Exercises Week 2

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Optimization in Energy Systems and Markets
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Contents

1 Exercise 1

Consider an electric power system formed exclusively by solar panels and an energy storage system (a battery or a pump storage plant).

(a) Model in detail the operational constraints of the energy storage system. An energy storage system is characterised by:

- A storage capacity E [MWh]: the maximum amount of energy that can be stored.
- A maximum charge and discharge power rate $P^{\max}$ [MW]: the maximum energy flow that can enter or leave the energy storage system. We assume this rate to be the same for charging and discharging.
- A round-trip efficiency ρ: during a full charging and discharging cycle, a percentage $1 - \rho$ of the energy is lost due to inefficiencies in the process.
- Limits in the states of charge allowed: minimum ($\underline{\mathrm{SOC}}$) and maximum ($\overline{\mathrm{SOC}}$). They are expressed as a percentage of the storage capacity E.
- Initial state of charge SOC_0 and (sometimes) final state of charge SOC_T.

Define the required decision variables and constraints that allow to model the feasible actions of the energy storage system and model its state of charge.

Proof. We shall first propose the following parameters:

- $\mathcal{T} = \{1, \ldots, T\}$: set of time periods.
- E : storage capacity of energy storage system [MWh].
- $P^{\max}$: maximum charge and discharge rate [MW].
- ρ: round-trip efficiency (%).
- $\underline{\mathrm{SOC}}$: minimum charge allowed (%).
- $\overline{\mathrm{SOC}}$: maximum charge allowed (%).
- SOC_0: initial stage of charge.
- SOC_T: final stage of charge.
- Δ_t: time elapsed between time periods $t - 1$ and t [h].

At any time period $t \in \mathcal{T}$ we can either charge or discharge the energy storage system a certain amount of energy [MWh]:

- x_t: theoretical amount of energy supplied to the system during time period t [MWh]
- y_t: theoretical amount of energy retrieved from the system during time period t [MWh]

Here, ”theoretical” means actual amount of energy if no energy was lost during energy transfer. Note that this means that supplying/retrieving a certain amount of energy translates to actually giving/taking less/more energy to/from the energy storage system.

We assume that it is feasible to perform supply and retrieval actions during a same time period $t \in \mathcal{T}$. Given the properties described, we can then characterize the energy storage system and its iteractions via the following constraints:

$$\underline{\mathrm{SOC}} \leq \mathrm{SOC}_0 + \sum_{t'=1}^{t} \rho x_{t'} - \frac{1}{\rho} y_{t'} \leq \overline{\mathrm{SOC}}, \quad t \in \mathcal{T} \tag{1}$$

$$\frac{\rho x_{t'}}{\Delta_t} \leq P^{\max}, \quad \frac{1/\rho \cdot y_{t'}}{\Delta_t} \leq P^{\max}, \quad t \in \mathcal{T} \tag{2}$$

$$\mathrm{SOC}_0 + \sum_{t'=1}^{T} \rho x_{t'} - \frac{1}{\rho} y_{t'} = \mathrm{SOC}_T \tag{3}$$

□

(b) How would you model the generation of solar panels?

Proof. Following the slides for the Economic Dispatch problem, we can provide a model similar to wind farms. Indeed, for the sake of our problem, solar panels simply correspond to renewable source of energy whose generation we cannot control and yields no direct production cost. For that reason, we shall only consider the parameter

- S_t: solar panel generation during time period $t \in \mathcal{T}$ [MW].

□

(c) Consider an Economic Dispatch (ED) problem with 4 periods (we can assume them to be 1 hour periods) in a power system formed exclusively by solar panels and an energy storage system.

The hourly total system demand and solar generation are given in Table 1.

Period	1	2	3	4
Demand [MW]	100	300	400	100
Solar [MW]	0	1000	500	100

Table 1: Total system demand and solar generation.

The technical characteristics and initial and final states of charge of the energy storage system are given in Table 2.

Obtain the fully parametrised mathematical formulation of this Economic Dispatch problem.

Proof. Under this framework note that no cost function is clear to be defined given that our solar power generation yields no actual production cost. However, in order to specify an Economic Dispatch problem, we shall propose utilization costs for both the energy storage system and the solar panels:

- C^{s}: solar panel power generation (degradation) cost per watt [W^{-1}].
- C^{e}: energy storage system utilization cost per watt [W^{-1}].

Moreover, consider the following parameters:

- S_t: solar power generation during time period $t \in \mathcal{T}$ [W].
- D_t: power demand during time period $t \in \mathcal{T}$ [W].

