly affects the timbre[17] and some aspects of playing the instrument. It does not affect the basic relationships in the harmonic series.)

[15]"Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/#p1c>
[16]"Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/#p1c>
[17]"Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>

3.1.4 Standing Waves in Other Objects

So far we have looked at two of the four main groups of musical instruments: chordophones and aerophones. That leaves membranophones[18] and idiophones[19]. **Membranophones** are instruments in which the sound is produced by making a membrane vibrate; drums are the most familiar example. Most drums do not produce tones; they produce rhythmic "noise" (bursts of irregular waves). Some drums do have pitch[20], due to complex-patterned standing waves on the membrane that are reinforced in the space inside the drum. This works a little bit like the waves in tubes, above, but the waves produced on membranes, though very interesting, are too complex to be discussed here.

Idiophones are instruments in which the body of the instrument itself, or a part of it, produces the original vibration. Some of these instruments (cymbals, for example) produce simple noise-like sounds when struck. But in some, the shape of the instrument - usually a tube, block, circle, or bell shape - allows the instrument to ring with a standing-wave vibration when you strike it. The standing waves in these carefully-shaped-and-sized idiophones - for example, the blocks on a xylophone - produce pitched tones, but again, the patterns of standing waves in these instruments are a little too complicated for this discussion. If a percussion instrument does produce pitched sounds, however, the reason, again, is that it is mainly producing harmonic-series overtones (Section 2.2).

> NOTE: Although percussion[21] specializes in "noise"-type sounds, even instruments like snare drums follow the basic physics rule of "bigger instrument makes longer wavelengths and lower sounds". If you can, listen to a percussion player or section that is using snare drums, cymbals, or other percussion of the same type but different sizes. Can you hear the difference that size makes, as opposed to differences in timbre[22] produced by different types of drums?

Exercise 3.1.2 ***(Solution on p. 77.)***

Some idiophones, like gongs, ring at many different pitches when they are struck. Like most drums, they don't have a particular pitch, but make more of a "noise"-type sound. Other idiophones, though, like xylophones, are designed to ring at more particular frequencies. Can you think of some other percussion instruments that get particular pitches? (Some can get enough different pitches to play a tune.)

3.2 Standing Waves and Wind Instruments[23]

3.2.1 Introduction

A wind instrument[24] makes a tone (p. 59) when a standing wave (Section 3.1) of air is created inside it. In most wind instruments, a vibration that the player makes at the mouthpiece[25] is picked up and amplified and given a pleasant timbre[26] by the air inside the tube-shaped body of the instrument. The shape and length of the inside of the tube give the sound wave its pitch[27] as well as its timbre.

You will find below a discussion of what makes standing waves in a tube (Section 3.2.2: What Makes the Standing Waves in a Tube), wind instruments and the harmonic series (Section 3.2.3: Harmonic Series in Tubes), and the types of tubes that can be used in musical instruments (Section 3.2.4: Basic Wind Instrument Tube Types). This is a simplified discussion to give you a basic idea of what's going on inside

[18]"Classifying Musical Instruments": Section Membranophones <http://cnx.org/content/m11896/latest/#s23>
[19]"Classifying Musical Instruments": Section Idiophones <http://cnx.org/content/m11896/latest/#s24>
[20]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>
[21]"Orchestral Instruments": Section Percussion <http://cnx.org/content/m11897/latest/#s14>
[22]"Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>
[23]This content is available online at <http://cnx.org/content/m12589/1.13/>.
[24]"Orchestral Instruments": Section The Sections of the Orchestra <http://cnx.org/content/m11897/latest/#s1>
[25]"Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/#p1b>
[26]"Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>
[27]"Pitch: Sharp, Flat, and Natural Notes" <http://cnx.org/content/m10943/latest/>

a wind instrument. Mathematical equations are avoided, and all the complications - for example, what happens to the wave when there are closed finger holes in the side of the tube - are ignored. Actually, the physics of what happens inside real wind instruments is so complex that physicists are still studying it, and still don't have all the answers. If you want a more in-depth or more technical discussion, there are some recommendations below (Section 3.2.5: Further Reading).

