Chat

User: Determine the value of $csc(\theta)$ given that the terminal side of angle θ intersects the unit circle in the first quadrant at $\left(\frac{7}{17}, y\right)$.

Assistant: To determine the value of $\csc(\theta)$ given that the terminal side of angle θ intersects the unit circle in the first quadrant at $\left(\frac{7}{17}, y\right)$, we can use the properties of the unit circle and the definition of the cosecant function.

The equation of the unit circle is:

$$x^2 + y^2 = 1$$

Given that $x = \frac{7}{17}$, we can solve for y using the equation of the unit circle.

$$y^2 = 1 - x^2$$

$$y^2 = 1 - \left(\frac{7}{17}\right)^2$$

Let's solve for y.

Assistant: From the equation $y^2 = \frac{240}{289}$, we have two solutions for y:
1) $y = -\frac{4\sqrt{15}}{17}$ 2) $y = \frac{4\sqrt{15}}{17}$
Since the point lies in the first quadrant, we will consider the positive value of y:

$$y = \frac{4\sqrt{15}}{17}$$

Now, the sine of angle θ is given by the y-coordinate of the point on the unit circle:

$$\sin(\theta) = \frac{4\sqrt{15}}{17}$$

The cosecant function is the reciprocal of the sine function:

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

Let's compute $\csc(\theta)$.

Assistant: The value of $\csc(\theta)$ is given by:

$$\csc(\theta) = \frac{17}{4\sqrt{15}}$$

This is the reciprocal of the sine value we found earlier. So, for the given point $\left(\frac{7}{17}, \frac{4\sqrt{15}}{17}\right)$ on the unit circle in the first quadrant, the value of $\csc(\theta)$ is $\frac{17}{4\sqrt{15}}$.

User: What is the pythagorean identity?

Assistant: The Pythagorean identity is a fundamental relationship between the sine and cosine functions in trigonometry. It is derived from the Pythagorean theorem and the definition of sine and cosine on the unit circle. The identity is:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

Where θ is any angle.

This identity states that for any angle θ, the square of the sine of θ plus the square of the cosine of θ will always equal 1. This relationship holds true for all values of θ and is a cornerstone of trigonometry.