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# ELEC4700 Assignment 4

## Table of Contents

Question 1 - Voltage Sweep of Assignment 3 .....	1
Question 2 - Getting R3 .....	4
Question 3 - Differential Equations and DC/AC sweeps .....	4
Question 4 Simulation of the Transient Circuit .....	8
Question 5 Simulation of the Circuit With Noise .....	12
Question 5 - Non Linearity .....	20

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## Question 1 - Voltage Sweep of Assignment 3

This section uses parts of the ode from the previous assignment to perform a linear fit of the output current after doing a voltage sweep from 0.1V to 10V

```
clear; close; clc;
set(0, 'DefaultFigureWindowStyle', 'docked')
%Setting Constants
mElec = 9.11e-31; % Electron rest mass (kg)
mEff = 0.26*mElec; % Effective mass (kg)
kb = 1.381e-23; % Boltzmann's Constant (J/K)
T = 300; % Room Temperature (K)
L = 200e-9; % Length (m)
W = 100e-9; % Width (m)
q = 1.60218e-19; % Elementary Charge (C)
V = 0.1:10;
ii = 1;
for v = 0.1:10
    Vvec = linspace(v, 0, 200);
    Vmat = zeros(100, 200);
    for k = 1:100
        Vmat(k,:) = Vvec;
    end

    [Ex,Ey] = gradient(-Vmat, 1e-9);

    Fx = q*Ex(1,1); % Force on the electrons (in Newtons)
    Fy = q*Ey(1,1); % Support for forces generated by E fields with
    vertical components

    ax = Fx/mEff; % Horizontal Acceleration
    ay = Fy/mEff; % Vertical Acceleration

    % Finding the thermal velocity
    Vth = sqrt((2*kb*T)/mEff); %Thermal Velocity in m/s
    mt = 0.2e-12; % mean time (s)
```

```
% Modeling the electron paths

numPar = 10; % Number of particles

% Assigning particle positions
posX = L.*rand(numPar,2);
posY = W.*rand(numPar,2);
posX(:,1) = posX(:,2);
posY(:,1) = posY(:,2);

spacialStep = sqrt(L^2+W^2)/1000;
stepTime = spacialStep/Vth;

% Assigning each particle a random velocity (acceleration will be
taken into account during the iteration)
Vx = randn(numPar,2)*sqrt((kb*T)/mEff);
Vy = randn(numPar,2)*sqrt((kb*T)/mEff);

% Displacement per step of each electron
dispX = stepTime*Vx(:,1);
dispY = stepTime*Vy(:,1);

colors = rand(numPar,3);

pScatter = 1 - exp(-stepTime/mt); % Probability of an electron
scattering

% Positions before collision
collX = posX(:,1);
collY = posY(:,1);

ic = 0; % iteration counter for animated plot

%  $J = n \cdot (q/\text{density}) \cdot \text{mean}(V_n)$ , We can get the drift current density
from
% this equation using the average velocity

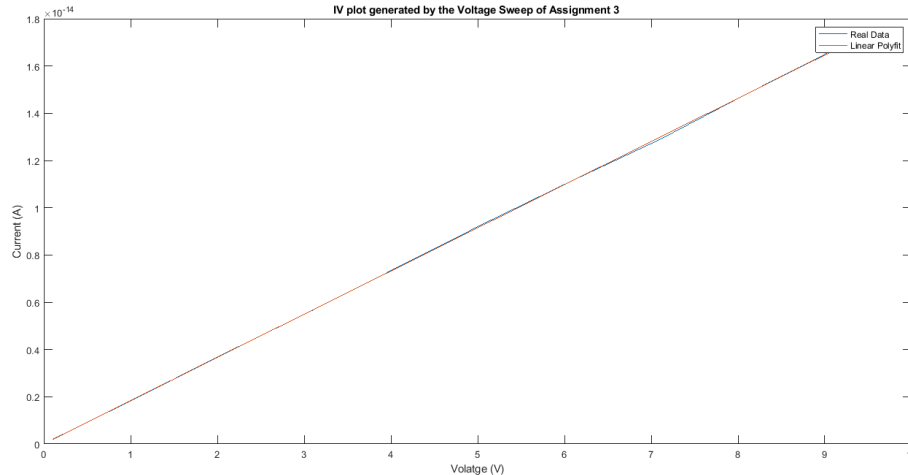
conc = 10^19; % concentration of electrons per m^2
numPar2 = 10000;
Vx = randn(numPar2,2)*sqrt((kb*T)/mEff);
Vy = randn(numPar2,2)*sqrt((kb*T)/mEff);
posX = L.*rand(numPar2,2);
posY = W.*rand(numPar2,2);
posX(:,1) = posX(:,2);
posY(:,1) = posY(:,2);
dispX = stepTime*Vx(:,1);
dispY = stepTime*Vy(:,1);
ic = 0;
avgVx = 0;
avgVy = 0;

for i = 1:1000
    Vx = Vx + ax*stepTime;
    Vy = Vy + ay*stepTime;
```

```
avgVx(i) = mean(Vx(:,2));
avgVy(i) = mean(Vy(:,2));

for j = 1:numPar2
    if(rand < pScatter)
        Vx(j,:) = randn(1)*sqrt((kb*T)/mEff) + ax*stepTime;
        Vy(j,:) = randn(1)*sqrt((kb*T)/mEff) + ay*stepTime;
        dispX(j,:) = stepTime*Vx(j,1);
        dispY(j,:) = stepTime*Vy(j,1);
        collX = posX(:,1);
        collY = posY(:,1);
    end
    if (posX(j,1)+dispX(j) > L)
        dispX(j,:) = stepTime*Vx(j,1);
        posX(j,2) = posX(j,1)+dispX(j)-L;
    elseif (posX(j,1)+dispX(j) < 0)
        dispX(j,:) = stepTime*Vx(j,1);
        posX(j,2) = posX(j,1)+dispX(j)+L;
    else
        dispX(j,:) = stepTime*Vx(j,1);
        posX(j,2) = posX(j,1)+dispX(j);
    end

    if ((posY(j,1)+dispY(j) > W) || (posY(j,1)+dispY(j) < 0))
        dispY(j) = -dispY(j);
        posY(j,2) = posY(j,1)+dispY(j);
    else
        posY(j,2) = posY(j,1)+dispY(j);
    end
end
end
J(ii) = mean(W*q*conc.*avgVx);
ii = ii + 1;
end
I = J*W*L;
poly = polyfit(V, I,1);
fit = poly(1)*V + poly(2);
figure(1)
plot(V, I)
hold on;
plot(V, fit)
title('IV plot generated by the Voltage Sweep of Assignment 3 ')
xlabel('Volatge (V)')
ylabel('Current (A)')
legend('Real Data','Linear Polyfit')
hold off;
```



## Question 2 - Getting R3

Normally the resistance would just be equal to the inverse of the slope of the linear fit. i.e  $R3 = 1/\text{poly}(1)$ ; However I am unsure of the value I'm getting from the previous assignment, the units might be off or I missed a step somewhere, so for the remainder of the assignment I'll be using  $R3 = 10$ , just so that the data for the rest of the assignment isn't skewed by R3.