If you can't follow the discussion below, try reviewing Acoustics for Music Theory[28], Standing Waves and Musical Instruments (Section 3.1), Harmonic Series I[29], and Wind Instruments: Some Basics[30]

3.2.2 What Makes the Standing Waves in a Tube

As discussed in Standing Waves and Musical Instruments (Section 3.1), instruments produce musical tones by trapping waves of specific lengths in the instrument. It's pretty easy to see why the standing waves on a string (Section 3.1.2: Standing Waves on Strings) can only have certain lengths; since the ends of the strings are held in place, there has to be a node (p. 61) in the wave at each end. But what is it that makes only certain standing waves possible in a tube of air?

To understand that, you'll have to understand a little bit about what makes waves in a tube different from waves on a string. Waves on a string are transverse waves[31]. The string is stretched out in one direction (call it "up and down"), but when it's vibrating, the motion of the string is in a different direction (call it "back and forth"). Take a look at this animation[32]. At the nodes (each end, for example), there is no back and forth motion, but in between the nodes, the string is moving back and forth very rapidly. The term for this back-and-forth motion is **displacement**. There is no displacement at a node; the most displacement happens at an antinode (p. 61).

Transverse Motion on a String

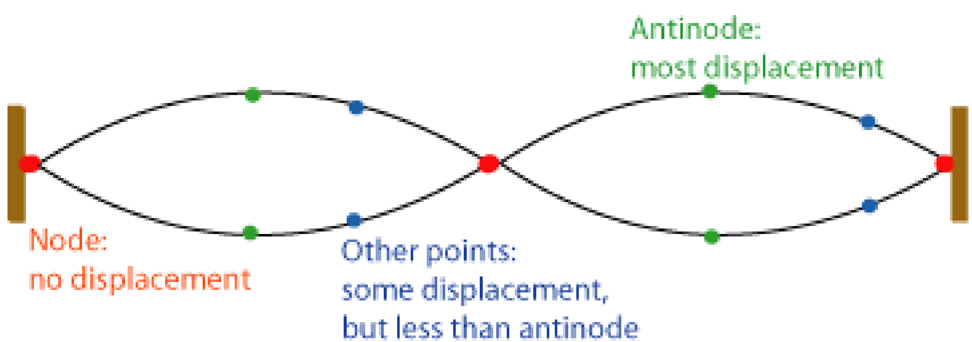

Figure 3.7

The standing waves of air in a tube are not transverse waves. Like all sound waves, they are longitudinal[33]. So if the air in the tube is moving in a certain direction (call it "left and right"), the vibrations in the air

[28]"Acoustics for Music Theory" <http://cnx.org/content/m13246/latest/>
[29]"Harmonic Series I: Timbre and Octaves" <http://cnx.org/content/m13682/latest/>
[30]"Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/>
[31]"Acoustics for Music Theory": Section Longitudinal and Transverse Waves <http://cnx.org/content/m13246/latest/#s11>
[32]See the file at <http://cnx.org/content/m12589/latest/TransverseNodes.swf>
[33]"Acoustics for Music Theory": Section Longitudinal and Transverse Waves <http://cnx.org/content/m13246/latest/#s11>

are going in that same direction (in this case, they are rushing "left and right").

But they are like the waves on a string in some important ways. Since they are standing waves, there are still nodes - in this case, places where the air is not rushing back and forth. And, just as on the string, in between the nodes there are antinodes, where the displacement is largest (the air is moving back and forth the most). And when one antinode is going in one direction (left), the antinodes nearest it will be going in the other direction (right). So, even though what is happening is very different, the end result of standing waves "trapped" in a tube will be very much like the end result of standing waves "trapped" on a string: a harmonic series[34] based on the tube length.

There will be more on that harmonic series in the next section (Section 3.2.3: Harmonic Series in Tubes). First, let's talk about why only some standing waves will "fit" in a tube of a particular length. If the tube were closed on both ends, it's easy to see that this would be a lot like the wave on the string. The air would not be able to rush back and forth at the ends, so any wave trapped inside this tube would have to have nodes at each end.

> NOTE: It's very difficult to draw air that is rushing back and forth in some places and standing still in other places, so most of the figures below use a common illustration method, showing the longitudinal waves as if they are simultaneously the two maximum positions of a transverse wave. Here is an animation[35] that may give you some idea of what is happening in a longitudinal standing wave. As of this writing, there was a nice Standing Waves applet[36] demonstration of waves in tubes. Also, see below (Figure 3.13: Displacement Waves) for more explanation of what the transverse waves inside the tubes really represent.

