Homework 2 Solutions

1. **Ex. 7**

$$
\begin{array}{lll}
 & ((-1) \times x) + x & \\
= & ((-1) \times x) + (x \times 1) & \{\times \text{ identity}\} \\
= & (x \times (-1)) + (x \times 1) & \{\times \text{ commutative}\} \\
= & x \times ((-1) + 1) & \{\times \text{ distributive law}\} \\
= & x \times 0 & \{+ \text{ complement}\} \\
= & 0 & \{\times \text{ null}\}
\end{array}
$$

2. **Ex. 8**

$$
\begin{array}{lll}
 & ((x + (((-1) \times (x + y)) + z)) + y) & \\
= & (x + ((((-1) \times x) + ((-1) \times y))) + z) + y) & \{\text{distributive law}\} \\
= & ((x + ((-1) \times x) + (((-1) \times y) + z)) + y) & \{+ \text{ associative}\} \\
= & ((x + ((-1) \times x)) + (((-1) \times y) + z) + y) & \{+ \text{ associative}\} \\
= & ((((-1) \times y) + z) + y) & \{+ \text{ complement}\} \\
= & ((z + ((-1) \times y)) + y) & \{+ \text{ commutative}\} \\
= & (z + (((-1) \times y) + y)) & \{+ \text{ associative}\} \\
= & z + 0 & \{\times \text{ negation}\} \\
= & z & \{+ \text{ identity}\}
\end{array}
$$

3. **Ex. 9**

$$
\begin{array}{lll}
 & (x \wedge y) \vee y & \\
= & y \vee (x \wedge y) & \{\vee \text{ commutative}\} \\
= & (y \vee x) \wedge (y \vee y) & \{\vee \text{ distributive}\} \\
= & (y \vee x) \wedge y & \{\vee \text{ idempotent}\} \\
= & (x \vee y) \wedge y & \{\vee \text{ commutative}\} \\
= & y & \{\wedge \text{ absorption}\}
\end{array}
$$

4. **Ex. 10**

Lemma: $x \wedge \text{True} = x$ $\{\wedge \text{ identity}\}$

$$
\begin{array}{lll}
 & x \wedge \text{True} & \\
= & (\neg(\neg x)) \wedge (\neg(\neg \text{True})) & \{\text{double negation}\} \times 2 \\
= & \neg((\neg x) \vee (\neg \text{True})) & \{\vee \text{ DeMorgan}\} \\
= & \neg((\neg x) \vee (\neg(\text{False} \to \text{False}))) & \{\text{self-implication}\} \\
= & \neg((\neg x) \vee (\neg((\neg \text{False}) \vee \text{False}))) & \{\text{implication}\} \\
= & \neg((\neg x) \vee (\neg(\neg \text{False}))) & \{\vee \text{ identity}\} \\
= & \neg((\neg x) \vee \text{False}) & \{\text{double negation}\} \\
= & \neg(\neg x) & \{\vee \text{ identity}\} \\
= & x & \{\text{double negation}\}
\end{array}
$$

Proof:

$$\begin{array}{lll}
 & ((\neg x) \wedge y) \vee (x \wedge (\neg y)) & \\
= & (((\neg x) \wedge y) \vee x) \wedge (((\neg x) \wedge y) \vee (\neg y)) & \{\text{distributive law}\} \\
= & (x \vee ((\neg x) \wedge y)) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\vee\,\text{commutative}\} \\
= & ((x \vee (\neg x)) \wedge (x \vee y))) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\text{distributive law}\} \\
= & (((\neg x) \vee x) \wedge (x \vee y))) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\vee\,\text{commutative}\} \\
= & ((x \rightarrow x) \wedge (x \vee y))) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\text{implication}\} \\
= & (\text{True} \wedge (x \vee y)) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\text{self-implication}\} \\
= & ((\neg(\neg\text{True})) \wedge (\neg(\neg(x \vee y)))) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\text{double negation}\} \times 2 \\
= & (\neg((\neg\text{True}) \vee (\neg(x \vee y)))) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\vee\,\text{DeMorgan}\} \\
= & (\neg((\neg(x \vee y)) \vee (\neg\text{True}))) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\vee\,\text{commutative}\} \\
= & ((\neg(\neg(x \vee y))) \vee (\neg(\neg\text{True}))) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\vee\,\text{DeMorgan}\} \\
= & ((x \vee y) \vee \text{True}) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\text{double negation}\} \times 2 \\
= & (x \vee y) \wedge ((\neg y) \vee ((\neg x) \wedge y)) & \{\wedge\,\text{identity}\} \\
= & (x \vee y) \wedge (((\neg y) \vee (\neg x)) \wedge ((\neg y) \vee y))) & \{\vee\,\text{distributive}\} \\
= & (x \vee y) \wedge (((\neg y) \vee (\neg x)) \wedge (y \rightarrow y)) & \{\text{implication}\} \\
= & (x \vee y) \wedge (((\neg y) \vee (\neg x)) \wedge \text{True}) & \{\text{self-implication}\} \\
= & (x \vee y) \wedge ((\neg y) \vee (\neg x)) & \{\wedge\,\text{identity}\} \\
= & (x \vee y) \wedge (\neg(x \wedge y)) & \{\vee\,\text{DeMorgan}\}
\end{array}$$

