

Thursday 22/10/20 \* First Meeting

Meeting with Soldini and the other research assistants involved.

Break-down between 4 tasks:

1. Mission Analysis: The need, the feasibility and the planning: Jack, Sam, Benedict.

2. Origami Robotics: self-disruptive.  
Robert.

3. Multi-body folding dynamics: The programmatical approach to simulation.  
Raymond.

Soldini introduced us to the tasks and research assistants like Alexander, who has developed a PYTHON-based tool for the analysis and simulation of unfolding panels. However, this is not fully developed as it only does 2d simulation, it is expected of me to further develop this to be used for the design.

4. Reflective Control devices: How the reflectivity of the panels may be controlled to this control momentum gained from solar exposure

It was deemed that meeting once a week on thursdays is the best option for everyone.

For next meeting, I will have familiarised myself with the subject, and read useful resources:

→ PhD thesis by Takaо Yuki:

"Improvement of sail storage  
and deployment mechanism  
for spin-type solar power sail"

→ PhD glasgow thesis "Multi body  
Dynamics" but with magnetic-  
field. → Where to find it?

→ Book "Large Space Structures: Dynamics  
and control"

→ Convince the library to  
get it?

\* There is a proposal due on the 1st of  
November. Does it have to be independent?

To do:

→ Read and summarise Takaо's thesis

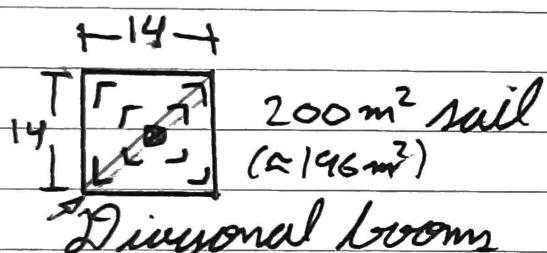
→ Find the Glasgow PhD project?

→ Find way to get book through the  
library

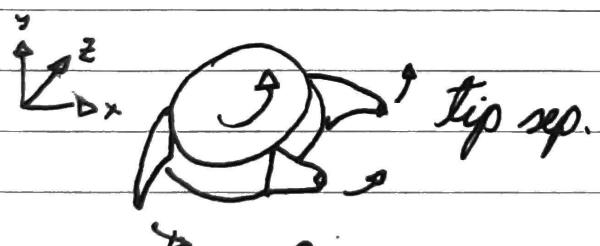
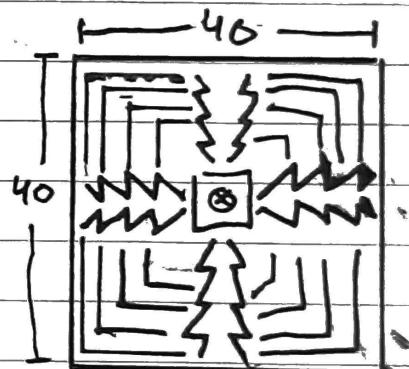
## 1st literature review:

O. Mor et al: "Improvement of sail storage..."

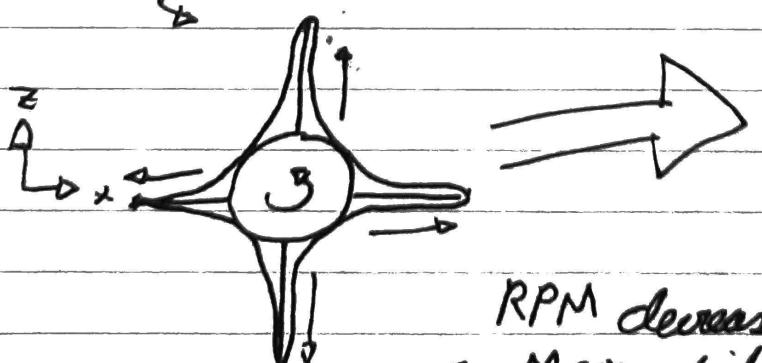
- The first mission to use sail technology uses the IKAROS in 2010 by JAXA. Sail probe is in Venus.
- IKAROS used a boom-supported sail to deploy its sail.
- For larger sails, booms and their mechanism become too heavy to be useful, so a centrifugal approach is taken.



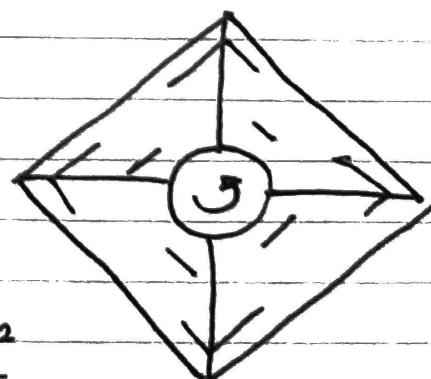
IKAROS



OKEANDS

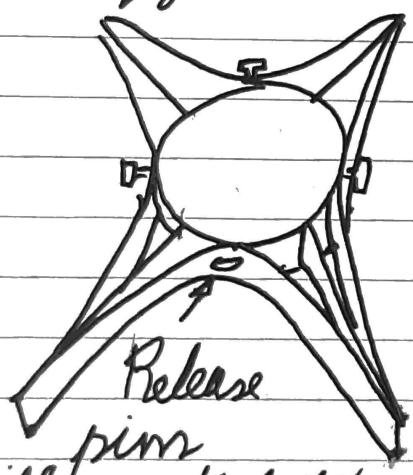


RPM decreases  
as MOI shifts.

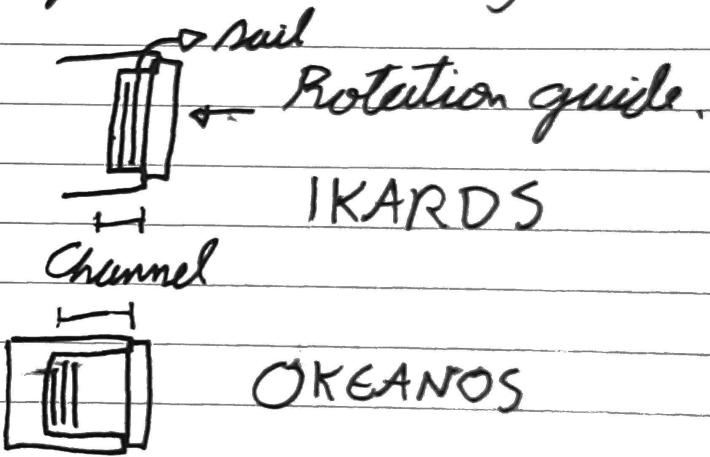


A problem in the deployment ensued when one of the sail petals did not roll out enough due to tape attachment/retainer not coming off. This retainer was ill-designed due to time and mass constraints, adding to the fact that IKAROS was a "piggyback" mission.

This problem could happen again with a bigger sail, so changes to the design were made to mit a more effective centrifugal deployment system including a 10x bigger sail.



will work better  
for synchronous  
deployment.



Due to the small channel in IKAROS, tape was used to avoid the sail from "Dropping Out".

A deeper channel in OKEANOS already prevents said dropout and thus the use of the unreliable tape/retainer is phased out.

# Pre-meeting 29/10/20

→ Discuss content for proposal, clearly outline and describe my deliverables.

→ Have Aleksander explain to me the above and possibly give me an introductory to the code.

✓ Get access to the online repository and give the code a skim to "orientate myself."

main.py:

$$\text{PAR1} = 7.5$$

$$\text{PAR2} = 4.5$$

$$\text{PAR3} = 0.055$$

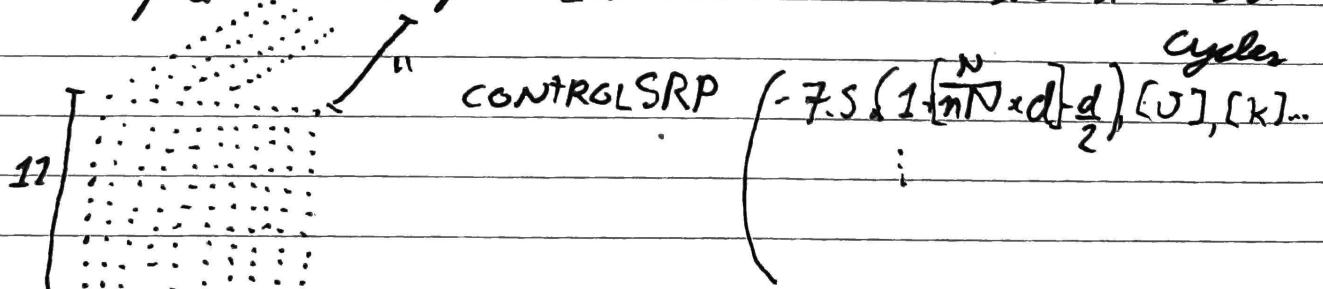
$$\text{CONTR Opt} = \begin{pmatrix} -7.5, -4.5, 0 \\ 7.5, -4.5, 0 \\ 7.5, 4.5, 0 \\ -7.5, 4.5, 0 \end{pmatrix}$$

$$\text{PAR Opt} = \text{PAR3}$$

$$\text{distance} = 0.1$$

$$\text{number of Nodes} = 10.$$

\* Triple for loop = 11 elements "Nodes" = ~~10<sup>3</sup>~~  $11^3 = 1331$



$$\text{CONTROLSRP} \left( -7.5 \left( 1 + \frac{N}{mN} \times d \right) \frac{d}{2}, [j], [k] \dots \right)$$

cycles

This seems to resemble an FEA operation where each one of the 1331 nodes receive a CONTROLSRP matrix.

