

2. c)

Define the potential cost $\Phi(D_i) = 2 \cdot (\text{\# of the elements in the enqueue stack})$, denoted as m_i in the following statement..

$\Phi(D_0) = 0$, the enqueue stack is empty initially.

➤ cost of operation ENQUEUE:

$$\begin{aligned}\hat{c}_i &= c_i + \Phi(D_i) - \Phi(D_{i-1}) \\ &= 1 + 2m_i - 2m_{i-1} \\ &= 1 + 2(m_{i-1} + 1) - 2m_{i-1} \\ &= 1 + 2 = 3 = O(1)\end{aligned}$$

➤ cost of operation DEQUEUE – dequeue stack is not empty:

$$\begin{aligned}\hat{c}_i &= c_i + \Phi(D_i) - \Phi(D_{i-1}) \\ &= 1 + 2m_i - 2m_{i-1} \\ &= 1 + 2(m_{i-1}) - 2m_{i-1} \\ &= 1 + 0 = 1 = O(1)\end{aligned}$$

➤ cost of operation DEQUEUE – dequeue stack is empty:

$$\begin{aligned}\hat{c}_i &= c_i + \Phi(D_i) - \Phi(D_{i-1}) \\ &= (2m_{i-1} + 1) + 2m_i - 2m_{i-1} \\ &= (2m_{i-1} + 1) + 0 - 2m_{i-1} \\ &= 1 = O(1)\end{aligned}$$

So, amortized cost of each operation is $O(1)$.

Amortized cost of a sequence of n operations is $O(n)$

※The definition of potential cost above is not the only solution.