Fully Closed Tube

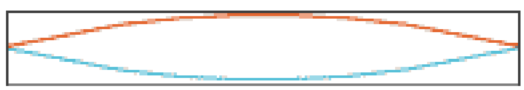

Figure 3.8: The standing waves inside the tube represent back-and-forth motion of the air. Since the air can't move through the end of the tube, a closed tube must have a node at each end, just like a string held at both ends.

Now, a closed tube wouldn't make a very good musical instrument; it wouldn't be very loud. Most of the sound you hear from an instrument is not the standing wave inside the tube; the sound is made at the open ends where the standing waves manage to create other waves that can move away from the instrument. Physicists sometimes study the acoustics of a tube closed at both ends (called a **Kundt tube**), but most wind instruments have at least one open end. An instrument that is open at both ends may be called **open-open**, or just an **open tube** instrument. An instrument that is only open at one end may be called **open-closed**, or a **closed tube** or **stopped tube** instrument (or sometimes **semi-closed** or **half-closed**). This is a little confusing, since such instruments (trumpets[37], for example) still obviously have one open end.

Now, there's nothing stopping the air from rushing back and forth at the open end of the tube. In fact, the waves that "fit" the tube are the ones that have antinodes at the open end, so the air is in fact rushing back and forth there, causing waves (at the same frequency[38] as the standing wave) that are not trapped in the instrument but can go out into the room.

[34] "Harmonic Series I: Timbre and Octaves" <http://cnx.org/content/m13682/latest/>
[35] See the file at <http://cnx.org/content/m12589/latest/PressureWaveNew.swf>
[36] http://www.physics.smu.edu/~olness/www/05fall1320/applet/pipe-waves.html
[37] "Trumpets and Cornets" <http://cnx.org/content/m12606/latest/>
[38] "Acoustics for Music Theory": Section Wavelength, Frequency, and Pitch <http://cnx.org/content/m13246/latest/#s2>

Open-Open and Open-Closed Tubes

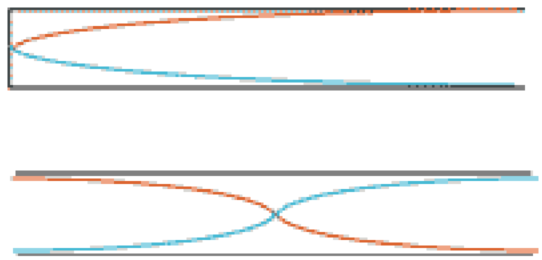

Figure 3.9: There must be a (displacement) antinode at any open end of a tube.

What is it that requires the waves to have an antinode at an open end? Look again at the animation[39] of what is happening to the air particles in the standing wave. The air at the nodes is not moving back and forth, but it is piling up and spreading out again. So the **air pressure** is changing a lot at the nodes. But at the antinodes, the air is moving a lot, but it is moving back and forth, not piling up and spreading out. In fact, you can imagine that same wave to be an air pressure wave instead of an air displacement wave. It really is both at the same time, but the pressure wave nodes are at the same place as the displacement antinodes, and the pressure antinodes are at the same place as the displacement nodes.

An Air Displacement Wave is also an Air Pressure Wave

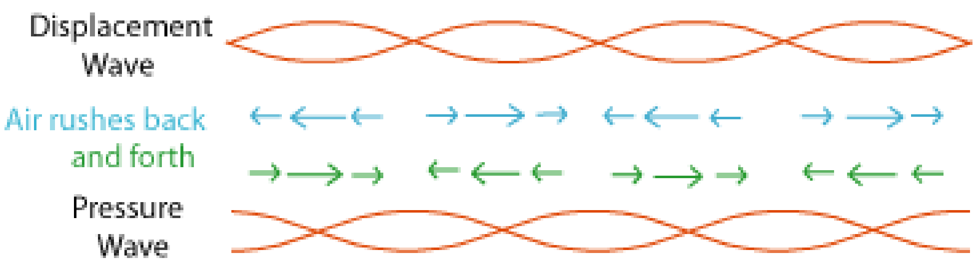

Figure 3.10: The nodes of the displacement wave, where the air is not rushing back-and-forth but is doing the most piling-up-and-spreading-out, are the antinodes of the pressure wave. The antinodes of the displacement wave, where the air is rushing back-and-forth the most, but is not piling up or spreading out at all, are the nodes of the pressure wave. Both waves must have exactly the same frequency, of course; they are actually just two aspects of the same sound wave.