For each matrix, an instance of ad.  
PROBLEM is called with pre-determined initial  
states and orientation.

PROPAGATE() → number the time-steps.  
linspace(0, propTime, propTime<sup>2</sup> \* 100)

For each of these time instances, a solution  
is calculated depending on the "control para-  
meters" which in these cases seem to be  
"SOLAR Sols"



Proposal stage report (for sunday 1st Nov)

- It is individual
- It is going to be similar among group members.

Structure planned:

- Intro. { → Front page.
- Project ↙ → 2-3 pages for background, aim and Obj.
- Plan { → 2-3 pages basically laying out the work
  - packager and tests
  - Gantt Chart.

## 1. Introduction

### 1.1 Background

- Satellites as a scientific instrument
- Common hardships of geocentric orbits
- Deep space exploration and constraints
- Sails as energy collector
- Sails as thrust mechanism
- Packing methods, folding techniques
- How origami is used and how it can be used in this field.

? → The importance of having a reliable system to numerically simulate multi-body dynamics (several panels)

### 1.2 Aim and Objectives ('4 of them!')

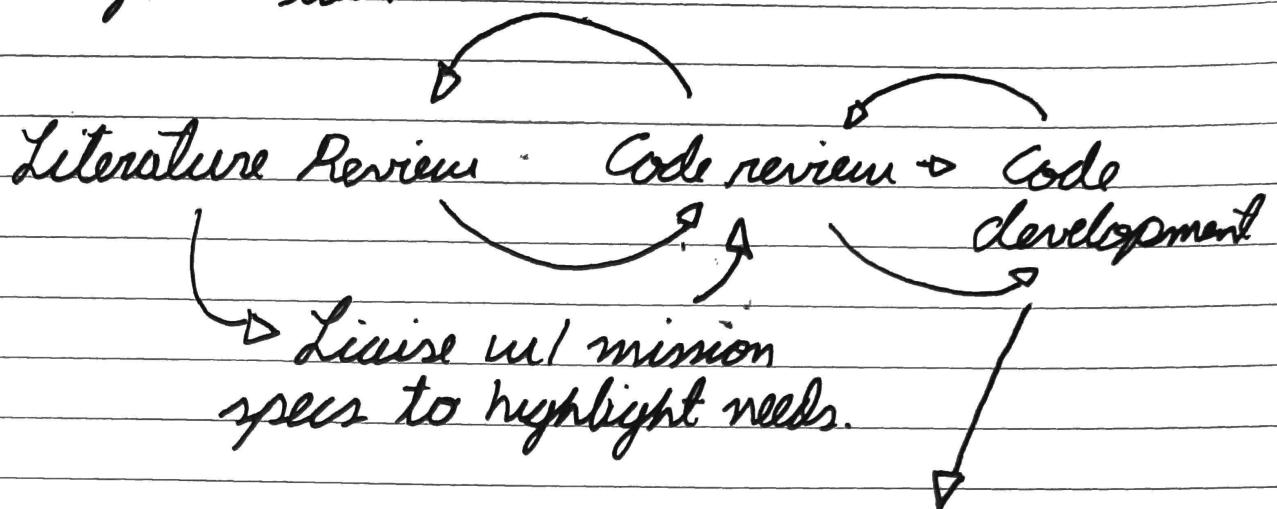
2.

Review Euler equations and ?

\* Robert is making simulations too active ... maneuvers ref 13

\* Alexander is to write a report on his code.

## 2. Project Plan.



Prove vulnerability  
through numerical  
simulation

Meeting 29/10/20

Foley Benedict:

- Design a mission design from scratch to de-orbiting
- Sam and Jack will also work in the above.
- Robert has been working on potential shapes for the organic sail.
- Aleksander will be making his internship report soon which will help me develop the first part.
- Stefania will be able to give me feedback for my proposal project on Friday 30th.

# Proposal writing

## - Introduction

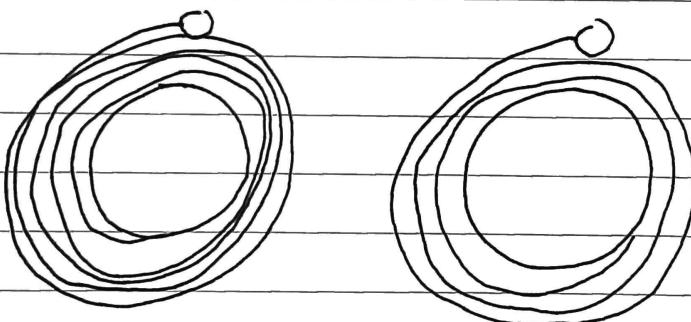
### - Background:

① Satellite story, first uses and sizes.

→ Sputnik

→ Vanguard I

↳ First use of solar energy, 1958



Expected:

2,000 year

Got:

240 years (Est.)

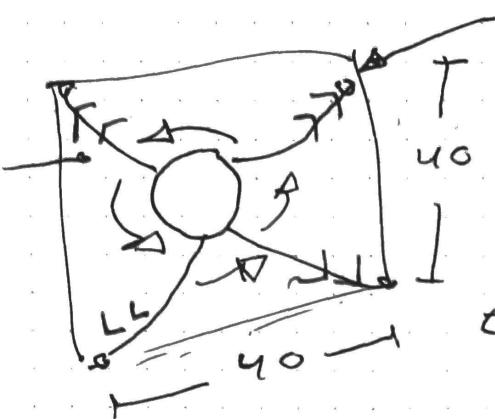
First scientific measurement  
of momentum change due to  
solar radiation pressure.



# IKAROS EXPERIMENT by JAXA.

$$W = 2 \text{ kg}$$

$$\begin{aligned} P &\approx \frac{2 \text{ kg}}{196 \text{ m}^2} \\ &= 10 \text{ g/m}^2 \end{aligned}$$



centrifugal mass

500g

$$t = 7.5 \text{ MM.}$$

polyimide.

$$\text{Polyimide} = 10 \text{ g/m}^2$$

$$F_{SRP} = 1.12 \cdot 10^3 \text{ N}$$

$$\therefore P_{SRP, eq} = 5.71 \cdot 10^{-6} \text{ N/m}^2$$

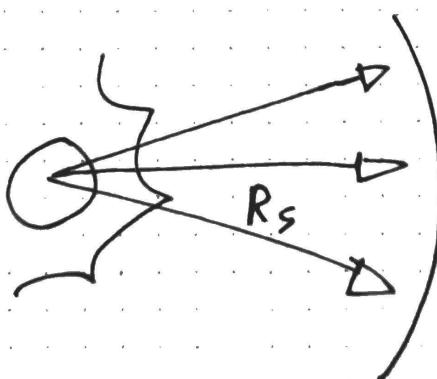
at a distance of: Venus?

$$\rightarrow 0.72 \text{ AU.}$$

Can I expect this?

$$\rightarrow \text{Solar constant: } 1361 \frac{\text{W}}{\text{m}^2}$$

$$\rightarrow \text{Pressure due to Gsc: } G_{sc} = 4.58 \cdot 10^{-6} \frac{\text{N}}{\text{m}^2}$$



$$d = 1 \text{ AU}$$

Reverse inverse law

$$P = 4.58 \cdot 10^{-6} \left( \frac{1 \text{ AU}}{0.72 \text{ AU}} \right)^2$$

$$P = 8.84 \cdot 10^{-6} \text{ N}$$

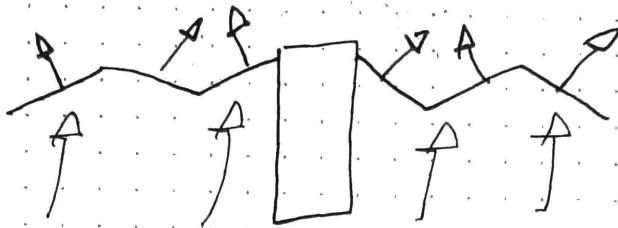
EXPECTED, VENUS.

$$\frac{5.71}{8.84} \cdot 0.65 \approx 64.6\%$$

?

## Simulation idea:

4



Study how much force, and time  
to deploy for these shapes.

- multidynamics, use visuale?
- my Py (Python?)
- WBS (so Gantt chart)

training needed? none that I can't do.  
technical needed? my computer.  
Financial need? none for the foreseeable  
future → Free open software solutions.

Feedback: Describe the apparatus  
used for controlling attitude

When was the "common ball" model  
made?

Submission date:

1st of November

angular restricted  
- three body problem

CR3BP

Newton's equation in inertial space.  
→ translatory

$$\mathbf{F} = (m I_3) \ddot{\mathbf{a}}_{cm}$$

$\begin{cases} \rightarrow \text{all cm} \\ \rightarrow \text{Identity } M \\ \rightarrow \text{mass} \end{cases}$

Euler eq. → rotational

$$\boldsymbol{\tau} = I_{cm} \ddot{\boldsymbol{\alpha}} + \bar{\boldsymbol{\omega}} \times I_{cm} \bar{\boldsymbol{\omega}}$$

$\begin{cases} \rightarrow \text{speed body ref.} \\ \rightarrow \text{not acc.} \end{cases}$

Aurtis Howard 2009 Orbital mech & Eng stud.

