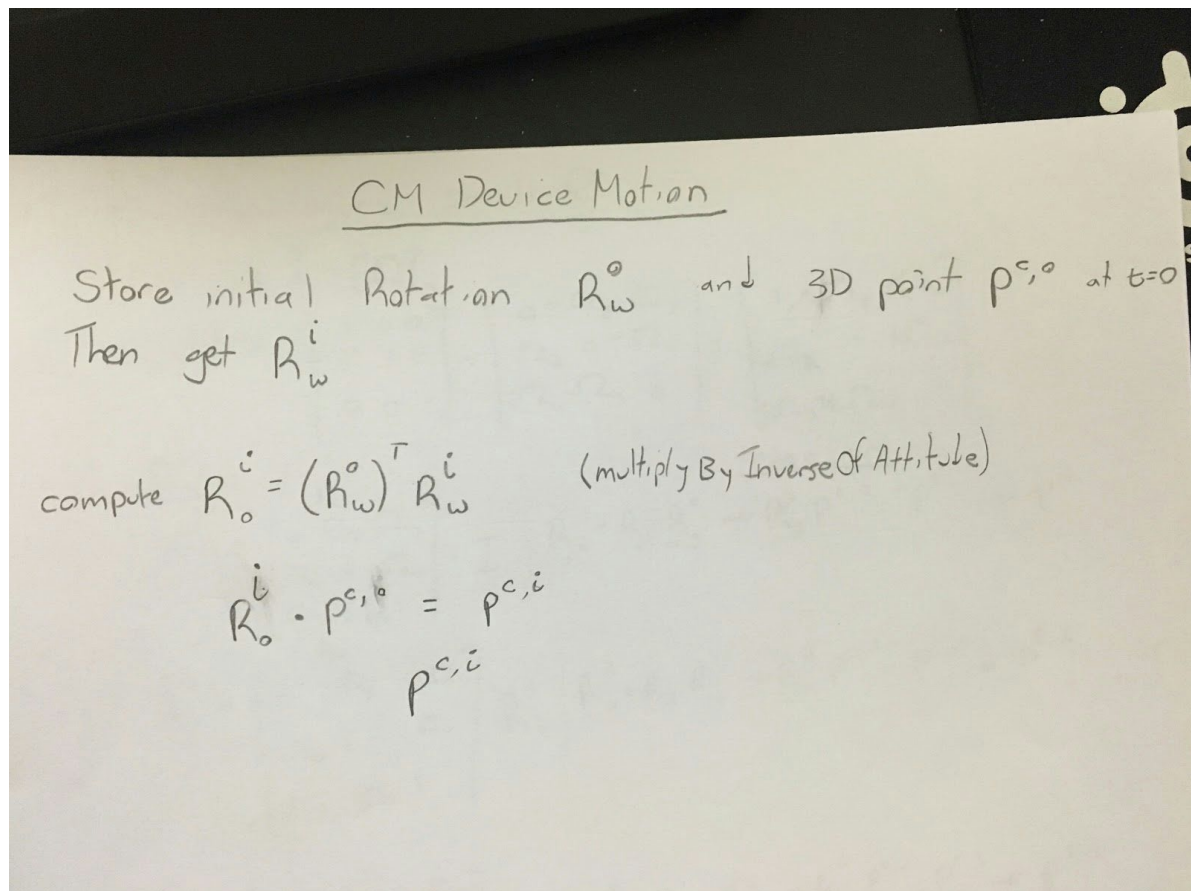


1) Describe the differences between the three modalities.

- Device Motion:
 - With the device motion mode we extract the phone's attitude from the handler when we start up the devicemotion. With that data we use the method `multiplyByInverseAttitude` to calculate the overall rotation matrix to then multiply that to the point to find the new pixel location.
- Rodriguez Formula Small Angle Approximation
 - In this mode we pulled data from the gyroscope of the iphone and created a rotational matrix based on this data. The matrix varied for each time it was pulled but we took the resulting matrix and multiplied it against the point in the camera reference frame. This gave us the new camera points and the only thing left was to convert it to the new pixel location. This differs from the regular Rodriguez formula because this is an approximation that is used for small angles of rotation.
- Rodriguez Formula Full
 - Same thing as the small angle formula except there are more calculations used to get a much more precise result.



Rodriguez small Θ

$$R(\Omega) \approx I - [\Omega]_x$$

$$R_0^t = R(\Omega) \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix} = \begin{bmatrix} 1 & t\Omega_z & -t\Omega_y \\ -t\Omega_z & 1 & t\Omega_x \\ t\Omega_y & -t\Omega_x & 1 \end{bmatrix}$$

$$t=0 \quad R_0^0 = I$$

$$t=1 \quad R_0^1 = \begin{bmatrix} 1 & \Omega_z & -\Omega_y \\ -\Omega_z & 1 & \Omega_x \\ \Omega_y & -\Omega_x & 1 \end{bmatrix} \cdot I \quad \left| \quad \underline{R}_0^1 = \underline{R}_1 \cdot \underline{R}_0^0 \rightarrow \underline{R}_0^1 p^{c,0} = p^{c,1} \rightarrow p^{s,1} \right.$$