## Question 3 - Differential Equations and DC/AC sweeps

a) The differential equations were found by performing KCL on each of the nodes in the circuit i)

$$\text{Node 1: } V1 = Vin \quad I1 = (V1-V2)/R1 + C(d(V1-V2))/dt$$

$$\text{Node 2: } (V1-V2)/R1 + C(d(V1-V2))/dt + V2/R2 + iL = 0$$

$$\text{Node 3: } V3/R3 = iL = I3 \quad V2 - V3 = L(d(iL))/dt$$

$$\text{Node 4: } V4 = aI3 \quad I4 = (V4-V5)/R4$$

$$\text{Node 5: } Vo/Ro = (V4-V5)/R4$$

ii) The differential equations in the frequency domain Node 1:  $V1 = Vin \quad I1 = G1(V1-V2) + j\omega C(d(V1-V2))$

$$\text{Node 2: } G1(V1-V2) + j\omega C(V1-V2) + V2G2 + j\omega L(V2-V3) = 0$$

$$\text{Node 3: } j\omega L(V2-V3) = V3G3$$

$$\text{Node 4: } V4 = aI3 \quad I4 = G4(V4-V5)$$

$$\text{Node 5: } VoGo = G4(V4-V5)$$

$$V1, \quad V2, \quad iL, \quad V3, \quad V4, \quad Vo$$

G:

1,	0,	0,	0,	0,	0
-1/R1,	1/R2+1/R1,	0,	0,	0,	0
0,	-1,	0,	1,	0,	0
0,	0,	-1,	1/R3,	0,	0
0,	0,	0,	-a/R3,	1,	0
0,	0,	0,	0,	-1/R4,	1/R4+1/Ro

C: 0, 0, 0, 0, 0, 0 -C, C, 0, 0, 0, 0 0, 0, L, 0, 0, 0 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0

b) i) DC Sweep

```

R1 = 1;
C = 0.25;
R2 = 2;
L = 0.2;
a = 100;
R3 = 10;
R4 = 0.1;
Ro = 1000;

G = [
    1, 0, 0, 0, 0, 0;
    -1/R1, 1/R2+1/R1, 0, 0, 0, 0;
    0, -1, 0, 1, 0, 0;
    0, 0, -1, 1/R3, 0, 0;
    0, 0, 0, -a/R3, 1, 0;
    0, 0, 0, 0, -1/R4, 1/R4+1/Ro
];

Cmat = zeros(6,6);
Cmat(2,1) = -C;
Cmat(2,2) = C;
Cmat(3,3) = L;

Vo = zeros(1,20);
V3 = zeros(1,20);
F = zeros(1,6);
Vin = -10:10;
for V = Vin
    F(1) = V;
    Vmat = G\F';
    Vo(V+1) = Vmat(6);
    V3(V+1) = Vmat(4);
end

figure(2)
plot(Vin,Vo)
hold on
plot(Vin,V3)
xlabel('Input Voltage')
legend('Vo', 'V3')
title('Input Voltage vs V0 and V3 for a DC sweep')

% ii) AC case plot

