

Simplify and Verify Trig Identities

Identities are statements that are true for all values of the input for which they are defined. For example, $2x + 6 = 2(x + 3)$ is an identity.

Identities enable us to **simplify** complicated expressions.

To **verify** trig identities, we usually start with the more complicated side and rewrite the expression until it has been transformed in the same expression as the other side. Things to try: apply identities, multiply, factor, common denominator, multiply by the conjugate, split fractions, and other algebraic strategies.

Reciprocal Identities

$$\begin{aligned}\sin \theta &= \frac{1}{\csc \theta} \Leftrightarrow \csc \theta = \frac{1}{\sin \theta} \\ \cos \theta &= \frac{1}{\sec \theta} \Leftrightarrow \sec \theta = \frac{1}{\cos \theta} \\ \tan \theta &= \frac{1}{\cot \theta} \Leftrightarrow \cot \theta = \frac{1}{\tan \theta}\end{aligned}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad 1 + \tan^2 \theta = \sec^2 \theta \qquad \cot^2 \theta + 1 = \csc^2 \theta$$

Even Functions

$$\begin{aligned}\cos(-x) &= \cos(x) \\ \sec(-x) &= \sec(x)\end{aligned}$$

Odd Functions

$$\begin{aligned}\sin(-x) &= -\sin(x) & \tan(-x) &= -\tan(x) \\ \csc(-x) &= -\csc(x) & \cot(-x) &= -\cot(x)\end{aligned}$$

Practice Problems

1. Simplify $\frac{\sec \theta}{\tan \theta}$.
2. Simplify $\cos t + \tan t \sin t$.
3. Simplify $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta}$.
4. Simplify $\frac{\cot x \sec x}{\csc x}$.

5. Verify $\frac{\cos x}{\sec x \sin x} = \csc x - \sin x$.

6. Verify $(1 + \sin x)(1 + \sin(-x)) = \cos^2 x$.

7. Verify $\frac{\tan y}{\csc y} = \frac{1}{\cos y} - \frac{1}{\sec y}$.

8. Verify $\frac{\cos^2 x}{\sin x} = \csc x - \sin x$.

9. Verify $\tan \theta + \cot \theta = \sec \theta \csc \theta$.

10. Verify $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$.

11. Verify $\csc x - \sin x = \cos x \cot x$.

12. Verify $\csc x \cos^2 x + \sin x = \csc x$.

13. Verify $\frac{\sec t - \cos t}{\sec t} = \sin^2 t$.

14. Verify $\frac{1 + \sec^2 x}{1 + \tan^2 x} = 1 + \cos^2 x$.

15. Verify $\frac{\csc x - \cot x}{\sec x - 1} = \cot x$.

16. Verify $\frac{\cos x}{1 + \sin x} = \frac{1 - \sin x}{\cos x}$.

17. Verify $\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$.

18. Verify $\frac{\cos u}{1 - \sin u} = \sec u + \tan u$.

Sum and Difference Formulas

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

19. Find the exact value of $\cos\left(\frac{5\pi}{4} - \frac{\pi}{6}\right)$.

20. Find the exact value of $\tan\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$.

21. Find the exact value of $\cos 75^\circ$.

22. Find the exact value of $\sin\left(\frac{11\pi}{12}\right)$.

23. Verify $\sin(x+y) - \sin(x-y) = 2 \cos x \sin y$.

24. Verify $3 \tan x - 3 \tan y = \frac{3 \sin(x-y)}{\cos x \cos y}$.

Double – Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\begin{aligned}\cos 2u &= \cos^2 u - \sin^2 u \\ &= 2 \cos^2 u - 1 \\ &= 1 - 2 \sin^2 u\end{aligned}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Half – Angle Formulas

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

Signs of $\sin \frac{u}{2}$ and $\cos \frac{u}{2}$ depends on the quadrant in which $\frac{u}{2}$ lies

25. Verify $\sin(2x) = \frac{2 \tan x}{1 + \tan^2 x}$.

26. Verify $\csc(2x) = \frac{\sec x \csc x}{2}$.

27. Verify $\cos^2(3x) - \sin^2(3x) = \cos(6x)$

28. Verify $\frac{\sin(4x)}{\sin x} = 4 \cos x \cos(2x)$.

29. Verify $\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}$.

30. Verify $\cot\left(\frac{x}{2}\right) = \csc x + \cot x$.

31. Find the exact value of $\sin 15^\circ$.

32. Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$.