INTRO TO ALGORITHMS FINAL EXAM

1 a) Chap 15, p. 47
  b) Chap 15, p. 49
  c) Chap 15, pp. 29,50

2 a) Chap 16, pp. 11~12
  b) Chap 16, pp. 13,14
  c) Chap 16, p. 7

3 a) Ex 17.1-3 (See the solution posted publicly)
  b) Ex 17.2-2 (See the solution posted publicly)

4 a) Chap 17, p.40
  b) Chap 17, pp.31,43
  c) Let the ith operation be the delete operation in question.
  Then, $num_i = size_i = 0$, and $\Phi(T_i) = 0$
  There are three cases.
  Case 1: $num_{i-1} = size_{i-1} = 1$
  $\Phi(T_{i-1}) = 2 \cdot num_{i-1} - size_{i-1} = 1$
  $\hat{c}_i = 1 + \Phi(T_i) - \Phi(T_{i-1}) = 1 + 0 - 1 = 0$
  Case 2: $num_{i-1} = 1,\ size_{i-1} = 2$
  $\Phi(T_{i-1}) = 2 \cdot num_{i-1} - size_{i-1} = 0$
  $\hat{c}_i = 1 + \Phi(T_i) - \Phi(T_{i-1}) = 1 + 0 - 0 = 1$
  Case 3: $num_{i-1} = 1,\ size_{i-1} = 4$
  $\Phi(T_{i-1}) = size_{i-1}/2 - num_{i-1} = 1$
  $\hat{c}_i = 1 + \Phi(T_i) - \Phi(T_{i-1}) = 1 + 0 - 1 = 0$

5 a) Probably true
  PRIME ∈ P is true. (Chap 34, p.10)
  But, P = NP may or may not be true.
  If P = NP, the statement is true.
  If P≠NP, the statement is false.
  b) Certainly true
  It is known that CLIQUE ∈ NPC. (Chap 34, p.71)
  Thus, if CLIQUE ∈ P, then P = NP. (Chap 34, p.44)

5 c) Certainly true

Since "the Hanoi Tower problem can be solved in polynomial time" is false, it follows that the statement is true. (Chap 34, p.37)

d) Certainly true

It is known that KNAPSACK $\in$ NPC.

Thus, if $\overline{\text{KNAPSACK}}$ $\in$ NP, then NP = co-NP. (Chap 34, p.47)

6 a) Chap 34, p.96

b) The time complexity is $O(nW) + O(1) = O(nW)$.

Chap 34, p.12

7 Chap 34, pp.82~84

8 HW#6, EX 34.1-1

9 a) Chap 35, pp.8,9

b) Chap 35, pp.10,11

9 Consider the knapsack optimization problem

$$\text{Maximize } \sum_{i=1}^{n} v_i x_i \quad \text{subject to} \sum_{i=1}^{n} w_i x_i \leq W, \qquad \text{where } x_i = 0 \text{ or } 1$$

As discussed in class, the dynamic programming solution of this problem takes a time in $O(nW)$.

Let $\text{KNAPSACK} = \left\{ \langle W, v_i, w_i, B \rangle \,\middle|\, \begin{array}{l} \text{There exists } x_i = 0 \text{ or } 1 \text{ such that} \\ \sum_{i=1}^{n} w_i x_i \leq W \text{ and } \sum_{i=1}^{n} v_i x_i \geq B \end{array} \right\}$

a) Show how to reduce KNAPSACK to the knapsack optimization problem, assuming that the latter is solved by dynamic programming.
Explain why this reduction doesn't yield a polynomial-time algorithm for KNAPSACK.

b) In fact, KNAPSACK is NPC.
In this part, you are asked to show that PARTITION $\leq_p$ KNAPSACK.
PARTITION