

# **Introduction to Applied Science**

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# 1 Preface

This is a Quarto book.

To learn more about Quarto books visit <https://quarto.org/docs/books>.

**1 + 1**

[1] 2

# **Part I**

## **Introduction**

## 2 Introduction

This is a book created from markdown and executable code.

See (**knuth84?**) for additional discussion of literate programming.

```
1 + 1
```

```
[1] 2
```

## 3 Modeling Literature

Bayesian Workflow Andrew Gelman, Aki Vehtari, Daniel Simpson, Charles C. Margossian, Bob Carpenter, Yuling Yao, Lauren Kennedy, Jonah Gabry, Paul-Christian Bürkner, Martin Modrák <https://arxiv.org/abs/2011.01808>

How to avoid machine learning pitfalls: a guide for academic researchers Michael A. Lones <https://arxiv.org/abs/2108.02497>

Cross-validation strategies for data with temporal, spatial, hierarchical, or phylogenetic structure David R. Roberts, Volker Bahn, Simone Ciuti, Mark S. Boyce, Jane Elith, Gurutzeta Guillera-Arroita, Severin Hauenstein, José J. Lahoz-Monfort, Boris Schröder, Wilfried Thuiller, David I. Warton, Brendan A. Wintle, Florian Hartig, Carsten F. Dormann <https://onlinelibrary.wiley.com/doi/full/10.1111/ecog.02881>

Effective Pandas <https://store.metasnake.com/effective-pandas-book>

Information geometry and divergences <https://franknielsen.github.io/IG/#bookIG>

Statistical Rethinking: A Bayesian Course with Examples in R and Stan (& PyMC3 & brms) <https://xcelab.net/rm/statistical-rethinking/> <https://www.youtube.com/playlist?list=PLDcUM9US4XdMROZOIRtIK0aOynbgZN>

**Part II**

**Mathematical Objects**



## 4 Set

Cites: [Wikipedia](#); [Wikidata](#); [PlanetMath](#)

## 5 List (Sequence)

AKA: Sequence,  $a_n$  where  $n$  is the  $n$ th element, (1,2,3, ....)

Distinct from: Set

Measure of:

Description: A list is a collection of objects with a specific ordering and where the same object can appear more than once. Call each object an element, and its location its index or rank. An index is a natural number counting upward from the first element in the list. Whether counting begins at 0 or 1 depends on local conventions.

Formalization:

Algorithm:

Cites: [Wikipedia](#) [Wikidata](#) [Encyclopedia Of Math](#) [Wolfram](#) [PlanetMath](#)

### 5.0.0.1 R

Documentation:

[list: Lists – Generic and Dotted Pairs](#)

Examples:

```
example_list = list(1,2,3)
example_list
```

```
[[1]]
[1] 1
```

```
[[2]]
[1] 2
```

```
[[3]]
[1] 3
```

### 5.0.0.2 Python

Documentation:

[More on Lists](#)

Examples:

```
example_list = [1,2,3]
example_list
```

```
[1, 2, 3]
```

### 5.0.0.3 SQL

```
library(DBI)
# Create an ephemeral in-memory RSQLite database
con <- dbConnect(RSQLite::SQLite(), dbname = ":memory:")
dbListTables(con)
```

```
character(0)
```

```
dbWriteTable(con, "mtcars", mtcars)
dbListTables(con)
```

```
[1] "mtcars"
```

```
create table StatisticalNumbers(
  value int
)
```

```
SELECT * FROM mtcars LIMIT 5;
```

Table 5.1: 5 records

mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
21.0	6	160	110	3.90	2.620	16.46	0	1	4	4
21.0	6	160	110	3.90	2.875	17.02	0	1	4	4

mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
22.8	4	108	93	3.85	2.320	18.61	1	1	4	1
21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
18.7	8	360	175	3.15	3.440	17.02	0	0	3	2

# 6 Tensor

Instance of: algebraic object

## 6.1 Frequentist

AKA:

Distinct from:

English:

Formalization:

Cites: [Wikipedia](#) ; Wikidata ; Wolfram

Code

### 6.1.0.1 R

Documentation: [mean](#): Arithmetic Mean

Examples:

### 6.1.0.2 Python

Documentation: [numpy.mean](#)

Examples:

### 6.1.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

```

library(DBI)
# Create an ephemeral in-memory RSQLite database
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#Configuration failed because libpq was not found. Try installing:
#* deb: libpq-dev libssl-dev (Debian, Ubuntu, etc)
#install.packages('RPostgres')
#remotes::install_github("r-dbi/RPostgres")
#Took forever because my file permissions were broken
#pg_lsclusters
require(RPostgres)

```

Loading required package: RPostgres

```

# Connect to the default postgres database
#I had to follow these instructions and create both a username and database that matched m
#https://www.digitalocean.com/community/tutorials/how-to-install-postgresql-on-ubuntu-20-0
con <- dbConnect(RPostgres::Postgres())

```

#### 6.1.0.4 Torch

```
import torch
```

## 6.2 Bayesian

English: Formalization:

Cites:

Code

# 7 Table

**Instance of:** arrangement of information or data

**AKA:** Dataframe

**Distinct from:**

**English:** A collection of rows and columns, where rows represent specific instances (AKA records, k-tuple, n-tuple, or a vector), and columns represent features (AKA variables, parameters, properties, attributes, or stanchions). The intersection of a row and column is called a sell.

**Formalization:**

**Cites:** [Wikipedia Table \(information\)](#) ; [Wikipedia Table Table \(database\)](#) ; Wikidata ; Wolfram

**Code**

## 7.0.0.1 R

**Documentation:** [data.frame: Data Frames](#)

Examples:

```
df=data.frame(a=c(1,2,3,4), b=c('a','b','c','d'))
df
```

```
  a b
1 1 a
2 2 b
3 3 c
4 4 d
```

### 7.0.0.2 Python

Documentation: [pandas.DataFrame](#)

Examples:

```
import pandas as pd
df = pd.DataFrame({'a': [1, 2,3,4], 'b': ['a','b','c','d']})
df
```

```
   a  b
0  1  a
1  2  b
2  3  c
3  4  d
```

### 7.0.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

```
library(DBI)
# Create an ephemeral in-memory RSQLite database
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#dbListTables(con)
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#dbListTables(con)

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```
# Connect to the default postgres database
#I had to follow these instructions and create both a username and database that matched m
#https://www.digitalocean.com/community/tutorials/how-to-install-postgresql-on-ubuntu-20-0
```



```

con <- dbConnect(RPostgres::Postgres())

DROP TABLE IF EXISTS df;

CREATE TABLE IF NOT EXISTS df (
  a INTEGER,
  b CHAR
);

INSERT INTO df (a, b)
VALUES
  (1, 'a'),
  (2, 'b'),
  (3, 'c'),
  (4, 'd');

SELECT * FROM df;

```

Table 7.1: 4 records

a	b
1	a
2	b
3	c
4	d

#### 7.0.0.4 Torch

```
import torch
```

**Part III**

**Operations of Arithmetic**

# 8 Addition

**Instance of:** operation of arithmetic

## 8.1 Frequentist

**AKA:** + ; add

**Distinct from:**

**English:**

**Formalization:**

**Cites:** [Wikipedia](#) ; [Wikidata](#) ; Wolfram

**Code**

### 8.1.0.1 R

**Documentation:** [mean](#): Arithmetic Mean

**Examples:**

### 8.1.0.2 Python

**Documentation:** [numpy.mean](#)

**Examples:**

### 8.1.0.3 SQL

**Documentation:** [PostgreSQL AVG Function](#)

```

library(DBI)
# Create an ephemeral in-memory RSQLite database
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```

Loading required package: RPostgres

```

# Connect to the default postgres database
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#https://www.digitalocean.com/community/tutorials/how-to-install-postgresql-on-ubuntu-20-0
con <- dbConnect(RPostgres::Postgres())

```

#### 8.1.0.4 Torch

```
import torch
```

## 8.2 Bayesian

English: Formalization:

Cites:

Code

# 9 Introduction

**Instance of:** operation of arithmetic

## 9.1 Frequentist

**AKA:** - ; minus

**Distinct from:**

**English:**

**Formalization:**

**Cites:** [Wikipedia](#) ; [Wikidata](#) ; Wolfram

**Code**

### 9.1.0.1 R

**Documentation:** [mean](#): Arithmetic Mean

**Examples:**

### 9.1.0.2 Python

**Documentation:** [numpy.mean](#)

**Examples:**

### 9.1.0.3 SQL

**Documentation:** [PostgreSQL AVG Function](#)