$b \rightarrow$  body ref

$c \rightarrow$  CR3BP ref

$i \rightarrow$  Fixed inertial ref.

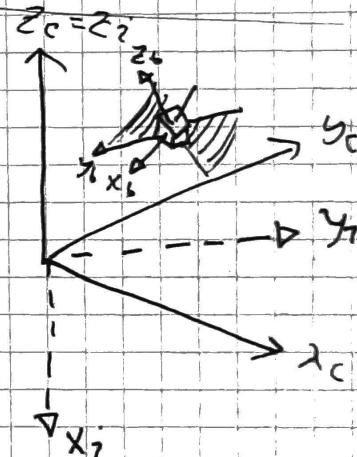
Eq. motion

$$\dot{x} = \frac{\partial U}{\partial z} - 2\dot{y}$$

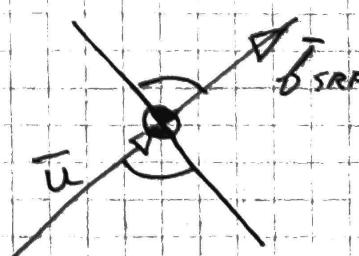
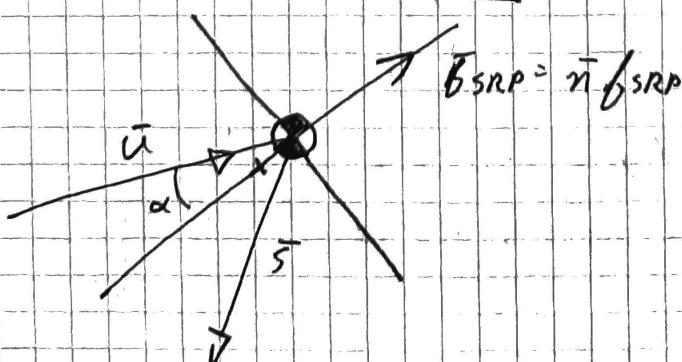
$$\dot{y} = \frac{\partial U}{\partial x} - 2\dot{x}$$

$$\ddot{z} = \frac{\partial U}{\partial z}$$

$U \rightarrow$  gravity field potential



ideal reflection vs pure absorption



P. 1

$$\bar{F}_{SRP} = \bar{F}_a = P A (\hat{u} \cdot \hat{n}) \hat{u} = F_a \hat{u}$$

$\hat{u}$  normalised radiation rays direction,  
 $\hat{n}$  dir. normal to sail.

Refresher: the dot product symbolises  
the amount of vector magnitude projected  
in the direction of another vector

$\hat{u} \cdot \hat{n} \rightarrow$  ratio of rays gone in direction  
of (normal to the sail),  $\hat{n}$

$P \rightarrow$  pressure

$A \rightarrow$  Area

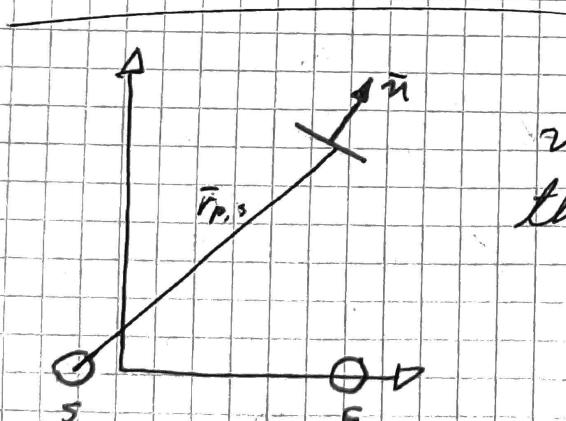
$$\frac{\bar{F}_{SRP}}{|F_{SRP}|} = \frac{\bar{F}_a}{F_a} = \hat{u} \text{ normalised.}$$
$$\therefore \bar{F}_a = F_a \hat{u}$$

I.R

$$\bar{F}_{SRP} = \bar{F}_R = 2PA(\hat{u} \cdot \hat{n})^2 \hat{n} = F_R \hat{n}$$

$\rightarrow$  see Inner Colin 1999 Solar sailing tech.

Dynamics and Mission Appli...



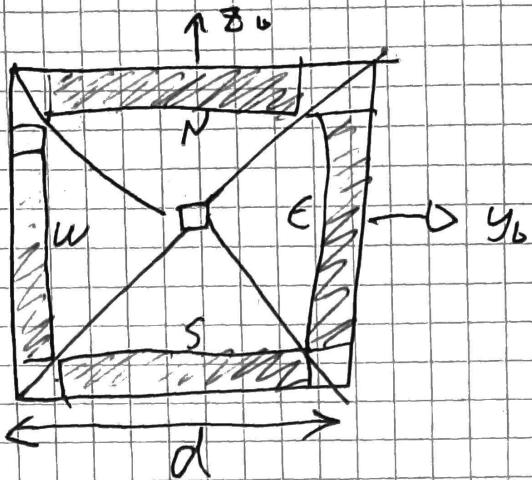
$\hat{u}$  unit vector passing through  
the S position vet os

$$\hat{u} = \frac{\bar{r}_{p,S}}{|\bar{r}_{p,S}|}$$

$$\bar{r}_{p,S} = \bar{r}_p - \bar{r}_S$$

$\hat{n} = -\hat{x} \rightarrow$  body frame axis + to sail  $\hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Let:



let state vector  $\bar{x} = [\bar{r}, \dot{\bar{r}}, \bar{q}, \bar{\omega}]$

$$\bar{q} = [q_3^T \ q_4^T]^T = \begin{matrix} \vdots & \vdots \\ \vdots & \vdots \\ z \times 3. \end{matrix} = \begin{pmatrix} \bar{q}_1 \\ \bar{q}_2 \\ \bar{q}_3 \\ \bar{q}_4 \end{pmatrix}$$

$$\bar{q} = \text{attitude quaternion} = (\dots)^T = \begin{matrix} \vdots & \vdots \\ \vdots & \vdots \\ z \times 3. \end{matrix} = \begin{pmatrix} \bar{q}_1 \\ \bar{q}_2 \\ \bar{q}_3 \\ \bar{q}_4 \end{pmatrix}$$

$$= [q_3^T \ q_4^T]^T \quad \bar{\omega} = \bar{\omega}_i$$

$$\dot{\bar{x}} = [\dot{\bar{r}} \ \ddot{\bar{r}} \ \dot{\bar{q}} \ \dot{\bar{\omega}}]$$

$$\frac{d}{dt} \left[ \begin{matrix} \bar{r} \\ \dot{\bar{r}} \end{matrix} \right] = \left[ \begin{matrix} \dot{\bar{r}} \\ \ddot{\bar{r}} \end{matrix} \right] = \left[ \begin{matrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \\ \dot{z} \\ \ddot{z} \end{matrix} \right]$$

Using quaternions  
prevents singularity

to find  $\dot{\bar{x}} \rightarrow$  algorithm Ex. 9.23

Auris Leonard (2009) Orbital mechanics for engineering students

$$\left\{ \begin{array}{l} \frac{dw_x}{dt} = \frac{M_{xNET}}{A} - \frac{C \cdot B}{A} w_x w_y \\ \frac{dw_y}{dt} = \frac{M_{yNET}}{B} - \frac{A \cdot C}{B} w_x w_z \\ \frac{dw_z}{dt} = \frac{M_{zNET}}{C} - \frac{B \cdot A}{C} w_y w_x \end{array} \right.$$

MOI along  
 $x_L$   
 $y_L$   
 $z_L$

$M_x$   
 $M_y$   
 $M_z$

$\bar{M} = \begin{bmatrix} M_{xNET} \\ M_{yNET} \\ M_{zNET} \end{bmatrix}$

The state of the reflectors is assumed to be binary. Leavissle step function

$$H(t) = \dots$$

Condition: the rail forms a normal plane against the direction of the run:

$$\hat{u} \cdot \hat{x} = -1$$

$$\hat{x}, \hat{y}, \hat{z} \rightarrow \text{body axes directions}$$

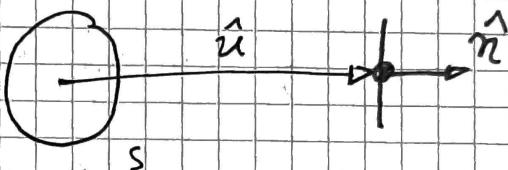
$$\hat{n} = -\hat{x}$$

$$-\hat{n} = \hat{x}$$

$$\hat{u} \cdot \hat{x} = -1$$

$$\hat{u} \cdot (-\hat{n}) = -1$$

$$\hat{u} \cdot \hat{n} = 1$$



As dot product uses

defined,  $\hat{u}$  is 1 times

the size of  $\hat{x}$  in the  $-\hat{1}$  direction (opposite) or

1 in the  $\hat{n}$  (vessel heading,

normal rail normal plane)

Further:

$$\hat{u} \cdot \hat{y} = 0$$

$$\hat{u} \cdot \hat{z} = 0$$

$\hat{u}$  projects itself

0 times on  $\hat{x}, \hat{y}$  and  $\hat{z}$

Assuming reaction wheels are ideal they can reach  $\alpha_{rw} = \infty$  and  $|T_{rw}| = \infty$

ail is released around  $\hat{n}$  because it wouldn't affect the orientation of the rail.