```

```

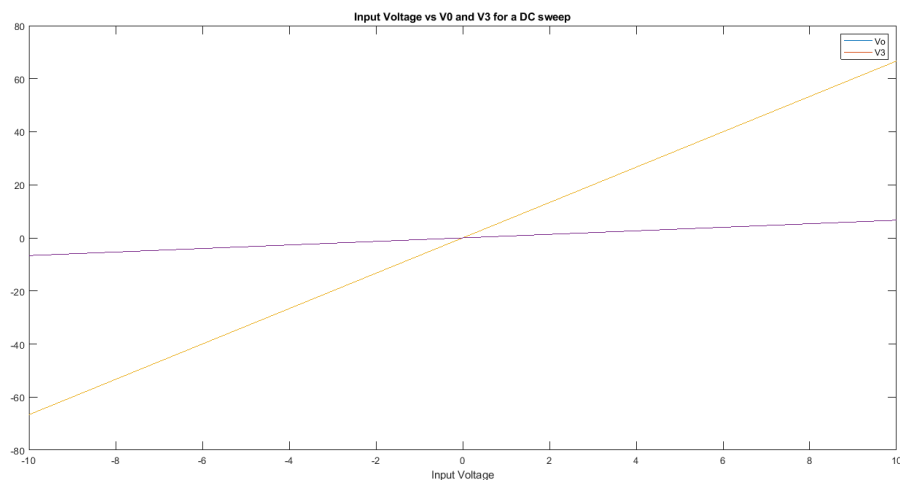
F(1) = 1;
Vo = zeros(1,1000);
w = 0:1000;
j = sqrt(-1);
for i = w
    G_ac = G + j*i*Cmat;
    Vmat = G_ac\F';
    Vo(i+1) = Vmat(6);
end
figure(3)
plot(w,abs(Vo))
xlabel('Frequency (rad/s)')
ylabel('Voltage')
title ('Output Voltage vs Frequency')

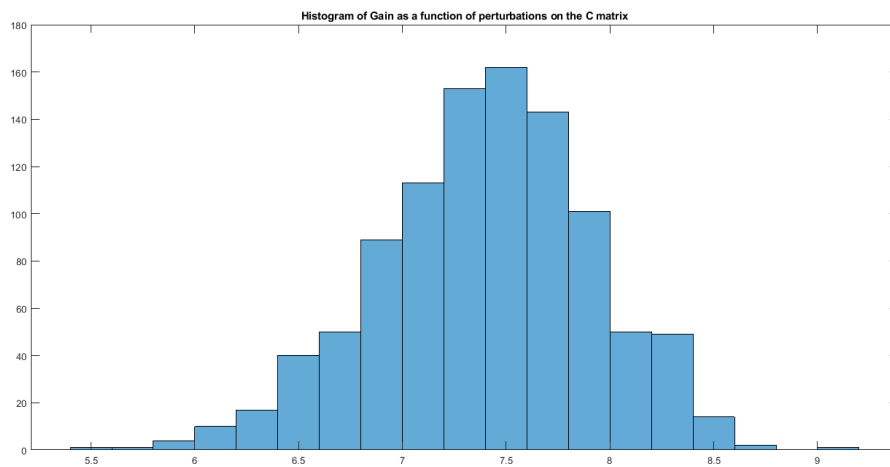
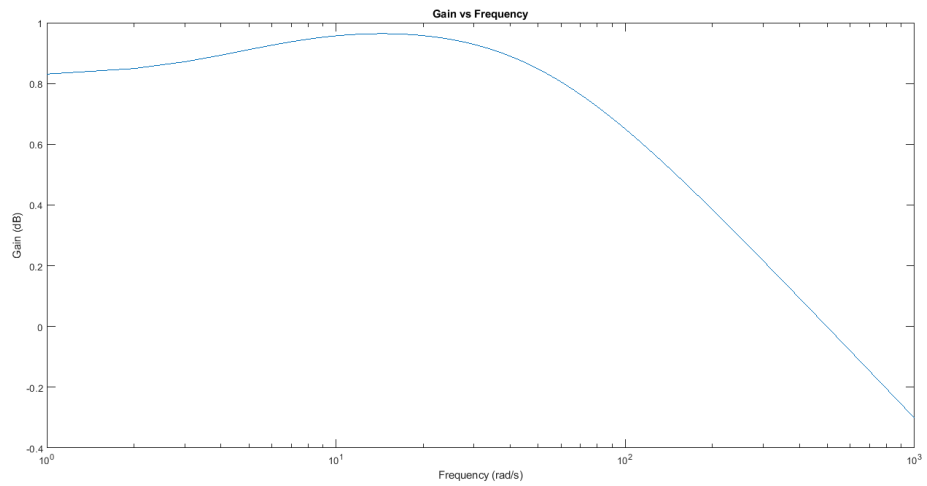
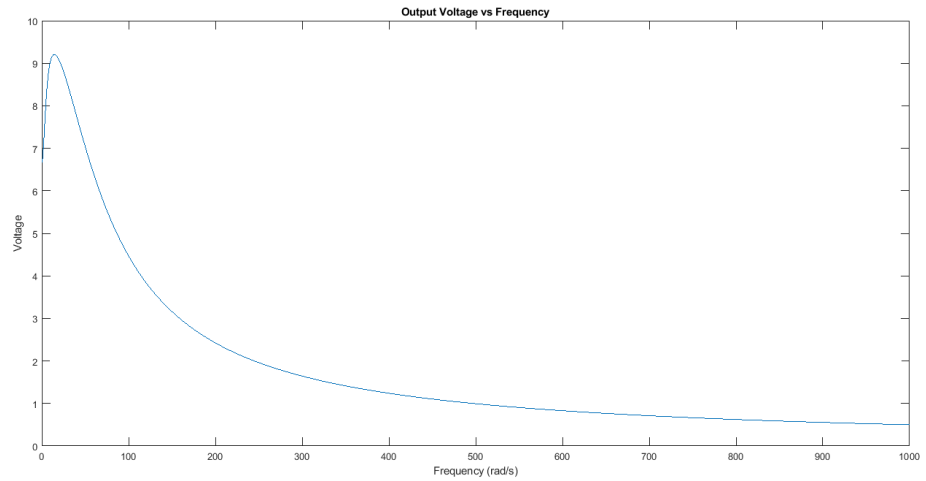
figure(4)
semilogx(w,log10(abs(Vo)))
xlabel('Frequency (rad/s)')
ylabel('Gain (dB)')
title ('Gain vs Frequency')

% iii) Gain as a function of random perturbations

AV = zeros(1,1000);
for i = 1:1000
    Cmat(2, 1) = normrnd(-C, 0.05);
    Cmat(2, 2) = normrnd(C, 0.05);
    Cmat(3, 3) = normrnd(L, 0.05);
    Vmat = (G+j*pi*Cmat)\F';
    AV(i) = Vmat(6)/F(1);
end
figure(5)
histogram(real(AV))
title('Histogram of Gain as a function of perturbations on the C
matrix')

```





## Question 4 Simulation of the Transient Circuit

The next part of the assignment is to perform a transient circuit simulation, which requires us to solve the time domain equation of the circuit.  $\frac{C(dV)}{dt} + GV = F$  A numerical solution of the equation can be done using the finite difference method.

```
% a) This circuit behaves like a low pass filter
% b) The gain should be constant and unattenuated at low frequencies
    and
% then there should be a constant drop in the gain starting from the
    circuit's
% cutoff frequency
% c) Using the Finite difference method we get
%  $\frac{C(dV)}{dt} + GV = F$ 

% d)
% In order to find plot the response of Vin and Vout in the time and
% frequency domain we need to simulate the circuit for each different
% type of input we use
% First we simulate the circuit with a step input changing the input
% voltage from 0 to 1 at t = 0.03
Vin = zeros(1, 1000); Vo = Vin; V3 = Vo;
