MAT 299 - Proofs and Problem Solving

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Homework 1 - Saturday, November 3rd, 2018

1.1 §1 Differential Equations

Problem 1 *Analyze the logical forms of the following statements. Use A to represent "Alice has a dog," B to represent "Bob has a dog," and C to represent "Carol has a cat" to write each as a symbolic statement.*

1. *Either Alice or Bob has a dog.*
   $A \vee B$
2. *Neither Alice nor Bob has a dog, but Carol has a cat.*
   $\neg(A \wedge B) \wedge C$
3. *Either Alice has a dog and Carol has a cat, or Bob has a dog and Carol does not have a cat.*
   $(A \wedge C) \vee (B \wedge \neg C)$

This is similar to Example 1.1.2 and to Exercise 2 in Section 1.1 of your SNHU MAT299 textbook.

Problem 2 *If D stands for "Doug is tall" and E stands for "Edie is short," what English sentences are represented by the following expressions?*

1. $(D \wedge E) \vee \neg D$
   Either Doug is tall and Edie is short, or Doug is not tall
2. $(D \vee \neg E) \wedge \neg(D \wedge E)$
   Either Doug is tall or Edie is not short and both Doug is not tall and Edie is not short

3. $\neg D \wedge ((E \wedge D) \vee \neg E)$
   Doug is not tall and either Edie is short and Doug is tall, or Eddie is not short

This is similar to Example 1.1.3 and to Exercise 6 in Section 1.1 of your SNHU MAT299 textbook.

Problem 3 *Make a truth table for the following formula.*

$$(G \vee \neg H) \wedge \neg(G \wedge L)$$

This is similar to Example 1.2.2 and to Exercise 2 in Section 1.2 of your SNHU MAT299 textbook.

G	H	L	$\neg H$	$G \vee \neg H$	$G \wedge L$	$\neg G \wedge L$	$(G \vee \neg H) \wedge \neg(G \wedge L)$
T	T	T	F	T	T	F	F
T	T	F	F	T	F	T	T
T	F	T	T	T	T	F	F
T	F	F	T	T	F	T	T
F	T	T	F	F	F	T	F
F	T	F	F	F	F	T	F
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

Problem 4 *Use truth tables to determine which of the following formulas are equivalent to each other.*

1. $(J \wedge K) \vee (\neg J \wedge \neg K)$

J	K	$J \wedge K$	$\neg J$	$\neg K$	$\neg J \wedge \neg K$	$(J \wedge K) \vee (\neg J \wedge \neg K)$
T	T	T	F	F	F	T
T	F	F	F	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	T	T

2. $J \vee K$

J	K	$J \vee K$
T	T	T
T	F	T
F	T	T
F	F	F

3. $J \wedge \neg K$

J	K	$\neg K$	$J \wedge \neg K$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

4. $\neg(\neg J \vee K)$

J	K	$\neg J$	$\neg J \vee K$	$\neg(\neg J \vee K)$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	F

5. $(J \wedge \neg K) \vee K$

J	K	$\neg K$	$J \wedge \neg K$	$(J \wedge \neg K) \vee K$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	F

This is similar to Example 1.2.4 and to Exercise 8 in Section 1.2 of your SNHU MAT299 textbook.

Problem 5 *Use truth tables to determine which of the following formulas are tautologies, which are contradictions, and which are neither.*

1. $(M \wedge \neg N) \vee (\neg M \wedge N)$

M	N	$\neg N$	$M \wedge \neg N$	$\neg M$	$\neg M \wedge N$	$(M \wedge \neg N) \vee (\neg M \wedge N)$
T	T	F	T	F	F	T
T	F	T	F	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	F	F

2. $(M \wedge \neg N) \wedge (\neg M \wedge N)$

M	N	$\neg N$	$M \wedge \neg N$	$\neg M$	$\neg M \wedge N$	$(M \wedge \neg N) \wedge (\neg M \wedge N)$
T	T	F	T	F	F	F
T	F	T	F	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	F	F

This is a Contradiction because the formula will always evaluate to false.

3. $(\neg M \wedge \neg N) \vee (\neg M \vee N) \vee (M \wedge \neg N)$

M	N	$\neg M$	$\neg N$	$\neg M \wedge \neg N$	$\neg M \vee N$	$M \wedge \neg N$	$(\neg M \wedge \neg N) \vee$ $(\neg M \vee N)$	$(\neg M \wedge \neg N) \vee$ $(\neg M \vee N) \vee$ $(M \wedge \neg N)$
T	T	F	F	F	T	F	T	T
T	F	F	T	F	F	T	F	T
F	T	T	F	F	T	F	T	T
F	F	T	T	T	T	F	T	T

This is similar to Example 1.2.6 and to Exercise 9 in Section 1.2 of your SNHU MAT299 textbook.

Problem 6 *Use the laws stated in the text to find simpler formulas equivalent to these formulas. Explain the reasoning that you used to find your solution.*

1. $\neg(\neg Q \vee (\neg P \wedge Q))$
2. $((P \wedge Q) \wedge \neg R) \vee (P \wedge \neg(Q \vee R))$

This is similar to Examples 1.2.5, 1.2.6, and 1.2.7 and to Exercise 12 in Section 1.2 of your SNHU MAT299 textbook

$\vee$ Or Vee
$\wedge$ and Wedge
$\neg \cup \vee\wedge$

$\square$