Homework 2

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Problem .

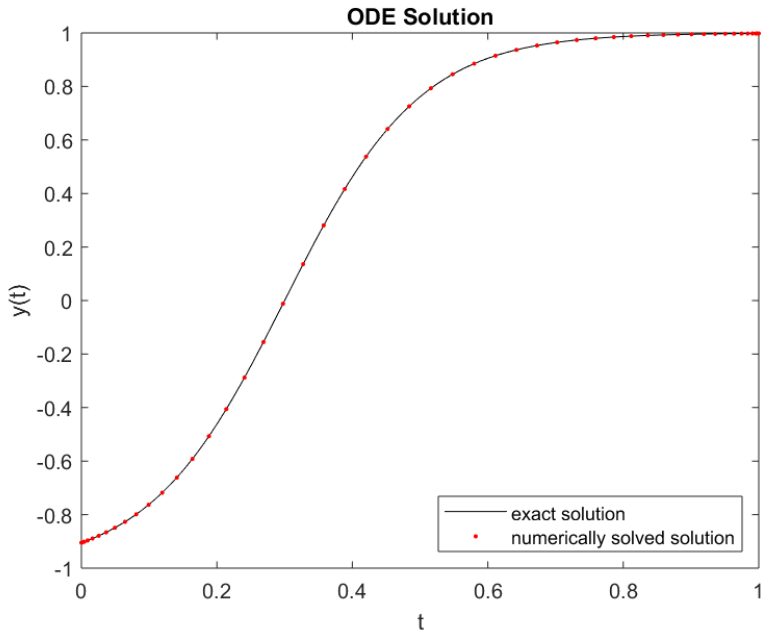
Use a Chebyshev method to solve the second order ordinary differential equation

$$u''(t) + au'(t) + bu(t) = f(t)$$

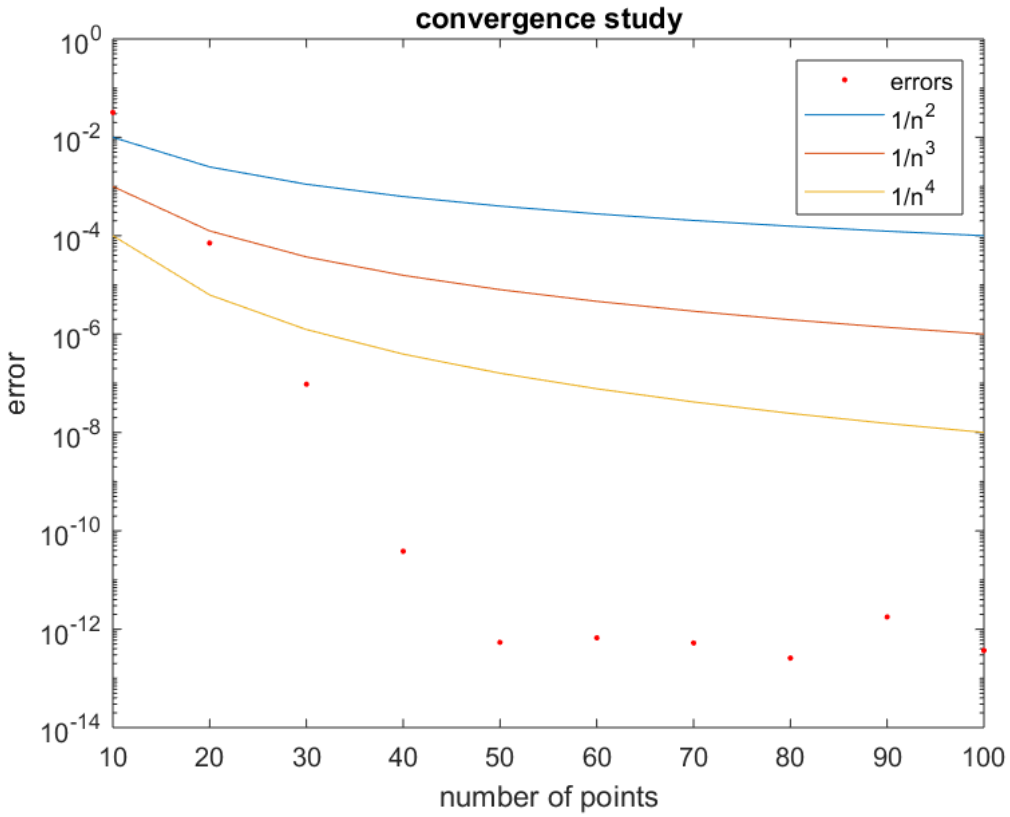
from $t = 0$ to $t = 1$ with initial conditions $u(0) = 1$ and $u'(0) = 0$.

1. Do a grid convergence study to test the accuracy of your method.
2. Setting $f(t) = 0$, experiment with the values a and b to identify two regimes with qualitatively different dynamics.

Solution I solved the ODE using the manufactured solution from the lecture $tanh(5(x-0.3))$ with 50 points.



Part 1): The errors show spectral convergence because they converge faster than any of the polynomial functions.



Part 2):
An unforced damped harmonic oscillator can be expresses as

$$u''(t) + au'(t) + bu(t) = 0$$

The roots are described when

$$\frac{-a \pm \sqrt{a^2 - 4b}}{2} = 0$$

There are three regimes of the solution that depend on the sign of the expression under the square root where

i $a^2 < 4b$ (when a is small compared to b, the system will be **underdamped**)

ii $a^2 = 4b$ (when a is is right between being over and underdamped the system is considered to be **critically damped**)

iii $a^2 > 4b$ (when a is relatively large compared to b, the system will be **overdamped**)