V = zeros(6,1); F = zeros(1,6);
t = linspace(0,1,1000);

for i = 1:1000
    if t(i) < 0.03
        Vin(i) = 0;
    else
        Vin(i) = 1;
    end
    F(1) = Vin(i);
    V = (Cmat/t(2) + G)\((Cmat*V/t(2)) + F');
    Vo(i) = V(6);
end

figure(6)
plot(t, Vo)
title('Circuit Simulation when using a step input at t = 0.03')
xlabel('Time')
ylabel('Voltage')
hold on
plot(t,Vin)
legend('Vo', 'Vin')
hold off;

f = fft(Vo);
fx = (-1000/2:1000/2-1);
p = abs(fftshift(f)).^2/1000;
figure(7)
plot(fx,p)
xlabel('Frequency')
ylabel('Magnitude')
```



```
title('Frequency content of the input and output Voltages for the step
input')
hold on;
p = abs(fftshift(fft(Vin))).^2/1000;
plot(fx,p)
legend('Vo', 'Vin')
xlim([-200 200])
hold off;

% Next, we repeat the simulation using a sinusoidal input with A = 1V
and
% T = 0.03s
V = zeros(6,1); F = zeros(1,6);
for i = 1:1000
    Vin(i) = sin(2*pi*(1/.03)*t(i));
    F(1) = Vin(i);
    V = (Cmat/t(2) + G)\((Cmat*V/t(2)) + F');
    Vo(i) = V(6);
end
figure(8)
plot(t, Vo)
title('Circuit Simulation when using a sinusoidal input with T =
0.03')
xlabel('Time')
ylabel('Voltage')
hold on
plot(t,Vin)
legend('Vo', 'Vin')
hold off;

f = fft(Vo);
fx = (-1000/2:1000/2-1);
p = abs(fftshift(f)).^2/1000;
figure(9)
plot(fx,p)
xlabel('Frequency')
ylabel('Magnitude')
title('Frequency content of the input and output Voltages for the
sinusoidal input')
hold on;
p = abs(fftshift(fft(Vin))).^2/1000;
plot(fx,p)
legend('Vo', 'Vin')
xlim([-200 200])
hold off;

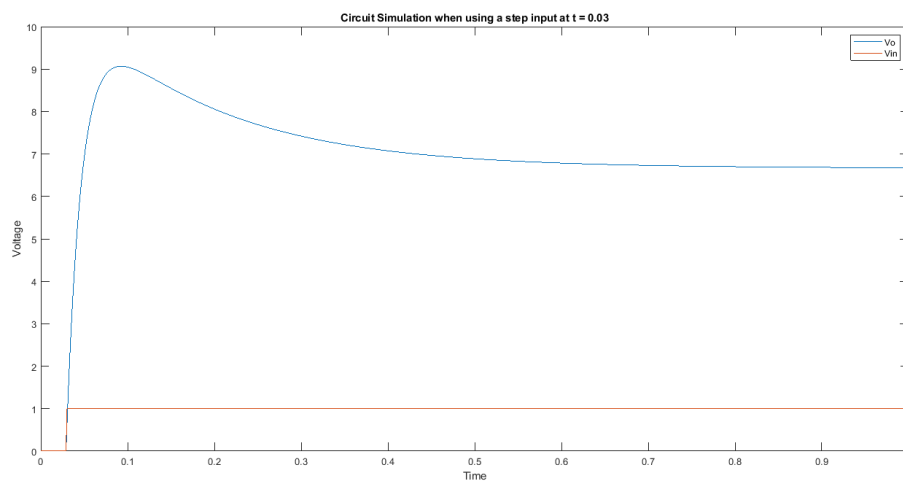
% Finally, we repeat the simulation one more time except this time
using a
% gaussian pulse of magnitude 1, 0.03s std deviation, and 0.06s delay
% Next, we repeat the simulation using a sinusoidal input with A = 1V
and
% T = 0.03s
V = zeros(6,1); F = zeros(1,6);
for i = 1:1000
```