At an open end of the tube, there is nothing to stop the air rushing in and out, and so it does. What the air cannot do at the open end is build up any pressure; there is nothing for the air to build up against, and any drop in pressure will just bring air rushing in from outside the tube. So the air pressure at an open end must remain the same as the air pressure of the room. In other words, that end must have a pressure node (where the air pressure doesn't change) and (therefore) a displacement antinode.

[39]See the file at <http://cnx.org/content/m12589/latest/PressureWaveNew.swf>

NOTE: Since being exposed to the air pressure outside the instrument is what is important, the "open end" of a wind instrument, as far as the sound waves are concerned, is the first place that they can escape - the first open hole. This is how woodwinds[40] change the length of the wave, and the pitch of the note. For more on this, please see Wind Instruments – Some Basics[41].

3.2.3 Harmonic Series in Tubes

As explained in the previous section (Section 3.2.2: What Makes the Standing Waves in a Tube), the standing waves in a tube must have a (displacement) node at a closed end and an antinode at an open end. In an open-open tube, this leads to a harmonic series[42] very similar to a harmonic series produced on a string (Section 3.1.2: Standing Waves on Strings) that's held at both ends. The **fundamental**, the lowest note possible in the tube, is the note with a wavelength twice the length of the tube (or string). The next possible note has twice the frequency (half the wavelength) of the fundamental, the next three times the frequency, the next four times, and so on.

Allowed Waves in an Open Tube

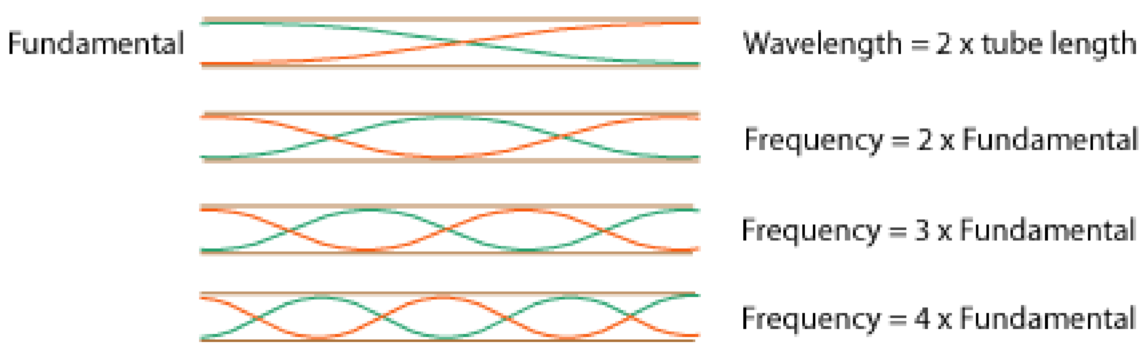

Figure 3.11: These are the first four harmonics allowed in an open tube. Any standing wave with a displacement antinode at both ends is allowed, but the lower harmonics are usually the easiest to play and the strongest harmonics in the timbre[43].

But things are a little different for the tube that is closed at one end and open at the other. The lowest note that you might be able to get on such a tube (a fundamental that is unplayable on many instruments) has a wavelength four times the length of the tube. (You may notice that this means that a stopped tube will get a note half the frequency[44] - an octave lower - than an open tube of the same length.) The next note that is possible on the half-closed tube has three times the frequency of the fundamental, the next five times,and so on. In other words, a stopped tube can only play the odd-numbered harmonics.

