MAT 350 - Applied Linear Algebra

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Class 1 - Thursday, September 19$^{\text{th}}$, 2017

1.1 §1 Differential Equations

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix}$$

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2.1 §2.8 Subspaces

Definition 1: Subspace

A subspace of rton is any set H in that has three properties:

- is in H
- For each u, v, in H u+v is also in H (Closure under addition)
- For each u in H and scalar C, cu is in H (Closure under scalar multiplication)

Example 1 *fdsaf Is H a subspace of rto2?*

- *is in h* ✓
- *u+v is not in H for all u,v in H*
- *Not closed under scalar multiplication either (-u not in H).*

H is not a subspace

Example 2

- *is in H* ✓
- *closed under addition* ✓

- *closed under scalar multiplication* ✓

  H is a subspace of rto2

Example 3 *Which of the following are subspaces of* $\mathbb{R}^2$*?*

- *not a subspace o is not in h*
- $\mathbb{R}^2$ *is a subspace of* $\mathbb{R}^2$
- $\mathbb{R}^2$ *is a subspace of* $\mathbb{R}^2$
-
  - ◇ *o is in H* ✓
  - ◇ *Closed under scalar multiplicaiton* ✓
  - ◇ *Not closed unfer addition x u+(-v) in in H.*

  not a subspace of $\mathbb{R}^2$
- *stuff*

So what are do subspaces of $\mathbb{R}^2$ look like?

- They are copies of $\mathbb{R}^0$, $\mathbb{R}^1$, $\mathbb{R}^2$... $\mathbb{R}^n$ that contain the zero vector.

Example 4 *In* $\mathbb{R}^3$*, possible subspaces are:*

- *Zero Subspaces*
- *Lines*
- *Planes*

Example 5 *If H=span*v_1*,*v_2 *(a*v_1*,b*v_2 *for any a,b), then H is a subspace of* $\mathbb{R}^n$

- *o is in H* ✓
  - ◇ *since o**v_1 *+o**v_2*=o*
- *closed under addition* ✓
  - ◇ $u = a_1v_1 + b_1v_2$
  - ◇ $w = a_2v_1 + b_2v_2$

- $\diamond$ $u + w = a_1v_1 + b_1v_2 + a_2v_1 + b_2v_2$
- $\diamond$ $u + w = (a_1 + a_2)v_1 + (b_1 + b_2)v_2$
- $\diamond$ *which is span*v_1,v_2

- *closed under scalar multiplication*
  - $\diamond$ $u = av_1 + bv_2$
  - $\diamond$ $c * u = c(av_1 + bv_2) = (c * a)v_1 + (c * b)v_2$

  which is spanv_1,v_2

Definition 2: Column Space

The column space of a matrix A, denoted col(A), is the set of all linear combinations of the columns of A. (col(a) is a subspace)

Example 6 *Let* $A = \begin{bmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{bmatrix}$ *and* $b = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$
is b in Col(A)?
if b is a linear combination of the columns of A, the Ax=b has a solution. We are asking whether [A—b] is consistant of not.
$\begin{bmatrix} 1 & -3 & -4 & | & 3 \\ -4 & 6 & -2 & | & 3 \\ -3 & 7 & 6 & | & -4 \end{bmatrix}$ *goes to* $\begin{bmatrix} 1 & -3 & -4 & | & 3 \\ 0 & -6 & -18 & | & 15 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$
No pivot in augmented column, so b is in col(A).
Note: b in col(A) for every b in $\mathbb{R}^m$

- *Ax=b has a solution for every b in* $\mathbb{R}^m$
- *The columns of A span* $\mathbb{R}^m$
- *A has a pivot in every row in REF*

Definition 3: Null Space

The null space of matrics A, denoted null(A), is the set of all solutions to Ax=$\vec{o}$

Note 1 *Any solution set of $Ax = \vec{o}$ can be written in parametric form.*

$x = x_2 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = span\{ \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \}$

so null(A) is a subspace.

Definition 4: Basis for a Subspace

A basis for a subspace H is a linearly independent set that spans H.

Example 7 *Find a basis for null(A), if* $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 & 0 & -1 & -3 & | & 0 \\ 0 & 0 & 1 & 2 & -2 & | & 0 \\ 0 & 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$

$$\begin{aligned} x_1 - 2x_2 - x_4 + 3x_5 &= 0 \\ x_2 &= x_2 \\ x_3 + 2x_4 - 2x_5 &= 0 \\ x_4 &= x_4 \\ x_5 &= x_5 \end{aligned}$$

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3.1 §2.8 Subspaces

Example 8 *Find a basis for Col(A) and Nul(A):*

$$
\begin{aligned}
A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} &\to \begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
&\to \begin{bmatrix} 1 & 0 & -4 & 7 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
x_1 - 4x_3 + 7x_4 &= 0 \\
x_2 + 5x_3 - 6x_4 &= 0 \\
x_3 &= x_3 \\
x_4 &= x_4
\end{aligned}
$$

Example 9