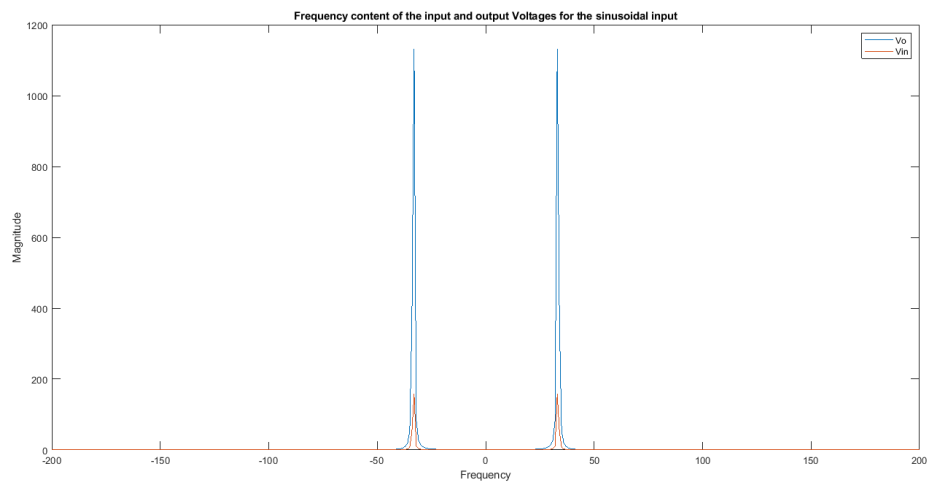
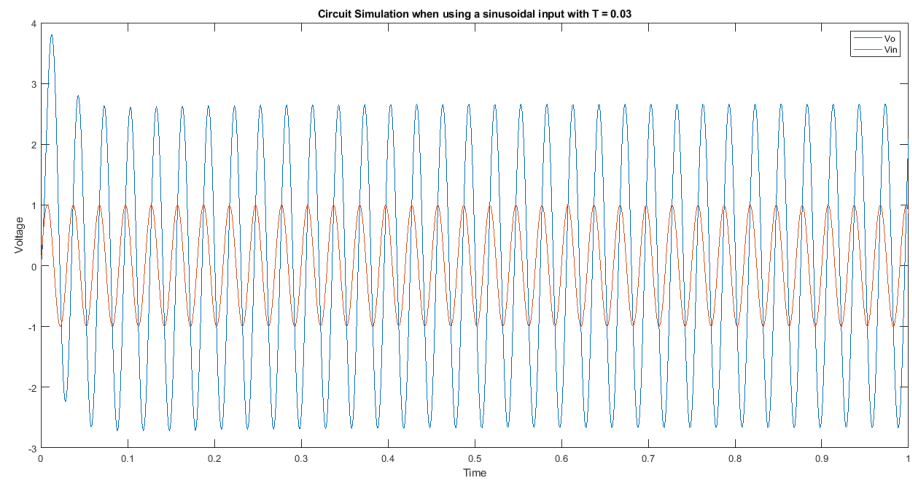
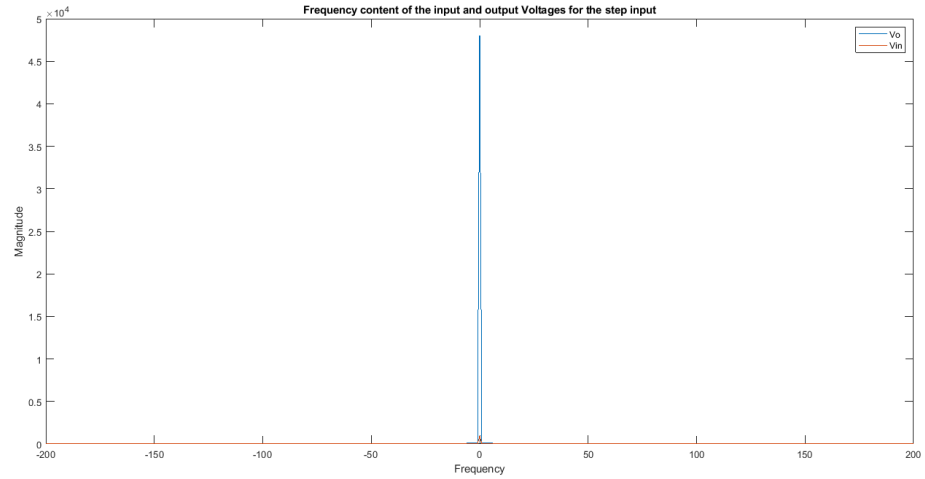
```

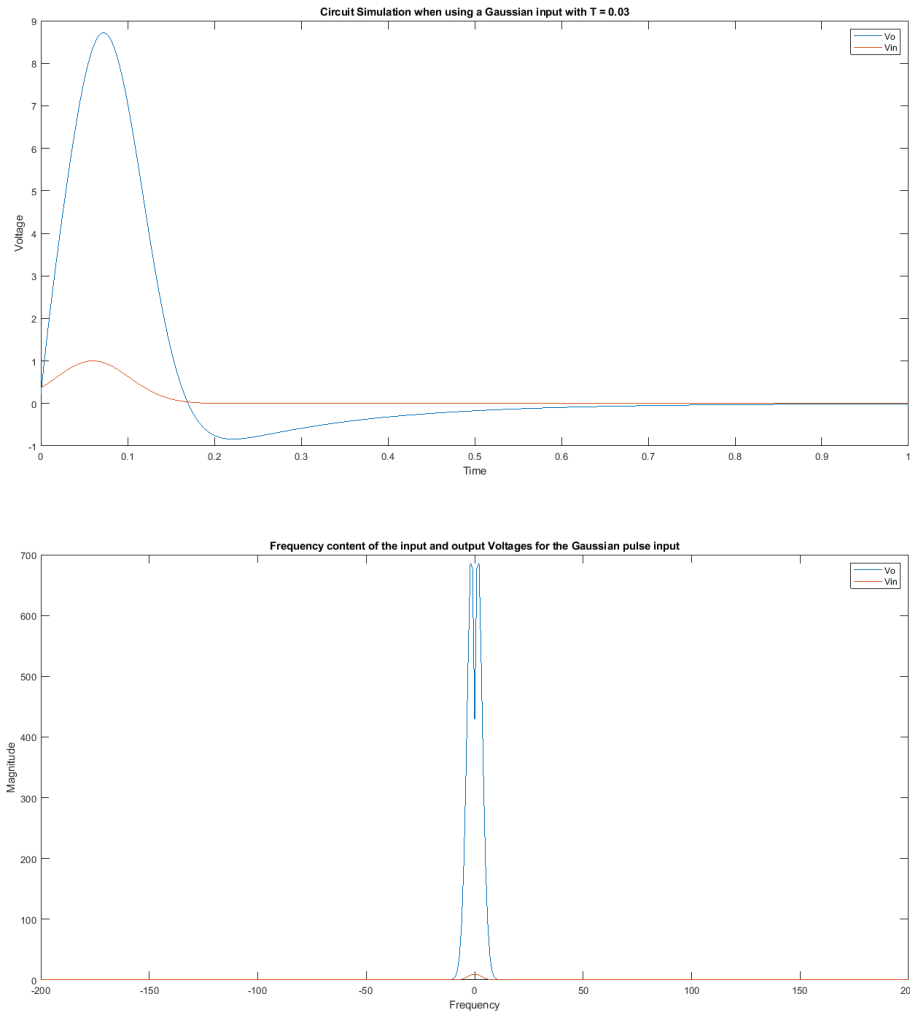
    Vin(i) = exp(-((t(i)-0.06)/(2*0.03))^2);
    F(1) = Vin(i);
    V = (Cmat/t(2) + G)\((Cmat*V/t(2)) + F');
    Vo(i) = V(6);
end
figure(10)
plot(t, Vo)
title('Circuit Simulation when using a Gaussian input with T = 0.03')
xlabel('Time')
ylabel('Voltage')
hold on
plot(t, Vin)
legend('Vo', 'Vin')
hold off;

f = fft(Vo);
fx = (-1000/2:1000/2-1);
p = abs(fftshift(f)).^2/1000;
figure(11)
plot(fx, p)
xlabel('Frequency')
ylabel('Magnitude')
title('Frequency content of the input and output Voltages for the
    Gaussian pulse input')
hold on;
p = abs(fftshift(fft(Vin))).^2/1000;
plot(fx, p)
legend('Vo', 'Vin')
xlim([-200 200])
hold off;

