Conic section Assignment

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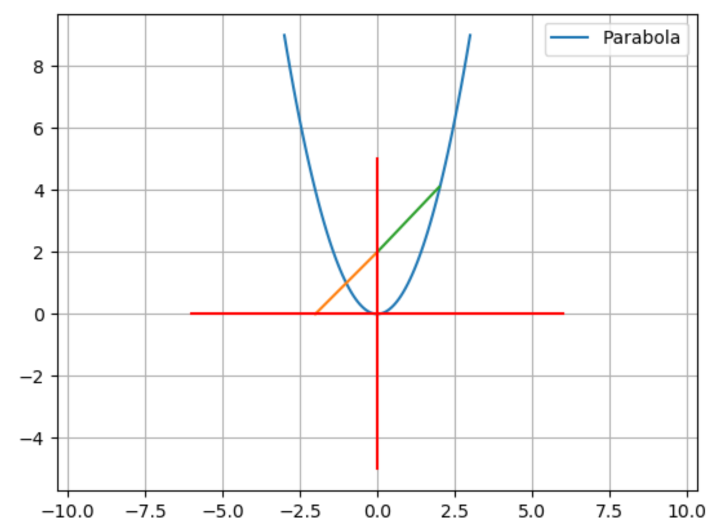
Problem Statement **- Find the area of the region bounded by the curve $x^2 = y$ and the lines y=x+2 and the x axis**

Solution



The given equation of parabola $x^2 = y$ can be written in the general quadratic form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix}, \tag{3}$$

$$f = 0 \tag{4}$$

The points of intersection of the line

$$L: \quad \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \quad \mu \in \mathbf{R} \tag{5}$$

with the conic section are given by

$$\mathbf{x}_i = \mathbf{q} + \mu_i \mathbf{m} \tag{6}$$

where

$$\mu_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}} \Big(-\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)} \Big) \tag{7}$$

From the line y=x+2 the vectors q,m are taken,

$$\mathbf{q} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{8}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{9}$$

by substituting eq(2),(3),(4),(8),(9) in eq(7)

$$\mu_i = -2 \tag{10}$$

substituting eq(8),(9),(10) in eq(6) the intersection points on the parabola are

$$\mathbf{a_0} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{11}$$

$$\mathbf{a_1} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{12}$$

Given line equation y=x+2

$$x - y = -2$$

$$\mathbf{n}^t\mathbf{x} = c$$

$$\mathbf{x} = \mathbf{A} + \lambda\mathbf{m}$$

$$\mathbf{x} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Substitute the x value in the quadratic equation then we get a quadratic equations

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{13}$$

$$\mu^2 - 3\mu + 2 = 0$$

$$\mu = 1, 2$$

$$\mu^2 - \mu$$

$$\mu = 1, 0$$

The resultant x values are

$$\mathbf{x} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

Area of the parabola in between the lines parabola and y=x+2 is given by

$$\implies A_1 = \int_{-2}^{-1} x + 2 \, dx \tag{14}$$

$$\implies A_2 = \int_{-1}^{0} x^2 \, dx \tag{15}$$

$$\implies A_1 + A_1 = \int_{-2}^{-1} x + 2 \, dx + \int_{-1}^{0} x^2 \, dx \tag{16}$$

$$\implies A_1 + A_2 = \frac{5}{6} squnits \tag{17}$$