Exercises Week 3

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Optimization in Energy Systems and Markets
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Contents

1 Exercise 1

Implement in the AML of your choice the first version of the Unit Commitment (UC) problem seen in class. This version of UC includes on/off status of thermal generators, detailed limits on the generation capacity of thermal generators, and start-up and shut-down costs.

Check your implementation with the instance seen in class. The technical parameters of thermal generators and demand are given in Table 1 and Table 2, respectively.

Gen.	C_i^q	C_i^l	C_i^b	C_i^{SU}	$P_i^{\min}$	$P_i^{\max}$	R_i	$P_{i,0}$	$U_{i,0}$
1	0.05	20	100	300	80	400	160	0	0
2	0.1	25	200	400	60	300	150	0	0
3	0.2	40	300	500	40	200	100	100	1

Table 1: Technical parameters of thermal generators.

Periodo	1	2	3	4
D_t	40	250	300	600

Table 2: Electricity Demand.

Proof. We will implement the Unit Commitment (UC) problem following the slides of the coURse, in AMPL using the CPLEX solver. The translation of the problem into this AML is straightforward and shall not be discussed in this exercise. However we note that we set the CPLEX option `qpmethod=4` which enforces the solver to use the Barrier optimizer in order to solve our Mixed-Integer Quadratic Program (MIQP). The results are as follows:

Cost Component	Amount
Total Production Cost	45,196.67
Total Startup/shutdown Cost	1,200.00
Total Cost	**46,396.67**

Table 3: Cost breakdown summary

Gen\Period	1	2	3	4
1	0	1	1	1
2	0	1	1	1
3	1	0	0	0

Table 4: Generator on/off status ($U_{i,t}$)

Gen\Period	1	2	3	4
1	0.00	160.00	236.67	396.67
2	0.00	90.00	63.33	203.33
3	40.00	0.00	0.00	0.00

Table 5: Power generation ($P_{i,t}$)

Gen\Period	1	2	3	4
1	0.00	300.00	0.00	0.00
2	0.00	400.00	0.00	0.00
3	0.00	500.00	0.00	0.00

Table 6: Startup/shutdown costs by generator and period

Period	Demand	Total Gen.	Diff.
1	40.00	40.00	0.00
2	250.00	250.00	0.00
3	300.00	300.00	0.00
4	600.00	600.00	0.00

Table 7: Demand balance verification (MW)

We observe that the optimal solution at time period $t = 1$ relies entirely on the already operational (at $t = 0$) generator $i = 3$ to provide the full demand. However, out of the three generators, it is the one with the highest operational and startup/shutdown costs, so at $t \geq 2$ we switch to generators $i = 1, 2$ to provide the rest of the energy. Table 5 and Table 7 show that this commitment is indeed feasible, given that the demand values are satisfied and the power generations is within the capacity constraints of every generation, at every time period.

Moreover, we compare our solution to the one present in Slide 20 of the Unit Commitment course slides, and we ideed see that it is the same solution. □

2 Exercise 2

Consider now the problem from Exercise 1, with the extra requirement to guarantee certain amounts of reserve at every time period. The upwards and downwards reserve requirements RU_t and RD_t, respectively, are given in Table 8.

Periodo	1	2	3	4
RU_t	10	50	80	150
RD_t	10	50	80	150

Table 8: Upwards and downwards reserve requirements.

1. Find the fully parametrised mathematical formulation of the UC problem with reserve (based on the first version of UC).

   Proof. We incorporate two new decision variables into the model in order to introduce reserve. Indeed, according to Slide 21 from Unit Commitment, upwards/downwards reserve corresponds to the "electric power available to be increased/decreased in a short time period after the activation instruction of the system operator". This means that we should model reserve as a capacity which the generator is committed to achieve under the rule of the system operator, but not actual energy scheduled to be produced at each time period.

