

Introduction to Applied Science

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Table of contents

| | |
|------------------------------------|-----------|
| Preface | 4 |
| 1 Introduction | 5 |
| 2 Modeling Literature | 6 |
| I mathematical object | 7 |
| 3 Set | 8 |
| 4 List (Sequence) | 9 |
| 5 Tensor | 12 |
| 5.1 Frequentist | 12 |
| 5.2 Bayesian | 13 |
| II operations of arithmetic | 14 |
| 6 Addition | 15 |
| 6.1 Frequentist | 15 |
| 6.2 Bayesian | 16 |
| 7 Introduction | 17 |
| 7.1 Frequentist | 17 |
| 7.2 Bayesian | 18 |
| 8 Multiplication | 19 |
| 8.1 Frequentist | 19 |
| 8.2 Bayesian | 20 |
| 9 Division | 21 |
| 9.1 Frequentist | 21 |
| 9.2 Bayesian | 22 |

| | |
|-------------------------------------|-----------|
| III operations of algebra | 23 |
| 10 Dot product | 24 |
| 10.1 Bayesian | 25 |
| IV moments of a distribution | 26 |
| 11 Mean | 27 |
| 11.1 Frequentist | 27 |
| 11.2 Bayesian | 31 |
| References | 32 |

Preface

This is a Quarto book.

To learn more about Quarto books visit <https://quarto.org/docs/books>.

1 + 1

[1] 2

1 Introduction

This is a book created from markdown and executable code.

See (**knuth84?**) for additional discussion of literate programming.

```
1 + 1
```

```
[1] 2
```

2 Modeling Literature

Bayesian Workflow Andrew Gelman, Aki Vehtari, Daniel Simpson, Charles C. Margossian, Bob Carpenter, Yuling Yao, Lauren Kennedy, Jonah Gabry, Paul-Christian Bürkner, Martin Modrák <https://arxiv.org/abs/2011.01808>

How to avoid machine learning pitfalls: a guide for academic researchers Michael A. Lones <https://arxiv.org/abs/2108.02497>

Cross-validation strategies for data with temporal, spatial, hierarchical, or phylogenetic structure David R. Roberts, Volker Bahn, Simone Ciuti, Mark S. Boyce, Jane Elith, Gurutzeta Guillera-Arroita, Severin Hauenstein, José J. Lahoz-Monfort, Boris Schröder, Wilfried Thuiller, David I. Warton, Brendan A. Wintle, Florian Hartig, Carsten F. Dormann <https://onlinelibrary.wiley.com/doi/full/10.1111/ecog.02881>

Effective Pandas <https://store.metasnake.com/effective-pandas-book>

Information geometry and divergences <https://franknielsen.github.io/IG/#bookIG>

Statistical Rethinking: A Bayesian Course with Examples in R and Stan (& PyMC3 & brms) <https://xcelab.net/rm/statistical-rethinking/> <https://www.youtube.com/playlist?list=PLDcUM9US4XdMROZOIRtIK0aOynbgZN>

Part I

mathematical object

3 Set

Cites: [Wikipedia](#); [Wikidata](#); [PlanetMath](#)

4 List (Sequence)

AKA: Sequence, a_n where n is the nth element, (1,2,3,)

Distinct from: Set

Measure of:

Description: A list is a collection of objects with a specific ordering and where the same object can appear more than once. Call each object an element, and its location its index or rank. An index is a natural number counting upward from the first element in the list. Whether counting begins at 0 or 1 depends on local conventions.

Formalization:

Algorithm:

Cites: [Wikipedia](#) [Wikidata](#) [Encyclopedia Of Math](#) [Wolfram](#) [PlanetMath](#)

4.0.0.1 R

Documentation:

[list: Lists – Generic and Dotted Pairs](#)

Examples:

```
example_list = list(1,2,3)
example_list
```

```
[[1]]
[1] 1
```

```
[[2]]
[1] 2
```

```
[[3]]
[1] 3
```

4.0.0.2 Python

Documentation:

[More on Lists](#)

Examples:

```
example_list = [1,2,3]
example_list
```

```
[1, 2, 3]
```

4.0.0.3 SQL

```
library(DBI)
# Create an ephemeral in-memory RSQLite database
con <- dbConnect(RSQLite::SQLite(), dbname = ":memory:")
dbListTables(con)
```

```
character(0)
```

```
dbWriteTable(con, "mtcars", mtcars)
dbListTables(con)
```

```
[1] "mtcars"
```

```
create table StatisticalNumbers(
  value int
)
```

```
SELECT * FROM mtcars LIMIT 5;
```

