Homework 3 Answers

1. **Ex. 7**

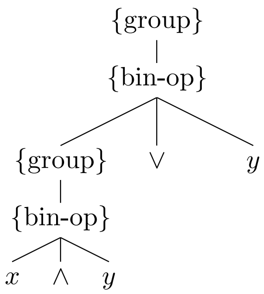
$$
\begin{array}{lll}
 & ((((-1) \times x) + (2 \times x)) + x) & \\
= & (((2 \times x) + ((-1) \times x)) + x) & \{+ \text{ commutative}\} \\
= & ((2 \times x) + (((-1) \times x) + x)) & \{+ \text{ associative}\} \\
= & ((2 \times x) + (((-1) \times x) + x \times 1)) & \{\times \text{ identity}\} \\
= & ((2 \times x) + ((x \times (-1)) + x \times 1)) & \{\times \text{ commutative}\} \\
= & ((2 \times x) + ((x \times (-1) + 1))) & \{\text{distributivelaw}\} \\
= & ((2 \times x) + ((x \times 0))) & \{+ \text{ complement}\} \\
= & ((2 \times x) + 0) & \{\times \text{ null}\} \\
= & (2 \times x) & \{+ \text{ identity}\}
\end{array}
$$

2. **Ex. 12**

x	y	z	$(\neg y)$	$(\neg z)$	$((\neg y) \wedge (\neg z))$	$(x \vee ((\neg y) \wedge (\neg z)))$
T	T	T	F	F	F	T
T	T	F	F	T	F	T
T	F	T	T	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	F	F
F	T	F	F	T	F	F
F	F	T	T	F	F	F
F	F	F	T	T	T	T

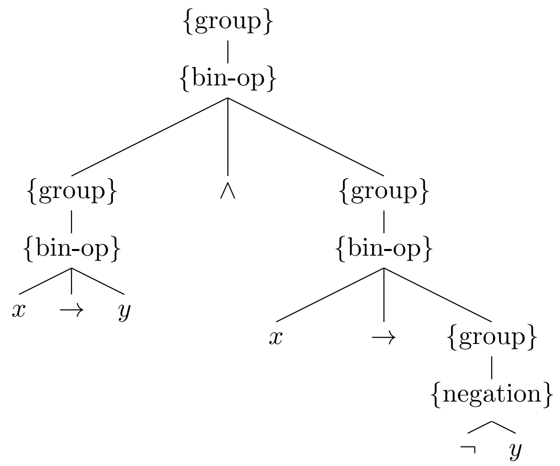
3. **Ex. 13**

(a)



x and y are atomic.

(b)



x and y are atomic.

(c) Not a well formed formula – $\neg$ used as a binary operator.

4. **Ex. 15**

(a)

$$\begin{array}{lll} & (x \rightarrow False) & \\ = & ((\neg x) \vee False) & \{\text{implication}\} \\ = & (\neg x) & \{\vee \text{ identity}\} \end{array}$$

(b)

$$\begin{array}{lll} & (\neg(x \wedge y)) & \\ = & (\neg((\neg(\neg x)) \wedge y)) & \{\text{double negation}\} \\ = & (\neg((\neg(\neg x)) \wedge (\neg(\neg y)))) & \{\text{double negation}\} \\ = & (\neg(\neg((\neg x) \vee (\neg y)))) & \{\vee \text{ DeMorgan}\} \\ = & ((\neg x) \vee (\neg y)) & \{\text{double negation}\} \end{array}$$

(c)

$$\begin{array}{lll} & (x \vee (\neg x)) & \\ = & ((\neg x) \vee x) & \{\vee \text{ commutative}\} \\ = & (x \rightarrow x) & \{\text{implication}\} \\ = & True & \{\text{self implication}\} \end{array}$$

(d)

$$\begin{array}{lll}
 & (x \wedge (\neg x)) & \\
= & ((\neg(\neg x)) \wedge (\neg x)) & \{\text{double negation}\} \\
= & (\neg((\neg x) \vee x)) & \{\vee \text{ DeMorgan}\} \\
= & (\neg(x \to x)) & \{\text{implication}\} \\
= & (\neg True) & \{\text{self implication}\} \\
= & (\neg(False \to False)) & \{\text{self implication}\} \\
= & (\neg((\neg False) \vee False)) & \{\text{implication}\} \\
= & (\neg(\neg False)) & \{\vee \text{ identity}\} \\
= & False & \{\text{double negation}\}
\end{array}$$

(e)

$$\begin{array}{lll}
 & (\neg True) & \\
= & (\neg(False \to False)) & \{\text{self implication}\} \\
= & (\neg((\neg False) \vee False)) & \{\text{implication}\} \\
= & (\neg(\neg False)) & \{\vee \text{ identity}\} \\
= & False & \{\text{double negation}\}
\end{array}$$

(f)

$$\begin{array}{lll}
 & (\neg False) & \\
= & (\neg(\neg True)) & \{\neg True\} \\
= & True & \{\text{double negation}\}
\end{array}$$

(g)

$$\begin{array}{lll}
 & (True \to x) & \\
= & ((\neg True) \vee x) & \{\text{implication}\} \\
= & (False \vee x) & \{\neg False\} \\
= & (x \vee False) & \{\vee \text{ commutative}\} \\
= & x & \{\vee \text{ identity}\}
\end{array}$$

(h)

$$\begin{array}{lll}
 & (x \wedge True) & \\
= & (\neg(\neg(x \wedge True))) & \{\text{double negation}\} \\
= & (\neg((\neg x) \vee (\neg True))) & \{\wedge \text{ DeMorgan}\} \\
= & (\neg((\neg x) \vee False)) & \{\neg True\} \\
= & (\neg(\neg x)) & \{\vee \text{ identity}\} \\
= & x & \{\text{double negation}\}
\end{array}$$