5. **Ex. 12**

$x \vee ((\neg y) \wedge (\neg z))$

(a) $(x = \text{False},\ y = \text{False},\ z = \text{False})$

$$\begin{array}{lll}
 & \text{False} \vee ((\neg\text{False}) \wedge (\neg\text{False})) & \\
= & ((\neg\text{False}) \wedge (\neg\text{False})) \vee \text{False} & \{\vee\,\text{commutative}\} \\
= & (\neg\text{False}) \wedge (\neg\text{False}) & \{\vee\,\text{identity}\} \\
= & \text{False} \vee \text{False} & \{\vee\,\text{DeMorgan}\} \\
= & \text{False} & \{\vee\,\text{identity}\}
\end{array}$$

(b) $(x = \text{False},\ y = \text{False},\ z = \text{True})$

$$\begin{array}{lll}
 & \text{False} \vee ((\neg\text{False}) \wedge (\neg\text{True})) & \\
= & ((\neg\text{False}) \wedge (\neg\text{True})) \vee \text{False} & \{\vee\,\text{commutative}\} \\
= & ((\neg\text{False}) \wedge (\neg\text{True})) & \{\vee\,\text{identity}\} \\
= & \text{False} \vee \text{True} & \{\vee\,\text{DeMorgan}\} \\
= & \text{True} & \{\vee\,\text{null}\}
\end{array}$$

(c) $(x = \text{False},\ y = \text{True},\ z = \text{False})$

$$\begin{array}{lll}
 & \text{False} \vee ((\neg\text{True}) \wedge (\neg\text{False})) & \\
= & ((\neg\text{True}) \wedge (\neg\text{False})) \vee \text{False} & \{\vee\,\text{commutative}\} \\
= & ((\neg\text{True}) \wedge (\neg\text{False})) & \{\vee\,\text{identity}\} \\
= & \text{True} \vee \text{False} & \{\vee\,\text{DeMorgan}\} \\
= & \text{True} & \{\vee\,\text{identity}\}
\end{array}$$

(d) $(x = \text{False},\ y = \text{True},\ z = \text{True})$

$$\begin{array}{lll}
 & \text{False} \vee ((\neg\text{True}) \wedge (\neg\text{True})) & \\
= & ((\neg\text{True}) \wedge (\neg\text{True})) \vee \text{False} & \{\vee\,\text{commutative}\} \\
= & ((\neg\text{True}) \wedge (\neg\text{True})) & \{\vee\,\text{identity}\} \\
= & \text{True} \vee \text{True} & \{\vee\,\text{DeMorgan}\} \\
= & \text{True} & \{\vee\,\text{null}\}
\end{array}$$

(e) $(x = \text{True},\ y = \text{False},\ z = \text{False})$

$$\begin{array}{cll} & \text{True} \lor ((\neg\text{False}) \land (\neg\text{False})) & \\ = & ((\neg\text{False}) \land (\neg\text{False})) \lor \text{True} & \{\lor\,\text{commutative}\} \\ = & \text{True} & \{\lor\,\text{null}\} \end{array}$$

(f) $(x = \text{True},\ y = \text{False},\ z = \text{True})$

$$\begin{array}{cll} & \text{True} \lor ((\neg\text{False}) \land (\neg\text{True})) & \\ = & ((\neg\text{False}) \land (\neg\text{True})) \lor \text{True} & \{\lor\,\text{commutative}\} \\ = & \text{True} & \{\lor\,\text{null}\} \end{array}$$

(g) $(x = \text{True},\ y = \text{True},\ z = \text{False})$

$$\begin{array}{cll} & \text{True} \lor ((\neg\text{True}) \land (\neg\text{False})) & \\ = & ((\neg\text{True}) \land (\neg\text{False})) \lor \text{True} & \{\lor\,\text{commutative}\} \\ = & \text{True} & \{\lor\,\text{null}\} \end{array}$$

(h) $(x = \text{True},\ y = \text{True},\ z = \text{True})$

$$\begin{array}{cll} & \text{True} \lor ((\neg\text{True}) \land (\neg\text{True})) & \\ = & ((\neg\text{True}) \land (\neg\text{True})) \lor \text{True} & \{\lor\,\text{commutative}\} \\ = & \text{True} & \{\lor\,\text{null}\} \end{array}$$