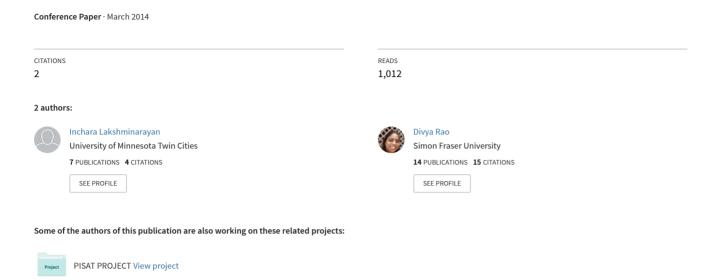
# Kalman Filter based estimation of constant angular rate bias for MEMS Gyroscope



# Kalman Filter based estimation of constant angular rate bias for MEMS Gyroscope

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Abstract— The reduction in size of sensor elements gives rise to challenges in attaining good measurement performance and high resolution. The presence of constant bias and varying noise contributes to a significant performance degradation of Micro Electro Mechanical Systems (MEMS) Gyroscopes. Here we propose a method to estimate the constant bias in noisy measurements using the recursive Kalman filter. The filter was tested for gyroscope readings from ADIS16405 tri axial inertial sensor. By estimating and eliminating the constant bias from gyroscope measurements, a significant reduction in error was observed.

#### I. INTRODUCTION

Gyroscopes are sensors that are used to detect and measure the angular velocity of rotating objects. They form the most essential component of an Inertial Navigation System. Gyroscopes based on Micro Electro Mechanical Systems (MEMS) technology are vibratory rate gyroscopes, which have no rotating parts that require bearings, and hence they can be easily miniaturized and batch fabricated using micromachining techniques. The rise of MEMS based gyroscopes has led to a radical reduction in the cost, size and power consumption of Inertial Measurement Units. However, this reduction in size of sensor components leads to sensitivity issues and an increase in noise levels. Fabrication imperfections also contribute to errors in sensing the rotation rate and weaken the performance of MEMS Gyroscopes.

Measurements from inertial sensors can be split into 3 components: true readings, constant bias and varying noise. The effect of constant bias can be seen as an identical vertical shift in the measurements from the true values. In other words, the observed mean of a set of sensor readings diverge from the expected mean of the measurements. Using the Kalman filtering algorithm, it is possible to accurately determine the constant bias introduced in sensor measurements and eliminate them. The Kalman filter is an "optimal estimator" and a very powerful tool for estimating the state of a dynamic system which is disturbed by some noise. Developed by Rudolf Emil Kalman in 1960, the Kalman filter is a recursive predictive filter that is based on the use of state space techniques and recursive algorithms [2].

#### II. STATE ESTIMATOR MODEL

The state estimator model proposed in [1] for magnetometer bias estimation has been extended and implemented for gyroscopes. The angular rate measurement  $G_{meas,k}$  by gyroscope at any time  $t_k$  is a combination of the true gyroscope measurement  $G_k$ , the constant bias b and the gyroscope noise  $\varepsilon_k$ . Thus the stochastic model of the gyroscope measurement is written as:

$$G_{meas, k} = G_k + b + \varepsilon_k, \qquad k = 1....N$$
 (1)

The effective measurement  $Z_k$  is given by Eq. (2) and measurement noise  $v_k$  with mean  $\mu_k$  is given by Eq. (3).

$$Z_k \equiv \left| G_{meas,k} \right|^2 \tag{2}$$

$$v_k \equiv 2(G_{meask} - b) \cdot \varepsilon_k - |\varepsilon_k|^2 \tag{3}$$

We assume that the measurement noise is white and Gaussian with covariance matrix  $\Sigma$  and the pseudo measurement covariance  $R_k$ .

$$\varepsilon_k \sim N(0, \Sigma) \tag{4}$$

$$v_k \sim N(\mu_k, R_k) \tag{5}$$

$$\mu_k \equiv E\{v_k\} = -tr(\Sigma) \tag{6}$$

Hence, the observation model is given by Eq. (7) and the pseudo measurement covariance is given by Eq. (8).