```







## Question 5 Simulation of the Circuit With Noise

since all the previous simulations used certain values for the C and G matrices, adding another current source and a capacitor to the circuit means we need another equation to be able to properly model the circuit. As a result the C and G matrices need to be reformulated to account for this.

```
Cn = 0.00001;

G = zeros(7); Cmat = G;
G = [
    1, 0, 0, 0, 0, 0, 0;
   -1/R1, 1/R2+1/R1, 0, 0, 0, 0, 0;
    0, -1, 0, 1, 0, 0, 0;
    0, 0, -1, 1/R3, 0, 0, 1;
    0, 0, 0, -a/R3, 1, 0, 0;
    0, 0, 0, 0, -1/R4, 1/R4+1/Ro, 0;
    0, 0, 0, 0, 0, 0, 1
];
```

```
Cmat(2,1) = -C;
Cmat(2,2) = C;
Cmat(3,3) = L;
Cmat(4,4) = Cn;

V = zeros(7,1); F = zeros(1,7);
for i = 1:1000
    Vin(i) = exp(-((t(i)-0.06)/(2*0.03))^2);
    F(1) = Vin(i);
    F(7) = randn*0.001;
    V = (Cmat/t(2) + G)\((Cmat*V/t(2)) + F');
    Vo(i) = V(6);
end
figure(12)
plot(t, Vo)
title('Circuit Simulation when using a Gaussian pulse input with
noise')
xlabel('Time')
ylabel('Voltage')
hold on
plot(t,Vin)
legend('Vo', 'Vin')
hold off;

f = fft(Vo);
fx = (-1000/2:1000/2-1);
p = abs(fftshift(f)).^2/1000;
figure(13)
plot(fx,p)
xlabel('Frequency')
ylabel('Magnitude')
title('Frequency content of the input and output Voltages for the
Gaussian pulse input')
hold on;
p = abs(fftshift(fft(Vin))).^2/1000;
plot(fx,p)
legend('Vo', 'Vin')
xlim([-200 200])
hold off;

% Next the Capacitance will gradually increase to see the effect of
% increased Cn on the response
Cmat(4,4) = 0.0001;
V = zeros(7,1); F = zeros(1,7);
for i = 1:1000
    Vin(i) = exp(-((t(i)-0.06)/(2*0.03))^2);
    F(1) = Vin(i);
    F(7) = randn*0.001;
    V = (Cmat/t(2) + G)\((Cmat*V/t(2)) + F');
    Vo(i) = V(6);
end
figure(14)
plot(t, Vo)
```

```
title('Circuit Simulation when using a Gaussian pulse input with noise
      (Cn = 0.0001)')
xlabel('Time')
ylabel('Voltage')
hold on
plot(t,Vin)
legend('Vo', 'Vin')
hold off;

f = fft(Vo);
fx = (-1000/2:1000/2-1);
p = abs(fftshift(f)).^2/1000;
figure(15)
plot(fx,p)
xlabel('Frequency')
ylabel('Magnitude')
title('Frequency content of the input and output Voltages for the
      Gaussian pulse input (Cn = 0.0001)')
hold on;
p = abs(fftshift(fft(Vin))).^2/1000;
plot(fx,p)
legend('Vo', 'Vin')
xlim([-200 200])
