Data Fusion

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Homework 2

Section 1: Discretizing the Continuous System:

Using the one-step Euler integrator, we can use equation (1) to discretize the system.

It should be noted that all the plots regarding the ||P|| values per iteration have been multiplied by e-03 to better depict the difference in estimated and corrected P values.

(1)

$$X_{k+1} = X_k + t * \dot{X}$$

and

$$\dot{X} = AX$$

By substituting the latter phrase into the former one, and factoring the state matrix X from the left-hand side, we will get equation (2).

(2)

$$X_{k+1} = X_k(I + t * A)$$

By substituting the given t and A in the above equation, we can see that the right-hand phrase or the multiplier of X, is as given in figure (1).

```
Discretization Matrix:
array([[ 1.  ,  0.02,  0.  ],
       [-0.03,  1.  ,  0.02],
       [ 0.1 ,  0.  ,  0.9 ]])
```

Figure 1: Discretizing multiplier of X

Section 2: RMSE Analysis

In this section, we will be calculating the RMSE measure between the three states and the three given sensor outputs. The real outputs y_1 to y_3 can be made by adding the given normal Gaussian noises to the multiplication of the three C vectors, into their corresponding true states, more specifically, the multiplication of C_1 into the true state matrix yields the first column, X_1, and then for C_2 and C_3, respectively X_2 and X_3. Then by calculating the difference between each of the yielded y vectors and the three sensor outputs, 9 different RMSE values are yielded which are shown in figure (3).

RMSE of sensor i and state j	Value of RMSE
RMSE11: sesnor 1 and state 1	10.534448760686589
RMSE12: sesnor 1 and state 2	73.56383856264694
RMSE13: sesnor 1 and state 3	16.282599392334067
RMSE21: sesnor 2 and state 1	72.24869098573045
RMSE22: sesnor 2 and state 2	5.576287233265098
RMSE23: sesnor 2 and state 3	77.0392450424218
RMSE31: sesnor 3 and state 1	13.538594988740696
RMSE32: sesnor 3 and state 2	77.74853074781069
RMSE33: sesnor 3 and state 3	5.284010187832365

Figure 2: RMSE Table of all states and all sensors

It is obvious that the lowest RMSE for each sensor, belongs to its corresponding state which it follows. Between the three corresponding sensor and state pairs, the third sensor follows the third state the best and also has the lowest RMSE for the other un-corresponding states.

Section 3: Estimating each state based on the three sensors with Kalman Filter

For this part we first implement an iterative KF algorithm as per the equations provided in the KF presentation of the course. In this algorithm we start from an initial point and an initial covariance matrix, and then iteratively complete time updates or predictions, and then measurement updates or corrections on the X vector and P matrix. Since we have the sensor outputs we use this data in the measurement update step of X, as a form of error correction. Thus we will provide 9 scenarios for each pair of sensor and state.

(a) **using sensor y_1 for state variable x_1:** In figure 3, the plot of the estimated and corrected states, against the actual sensor output, and the norm of the estimated and corrected P matrices per iteration, have been depicted.

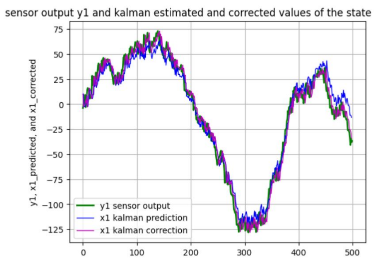

Figure 3: corrected and estimated x1 values tracking the sensor y1

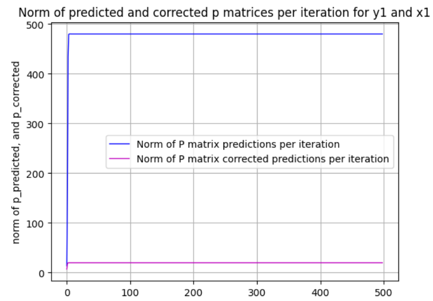

Figure 4: Norm of the estimated and corrected P matrices per iteration for state variable x1

the RMSE of the corrected and estimated state variable x_1 with regards to the sensor output y_1 has been reported in figure 5.

```
RMSE of Y1 and Estimated X1: 8.433986012708582 while RMSE of Y1 and Corrected X1: 0.1088049297066677
```

Figure 5: RMSE values for estimated and corrected x1 using output y1

(b) **using sensor y_1 for state variable x_2:**

In figures 6 and 7, the plot of the estimated and corrected states, against the actual sensor output, and the norm of the estimated and corrected P matrices per iteration, have been depicted.

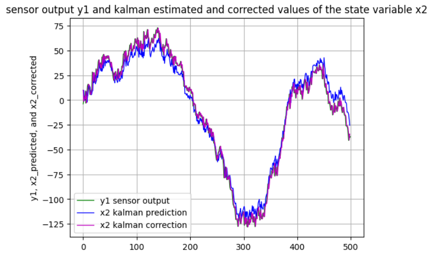

Figure 6: corrected and estimated x2 values tracking the sensor y1

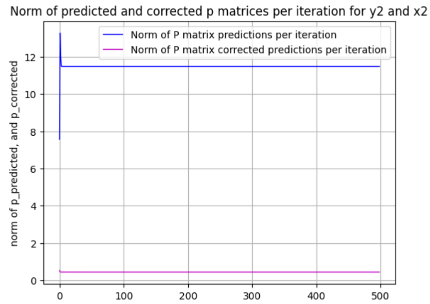

Figure 7: Norm of the estimated and corrected P matrices per iteration for state variable x2

the RMSE of the corrected and estimated state variable x_2 with regards to the sensor output y_1 has been reported in figure 8.

```
RMSE of Y1 and Estimated X2: 8.75372608737757 while RMSE of Y1 and Corrected X2: 0.14642431306328882
```

Figure 8: RMSE values for estimated and corrected x2 using output y1

(c) **using sensor y_1 for state variable x_3:**

In figures 9 and 10, the plot of the estimated and corrected states, against the actual sensor output, and the norm of the estimated and corrected P matrices per iteration, have been depicted.