```

library(DBI)
# Create an ephemeral in-memory RSQLite database
#con <- dbConnect(RSQLite::SQLite(), dbname = ":memory:")
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#dbWriteTable(con, "mtcars", mtcars)
#dbListTables(con)

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require(RPostgres)

```

Loading required package: RPostgres

```

# Connect to the default postgres database
#I had to follow these instructions and create both a username and database that matched m
#https://www.digitalocean.com/community/tutorials/how-to-install-postgresql-on-ubuntu-20-0
con <- dbConnect(RPostgres::Postgres())

```

#### 9.1.0.4 Torch

```
import torch
```

## 9.2 Bayesian

English: Formalization:

Cites:

Code

# 10 Multiplication

**Instance of:** operation of arithmetic

## 10.1 Frequentist

**AKA:** \* ;  $\times$  ; ; multiply

**Distinct from:**

**English:**

**Formalization:**

**Cites:** Wikipedia ; Wikidata ; Wolfram

**Code**

### 10.1.0.1 R

**Documentation:** [mean](#): Arithmetic Mean

**Examples:**

### 10.1.0.2 Python

**Documentation:** [numpy.mean](#)

**Examples:**

### 10.1.0.3 SQL

**Documentation:** [PostgreSQL AVG Function](#)

```

library(DBI)
# Create an ephemeral in-memory RSQLite database
#con <- dbConnect(RSQLite::SQLite(), dbname = ":memory:")
#dbListTables(con)
#dbWriteTable(con, "mtcars", mtcars)
#dbListTables(con)

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#install.packages('RPostgres')
#remotes::install_github("r-dbi/RPostgres")
#Took forever because my file permissions were broken
#pg_lsclusters
require(RPostgres)

```

Loading required package: RPostgres

```

# Connect to the default postgres database
#I had to follow these instructions and create both a username and database that matched m
#https://www.digitalocean.com/community/tutorials/how-to-install-postgresql-on-ubuntu-20-0
con <- dbConnect(RPostgres::Postgres())

```

#### 10.1.0.4 Torch

```
import torch
```

## 10.2 Bayesian

English: Formalization:

Cites:

Code



# 11 Division

Instance of: operation of arithmetic

## 11.1 Frequentist

AKA:  $/$  ;  $\frac{numerator}{denominator}$  ;  $\div$

Distinct from:

English:

Formalization:

Cites: [Wikipedia](#) ; [Wikidata](#) ; [Wolfram](#)

Code

### 11.1.0.1 R

Documentation: [mean](#): [Arithmetic Mean](#)

Examples:

### 11.1.0.2 Python

Documentation: [numpy.mean](#)

Examples:

### 11.1.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

```

library(DBI)
# Create an ephemeral in-memory RSQLite database
#con <- dbConnect(RSQLite::SQLite(), dbname = ":memory:")
#dbListTables(con)
#dbWriteTable(con, "mtcars", mtcars)
#dbListTables(con)

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#Took forever because my file permissions were broken
#pg_lsclusters
require(RPostgres)

```

Loading required package: RPostgres

```

# Connect to the default postgres database
#I had to follow these instructions and create both a username and database that matched m
#https://www.digitalocean.com/community/tutorials/how-to-install-postgresql-on-ubuntu-20-0
con <- dbConnect(RPostgres::Postgres())

```

#### 11.1.0.4 Torch

```
import torch
```

## 11.2 Bayesian

English: Formalization:

Cites:

Code

**Part IV**

**Operations of Algebra**

# 12 Dot product

**Instance of:** algebraic operation

**AKA:** scalar product; inner product ; projection product ;  $\$ \cdot \$$

**Distinct from:**

**English:**

**Formalization:**

$$a \cdot b$$

**Cites:** [Wikipedia](#) ; Wikidata ; Wolfram

**Code**

## 12.0.0.1 R

**Documentation:** [mean](#); [Arithmetic Mean](#)

**Examples:**

## 12.0.0.2 Python

**Documentation:** [numpy.mean](#)

**Examples:**

## 12.0.0.3 SQL

**Documentation:** [PostgreSQL AVG Function](#)

```
library(DBI)
# Create an ephemeral in-memory RSQLite database
#con <- dbConnect(RSQLite::SQLite(), dbname = ":memory:")
#dbListTables(con)
```

```
#dbWriteTable(con, "mtcars", mtcars)
#dbListTables(con)