hold off;

Cmat(4,4) = 0.001;
V = zeros(7,1); F = zeros(1,7);
for i = 1:1000
    Vin(i) = exp(-((t(i)-0.06)/(2*0.03))^2);
    F(1) = Vin(i);
    F(7) = randn*0.001;
    V = (Cmat/t(2) + G)\((Cmat*V/t(2)) + F');
    Vo(i) = V(6);
end
figure(16)
plot(t, Vo)
title('Circuit Simulation when using a Gaussian pulse input with noise
      (Cn = 0.001)')
xlabel('Time')
ylabel('Voltage')
hold on
plot(t,Vin)
legend('Vo', 'Vin')
hold off;

f = fft(Vo);
fx = (-1000/2:1000/2-1);
p = abs(fftshift(f)).^2/1000;
figure(17)
plot(fx,p)
xlabel('Frequency')
ylabel('Magnitude')
title('Frequency content of the input and output Voltages for the
      Gaussian pulse input (Cn = 0.001)')
```

```
hold on;
p = abs(fftshift(fft(Vin))).^2/1000;
plot(fx,p)
legend('Vo', 'Vin')
xlim([-200 200])
hold off;

Cmat(4,4) = 0.01;
V = zeros(7,1); F = zeros(1,7);
for i = 1:1000
    Vin(i) = exp(-((t(i)-0.06)/(2*0.03))^2);
    F(1) = Vin(i);
    F(7) = randn*0.001;
    V = (Cmat/t(2) + G)\((Cmat*V/t(2)) + F');
    Vo(i) = V(6);
end
figure(18)
plot(t, Vo)
title('Circuit Simulation when using a Gaussian pulse input with noise
      (Cn = 0.01)')
xlabel('Time')
ylabel('Voltage')
hold on
plot(t,Vin)
legend('Vo', 'Vin')
hold off;

f = fft(Vo);
fx = (-1000/2:1000/2-1);
p = abs(fftshift(f)).^2/1000;
figure(19)
plot(fx,p)
xlabel('Frequency')
ylabel('Magnitude')
title('Frequency content of the input and output Voltages for the
      Gaussian pulse input (Cn = 0.01)')
hold on;
p = abs(fftshift(fft(Vin))).^2/1000;
plot(fx,p)
legend('Vo', 'Vin')
xlim([-200 200])
hold off;

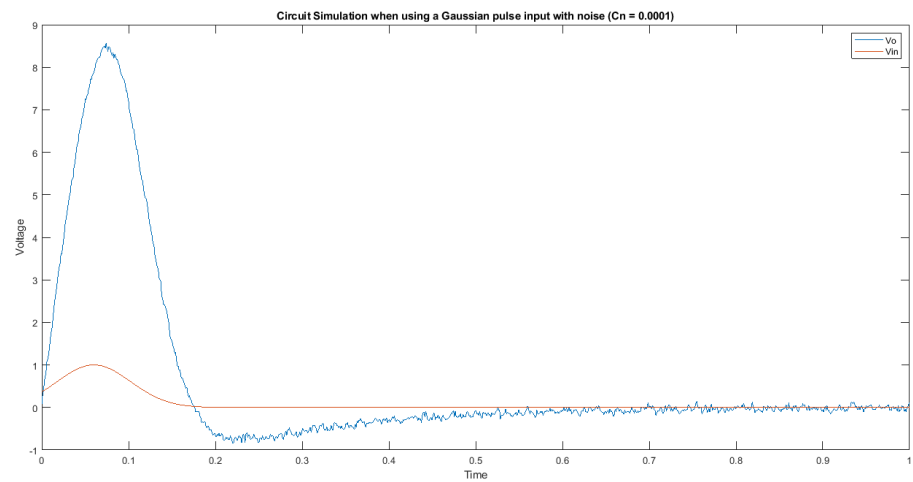
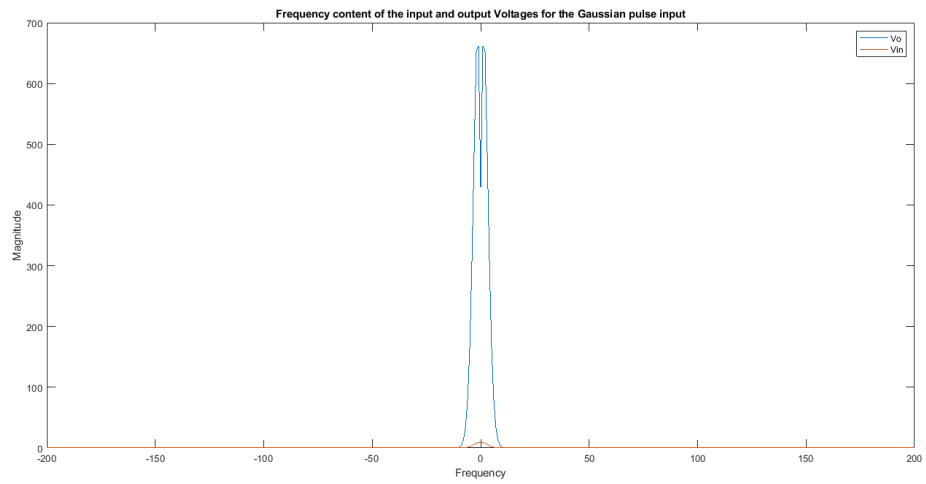
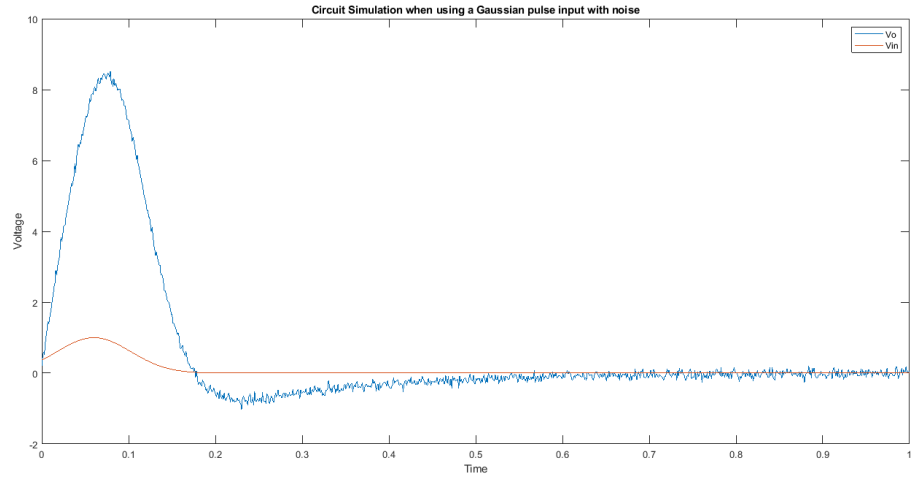
% As shown in the above plots, the bandwidth of the plot in the
% frequency domain and the frequency
% in the time domain change more drastically with a greater noise
% capacitance. The frequency of the plots in the time domain decreases
% with
% increased capacitance and distance between the peaks in the
% frequency
% domain increases with increased noise capacitance

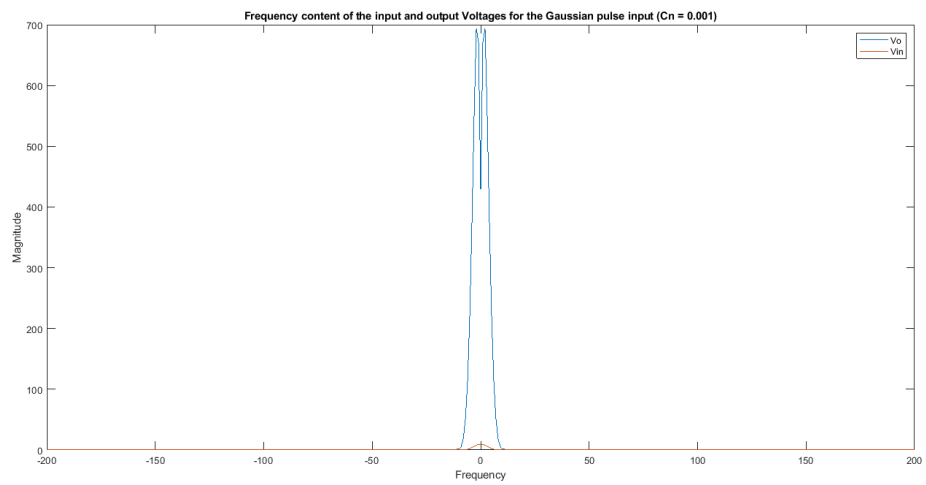
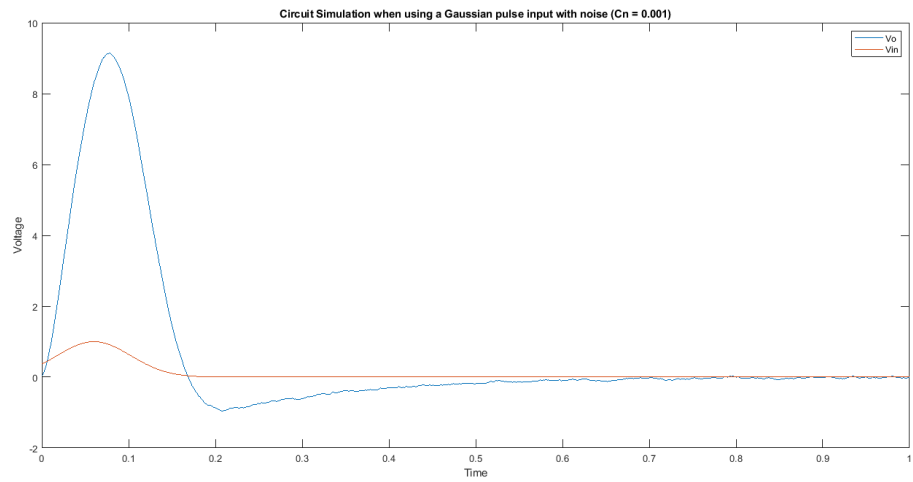
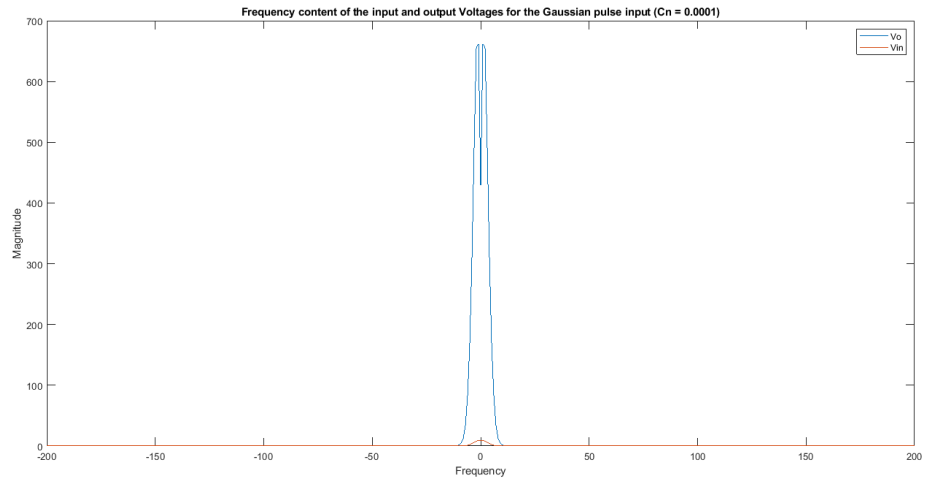
% The next step was to examine the how changing the number of
% timesteps
```