Table 4.1: 5 records

| mpg | cyl | disp | hp | drat | wt | qsec | vs | am | gear | carb |
|------|-----|------|-----|------|-------|-------|----|----|------|------|
| 21.0 | 6 | 160 | 110 | 3.90 | 2.620 | 16.46 | 0 | 1 | 4 | 4 |
| 21.0 | 6 | 160 | 110 | 3.90 | 2.875 | 17.02 | 0 | 1 | 4 | 4 |

| mpg | cyl | disp | hp | drat | wt | qsec | vs | am | gear | carb |
|------|-----|------|-----|------|-------|-------|----|----|------|------|
| 22.8 | 4 | 108 | 93 | 3.85 | 2.320 | 18.61 | 1 | 1 | 4 | 1 |
| 21.4 | 6 | 258 | 110 | 3.08 | 3.215 | 19.44 | 1 | 0 | 3 | 1 |
| 18.7 | 8 | 360 | 175 | 3.15 | 3.440 | 17.02 | 0 | 0 | 3 | 2 |

5 Tensor

Instance of: algebraic object

5.1 Frequentist

AKA:

Distinct from:

English:

Formalization:

Cites: [Wikipedia](#) ; Wikidata ; Wolfram

Code

5.1.0.1 R

Documentation: [mean](#): [Arithmetic Mean](#)

Examples:

5.1.0.2 Python

Documentation: [numpy.mean](#)

Examples:

5.1.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

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#* deb: libpq-dev libssl-dev (Debian, Ubuntu, etc)
#install.packages('RPostgres')
#remotes::install_github("r-dbi/RPostgres")
#Took forever because my file permissions were broken
#pg_lsclusters
require(RPostgres)

```

Loading required package: RPostgres

```

# Connect to the default postgres database
#I had to follow these instructions and create both a username and database that matched m
#https://www.digitalocean.com/community/tutorials/how-to-install-postgresql-on-ubuntu-20-0
con <- dbConnect(RPostgres::Postgres())

```

5.1.0.4 Torch

```
import torch
```

5.2 Bayesian

English: Formalization:

Cites:

Code

Part II

operations of arithmetic

6 Addition

Instance of: operation of arithmetic

6.1 Frequentist

AKA: + ; add

Distinct from:

English:

Formalization:

Cites: [Wikipedia](#) ; [Wikidata](#) ; Wolfram

Code

6.1.0.1 R

Documentation: [mean](#): [Arithmetic Mean](#)

Examples:

6.1.0.2 Python

Documentation: [numpy.mean](#)

Examples:

6.1.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

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```

6.1.0.4 Torch

```
import torch
```

6.2 Bayesian

English: Formalization:

Cites:

Code

7 Introduction

Instance of: operation of arithmetic

7.1 Frequentist

AKA: - ; minus

Distinct from:

English:

Formalization:

Cites: [Wikipedia](#) ; [Wikidata](#) ; Wolfram

Code

7.1.0.1 R

Documentation: [mean](#): Arithmetic Mean

Examples:

7.1.0.2 Python

Documentation: [numpy.mean](#)

Examples:

7.1.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

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con <- dbConnect(RPostgres::Postgres())

```

7.1.0.4 Torch

```
import torch
```

7.2 Bayesian

English: Formalization:

Cites:

Code

8 Multiplication

Instance of: operation of arithmetic

8.1 Frequentist

AKA: * ; \times ; ; multiply

Distinct from:

English:

Formalization:

Cites: Wikipedia ; Wikidata ; Wolfram

Code

8.1.0.1 R

Documentation: [mean](#): [Arithmetic Mean](#)

Examples:

8.1.0.2 Python

Documentation: [numpy.mean](#)

Examples:

8.1.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

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con <- dbConnect(RPostgres::Postgres())

```

8.1.0.4 Torch

```
import torch
```

8.2 Bayesian

English: Formalization:

Cites:

Code

9 Division

Instance of: operation of arithmetic

9.1 Frequentist

AKA: $/$; $\frac{numerator}{denominator}$; \div

Distinct from:

English:

Formalization:

Cites: [Wikipedia](#) ; [Wikidata](#) ; [Wolfram](#)

Code

9.1.0.1 R

Documentation: [mean](#): [Arithmetic Mean](#)

Examples:

9.1.0.2 Python

Documentation: [numpy.mean](#)

Examples:

9.1.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

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con <- dbConnect(RPostgres::Postgres())