Energy Capacity [MWh]	E	1000
Maximum charge and discharge power rate [MW]	$P^{\max}$	500
Round-trip efficiency	ρ	80%
Maximum state of charge	$\overline{\text{SOC}}$	90%
Minimum state of charge	$\underline{\text{SOC}}$	10%
Initial state of charge	SOC_0	90%
Final state of charge	SOC_T	90%

Table 2: Technical characteristics and initial and final states of charge of the energy storage systems.

We shall take into account all the decisions and constraints described previously concerning the energy storage system. At any time period $t \in \mathcal{T}$ we can also perform another type of decision:

- s_t: solar power energy served directly during time period $t \in \mathcal{T}$ [MWh].

This way, we can distribute S_t into supply for the demand via s_t or store in our unit via x_t.
Given the hour time periods we can simplify $\Delta_t = 1$ for all $t \in \mathcal{T}$. Therefore, we can model our Economic Dispatch (ED) problem as follows:

$$
\begin{aligned}
\text{(ED)} = \quad \min_{s_t, x_t, y_t \in \mathbb{R}^{3\times 4}} \quad & \sum_{t=1}^{T} C^s s_t + C^e \left(x_t + y_t\right) \\
\text{s.t.} \quad & s_t + x_t \leq S_t, \quad t \in \mathcal{T} \\
& s_t + y_t = D_t, \quad t \in \mathcal{T} \\
& \underline{\text{SOC}} \leq \text{SOC}_0 + \sum_{t'=1}^{t} \rho x_{t'} - \frac{1}{\rho} y_{t'} \leq \overline{\text{SOC}}, \quad t \in \mathcal{T} \\
& \rho \cdot x_{t'} \leq P^{\max}, \quad 1/\rho \cdot y_{t'} \leq P^{\max}, \quad t \in \mathcal{T} \\
& \text{SOC}_0 + \sum_{t'=1}^{T} \rho x_{t'} - \frac{1}{\rho} y_{t'} = \text{SOC}_T
\end{aligned}
$$

□

(d) Implement and solve it in AMPL or the Algebraic Modelling Language of your choice. Upload your model files with the name `Surnames_wk2.zip`. How much does it cost to serve demand in this electric power system?

Proof. We shall not describe the code implementation in this report. Our model in particular needs the specification of values for C^s and C^e. We have found that the magnitude of these parameters only affects in the value of the objective value; the set of decisions is always the same for the two cases $C^s \leq C^e$ and $C^s > C^e$:

Case $C^s \leq C^e$

We set $C^s = C^e = 0.10\ \text{W}^{-1}$. The optimal solution has a total cost of **105.62**.

Period	Solar Used [MWh]	Charged [MWh]	Discharged [MWh]	Demand [MWh]	SOC [%]
1	0.00	0.00	100.00	100.00	77.50
2	300.00	56.25	0.00	300.00	82.00
3	400.00	100.00	0.00	400.00	90.00
4	100.00	0.00	0.00	100.00	90.00

Table 3: Hourly operation of the solar panels and energy storage system ($C^s \leq C^e$).

Metric	Value
Initial SOC	90.00%
Final SOC	90.00%
Total Energy Charged	156.25 MWh
Total Energy Discharged	100.00 MWh
Total Solar Generation	1600.00 MWh
Total Demand Served	900.00 MWh

Table 4: Summary statistics for the energy storage system and power generation ($C^s \leq C^e$).

Case $C^s > C^e$

We set $C^s = 0.10\ \mathrm{W}^{-1}$ and $C^e = 0.01\ \mathrm{W}^{-1}$. The optimal solution has a total cost of **47.03**.

Period	Solar Used [MWh]	Charged [MWh]	Discharged [MWh]	Demand [MWh]	SOC [%]
1	0.00	0.00	100.00	100.00	77.50
2	0.00	625.00	300.00	300.00	90.00
3	222.22	277.78	177.78	400.00	90.00
4	100.00	0.00	0.00	100.00	90.00

Table 5: Hourly operation of the solar panels and energy storage system ($C^s > C^e$)

Metric	Value
Initial SOC	90.00%
Final SOC	90.00%
Total Energy Charged	902.78 MWh
Total Energy Discharged	577.78 MWh
Total Solar Generation	1600.00 MWh
Total Demand Served	900.00 MWh

Table 6: Summary statistics for the energy storage system and power generation ($C^s > C^e$).

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