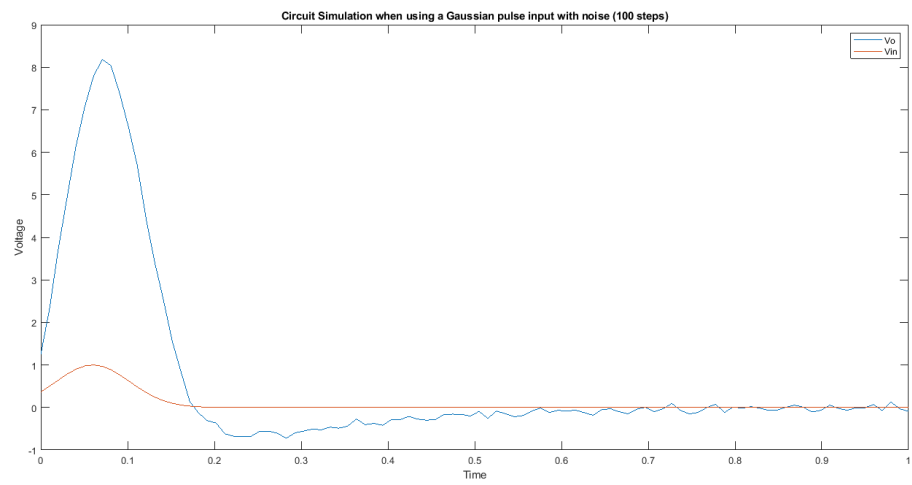
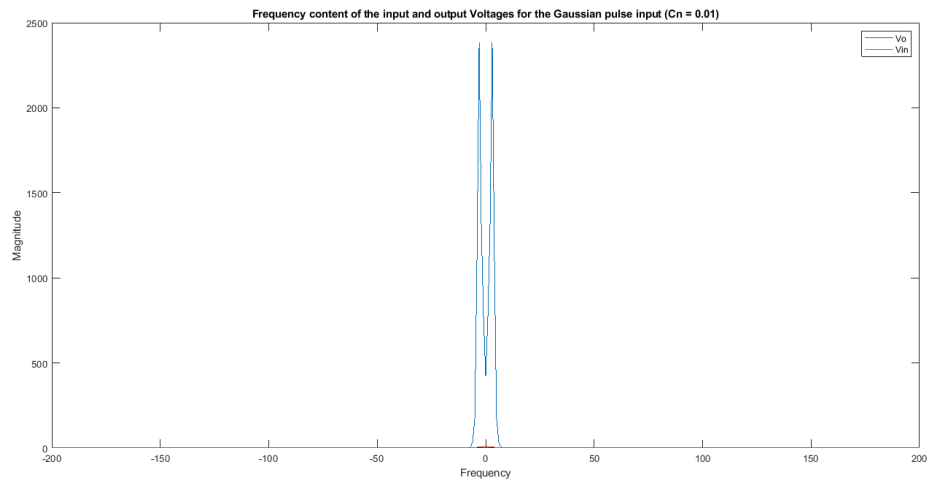
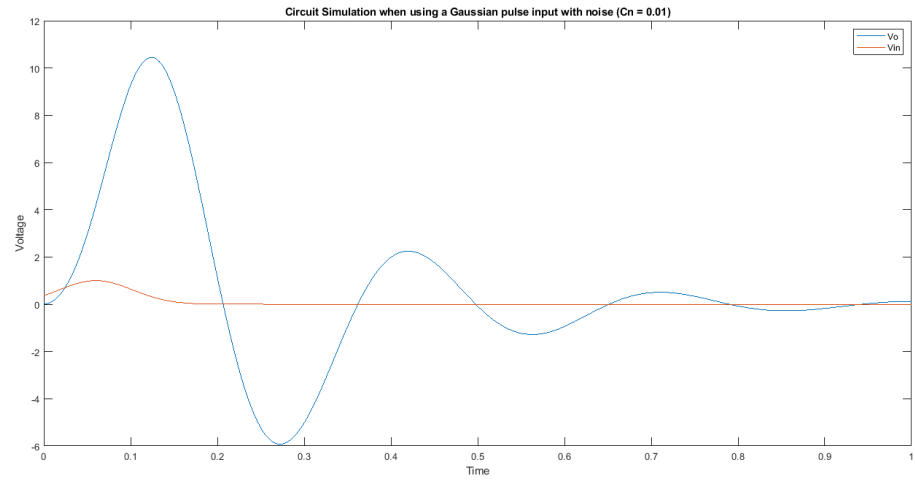
```
% affected the simulation
t = linspace(0,1,100);
Vin = zeros(1,100); Vo = Vin;
Cmat(4,4) = Cn;
V = zeros(7,1); F = zeros(1,7);
for i = 1:100
    Vin(i) = exp(-((t(i)-0.06)/(2*0.03))^2);
    F(1) = Vin(i);
    F(7) = randn*0.001;
    V = (Cmat/t(2) + G)\((Cmat*V/t(2)) + F');
    Vo(i) = V(6);
end
figure(20)
plot(t, Vo)
title('Circuit Simulation when using a Gaussian pulse input with noise
(100 steps)')
xlabel('Time')
ylabel('Voltage')
hold on
plot(t,Vin)
legend('Vo', 'Vin')
hold off

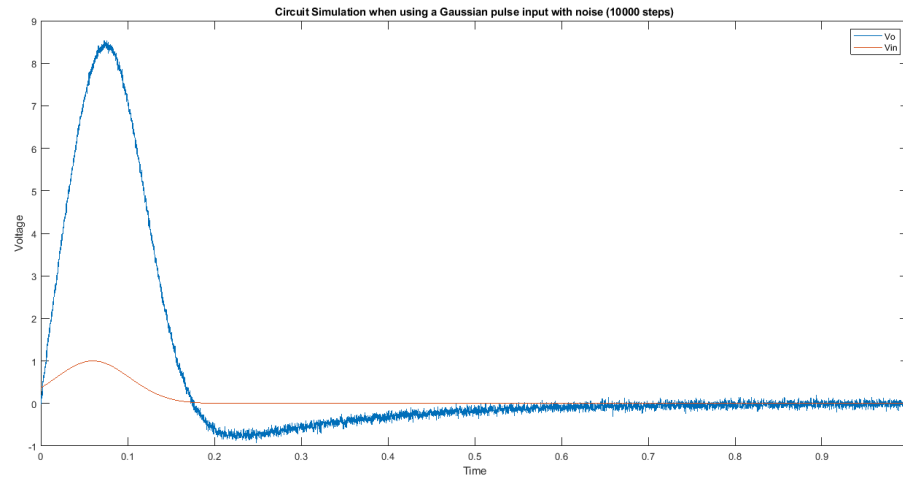
t = linspace(0,1,10000);
Vin = zeros(1,10000); Vo = Vin;
V = zeros(7,1); F = zeros(1,7);
for i = 1:10000
    Vin(i) = exp(-((t(i)-0.06)/(2*0.03))^2);
    F(1) = Vin(i);
    F(7) = randn*0.001;
    V = (Cmat/t(2) + G)\((Cmat*V/t(2)) + F');
    Vo(i) = V(6);
end
figure(21)
plot(t, Vo)
title('Circuit Simulation when using a Gaussian pulse input with noise
(10000 steps)')
xlabel('Time')
ylabel('Voltage')
hold on
plot(t,Vin)
legend('Vo', 'Vin')
hold off;
```











## Question 5 - Non Linearity

If the voltage were changed the equations used to formulate the  $G$  matrix would have to be changed to account for this change, as a result the  $G$  matrix would also have to change to reflect this. Due to the nonlinearity of this voltage, the matrix equation would also have to be altered by adding a  $B$  vector to the left hand side to account for this nonlinearity. The algorithm used to solve this equation would also have to change as gaussian elimination wouldn't work for a nonlinear system. It might be better to use another iterative numerical method like the Newton Raphson method find estimates of the roots.

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