$$H_k = 2\left(G_{meask}^T - b^T\right) \tag{7}$$

$$R_k \equiv E\{v_k^2\} - \mu_k^2 = 4(G_{meask} - b)^T \cdot \Sigma \cdot (G_{meask} - b) + 2(tr\Sigma^2)$$
 (8)

Now, the Kalman filter algorithm is used to update the covariance matrices and the bias estimate. The measurement residual  $Y_k$  is given by Eq. (9)

$$Y_k = Z_k - H_k \cdot b \tag{9}$$

The residual covariance  $S_k$  is given by Eq. (10) where  $P_k$  is the predicted bias covariance.

$$S_k = H_k \cdot P_k \cdot H_k^T + R_k \tag{10}$$

The Kalman gain  $K_k$  is computed and used to update the bias estimate and the bias covariance.

$$K_k = P_k \cdot H_k^T \cdot S_k^{-1} \tag{11}$$

$$b = b + K_k \cdot Y_k \tag{12}$$

$$P_k = (I - K_k \cdot H_k) \cdot P_k \tag{13}$$

#### III. SIMULATION RESULTS

The tri axial inertial sensor, ADIS16405, was kept on rate table and gyroscope readings were recorded for zero rate condition under normal room temperature. 200000 data samples were collected with a sampling rate of 819.2 SPS by running the setup for 4 minutes. The Kalman filtering algorithm was run on the obtained measurements for constant angular rate bias estimation.

Table I. shows the initializations given for the simulation parameters. The variance of the noisy readings was used to define the covariance matrices.

TABLE I. SIMULATION PARAMETERS

	Initialization				
b	[0;0;0]				
ъ	0.0	282 0	0 ]		
$P_k$ , $R_k$ , $\Sigma$	(	0.024	47 0		
	Ĺ	0	0.0328		

The Fig. 1 shows the gyroscope bias estimate and covariance convergence about all three axes.

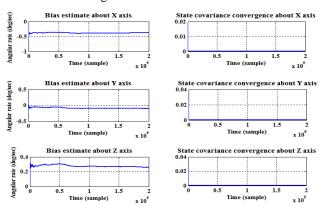


Fig. 1. Gyroscope bias estimate and covariance convergence about X, Y and Z axes in order

The distribution of the tri axial gyroscope data before and after filtering is compared in Fig. 2, Fig. 3 and Fig. 4.

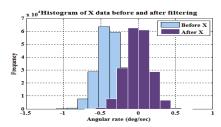


Fig. 2. Distribution of X axis gyroscope readings before and after filtering

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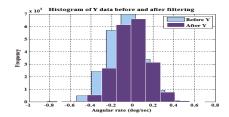


Fig.3. Distribution of Y axis gyroscope readings before and after filtering

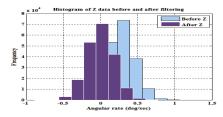


Fig.4. Distribution of Z axis gyroscope readings before and after filtering

Table II. shows the mean of measurements along all three axes before and after Kalman filtering.

TABLE II. COMPARISON OF MEAN BEFORE AND AFTER FILTERING

	Mean of X data	Mean of Y data	Mean of Z data
Before filtering	-0.3832	-0.0906	0.2718
After filtering	0.00028	-0.0019	-0.0031

## IV. CONCLUSION

A Kalman filter based state estimator model was used to estimate the constant bias in the measurements along all the three axes of the tri axial gyroscope in ADIS16405 inertial sensor. The covariance was found to converge quickly by 5000 iterations. The filter was tested for 200000 data samples and the estimated constant bias with rotation along X, Y and Z axes were -0.3823, -0.1035 and 0.2587 respectively. A significant improvement was seen in the filtered gyroscope measurements after eliminating the estimated constant bias, which is observed as a shift in the histogram plots before and after filtering in Fig.2, Fig. 3, Fig. 4, and the same is reflected by mean values in Table II.

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