PYTHON PROGRAMMING ON MATRICES

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FWC220   Matrix:Circle

1 Problem

If the two circles

$$(x-1)^2 + (y-3)^2 = r^2$$

and

$$x^2 + y^2 - 8x + 2y + 8 = 0$$

intersect in two distinct points, then

$$\|\mathbf{c}_1 - \mathbf{c}_2\| = \sqrt{(\mathbf{c}_1 - \mathbf{c}_2)^T(\mathbf{c}_1 - \mathbf{c}_2)} = 5$$

Two circles intersect in two distinct point then

$$r_1 - r_2 < \|\mathbf{c}_1 - \mathbf{c}_2\| < r_1 + r_2$$
$$r - 3 < 5 < r + 3$$
$$2 < r < 8$$

2 Solution

Given that: The circle equation is

$$\mathbf{x}^T V \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$$

$$r = \sqrt{\|\mathbf{u}\|^2 - f}$$

Comparing the given equations 1 and 2 with the circle equation to find the centre and radius
eq(1) can be expressed as

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} -1 & -3 \end{pmatrix} \mathbf{x} + 10 = r^2$$

centre and radius are

$$\mathbf{c} = -\mathbf{u}$$
$$\mathbf{c} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad r_1 = r$$

eq(2) can be expressed as

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} -4 & 1 \end{pmatrix} \mathbf{x} + 8 = 0$$

centre and radius are

$$\mathbf{c} = -\mathbf{u}$$

$$\mathbf{c} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$r_2 = 3$$

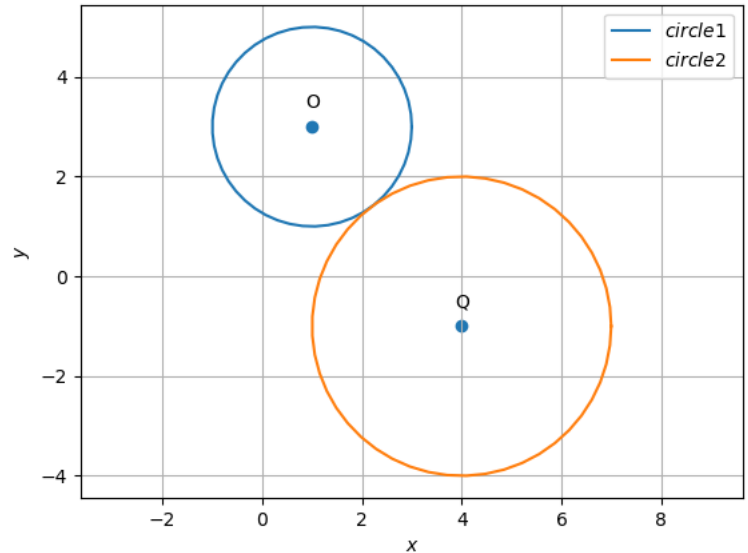Distance between two points (1,3)=c_1 and (4,-1)=c_2
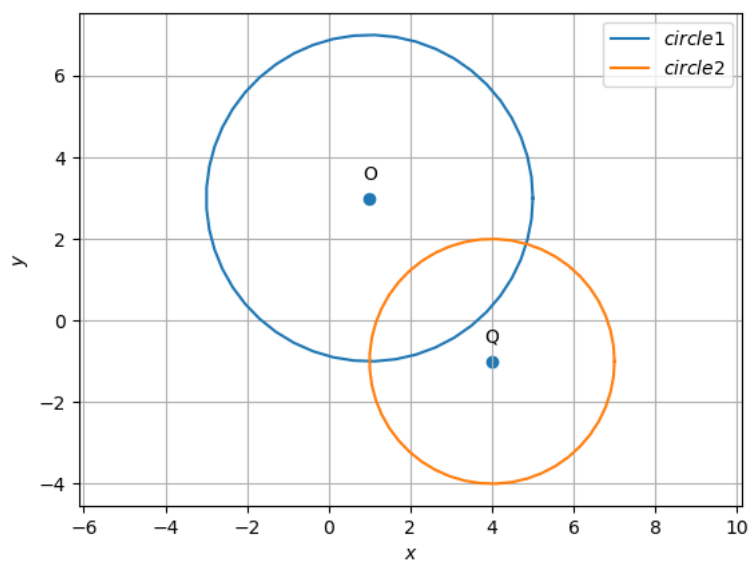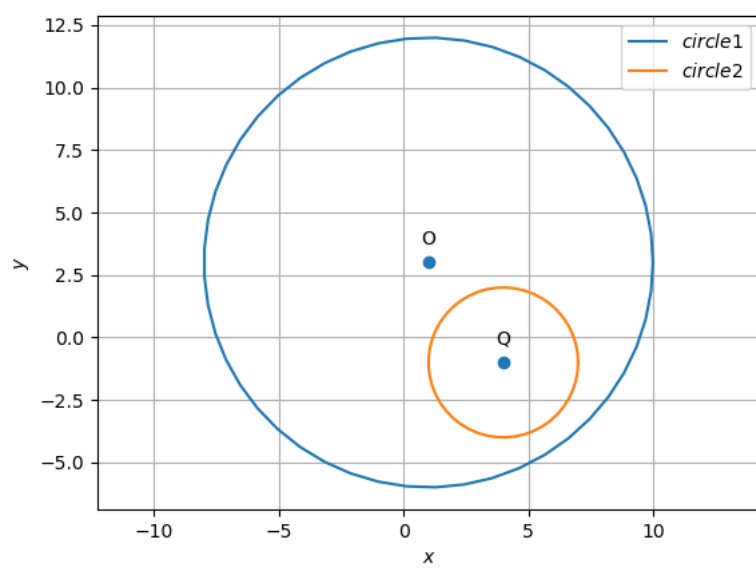

Figure 1: $r_1 = 2, r_2 = 3$

Figure 2: $r_1 = 4, r_2 = 3$

Figure 3: $r_1 = 9, r_2 = 3$