[40]"Orchestral Instruments": Section Woodwinds <http://cnx.org/content/m11897/latest/#s12>
[41]"Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/>
[42]"Harmonic Series I: Timbre and Octaves" <http://cnx.org/content/m13682/latest/>
[43]"Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>
[44]"Acoustics for Music Theory": Section Wavelength, Frequency, and Pitch <http://cnx.org/content/m13246/latest/#s2>

Allowed Wavelengths in a Stopped Tube

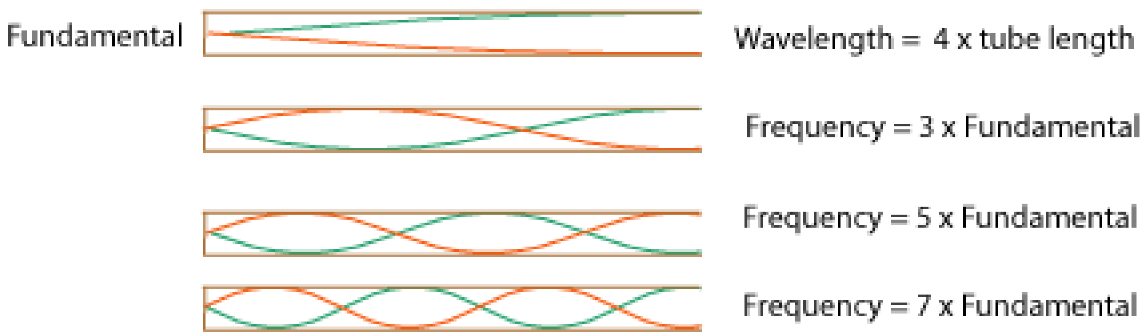

Figure 3.12: Again, these are the lowest (lowest pitch and lowest frequency) four harmonics allowed. Any wave with a displacement node at the closed end and antinode at the open end is allowed. Note that this means only the odd-numbered harmonics "fit".

Reminder: All of the transverse waves in Figure 3.8 (Fully Closed Tube), Figure 3.9 (Open-Open and Open-Closed Tubes), Figure 3.11 (Allowed Waves in an Open Tube), and Figure 3.12 (Allowed Wavelengths in a Stopped Tube) represent longitudinal displacement waves, as shown in Figure 3.13 (Displacement Waves). All of the harmonics would be happening in the tube at the same time, and, for each harmonic, the displacement (Figure 3.13 (Displacement Waves)) and pressure waves (Figure 3.14 (Pressure Waves)) are just two different ways of representing the same wave.