$$t=2 \quad R_0^2 = \begin{bmatrix} 1 & \Omega_z & -\Omega_y \\ \Omega_z & 1 & \Omega_x \\ \Omega_y & -\Omega_x & 1 \end{bmatrix} R_0^1 \quad \left| \quad \underline{R}_0^2 = \underline{R}_2 \cdot \underline{R}_0^1 \rightarrow \underline{R}_0^2 p^{c,1} = p^{c,2} \rightarrow p^{s,2} \right.$$

$$t=3 \quad R_0^3 = \begin{bmatrix} 1 & \Omega_z & -\Omega_y \\ \Omega_z & 1 & \Omega_x \\ \Omega_y & -\Omega_x & 1 \end{bmatrix} R_0^2 \quad \left| \quad \underline{R}_0^3 = \underline{R}_3 \cdot \underline{R}_0^2 \rightarrow \underline{R}_0^3 p^{c,2} = p^{c,3} \rightarrow p^{s,3} \right.$$

trace off

$$p^{c,0} \rightarrow \underline{p^{s,0}}$$

Rodriguez Formula Full

$$R_o^t = R(\Omega) = I - [\Omega]_x \frac{(\sin \|\Omega\|)}{\|\Omega\|} + [\Omega]_x^2 \frac{(1 - \cos \|\Omega\|)}{\|\Omega\|^2}$$

$$R(\Omega) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix} \frac{\|\Omega\|}{\|\Omega\|} \frac{(\sin \|\Omega\|)}{\|\Omega\|} + \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix}^2 \frac{\|\Omega\|^2}{\|\Omega\|^2} \frac{(1 - \cos \|\Omega\|)}{\|\Omega\|^2}$$

$$R(\Omega) = \begin{bmatrix} 1 & \Omega_z \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) & -\Omega_y \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) \\ -\Omega_z \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) & 1 & \Omega_x \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) \\ \Omega_y \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) & -\Omega_x \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) & 1 \end{bmatrix} + \begin{bmatrix} (\Omega_z^2 - \Omega_y^2) \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & (\Omega_z \Omega_y) \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & \Omega_z \Omega_x \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) \\ \Omega_x \Omega_y \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & (-\Omega_z^2 - \Omega_x^2) \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & \Omega_z \Omega_x \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) \\ \Omega_x \Omega_z \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & \Omega_y \Omega_z \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & -\Omega_y^2 - \Omega_z^2 \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) \end{bmatrix}$$

$$R(\Omega) = \begin{bmatrix} 1 + (\Omega_z^2 - \Omega_y^2) \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & \Omega_z \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) + (\Omega_y \Omega_x) \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & -\Omega_y \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) + \Omega_z \Omega_x \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) \\ -\Omega_z \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) + \Omega_x \Omega_y \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & 1 + (-\Omega_z^2 - \Omega_x^2) \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & \Omega_x \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) + \Omega_z \Omega_y \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) \\ \Omega_y \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) + \Omega_x \Omega_z \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & -\Omega_x \left(\frac{\sin \|\Omega\|}{\|\Omega\|} \right) + \Omega_y \Omega_z \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) & 1 + (-\Omega_y^2 - \Omega_z^2) \left(\frac{1 - \cos \|\Omega\|}{\|\Omega\|^3} \right) \end{bmatrix}$$

Computing Points

1) When we tap the phone screen we get

$$p^{s,0} = \begin{bmatrix} p_x^{s,0} \\ p_y^{s,0} \end{bmatrix}$$

• We translate these pixel points to the camera reference frame

$$p^{c,0} = p^{s,0} = \begin{bmatrix} p_x^{s,0} - c_x \\ p_y^{s,0} - c_y \\ f \end{bmatrix}$$

$$c_x = 240$$

$$c_y = 320$$

$$f = 611$$

• to get the $p^{c,t}$ after rotation

$$p^{c,t} = R_o^t p^{c,0}$$

• convert $p^{c,t}$ to pixel point

$$p^{s,t} = \begin{bmatrix} p_x^{s,t} \\ p_y^{s,t} \end{bmatrix} = \begin{bmatrix} f \cdot \frac{p_x^{c,t}}{p_z^{c,t}} + c_x \\ f \cdot \frac{p_y^{c,t}}{p_z^{c,t}} + c_y \end{bmatrix}$$

• this is the points we use to draw the new circle

2)

3) The initial state of our system is just the camera with the slider on and the segmented control at none. You can touch anywhere and a circle will be created where you touch regardless of the state you're in. If you change the segmented control and touch again, this will start that mode in the iphone, if you touch anywhere on the screen while the Gyroscope option is selected, the gyroscope will start, etc. The reset button can be pressed at any point to stop any background

process as well as reset the circles on the screen. The slider toggles whether the circles will trace, leave a path behind them, or just have the one circle bouncing around the screen as you move your device. Further, the second segmented control below the main one will toggle whether you would like to compute the data with the full Rodriguez formula or the small angle formula. Unfortunately, my partner and I were only able to get the small angle to work (with trace) and the device motion was giving us a lot of problems. We tried to get the full Rodriguez formula to work but we both of us are exhausted from studying extensively for this upcoming week of finals and couldn't perform as well as he had hoped. Sorry about the inconvenience, but the only functions that work are the small angle and the device motion to an extent.