Glasgow PhD student  $\rightarrow$  deals with attitude control but with EMF

mass & size  $\leftrightarrow$

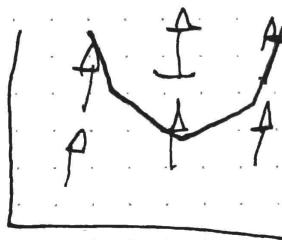
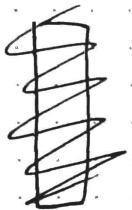
torque is located at the edges.

2D  $\rightarrow$  3D

start from welding will send one  
next  
The presentation //

meeting 12/11/20

- Robert has made a simulation to show folds required.
- Benedict wasn't in the meeting.
- the student in Glasgow, although he hasn't published it. I can use it to guide myself.



→ Some panels have values of 1 or 0 and reflect more or less respect.

+ momentum  
- momentum

meeting 18/11/20

- Clarifying the need for countermeasures  
Alexander reports in "Gumball  
Lock"
- First start with transformation  
matrices

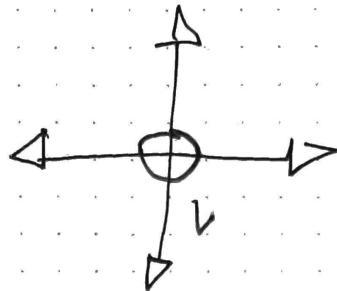
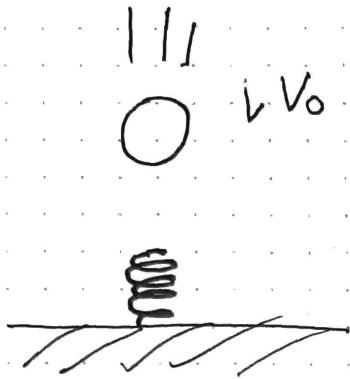
meeting 11/12/20

1-on-1 w/ Soldini

→ Explanation of the influence of the LCD's on the panels' reflection.

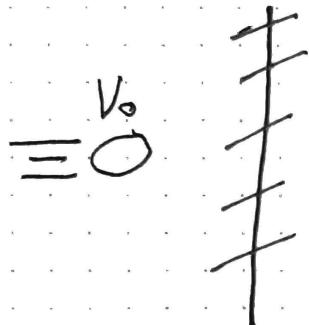
→ Deliverable for interim report:  
MBD simulation of panels on SRP.

→ check ref 1. for SRP formulas.



$$K_o = m v \frac{1}{2} \equiv K_o = I \omega \frac{1}{2}$$

$$F_{max} \approx M = I\alpha$$



time-to-stop: kinetical energy is dinned.

$$\frac{K_o}{s} = I \omega \frac{1}{2} = -I\alpha \frac{1}{2} = \frac{M}{s}$$

$$s = \frac{M}{-I\alpha \frac{1}{2}} ?$$

~~$$\alpha_s t_s = W_{ASRP}$$~~

$$\alpha_s = \frac{M_{SRP} - M_{SPR}}{I} \quad \left( \frac{M_{SRP} - M_{SPR}}{I} \right) t_s = W_{SRP}$$

On default model  $\frac{-M_{SPR} t_s}{I} = W_{SRP} + \frac{M_{SRP} t_s}{I}$

$0 \rightarrow 0$  is achieved  $M_{SRP} = \frac{W_{SRP} I}{I} + \frac{M_{SRP} J}{J}$   
in 300 s, therefore

$$t_s = C(\theta) = -\frac{\alpha I}{300}$$

$$M = I\alpha$$

$$\alpha = \frac{W}{300} =$$

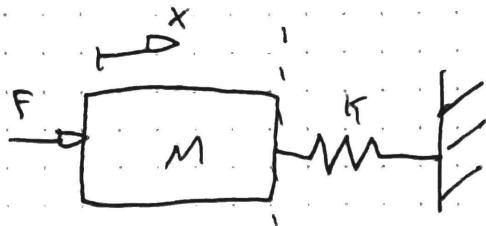
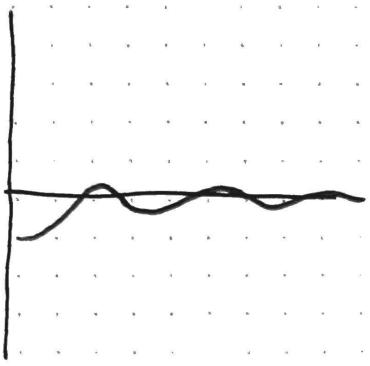
$$M_{SPR} = M_{SRP} - \frac{W_{SRP} I}{T_s}$$

$$C(\theta) = M_{SRP} - \frac{W_{SRP} I}{T_s}$$

$$\lim_{T_s \rightarrow 0} C(\theta) = -\frac{W_{SRP} I}{T_s}$$

$$C(\theta) = -\infty$$

$$\lim_{T_s \rightarrow \infty} C(\theta) = M_{SRP}$$



$$am = F - Kx$$

$$am = F - Kx$$

$$\ddot{x}m = u - Kx$$

$$\ddot{x}m + Kx = u$$

$$\ddot{x}m = F - Kx$$

$$\frac{dx}{dt} = v = \dot{x}$$

$$x = F_0 - \dot{x}m$$

$$\frac{dv}{dt} = a = \ddot{x}$$

$$m\ddot{x} - F(t) + x = 0$$

$$ma'' -$$

$$a = \ddot{v} = \ddot{x}$$

$$\ddot{x} - F(t) + m$$

$$\text{at } u=0 \quad x=0$$

$$\dot{x}m = 5N - 0$$

$$\frac{du}{dt} u = 5N$$

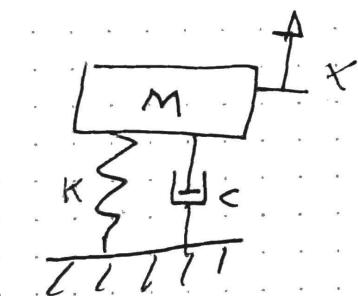
$$\ddot{x} = 0.025 \text{ kg} \cdot m/s^2$$

$$-5N = Kx$$

$$v = 0$$

$$x = 1.67$$

$$w = 0.10 \text{ rad/s}$$



$$m\ddot{x} = -Kx - C\dot{x} + F_{ext}$$

$$m\ddot{x} + Kx + C\dot{x} - F_{ext} = 0 \Rightarrow$$

$$\ddot{x} + \frac{K}{m}x + \frac{C}{m}\dot{x} - \frac{F_{ext}}{m} = 0$$

$$2\xi W_n = \frac{C}{m} \quad W_n = \sqrt{\frac{K}{m}}$$

$$\xi = \frac{C}{2m\sqrt{\frac{K}{m}}}$$

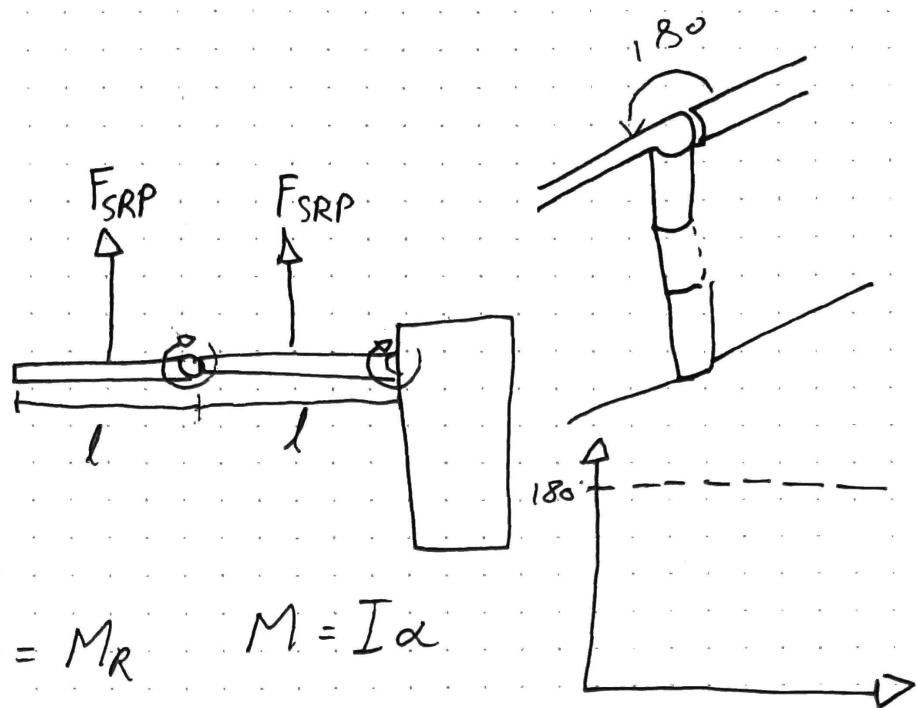
$$m\ddot{x} + Kx + C\dot{x} - F_{ext} = 0$$

$$\xi = \frac{C}{2\sqrt{Km}}$$

$$\ddot{x} + \frac{C}{m}\dot{x} + \frac{K}{m}x = \frac{F_{ext}}{m} \quad \therefore = \sqrt{\frac{K}{I}}$$

$$2\xi W_n = 0 \quad ; \quad W_n = \sqrt{\frac{K}{m}}$$

$\xi = 0 \rightarrow$  Ever oscillating

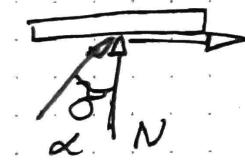


$$at n=1$$

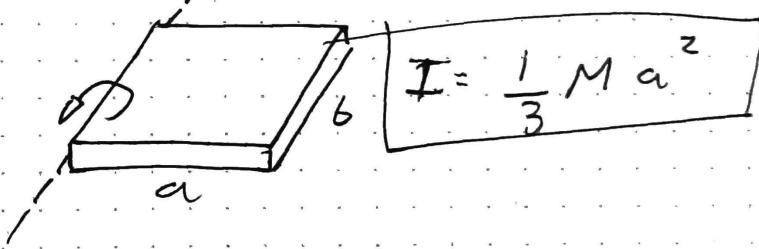
$$F_{SRP} \cdot \frac{l}{2} = M_R \quad M = I\alpha$$

$$\Rightarrow [NM] = [? \frac{\text{rad}}{\text{s}}]$$

$$[\frac{NMS}{\text{rad}}] = [\frac{\text{kg m s}^2}{\text{rad}}] = [\frac{\text{kg m}}{\text{s rad}}]$$

