for those who handed 1st hwk late (latecomers ~~to~~ for the class) (on 16/7)

1.2.19 In augmented form, the system corresponds to

$$\left[\begin{array}{cc|c} 1 & h & 2 \\ 4 & 8 & k \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & h & 2 \\ 0 & 8-4h & k-8 \end{array}\right]$$

(a) if $h=2$, $k \neq 8 \Rightarrow$ inconsistent

(b) $8-4h = 1 \Rightarrow 4h = -7 \Rightarrow h = -\frac{7}{4}$ gives a unique solution to the system $\forall k \in \mathbb{R}$ (even if $k=8$).

and as long as $h \neq 2$, $\forall h \in \mathbb{R}$

~~(c)~~ This is a solution to the system

(c) $h=2$, $k=8$ gives $\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & 0 \end{array}\right]$

gives infinitely many solutions.

We just need a row of zeros in the 2nd row of the matrix.

( 8 ; (a) : 3 for general solutions
(b) : 3 for general solutions
(c) : 2 for correct numbers )

1.3.22 When we have an $A \in M_{3\times3}$ without 3 linearly independent column vectors, the $C(A)$ does not span $\mathbb{R}^3$. Hence, we can always find $b \in \mathbb{R}^3$ s.t. $b \notin C(A)$.

An example: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.

(6; 2 for the realization that A needs 3 linearly independent column vectors, 1 for attempt at example, 2 for correct examples, 1 for all of the above)

1.4.21 Only a set of 4 linearly independent vectors $\in \mathbb{R}^4$ spans $\mathbb{R}^4$.

There are 3 linearly independent vectors in this set — so this cannot form a basis for $\mathbb{R}^4$.

(6; 1 for attempting the problem; 2 for realization above, 2 for correct subsequent explanations, 1 for having all of the above)

1.5.23 (a) TRUE the trivial solution is always a solution

(b) FALSE the solution set is given

$$S = \{ x_p + \lambda x_h : \lambda \in \mathbb{R} \}$$

Where $x_p$ is particular solution and $x_h$ is homogenous solution

The equation gives an implicit description of solution set to $Ax = b$.

(c) FALSE the trivial solution is always to $Ax = 0$

(d) FALSE The line goes through $p$ that is parallel to $v$.

(e) FALSE ~~this~~ see above; this is only true if $p$ is a particular solution, ie. if $Ap = b$. holds.

(10; 2 each, 1 for T/F, 1 for correct explanation)

1.7.22 (a) TRUE if 2 points lie on the same line passing through the origin then the vectors are scalar multiples of one another.

(b) FALSE Counterexample: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \right\}$.

(c) TRUE by def.

(d) FALSE not always true. Counterexample:
$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \right\}$ is a set of ~~one~~ linearly independent vectors, but has fewer vectors then there are vectors.

( 8 ; 2 each, 1 for T/F, 1 for correct explanation )

<u>Proof</u> let $T(x_1, x_2) = (2x_1 - 3x_2,\ x_1 + 4,\ 5x_2)$

<u>1.8.33</u> To see $T$ is not a linear transformation we need just one counterexample.

<u>ie</u> let $x := (1,1)$, & $y := (1, 0)$.

Then $T(x+y) = T((2,\cancel{1}1))$

$= (1, 6, 5)$

<u>but</u> $T(x) + T(y) = (-1, 5, 5)$

$+ (2, 5, 0)$

$= (1, 10, 5)$

so $T(x+y) \neq T(x) + T(y) \implies$

$T$ is not a linear map! $\square$

($\underline{8}$; 1 for attempting the problem, 2 for realizing exactly what to do, 4 for a correct counterexample & subsequent justification, 1 for all of the above)

1.9.24 (a) FALSE For a linear transformation, we see the image of $\mathcal{B} := \{e_1, \ldots, e_n\}$ under $T$.

(b) TRUE. By def.

(c) TRUE. Check matrix multiplication!

(d) FALSE injective function $f: X \mapsto Y$ is where $f(x) = f(y) \Rightarrow x = y$.
So every vector $x \in \mathbb{R}^m$ is mapped.

(e) ~~FALSE~~ TRUE ~~the~~ any map from $\mathbb{R}^n \mapsto \mathbb{R}^{n+1}$ cannot be onto (surjective)

(10; 2 each, 1 for T/F, 1 for correct explanation)