Displacement Waves

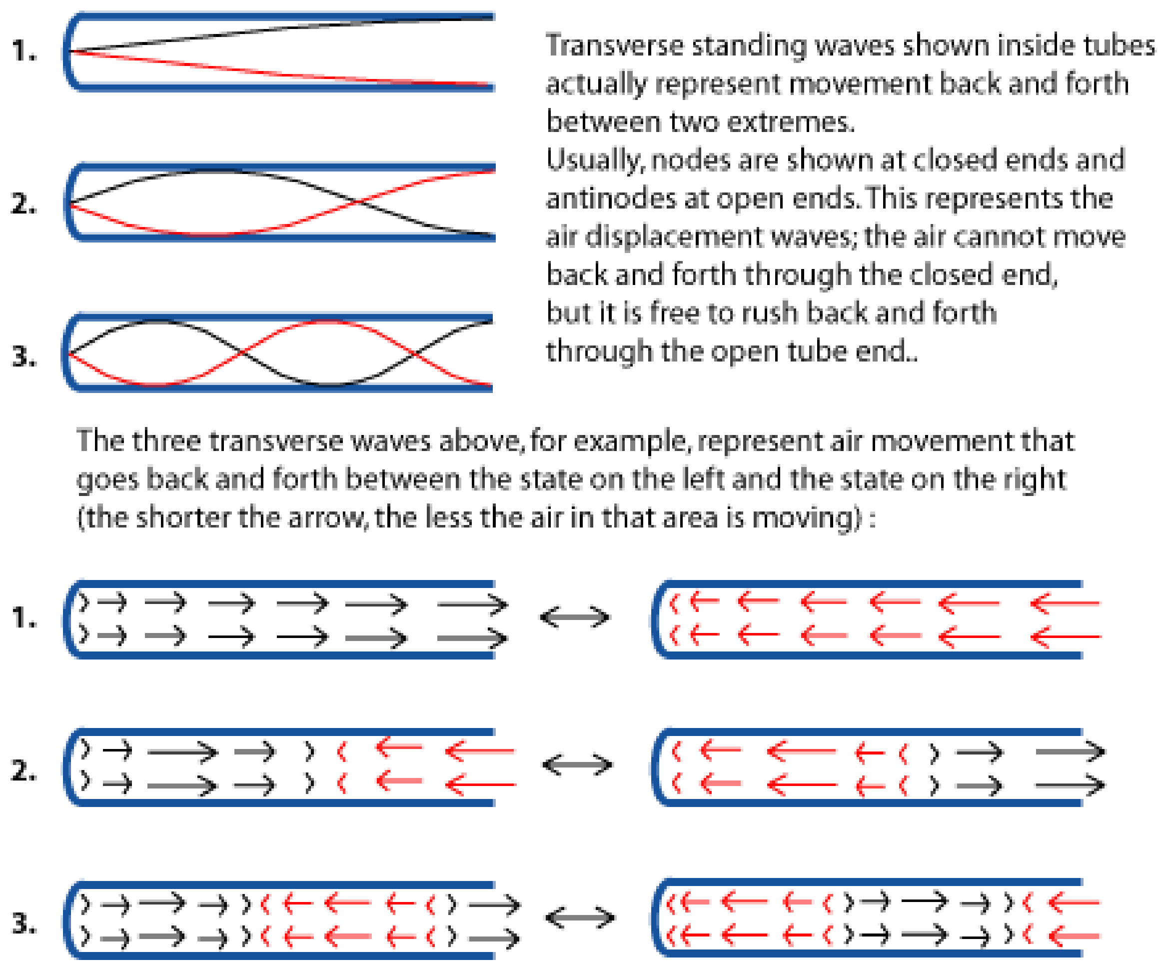

Figure 3.13: Here are the first three possible harmonics in a closed-open tube shown as longitudinal displacement waves.

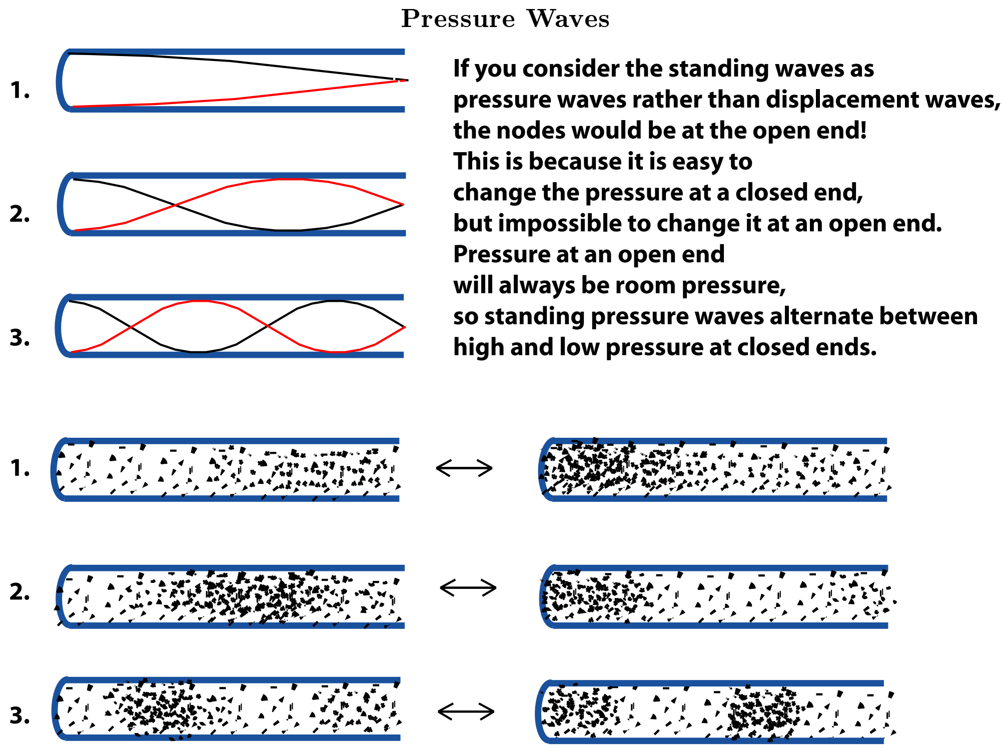

Figure 3.14: Here are those same three waves shown as pressure waves.