```

9.1.0.4 Torch

```
import torch
```

9.2 Bayesian

English: Formalization:

Cites:

Code

Part III

operations of algebra

10 Dot product

Instance of: algebraic operation

AKA: scalar product; inner product ; projection product ; $\$ \cdot \$$

Distinct from:

English:

Formalization:

$$a \cdot b$$

Cites: [Wikipedia](#) ; Wikidata ; Wolfram

Code

10.0.0.1 R

Documentation: [mean](#); [Arithmetic Mean](#)

Examples:

10.0.0.2 Python

Documentation: [numpy.mean](#)

Examples:

10.0.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

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con <- dbConnect(RPostgres::Postgres())
```

10.0.0.4 Torch

```
import torch
```

10.1 Bayesian

English: Formalization:

Cites:

Code

Part IV

moments of a distribution

11 Mean

Measure of: Central tendency

11.1 Frequentist

AKA: Arithmetic mean; average; \bar{x} (sample mean); μ (population mean); μ_x (population mean)

Distinct from: Geometric mean (GM); Harmonic mean (HM); generalized mean/ Power mean; weighted arithmetic mean

English: Take a list of numbers, sum those numbers, and then divide by the number of numbers.

Formalization:

$$\bar{x} = \frac{1}{n} \left(\sum_{i=1}^n x_i \right) = \frac{x_1 + x_2 + \dots + x_n}{n}$$

Cites: [Wikipedia](#) ; [Wikidata](#) ; [Wolfram](#)

Code

11.1.0.1 R

Documentation: [mean: Arithmetic Mean](#)

Examples:

```
x = c(1,2,3,4)
x
```

```
[1] 1 2 3 4
```

```
#Algorithm
x_bar = sum(x, na.rm=T)/length(x)
x_bar
```

[1] 2.5

```
#Base Function
x_bar = mean(x, na.rm=T)
x_bar
```

[1] 2.5

11.1.0.2 Python

Documentation: [numpy.mean](#)

Examples:

```
x = [1,2,3,4]
print(x)
```

[1, 2, 3, 4]

```
#Algorithm
x_bar= sum(x)/len(x)
x_bar
```

2.5

```
#statistics Function
import statistics
x_bar = statistics.mean(x)
x_bar
```

2.5

```
#scipy Function
#<string>:1: DeprecationWarning: scipy.mean is deprecated and will be removed in SciPy 2.0
import scipy
x_bar = scipy.mean(x)
```

<string>:1: DeprecationWarning: scipy.mean is deprecated and will be removed in SciPy 2.0.0,

```
x_bar
```

2.5

```
#numpy Function
import numpy as np
x = np.array(x)
x_bar = x.mean()
x_bar
```

2.5

11.1.0.3 SQL

Documentation: [PostgreSQL AVG Function](#)

```
library(DBI)
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con <- dbConnect(RPostgres::Postgres())
```

```
DROP TABLE IF EXISTS t1;
```

```
CREATE TABLE IF NOT EXISTS t1 (
  id serial PRIMARY KEY,
  amount INTEGER
);
```

```
INSERT INTO t1 (amount)
VALUES
  (10),
  (NULL),
  (30);
```

```
SELECT * FROM t1;
```

Table 11.1: 3 records

| id | amount |
|----|--------|
| 1 | 10 |
| 2 | NA |
| 3 | 30 |

```
SELECT AVG(amount)::numeric(10,2)
FROM t1;
```

Table 11.2: 1 records

| avg |
|-----|
| 20 |

11.1.0.4 Torch

```
import torch
```

11.2 Bayesian

Bayesian average; Solving an age-old problem using Bayesian Average; Of bayesian average and star ratings; Bayesian Average Ratings ;

English: The Bayesian average is the weighted average of a prior and the observed sample average. When would you want this? When you have strong beliefs about the true mean, or when sample size is too small to reliably calculate a mean. For example a movie rating website where a movie may have only a single 5 star rating and so would rank higher than the Godfather with over a 100 almost all 5 star ratings.

Formalization:

$$\bar{x} = \frac{C * m + (\sum_{i=1}^n x_i)}{c + n}$$

Where m is a prior for true mean, and C is a constant representing how many elements would be necessary to reliably estimate a sample mean.

Code

References