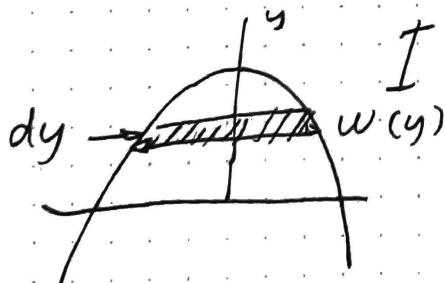


$$\alpha \cos \alpha = \frac{A}{M}$$

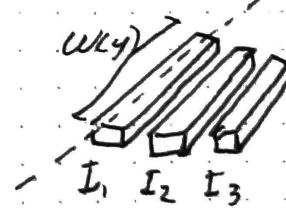


$$F_{SRP} \cos \alpha = F_{SRP_N}$$

$F_{SRP} \sin \alpha = F_{SRP_T}$  & resisted by joint



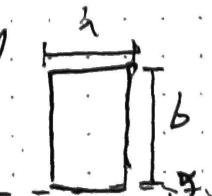
$$I = \int \rho y^2 w(y) dy$$



$$I = \sum I_N$$

$$I_N = \underbrace{w(y) dy}_\text{area} \rho y$$

$$I_N = \int w(y) y^2 \rho dy$$



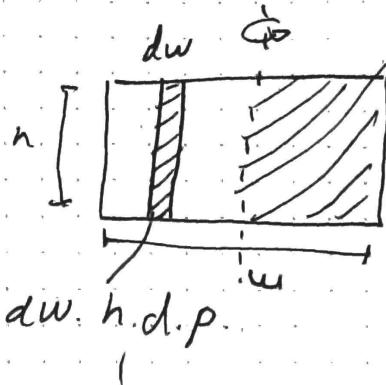
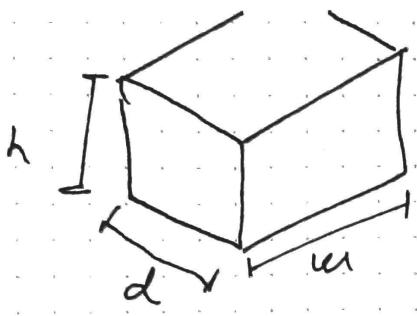
For a rectangle

slab

$$pb.b.b/2 = \frac{1}{2} \rho b^2 h$$

mass at point

inertia.

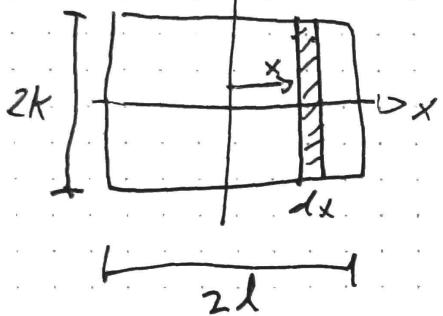


$$M = \frac{\omega}{2} h d p$$

$$I = \frac{h}{2} d p \cdot \frac{w}{4}$$

$$I = \frac{\omega^2}{5} h d p$$

$$T_h = \frac{1}{12} \rho h d w^3 + \frac{1}{12} \rho h d^3 w$$



$$\Delta m x^2$$

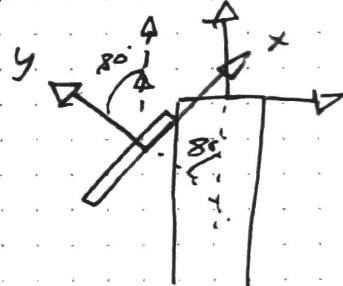
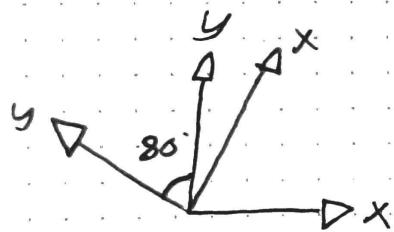
$$P = C P$$

$$\Delta m = d \times 2k \times p \times z^2$$

$$I_y = \frac{1}{2} k d p$$

$F_{SRP}$

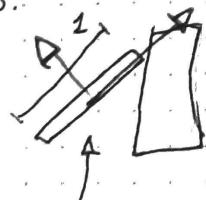
$$F_{SRP} = (0, S, 0, 0)$$



$$C = 0.5$$

$$0.87 N$$

$$4.92 N$$



$$\text{torque} = 0.87 \times 0.5 = 0.435 \text{ NM}$$

$$M = I \alpha$$

$$\alpha = \frac{M}{J}$$

$$I = \frac{1}{3} \rho a^2 \cdot 2$$

$$\alpha = 0.2175 \text{ rad/s/s}^2$$

$$[\alpha] = \left[ \frac{\text{Nm}}{\text{kg m}^2} \right] = \frac{\text{N}}{\text{kg m}} = \frac{\text{kg m}}{\text{kg s}^2} = \frac{1}{\text{s}^2}$$

$$\alpha = \ddot{\theta}(t)$$

$$\omega = \alpha t$$

$$\omega =$$

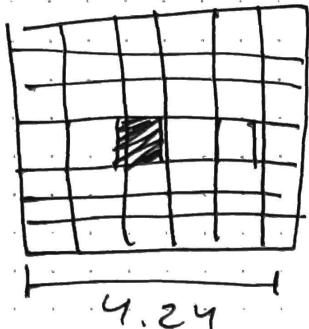
$$X = V_0 \cdot t + \frac{\alpha t^2}{2}$$

$$V_F = V_0 + \alpha \cdot t$$

$$\theta = \omega_0 t + \frac{\alpha t^2}{2}$$

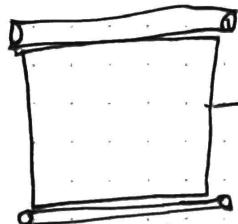
$$\omega_{s_2} = \omega + \alpha t$$

0.70 I



4.24

1.8 m<sup>2</sup>



[1.5 kg/m<sup>2</sup>]

$$\rightarrow I = 0.49 \\ m = 7.35 \text{ kg}$$

Foil: 2.71 g/cm<sup>3</sup>

IKAROS:  $\frac{2 \text{ kg}}{196 \text{ m}^2} = \boxed{10.2 \cdot 10^{-3} \frac{\text{kg}}{\text{m}^2}}$

7.5 mm thick

For a PLA panel, 50 μm thick,  $\rho = 1.24 \text{ g/cm}^3$

$$V = 2.45 \cdot 10^{-5} \text{ m}^3 \\ V = 24.5 \text{ cm}^3 \\ m = 30.38 \text{ g}$$

I 50 μm

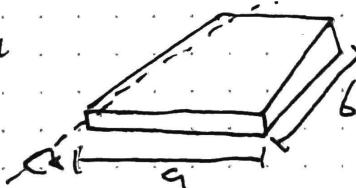
$$20 \text{ cm} \quad \rho = 6.2 \cdot 10^{-3} \text{ g/cm}^3$$

$$\rho = 1.24 \text{ g/cm}^3 = 1240 \text{ kg/m}^3$$

$$\rho = 6.2 \cdot 10^{-3} \frac{\text{g}}{\text{cm}^3} \cdot \frac{1.10^6 \text{ cm}^3}{\text{m}^3}$$

$$P_{\text{panel}} = 0.62 \text{ kg/m}^2 \quad \rho = 62 \text{ g/m}^2 \\ = 62 \text{ g/m}^3 //$$

$$I = \frac{1}{3} Ma$$



$$\therefore I = \frac{1}{3} 0.03 \cdot 0.7 = 7 \cdot 10^{-3} \frac{\text{kg}}{\text{m}}$$

