Define:

|  |
| --- |
|  |

Equation 1

Where:

|  |
| --- |
|  |

Equation 2

(these are lag-day returns)

Where:

|  |
| --- |
|  |

Equation 3

Define:

|  |
| --- |
|  |

Equation 4

Then if , otherwise, :

|  |
| --- |
|  |

Equation 5

And since we can easily prove that:

|  |
| --- |
|  |

Equation 6

With:

|  |
| --- |
|  |

Equation 7

So for small lag, we can analytically evaluate , which is the multivariate correlated normal cdf, evaluated in +K or – K depending on .

Then define:

|  |
| --- |
|  |

Equation 8

Which can also be evaluated analytically if lag+1 remains tractable.

We then can easily see a way to compute:

|  |
| --- |
|  |

Equation 9

By simply taking the correct combinations of in the distribution of the Equation 8.

Once we have the above distribution, we see that we can compute:

|  |
| --- |
|  |

Equation 10

By the following property:

|  |
| --- |
|  |

Equation 11

And:

|  |
| --- |
|  |

Equation 12

Since , and:

|  |
| --- |
|  |

Equation 13

Which can be evaluated since we know the numerator (distributed identically as in Equation 5), and the denominator (can also easily be computed from the distribution of Equation 5).

For which the numerator was given above (Equation 9) and the denominator can be computed by:

|  |
| --- |
|  |

Equation 14

Now we have evaluated:

|  |
| --- |
|  |

Equation 15

And we can again, group the first two random variables, and evaluate:

|  |
| --- |
|  |

Equation 16

From which we can evaluate:

|  |
| --- |
|  |

Equation 17

In the same manner as we went from Equation 10 to Equation 16.

And we can iterate and find the distribution of .

Confidence intervals are then easy to evaluate.

In total (not very readable), we have:

mvncdf with structure in correl matrix:

https://arxiv.org/pdf/1809.08315.pdf