(i)

$$
\begin{array}{lll}
 & (x \wedge y) & \\
= & ((\neg(\neg x)) \wedge y) & \{\text{double negation}\} \\
= & ((\neg(\neg x)) \wedge (\neg(\neg y))) & \{\text{double negation}\} \\
= & (\neg((\neg x) \vee (\neg y))) & \{\vee \text{ DeMorgan}\} \\
= & (\neg((\neg y) \vee (\neg x))) & \{\vee \text{ commutative}\} \\
= & ((\neg(\neg y) \wedge (\neg(\neg x)))) & \{\vee \text{ DeMorgan}\} \\
= & (y \wedge (\neg(\neg x)))) & \{\text{double negation}\} \\
= & (y \wedge x) & \{\text{double negation}\}
\end{array}
$$

(j)

$$
\begin{array}{lll}
 & (x \wedge (y \wedge z)) & \\
= & ((\neg(\neg x)) \wedge ((\neg(\neg y)) \wedge (\neg(\neg z)))) & \{\text{double negation} \times 3\} \\
= & ((\neg(\neg x)) \wedge (\neg((\neg y) \vee (\neg z)))) & \{\vee \text{ DeMorgan}\} \\
= & (\neg((\neg x) \vee ((\neg y) \vee (\neg z)))) & \{\vee \text{ DeMorgan}\} \\
= & (\neg(((\neg x) \vee (\neg y)) \vee (\neg z))) & \{\vee \text{ associative}\} \\
= & (\neg((\neg(x \wedge y)) \vee (\neg z))) & \{\wedge \text{ DeMorgan}\} \\
= & (\neg(\neg((x \wedge y) \wedge z))) & \{\wedge \text{ DeMorgan}\} \\
= & ((x \wedge y) \wedge z) & \{\text{double negation}\}
\end{array}
$$

(k)

$$
\begin{array}{lll}
 & (x \wedge (y \vee z)) & \\
= & (\neg(\neg(x \wedge (y \vee z)))) & \{\text{double negation}\} \\
= & (\neg((\neg x) \vee (\neg(y \vee z)))) & \{\wedge \text{ DeMorgan}\} \\
= & (\neg((\neg x) \vee ((\neg y \wedge (\neg z))))) & \{\vee \text{ DeMorgan}\} \\
= & (\neg(((\neg x) \vee (\neg y)) \wedge ((\neg x \vee (\neg z))))) & \{\vee \text{ distributive}\} \\
= & (\neg(\neg(x \wedge y)) \wedge ((\neg x \vee (\neg z)))) & \{\wedge \text{ DeMorgan}\} \\
= & (\neg(\neg(x \wedge y)) \wedge (\neg(x \wedge z))) & \{\wedge \text{ DeMorgan}\} \\
= & ((x \wedge y) \vee (x \wedge z)) & \{\vee \text{ DeMorgan}\}
\end{array}
$$

(l)

$$
\begin{array}{lll}
 & (x \wedge x) & \\
= & (\neg(\neg(x \wedge x))) & \{\text{doublenegation}\} \\
= & (\neg((\neg x) \vee (\neg x))) & \{\wedge \text{ DeMorgan}\} \\
= & (\neg(\neg x)) & \{\vee \text{ idempotent}\} \\
= & x & \{\text{double negation}\}
\end{array}
$$

(m)

$$
\begin{array}{lll}
 & (x \rightarrow y) & \\
= & ((\neg x) \vee y) & \{\text{implication}\} \\
= & (y \vee (\neg x)) & \{\vee \text{ commutative}\} \\
= & ((\neg(\neg y)) \vee (\neg x)) & \{\text{double negation}\} \\
= & ((\neg y) \rightarrow (\neg x)) & \{\text{implication}\}
\end{array}
$$

(n)

$$
\begin{array}{lll}
 & (x \to (y \to z)) & \\
= & ((\neg x) \vee ((\neg y) \vee z)) & \{\text{implication} \times 2\} \\
= & (((\neg x) \vee (\neg y)) \vee z) & \{\vee \text{ associative}\} \\
= & ((\neg (x \wedge y)) \vee z) & \{\wedge \text{ DeMorgan}\} \\
= & ((x \wedge y) \to z) & \{\text{implication}\}
\end{array}
$$

(o)

$$
\begin{array}{lll}
 & ((x \wedge y) \vee y) & \\
= & (y \vee (x \wedge y)) & \{\vee \text{ commutative}\} \\
= & (y \vee (y \wedge x)) & \{\wedge \text{ commutative}\} \\
= & ((y \wedge True) \vee (y \wedge x)) & \{\vee \text{ commutative}\} \\
= & (y \wedge (x \vee True)) & \{\wedge \text{ distributive}\} \\
= & (y \wedge True) & \{\vee \text{ null}\} \\
= & y & \{\wedge \text{ identity}\}
\end{array}
$$

(p)

$$
\begin{array}{lll}
 & ((x \to y) \wedge (x \to (\neg y))) & \\
= & (((\neg x) \vee y) \wedge ((\neg x) vee (\neg y))) & \{\text{implication}\} \\
= & ((\neg x) \vee (y \wedge (\neg y))) & \{\vee \text{ distributive}\} \\
= & ((\neg x) \vee False) & \{\wedge \text{ complement}\} \\
= & (\neg x) & \{\vee \text{ identity}\}
\end{array}
$$

5. **Ex. 19**