$$[\text{Nm}] = [\text{kg m}] \left[ \frac{\text{m}^2}{\text{s}^2} \right]$$

$$= \left[ \frac{\text{kg m}^2}{\text{s}^2} \right] = [\text{Nm}]$$

turn  $P_1$  0.52°  
BUS  $P_1$   $P_2$

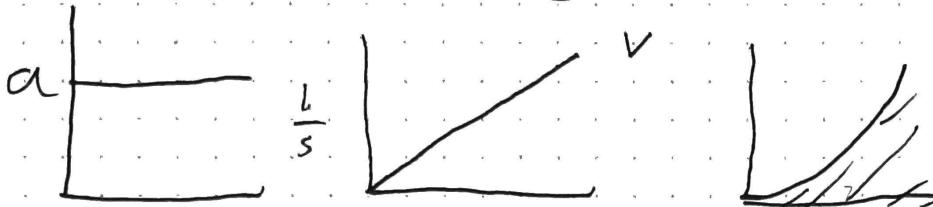
turn  $P_2$  0.52°  
BUS  $P_1$   $P_2$

$P_C$  should be 1.647° ccw  
from Bus

$$\alpha = \dot{\omega} = \ddot{\theta}$$

$$V_F = V_0 t + \frac{a t^2}{2}$$

$$d_F = V_0 t^2 + \frac{a \cdot t^3}{2}$$



$$\theta = \omega_0 t + \frac{\alpha t^2}{2} \quad V_F = V_0 t + \frac{a t^2}{2} \quad V_F = \frac{d}{t}$$

$$2\theta = 2\omega \quad d = V_0 t^2 + \frac{a t^3}{2}$$

$$2d - 2V_0 t^2 - \frac{a t^3}{2}$$

$$\begin{aligned} t &= 5 \\ d &= 50 \\ a &= 2 \end{aligned}$$

B  $d = V_F \quad \sqrt{\frac{d}{a}} = t \quad V_F = 10 \text{ m/s}$

$$d = a t^2$$

$$10 \text{ m/s} = 0 + \frac{a t^2}{2} \quad t =$$

$$2\theta = 2\omega_0 t + \alpha t^2 \quad \alpha t^2 + 2\omega_0 t - 2\theta$$

$$2\theta = (2\omega_0 + \alpha t)t \quad -2\theta \pm \sqrt{4\theta^2 + 4\alpha \cdot 2\theta}$$

$$\alpha = 2 \text{ rad/s}^2$$

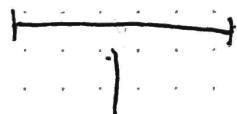
$$t = 2\theta \quad 2\alpha$$

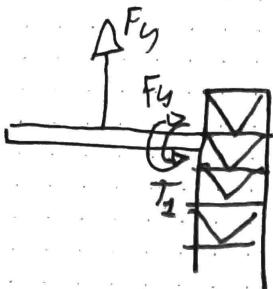
$$\theta = 0,349^\circ$$

$$t = 1.78 \text{ s}, 0.59 \text{ s}$$

$$t = \frac{\sqrt{4\alpha \cdot 2\theta}}{2\alpha} = \frac{\sqrt{8\alpha \theta}}{2\alpha}$$

$$t = 2 \text{ s}, 1.18 \text{ s}$$



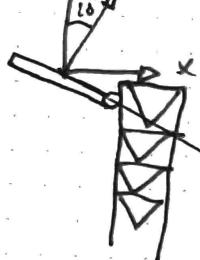


a torsional spring acts at the root, as such:

$$M = T_R - F_y \cdot r = 0$$

$$T_R = C(\theta - \theta_r) \text{ where}$$

$\theta_r$  is the reference at which the spring will not deliver moment  
 $C$  is the spring constant at which with units  $\frac{N}{rad}$ ,



with a sun-bus angle of  $30^\circ$  and distance 1.5 AL,

$$P = 1.52 \cdot 10^{-5} N/m^2$$

$$P = 1.52 \cdot 10^{-5} N/m^2$$



$$\frac{P}{\cos \theta}$$

$$M = F_{SRP} \cdot 0.5m$$

$$= 7.4846 \cdot 10^{-7} NM$$

$$M_T = M_{SRP} - M_C = 0$$

$$M_C = M_{SRP}$$

$$C(\theta - \theta_r) = M_C = +7.48 \cdot 10^{-7} NM$$

$$C(0.1745 - \theta) = 7.48 \cdot 10^{-7} NM$$

Assuming  $\theta_r = 0^\circ, 0^\circ \rightarrow C = +4.29 \cdot 10^{-6} \frac{NM}{rad}$

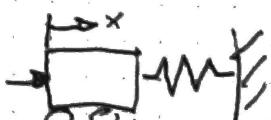
BUS is  $30^\circ$  ACW, sun

FRAME is  $10^\circ$  CW, BUS

$$P_{SRP} = 1.995 \cdot 10^{-6} N/m^2$$



$$mx = F - cx$$

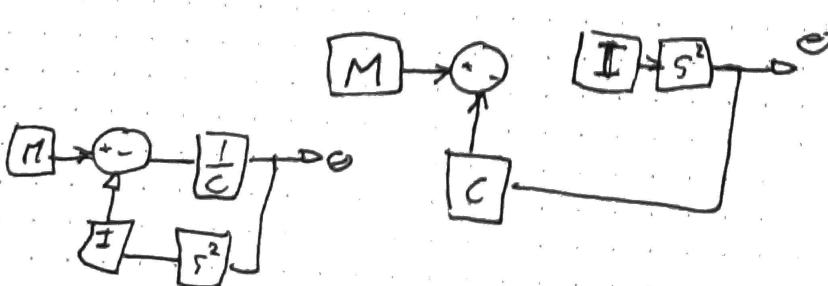


$$mx = F - xc$$

$$I\ddot{\theta} = M - c\dot{\theta}$$

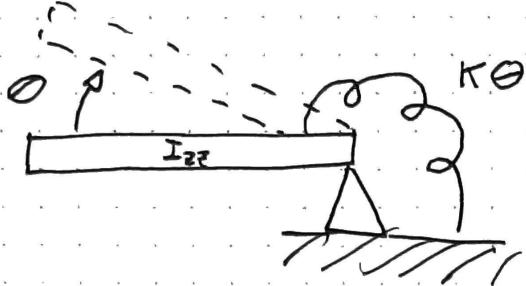
$$I\ddot{\theta} = M - c\dot{\theta}$$

$$\theta = \frac{M - I\ddot{\theta}}{c}$$



Meeting with Aloisa 29/1/21

→ In regards to the dynamic system I found:



$$M = I\alpha - I\ddot{\theta}$$

$$\therefore \ddot{\theta} = \frac{M}{I}$$

$$M = F_{eq, SRP} \cdot \frac{l_z}{2}$$

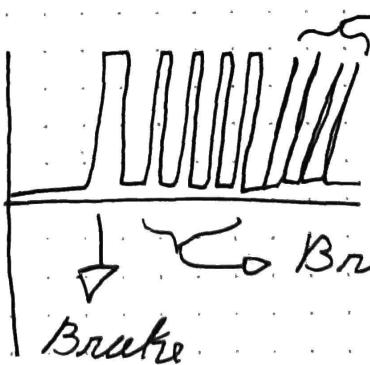
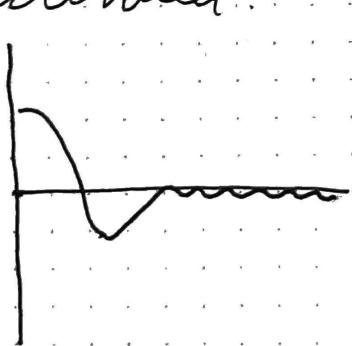
$$\therefore I\alpha = I\alpha_{SRP} + (-K)\theta$$

$$I\ddot{\theta} = F_{eq}(\theta) \frac{l_z}{2} - K\theta \quad \text{where } F_{eq} = P_{SRP} A \cos \theta$$



← No dampening?

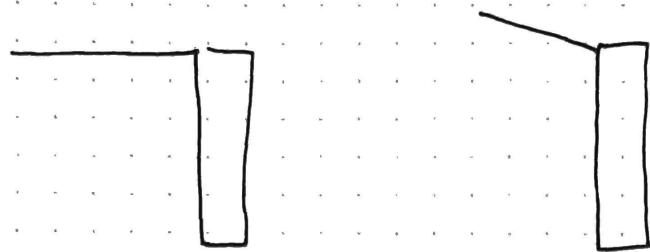
→ Aloisa says this doesn't have to be permanently on, spring can be electrically activated.



→ maintain normal to the sun

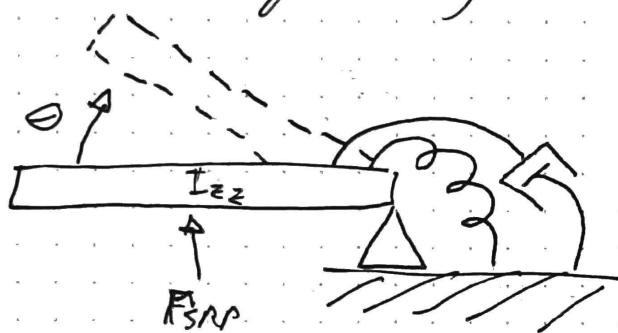
↓ To Bring to 0°  
Brake

The spring could activate when the panel goes beyond a point, and should help to maintain the panel at an optimum angle.



$0^\circ \rightarrow$  maximum force       $\sim 10^\circ$  Equilibrium point, less energy used on spring.