#Configuration failed because libpq was not found. Try installing:
## deb: libpq-dev libssl-dev (Debian, Ubuntu, etc)
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#Took forever because my file permissions were broken
#pg_lsclusters
require(RPostgres)
```

Loading required package: RPostgres

```
# Connect to the default postgres database
#I had to follow these instructions and create both a username and database that matched m
#https://www.digitalocean.com/community/tutorials/how-to-install-postgresql-on-ubuntu-20-0
con <- dbConnect(RPostgres::Postgres())
```

#### 12.0.0.4 Torch

```
import torch
```

## 12.1 Bayesian

English: Formalization:

Cites:

Code

## **Part V**

# **Moments of a Distribution**

# 13 Mean

**Measure of:** Central tendency

## 13.1 Frequentist

**AKA:** Arithmetic mean; average;  $\bar{x}$  (sample mean);  $\mu$  (population mean);  $\mu_x$  (population mean)

**Distinct from:** Geometric mean (GM); Harmonic mean (HM); generalized mean/ Power mean; weighted arithmetic mean

**English:** Take a list of numbers, sum those numbers, and then divide by the number of numbers.

**Formalization:**

$$\bar{x} = \frac{1}{n} \left( \sum_{i=1}^n x_i \right) = \frac{x_1 + x_2 + \dots + x_n}{n}$$

**Cites:** [Wikipedia](#) ; [Wikidata](#) ; [Wolfram](#)

**Code**

### 13.1.0.1 R

**Documentation:** [mean](#): [Arithmetic Mean](#)

Examples:

```
x = c(1,2,3,4)
x
```

```
[1] 1 2 3 4
```

```
#Algorithm
x_bar = sum(x, na.rm=T)/length(x)
x_bar
```

[1] 2.5

```
#Base Function
x_bar = mean(x, na.rm=T)
x_bar
```

[1] 2.5

### 13.1.0.2 Python

**Documentation:** [numpy.mean](#)

Examples:

```
x = [1,2,3,4]
print(x)
```

[1, 2, 3, 4]

```
#Algorithm
x_bar= sum(x)/len(x)
x_bar
```

2.5

```
#statistics Function
import statistics
x_bar = statistics.mean(x)
x_bar
```

2.5



```
#scipy Function
#<string>:1: DeprecationWarning: scipy.mean is deprecated and will be removed in SciPy 2.0
import scipy
x_bar = scipy.mean(x)
```

<string>:1: DeprecationWarning: scipy.mean is deprecated and will be removed in SciPy 2.0.0,

```
x_bar
```

2.5

```
#numpy Function
import numpy as np
x = np.array(x)
x_bar = x.mean()
x_bar
```

2.5

### 13.1.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

```
library(DBI)
# Create an ephemeral in-memory RSQLite database
#con <- dbConnect(RSQLite::SQLite(), dbname = ":memory:")
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```
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#https://www.digitalocean.com/community/tutorials/how-to-install-postgresql-on-ubuntu-20-0
con <- dbConnect(RPostgres::Postgres())
```

```
DROP TABLE IF EXISTS t1;
```

```
CREATE TABLE IF NOT EXISTS t1 (
  id serial PRIMARY KEY,
  amount INTEGER
);
```

```
INSERT INTO t1 (amount)
VALUES
  (10),
  (NULL),
  (30);
```

```
SELECT * FROM t1;
```

Table 13.1: 3 records

id	amount
1	10
2	NA
3	30

```
SELECT AVG(amount)::numeric(10,2)
FROM t1;
```

Table 13.2: 1 records

—
avg
—
20
—

#### 13.1.0.4 Torch

```
import torch
```

## 13.2 Bayesian

Bayesian average; Solving an age-old problem using Bayesian Average; Of bayesian average and star ratings; Bayesian Average Ratings ;

**English:** The Bayesian average is the weighted average of a prior and the observed sample average. When would you want this? When you have strong beliefs about the true mean, or when sample size is too small to reliably calculate a mean. For example a movie rating website where a movie may have only a single 5 star rating and so would rank higher than the Godfather with over a 100 almost all 5 star ratings.

**Formalization:**

$$\bar{x} = \frac{C * m + (\sum_{i=1}^n x_i)}{c + n}$$

Where  $m$  is a prior for true mean, and  $C$  is a constant representing how many elements would be necessary to reliably estimate a sample mean.

**Code**

## References