3.2.4 Basic Wind Instrument Tube Types

The previous section shows why only the odd-numbered harmonics "fit" in a cylinder-shaped tube, but that is not the whole story. There is one other tube shape that works well for wind instruments, and it abides by slightly different rules.

Just as on a string (Section 3.1.2: Standing Waves on Strings), the actual wave inside the instrument is a complex wave that includes all of those possible harmonics[45]. A cylinder makes a good musical instrument because all the waves in the tube happen to have simple, harmonic-series-type relationships. This becomes very useful when the player **overblows** in order to get more notes. As mentioned above, woodwind players get different notes out of their instruments by opening and closing finger holes, making the standing wave tube longer or shorter. Once the player has used all the holes, higher notes are played by **overblowing**, which causes the next higher harmonic of the tube to sound. In other words, the fundamental of the tube is not heard when the player "overblows"; the note heard is the pitch of the next available harmonic (either harmonic two or three). Brass players can get many different harmonics from their instruments, and so do not need as many fingerings. (Please see Harmonic Series[46] and Wind Instruments – Some Basics[47] for more on this.)

[45]"Harmonic Series I: Timbre and Octaves" <http://cnx.org/content/m13682/latest/>
[46]"Harmonic Series I: Timbre and Octaves" <http://cnx.org/content/m13682/latest/>
[47]"Wind Instruments: Some Basics" <http://cnx.org/content/m12364/latest/>

For most possible tube shapes, a new set of holes would be needed to get notes that are in tune with the lower set of notes. But a couple of shapes, including the cylinder, give higher notes that are basically in tune with the lower notes using the same finger holes (or valves). (Even so, some extra finger holes or an extra slide or valve is sometimes necessary for good tuning.) One other possible shape is basically not used because it would be difficult to build precisely and unwieldy to play. (Basically, it has to flare rapidly, at a very specific rate of flare. The resulting instrument would be unwieldy and impractical. Please see John S. Rigden's *Physics and the Sound of Music*, as cited below (Section 3.2.5: Further Reading) for more on this.)

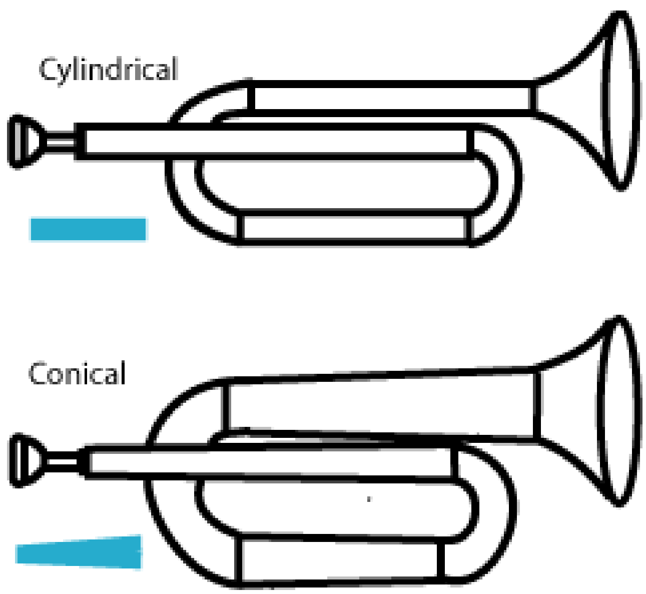

Figure 3.15: The two shapes that are useful for real wind instruments are the cylinder and the cone. Most real wind instruments are a combination of cylindrical and conical sections, but most act as (and can be classified as) either **cylindrical bore** or **conical bore** instruments.

The other tube shape that is often used in wind instruments is the cone. In fact, most real wind instruments are tubes that are some sort of combination of cylindrical and conical tubes. But most can be classified as either cylindrical or conical instruments.

The really surprising thing is that stopped-tube instruments that are basically conical act as if they are open-tube cylindrical instruments.

> NOTE: The math showing why this happens has been done, but I will not go into it here. Please see the further reading (Section 3.2.5: Further Reading), below for books with a more rigorous and in-depth discussion of the subject.