→ Passive dampening



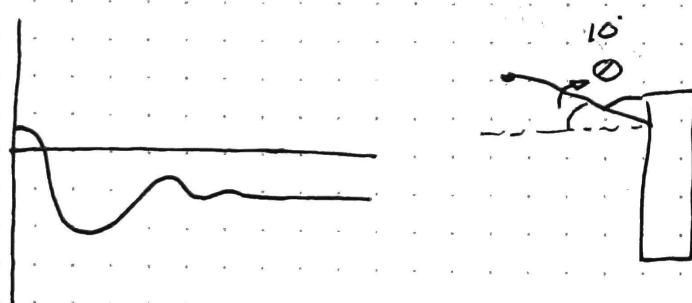
$$I\ddot{\theta} = P_{SRP}(\theta) - K\theta - CW$$

$$I\ddot{\theta} + C\dot{\theta} + K\theta = P_{SRP}(\theta)$$

$$\ddot{\theta} + \frac{C}{I}\dot{\theta} + \frac{K}{I}\theta = \frac{M_{SRP}}{I}(\theta)$$

where  $W_n = \sqrt{\frac{K}{I}}$  and  $\xi = \frac{C}{I_2 W_n}$

$$\xi = \frac{C}{2 \pm \sqrt{\frac{K}{I}}} = \frac{C}{2\sqrt{KI}}$$



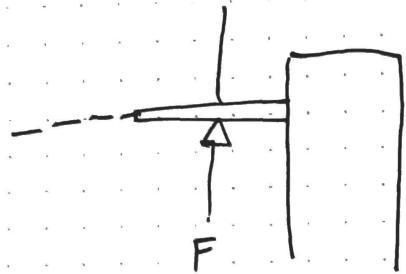
Will converge in a less-than-optimal angle.

ADJUST

$M_{SPRING}$ ?

$$\theta_{con.} = M_{SPRING}(\theta) + M_{SRP}(\theta) = 0$$

EQUILIBRIUM



$$\alpha_x = 0.5 \text{ m}$$

$$F_{SRP} = 4.56 \cdot 10^6 \text{ N}$$

$$F_{SRP}_{\text{LOCAL}} = 1.52 \cdot 10^6 \text{ N}$$

$$M = F \cos \theta \alpha_x$$

$$\theta = M_g = 4.29 \cdot 10^6 \frac{\text{Nm}}{\text{rad}} \theta$$

$$M_s = C(\theta)$$

$$M_r = M_{SRP} + M_{SPR}$$

$$I \ddot{\alpha}_x = F \cos \theta \alpha_x + C \theta$$

$$I \ddot{\theta} = F \cos \theta \alpha_x + C \theta$$

$$\boxed{\theta = \frac{I \ddot{\theta} - F \cos \theta}{C}}$$

$$x = \frac{\dot{x} - F_{SRP} \cos(x)}{C}$$

$$M_r = M_p + M_s$$

$$I_r \ddot{\alpha}_r = F_{SRP} \cos \theta \alpha_x + C \theta$$

$$x = \frac{\dot{x} - \cos(x) F_{SRP}}{C}$$

$$\boxed{\frac{I_r \ddot{\alpha}_r - F_{SRP} \cos \theta \alpha_x}{C} = \theta}$$

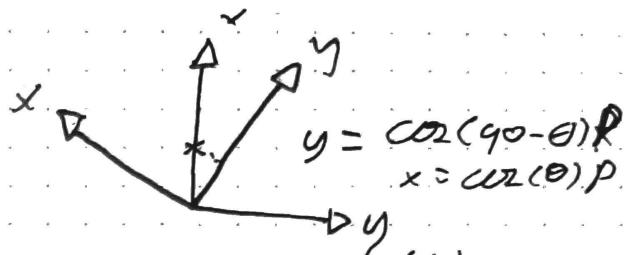
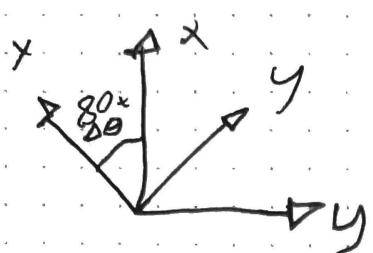
$$a^s = \beta \frac{M_{\text{SUN}}}{r^2}$$

#15 reference.

IKAROS  $\rightarrow$  travelled with normal engines

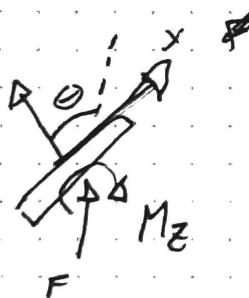
or

→ get angle between panel to bus  
 → re-orient frame  $\Delta\theta$  per iteration



$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} (1, 0, 0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{if } v = \boxed{\hat{v} = (0, 0, 0)_N} \quad \hat{v} = (0, -1, 0)_A \quad \text{when } A_\theta = 90^\circ$$



F. express (PANEL FRAME)

$$(5 \sin(0.444\pi), 0, 5 \cos(0.444\pi), 0)$$

$$(0.12, 4.97)$$

$$\alpha = \frac{M}{I} \quad \alpha = MI^{-1}$$

$$M = I\alpha \quad \begin{pmatrix} -0.217 \\ 0 \\ 0 - 0.217 \end{pmatrix} = \begin{pmatrix} L \\ M \\ N \end{pmatrix} \begin{pmatrix} 1/I_{xx} \\ 1/I_{xy} \\ 1/I_{zz} \end{pmatrix}$$

$$M_z = l_x \cdot F_y$$

$$M = F \cdot l_F = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \begin{pmatrix} 45^\circ \\ 0 \\ 0 \end{pmatrix}$$

moments

$$\begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix}$$

$$\begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}^{-1} \begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}$$

$$M = F dl \quad (\tau_x) \quad ($$

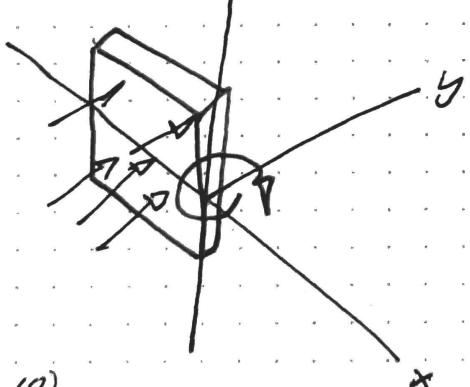
$$\alpha = \frac{M}{I}$$

$M$  must be  
-ve.

$$\tau_x = F_y \cdot l_x + 0$$

$$\tau_y = F_x \cdot l_y + 0 \quad \tau_z = \alpha_x I_{2x} + \alpha_y I_{2y} + \alpha_z I_{2z}$$

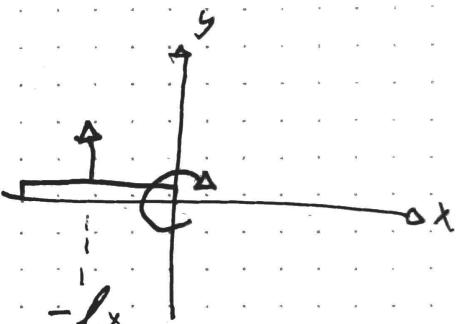
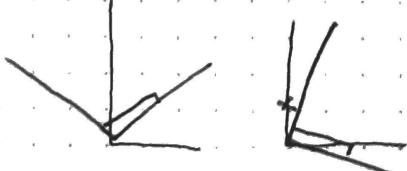
$$\tau_z = F_y$$



$$+ c x + s x$$



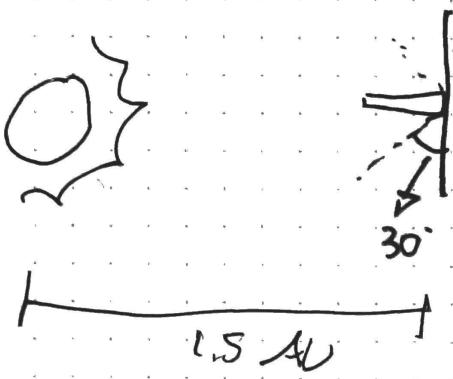
$$+ c \theta + s \theta \quad c \theta - s \theta$$



$$\tan$$

$$\theta^2 = A^2 + B^2 + 2ac \cos \alpha$$

$$\underline{a} \cdot \underline{b} = |a||b| \cos \theta$$



$$P = 4.56 \cdot 10^{-6} \frac{1}{1.5^2} \cos(\text{SUN-BUS})$$

$$P = 2.03 \cdot 10^{-6} \text{ N/m}^2$$

$$A = 0.49 \text{ m}^2$$

$$\text{State : 1} \quad F_T = 1.99 \cdot 10^6 \text{ N}$$

$$M_{GR} = -k\theta$$

$$= -4.29 \cdot 10^{-6} \cdot 30$$

rad

$$= -2.25 \cdot 10^{-6} \text{ NM}$$

$$F_{PA_{NGC}} = 1.72 \cdot 10^6 \text{ N}$$



$$M_{SRP} = 6.03 \cdot 10^7 \text{ NM} \quad 2.85 \cdot 10^6$$

$$M_T = \underline{-1.64 \cdot 10^6 \text{ NM}} = \underline{8.28 \cdot 10^6}$$

$$I = 7.09 \cdot 10^{-3}; \alpha = -2.32 \cdot 10^4 \frac{\text{rad}}{\text{s}^2}$$