   - $w^{\mathrm{UR}}_{i,t} \in \mathbb{R}_{\geq 0}$: Upward reserve capacity of generator $i \in \mathcal{G}$ in period $t \in \mathcal{T}$ [MW]
   - $w^{\mathrm{DR}}_{i,t} \in \mathbb{R}_{\geq 0}$: Downward reserve capacity of generator $i \in \mathcal{G}$ in period $t \in \mathcal{T}$ [MW]

   For this reason, these new decision variables will not participate in the objective function, but only in capacity constraints (another reason is the fact that reserve energy is traded in a different market, as we have seen in the course).

$$
\begin{aligned}
(\text{UC-R}) = \min_{p,u,w^{\mathrm{UR}},w^{\mathrm{DR}},v^{\mathrm{SU}} \in \mathbb{R}^{|\mathcal{G}|\times|\mathcal{T}|}} \quad & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{G}} \left(C^q_i p^2_{i,t} + C^l_i p_{i,t} + C^b_i u_{i,t} \right) + \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{G}} v^{\mathrm{SU}}_{i,t} && (1)\\
\text{s.t.} \quad & \sum_{i\in\mathcal{G}} p_{i,t} = D_t, \quad \forall t \in \mathcal{T} && (2)\\
& \sum_{i\in\mathcal{G}} w^{\mathrm{UR}}_{i,t} \geq \mathrm{RU}_t, \quad \forall t \in \mathcal{T} && (3)\\
& \sum_{i\in\mathcal{G}} w^{\mathrm{DR}}_{i,t} \geq \mathrm{RD}_t, \quad \forall t \in \mathcal{T} && (4)\\
& u_{i,t} P^{\min}_i \leq p_{i,t} - w^{\mathrm{DR}}_{i,t}, \quad \forall i \in \mathcal{G}, \forall t \in \mathcal{T} && (5)\\
& p_{i,t} + w^{\mathrm{UR}}_{i,t} \leq u_{i,t} P^{\max}_i, \quad \forall i \in \mathcal{G}, \forall t \in \mathcal{T} && (6)\\
& -R_i \leq p_{i,1} - P_{i,0} \leq R_i, \quad \forall i \in \mathcal{G} && (7)\\
& -R_i \leq p_{i,t} - p_{i,t-1} \leq R_i, \quad \forall i \in \mathcal{G}, \forall t \in \mathcal{T}, t > 1 && (8)\\
& v^{\mathrm{SU}}_{i,1} \geq C^{\mathrm{SU}}_i (u_{i,1} - U_{i,0}), \quad \forall i \in \mathcal{G} && (9)\\
& v^{\mathrm{SU}}_{i,t} \geq C^{\mathrm{SU}}_i (u_{i,t} - u_{i,t-1}), \quad \forall i \in \mathcal{G}, \forall t \in \mathcal{T}, t > 1 && (10)\\
& v^{\mathrm{SU}}_{i,1} \geq C^{\mathrm{SU}}_i (U_{i,0} - u_{i,1}), \quad \forall i \in \mathcal{G} && (11)\\
& v^{\mathrm{SU}}_{i,t} \geq C^{\mathrm{SU}}_i (u_{i,t-1} - u_{i,t}), \quad \forall i \in \mathcal{G}, \forall t \in \mathcal{T}, t > 1 && (12)\\
& v^{\mathrm{SU}}_{i,t} \geq 0, \quad \forall i \in \mathcal{G}, \forall t \in \mathcal{T} && (13)\\
& u_{i,t} \in \{0, 1\}, \quad \forall i \in \mathcal{G}, \forall t \in \mathcal{T} && (14)\\
& p_{i,t}, w^{\mathrm{UR}}_{i,t}, w^{\mathrm{DR}}_{i,t} \geq 0, \quad \forall i \in \mathcal{G}, \forall t \in \mathcal{T} && (15)
\end{aligned}
$$

   Observe that constraints (5), (6) are the only ones which actually compromise the generation and depend on the technical requirements of the generator.