Compare, for example, the clarinet and the saxophone, woodwinds with very similar mouthpieces. Both instruments, like any basic woodwind, have enough finger holes and keys to play all the notes within an octave. To get more notes, a woodwind player **overblows**, blowing hard enough to sound the next harmonic of the instrument. For the saxophone, a very conical instrument, the next harmonic is the next octave[48] (two times the frequency of the fundamental), and the saxophonist can continue up this next octave by essentially repeating the fingerings for the first octave. Only a few extra keys are needed to help with tuning.

[48]"Octaves and the Major-Minor Tonal System" <http://cnx.org/content/m10862/latest/>

The clarinet player doesn't have it so easy. Because the clarinet is a very cylindrical instrument, the next harmonic available is three times the frequency, or an octave and a fifth[49] higher, than the fundamental. Extra holes and keys have to be added to the instrument to get the notes in that missing fifth, and then even more keys are added to help the clarinetist get around the awkward fingerings that can ensue. Many notes have several possible fingerings, and the player must choose fingerings based on tuning and ease of motion as they change notes.

So why bother with cylindrical instruments? Remember that an actual note from any instrument is a very complex sound wave that includes lots of harmonics. The pitch that we hear when a wind instrument plays a note is (usually) the lowest harmonic that is being produced in the tube at the time. The higher harmonics produce the timbre[50], or sound color, of the instrument. A saxophone-shaped instrument simply can't get that odd-harmonics clarinet sound.

The shapes and sounds of the instruments that are popular today are the result of centuries of trial-and-error experimentation by instrument-makers. Some of them understood something of the physics involved, but the actual physics of real instruments - once you add sound holes, valves, keys, mouthpieces, and bells - are incredibly complex, and theoretical physicists are still studying the subject and making new discoveries.

3.2.5 Further Reading

- Alexander Wood's *The Physics of Music* (1944, The Sherwood Press) is a classic which includes both the basics of waves in a pipe and information about specific instruments.
- John Backus' *The Acoustical Foundations of Music* (1969, W.W. Norton and Company) also goes into more detail on the physics of specific instruments.
- John S. Rigden's *Physics and the Sound of Music* (1977, John Wiley and Sons) includes most of the math necessary for a really rigorous, complete explanation of basic acoustics, but is (in my opinion) still very readable.
- Arthur H. Benade's *Fundamentals of Musical Acoustics* is a more technical textbook that gives some idea of how acoustical experiments on instruments are designed and carried out. Those who are less comfortable with the science/engineering aspect of the subject may prefer the two very thorough articles by Benade in:
- *The Physics of Music* (W. H. Freeman and Co.), a collection of readings from the periodical *Scientific American*.

[49] "Interval" <http://cnx.org/content/m10867/latest/#p21b>
[50] "Timbre: The Color of Music" <http://cnx.org/content/m11059/latest/>

Solutions to Exercises in Chapter 3

Solution to Exercise 3.1.1 (p. 63)

1. The part of the string that can vibrate is shorter. The finger becomes the new "end" of the string.
2. The new sound wave is shorter, so its frequency is higher.
3. It sounds higher; it has a higher pitch.

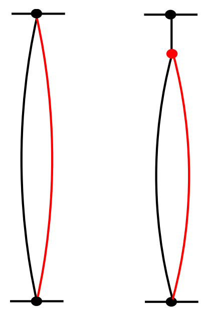

Figure 3.16: When a finger holds the string down tightly, the finger becomes the new end of the vibrating part of the string. The vibrating part of the string is shorter, and the whole set of sound waves it makes is shorter.

Solution to Exercise 3.1.2 (p. 67)

There are many, but here are some of the most familiar:

- Chimes
- All xylophone-type instruments, such as marimba, vibraphone, and glockenspiel
- Handbells and other tuned bells
- Steel pan drums

Index of Keywords and Terms

Keywords are listed by the section with that keyword (page numbers are in parentheses). Keywords do not necessarily appear in the text of the page. They are merely associated with that section. *Ex.* apples, § 1.1 (1) **Terms** are referenced by the page they appear on. *Ex.* apples, 1

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For middle school and up, a short course that explains acoustics (the physics of sound waves) as it relates to music and musical instruments. Suggestions for presenting some of the concepts to younger students are included.

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