Homework 6 Solutions

1. **Ex. 27**

$$P(n) \equiv \texttt{(append xs (append ys zs))} = \texttt{(append (append xs ys) zs)}$$

$$\text{where } \texttt{(len xs)} = n$$

Base case: $P(0)$

$$\begin{array}{lll} & \texttt{(append nil (append ys zs))} & \\ = & \texttt{(append ys zs)} & \{\mathit{app0}\} \\ = & \texttt{(append (append nil ys) zs)} & \{\mathit{app0}\} \end{array}$$

Inductive case: $P(n) \to P(n+1)$

$$\begin{array}{lll} & \texttt{(append (cons x xs) (append ys zs))} & \\ = & \texttt{(cons x (append xs (append ys zs)))} & \{\mathit{app1}\} \\ = & \texttt{(cons x (append (append xs ys) zs))} & \{P(n)\} \\ = & \texttt{(append (cons x (append xs ys)) zs)} & \{\mathit{app1}\} \\ = & \texttt{(append (append (cons x xs) ys) zs)} & \{\mathit{app1}\} \end{array}$$

2. **Ex. 28**

$$E(n) \equiv \texttt{(expt x n)} = x^n$$

Base case: $E(0)$

$$\begin{array}{lll} & \texttt{(expt x 0)} & \\ = & \texttt{1} & \{\mathit{expt0}\} \\ = & x^0 & \{\mathit{algebra}\} \end{array}$$

Inductive case

$$\begin{array}{lll} & \texttt{(expt x (+ n 1))} & \\ = & \texttt{(* x (expt x n))} & \{\mathit{expt1}\} \\ = & \texttt{(* x } x^n\texttt{)} & \{E(n)\} \\ = & x^{n+1} & \{\mathit{algebra}\} \end{array}$$

3. **Ex. 29**

$$R(n) \equiv \texttt{(len (rep n x))} = \texttt{n}$$

Base case: $R(0)$

$$\begin{array}{lll} & \texttt{(len (rep 0 x))} & \\ = & \texttt{(len nil)} & \{\mathit{rep0}\} \\ = & \texttt{0} & \{\mathit{len0}\} \end{array}$$

Inductive case

$$\begin{array}{lll} & \texttt{(len (rep (+ n 1) x))} & \\ = & \texttt{(len (cons x (rep n x)))} & \{\mathit{rep1}\} \\ = & \texttt{(+ 1 (len (rep n x)))} & \{\mathit{len1}\} \\ = & \texttt{(+ 1 n)} & \{R(n)\} \end{array}$$

4. **Ex. 30**

$M(n) \equiv$ `(member-equal y (rep n x))` $\rightarrow$ `(member-equal y (list x))`

$n \in \mathbb{N}$

Base case: $M(0)$

$$
\begin{array}{lll}
 & \texttt{(member-equal y (rep 0 x))} \rightarrow \texttt{(member-equal y (list x))} & \\
= & \texttt{(member-equal y nil)} \rightarrow \texttt{(member-equal y (list x))} & \{\textit{rep0}\} \\
= & \texttt{nil} \rightarrow \texttt{(member-equal y (list x))} & \{\textit{mem0}\} \\
= & (\neg\text{False}) \vee \texttt{(member-equal y (list x))} & \{\text{implication}\} \\
= & \text{True} \vee \texttt{(member-equal y (list x))} & \{\neg\text{False}\} \\
= & \texttt{(member-equal y (list x))} \vee \text{True} & \{\vee \text{ commutative}\} \\
= & \text{True} & \{\vee \text{ null}\}
\end{array}
$$

Inductive case

$$
\begin{array}{lll}
 & \texttt{(member-equal y (rep (+ n 1) x))} \rightarrow & \\
 & \quad \texttt{(member-equal y (list x))} & \\
= & \texttt{(member-equal y (cons x (rep n x))} \rightarrow & \\
 & \quad \texttt{(member-equal y (list x))} & \{\textit{rep1}\} \\
= & \texttt{(member-equal y (cons x (rep n x))} \rightarrow & \\
 & \quad \texttt{(member-equal y (cons x nil))} & \{\textit{list}\} \\
= & (\texttt{(equal y x)} \vee \texttt{(member-equal y (rep n x))}) \rightarrow & \\
 & \quad (\texttt{(equal y x)} \vee \texttt{(member-equal y nil)}) & \{\textit{mem1}\} \times 2 \\
= & (\texttt{(equal y x)} \vee \texttt{(member-equal y (rep n x))}) \rightarrow & \\
 & \quad (\texttt{(equal y x)} \vee \texttt{nil}) & \{\textit{mem0}\} \\
= & (\texttt{(equal y x)} \vee \texttt{(member-equal y (rep n x))}) \rightarrow & \\
 & \quad \texttt{(equal y x)} & \{\vee \text{ id}\}
\end{array}
$$

5. **Ex. 31**

$C(n) \equiv$ `(len (nthcdr (len xs) xs))` $= 0$

where `(len xs)` $= n$

Base case: $C(0)$

$$
\begin{array}{lll}
 & \texttt{(len (nthcdr (len nil) nil))} & \\
= & \texttt{(len (nthcdr 0 nil))} & \{\textit{len0}\} \\
= & \texttt{(len nil)} & \{\textit{sfx0}\} \\
= & \texttt{0} & \{\textit{len0}\}
\end{array}
$$

Inductive case

$$
\begin{array}{lll}
 & \texttt{(len (nthcdr (len (cons x xs)) (cons x xs)))} & \\
= & \texttt{(len (nthcdr (+ 1 (len xs)) (cons x xs)))} & \{\textit{len1}\} \\
= & \texttt{(len (nthcdr (len xs) xs))} & \{\textit{sfx1}\} \\
= & \texttt{0} & \{C(n)\}
\end{array}
$$

6. **Ex. 32**

$D(n) \equiv$ `(len (nthcdr (+ (len xs) n)))`

Base case

$$
\begin{array}{ll}
 & \texttt{(len (nthcdr (+ (len xs) 0) xs))} \\
= & \texttt{(len (nthcdr (len xs) xs))}\{+ \text{ identity}\} \\
= & \texttt{0}\{\mathit{drop} - \mathit{all0}\}
\end{array}
$$

Inductive case

$$
\begin{array}{lll}
 & \texttt{(len (nthcdr (+ (len xs) (+ n 1)) xs))} & \\
= & \texttt{(len (nthcdr (+ (+ (len xs) n) 1) xs))} & \{+ \text{ associative}\} \\
= & \texttt{(len (nthcdr (+ (len xs) n) (rest xs)))} & \{\mathit{sfx1}\}
\end{array}
$$

Case 1: `xs` $=$ `nil`

$$
\begin{array}{lll}
= & \texttt{(len (nthcdr (+ (len nil) n) (rest nil)))} & \\
= & \texttt{(len (nthcdr (+ (len nil) n) nil))} & \{\mathit{rest0}\} \\
= & \texttt{0} & \{D(n)\}
\end{array}
$$

Case 2: `xs` $=$ `(cons y ys)`

$$
\begin{array}{lll}
= & \texttt{(len (nthcdr (+ (len (cons y ys)) n) (rest (cons y ys))))} & \\
= & \texttt{(len (nthcdr (+ (len (cons y ys)) n) ys))} & \{\mathit{rest}\} \\
= & \texttt{0} & \{D(n)\}
\end{array}
$$