$$\theta = \frac{\alpha I^2}{2} = 2.602 \cdot 10^{-3} \quad \alpha = 4.02$$

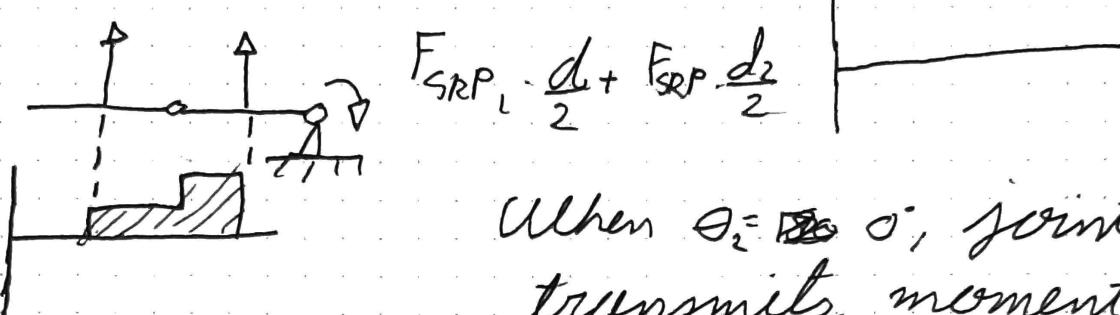
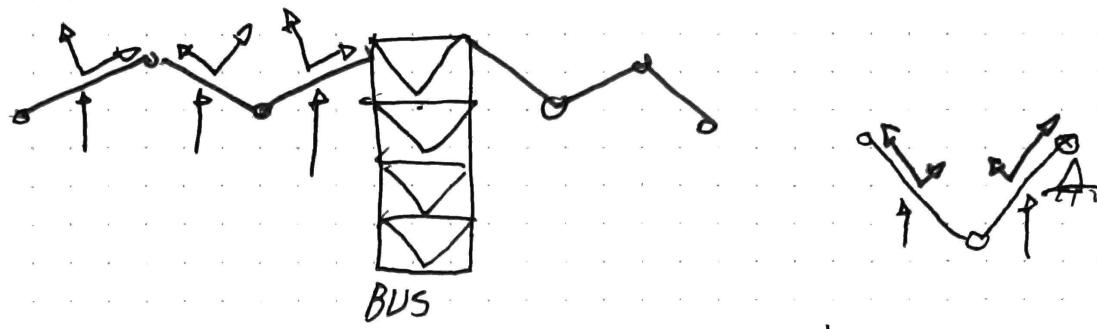
(I) NO FINAL RESULT

(II) SPRING ACTUATED

$$F_T =$$

(III) DAMPED at  $\xi = 0.45$

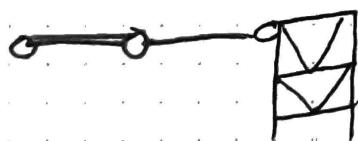
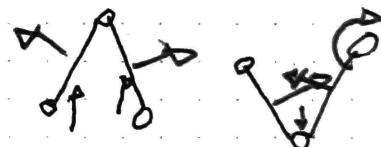
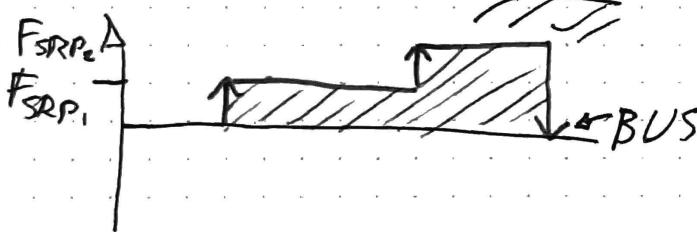
$$F_T = 3.92 \text{ MN}$$



$$\text{where } d_{\theta_1} = 2\theta - \frac{\alpha}{2}$$

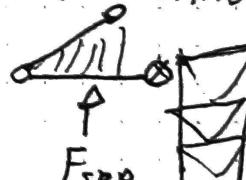


$$ma - a \\ 3.1 - \frac{2}{2} = 3 - \frac{1}{2} \\ = 2.5$$



$$P_{SRP} = P_0 (\sigma_1 + \rho) \left( \frac{R_{S0}}{R_s} \right)^2 \cos \alpha$$

$$R_{S0}$$

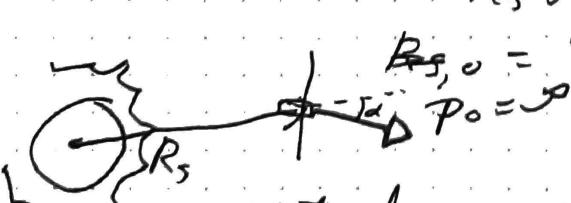


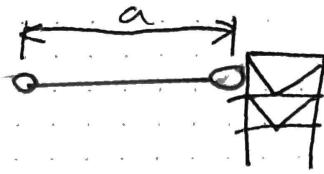
attitude control of large  
gossamer spacecraft

$$\text{dt } R_s = 2 \text{ AU} \\ \text{and } \alpha = 45^\circ$$

$$F_{SRP} = \frac{1}{8} P_0$$

$$R_{S0} = 4.56 \cdot 10^6 \frac{N}{m^2} \rightarrow SRP \text{ at 1au.}$$





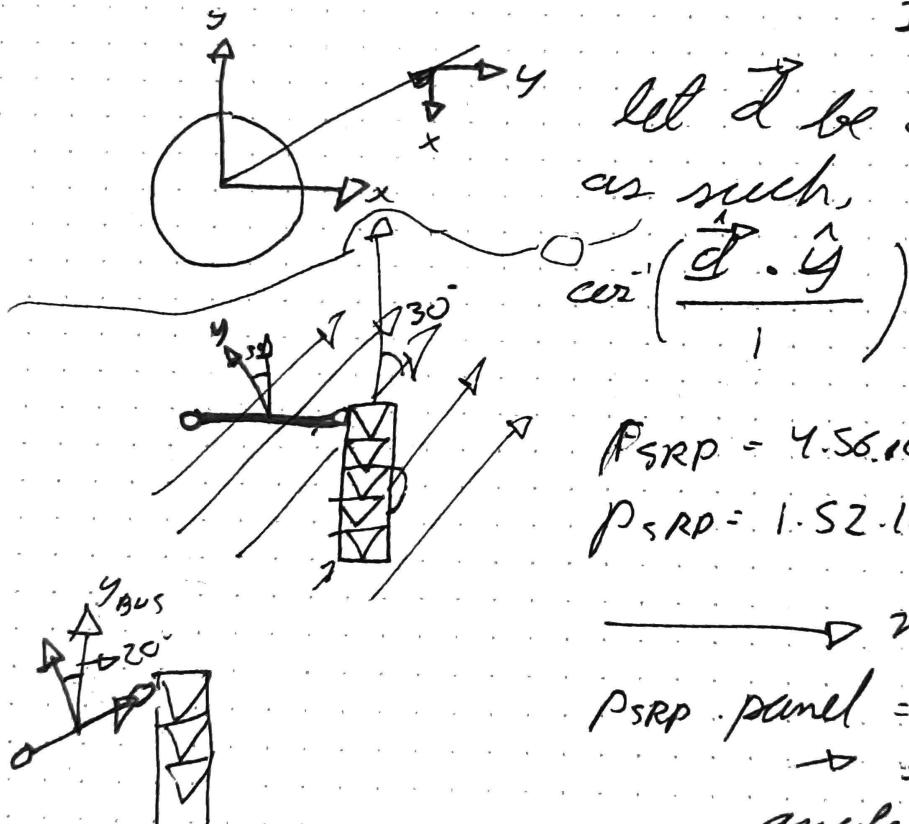
let  $a = 1\text{m}$   
and a panel area is  $a^2$   
at a distance of 1.5AU  
 $\theta = 0^\circ$  (Normal to the sun)  
state = 1 (on, reflection)

$$P_{SRP} = P_0(1+A)\left(\frac{R_{Sc}}{R_s}\right)^2 \cos^2\theta$$

$$P_{SRP} = 4.56 \cdot 10^{-6} \cdot 2 \cdot (0.444) \cdot 1 = 4.05 \cdot 10^{-6} \frac{\text{N}}{\text{m}}$$

$$F_{SRP} = 4.05 \cdot 10^{-6} \text{ N}_\parallel$$

$$M = I_{zz} \alpha \quad \alpha = \frac{M}{I_{zz}} = \ddot{\theta}$$



Let  $\vec{r}$  be a position vector  
as such,  
 $\cos\left(\frac{\vec{r} \cdot \vec{g}}{1}\right)$

$$P_{SRP} = 4.56 \cdot 10^{-6} (1+0) \left(\frac{R_{Sc}}{R_s}\right)^2 \cos 30^\circ$$

$$P_{SRP} = 1.52 \cdot 10^{-6} \frac{\text{N}}{\text{m}^2}$$

normal to bus  
P<sub>SRP</sub> · panel = force

$$\rightarrow = 1.52 \cdot 10^{-6} \text{ N} \quad \checkmark$$

angled force

$$y = 1.43 \cdot 10^{-6} \quad x = 5.197 \cdot 10^{-7}$$

Force acting normal  
to surface:

$$F_y = 1.428 \cdot 10^{-6} \text{ N}$$

$$M = F_y \cdot dy = 1.43 \cdot 10^{-6} \cdot 0.5\text{m}$$

$$M = 7.15 \cdot 10^{-7} \text{ NM}$$

$$\alpha = \frac{M}{I} = \frac{7.15 \cdot 10^{-9} \text{ NM}}{2.0 \text{ rad/s}^2} = 3.58 \cdot 10^{-7}$$