   □

2. Implement it in the AML of your choice and compare the solution with the solution from Exercise 1.

Proof. Again, the implementation is straightforward and will not be described. For this particular instance, the problem is declared as unfeasible by the solver. Indeed:

Gen\Period	1	2	3	4
1	0	0	0	1
2	0	0	0	0
3	1	0	0	0

Table 9: Generator on/off status ($u_{i,t}$)

Gen\Period	1	2	3	4
1	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00
3	40.00	0.00	0.00	0.00

Table 10: Scheduled power output ($p_{i,t}$)

Gen\Period	1	2	3	4
1	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00

Table 11: Startup/shutdown costs

Period	Demand	Total Sched.	Diff.
1	40.00	40.00	0.00
2	250.00	0.00	-250.00
3	300.00	0.00	-300.00
4	600.00	0.00	-600.00

Table 12: Demand balance check (MW)

Period	RU_t Req.	Total UR	OK	RD_t Req.	Total DR	OK
1	10.00	10.00	YES	10.00	0.00	NO
2	50.00	0.00	NO	50.00	0.00	NO
3	80.00	0.00	NO	80.00	0.00	NO
4	150.00	0.00	NO	150.00	0.00	NO

Table 13: Reserve requirements check

Generator 3 is the only economically viable unit to satisfy the low demand of 40 MW in period 1. However, this creates an infeasible situation:

(a) Generator 3 must produce $p_{3,1} = 40$ MW to meet demand.

(b) Since Generator 3 is committed ($u_{3,1} = 1$), it operates at its minimum:

$$p_{3,1} = P_3^{\min} = 40 \text{ MW}$$

(c) The capacity lower bound requires:

$$\begin{aligned} p_{3,1} - w_{3,1}^{\mathrm{DR}} &\geq P_3^{\min} \\ 40 - w_{3,1}^{\mathrm{DR}} &\geq 40 \\ w_{3,1}^{\mathrm{DR}} &\leq 0 \Rightarrow w_{3,1}^{\mathrm{DR}} = 0 \end{aligned}$$

Hence, Generator 3 cannot provide downward reserve.

(d) The system requires $\sum_{i\in\mathcal{G}} w_{i,1}^{\mathrm{DR}} \geq 10$ MW, but only 0 MW is available.

For the sake of solving a problem instance, we propose to decrease the value of $P_3^{\min} = 20$ in order to make the problem feasible. The results are as follows:

Cost Component	Amount
Quadratic Cost	15,548.44
Linear Cost	27,978.12
Fixed Cost	2,100.00
Startup/Shutdown Cost	700.00
Total Cost	**46,326.56**

Table 14: Cost breakdown summary

Gen\Period	1	2	3	4
1	0	1	1	1
2	0	1	1	1
3	1	1	1	1

Table 15: Generator on/off status ($u_{i,t}$)

Gen\Period	1	2	3	4
1	0.00	160.00	213.12	373.12
2	0.00	70.00	66.88	176.25
3	40.00	20.00	20.00	50.63

Table 16: Scheduled power output ($p_{i,t}$)

Gen\Period	1	2	3	4
1	0.00	240.00	186.88	26.88
2	0.00	230.00	233.12	123.75
3	10.00	180.00	180.00	149.38

Table 17: Upward reserve capacity ($w_{i,t}^{UR}$)

Gen\Period	1	2	3	4
1	0.00	80.00	133.12	293.12
2	0.00	10.00	6.88	116.25
3	10.00	0.00	0.00	30.63

Table 18: Downward reserve capacity ($w_{i,t}^{DR}$)

Gen\Period	1	2	3	4
1	0.00	300.00	0.00	0.00
2	0.00	400.00	0.00	0.00
3	0.00	0.00	0.00	0.00

Table 19: Startup/shutdown costs

Period	Demand	Total Sched.	Diff.
1	40.00	40.00	0.00
2	250.00	250.00	0.00
3	300.00	300.00	0.00
4	600.00	600.00	0.00

Table 20: Demand balance check (MW)

Period	RU_t Req.	Total UR	OK	RD_t Req.	Total DR	OK
1	10.00	10.00	YES	10.00	10.00	YES
2	50.00	650.00	YES	50.00	90.00	YES
3	80.00	600.00	YES	80.00	140.00	YES
4	150.00	300.00	YES	150.00	440.00	YES

Table 21: Reserve requirements check

□