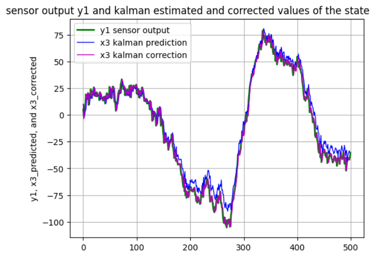

Figure 9: corrected and estimated x3 values tracking the sensor y1

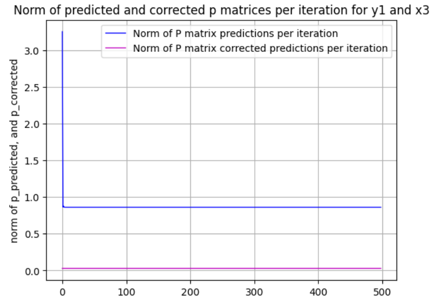

Figure 10: Norm of the estimated and corrected P matrices per iteration for state variable x3

the RMSE of the corrected and estimated state variable x_3 with regards to the sensor output y_1 has been reported in figure 11.

```
RMSE of Y1 and Estimated X2: 7.646328753421114 while RMSE of Y1 and Corrected X2: 0.09405053297126252
```

Figure 11: RMSE values for estimated and corrected x3 using output y1

(d) **using sensor y_2 for state variable x_1:**

From the formulation of the Iterative KF algorithm, we know that by changing the used actual output y_2 or z_k, the the value of the corrected state x changes directly and this change indirectly changes the gain K and the covariance matrix p (both estimated and corrected) through the iterations. So it is expected that the plot of the norm of p, also changes. By comparing the following figure to the plot of norm of p from section (a) of this question, we can see that the magnitude of the norm has decreased while using the output of the second sensor for correcting the values of the first state.

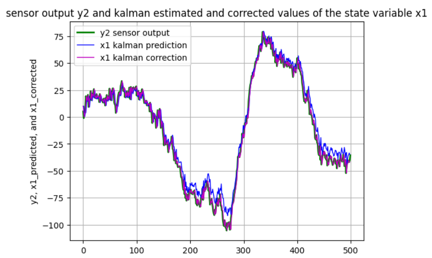

Figure 12: corrected and estimated x1 values tracking the sensor y2

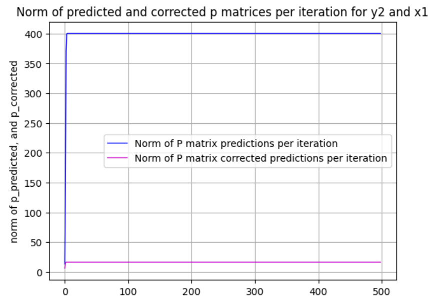

Figure 13: Norm of the estimated and corrected P matrices per iteration for state variable x1

Also the RMSE of the corrected and estimated state variable x_1 while using the sensor y_2 has been given in figure 14 and it can be seen that both states have a lower RMSE than that of figure 5, meaning that sensor y_2 is better than y_1 for correcting the values of the estimated state x_1.

```
RMSE of Y2 and Estimated X1: 8.348986929905315 while RMSE of Y3 and Corrected X1: 0.06925951305266241
```

Figure 14: RMSE values for estimated and corrected x1 using output y2

(e) **using sensor y_2 for state variable x_2:**

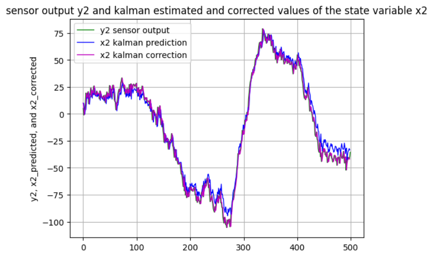

Figure 15: corrected and estimated x2 values tracking the sensor y2

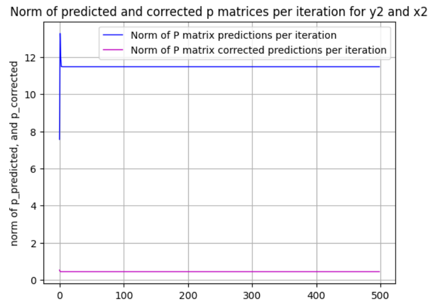

Figure 16: Norm of the estimated and corrected P matrices per iteration for state variable x2

The RMSE of the corrected and estimated state variable x_2 while using the sensor y_2 has been given in the following figure.

```
RMSE of Y2 and Estimated X2: 6.620673274387038 while RMSE of Y2 and Corrected X2: 0.07089102615892563
```

Figure 17: RMSE values for estimated and corrected x1 using output y2

(f) **using sensor y_2 for state variable x_3:**

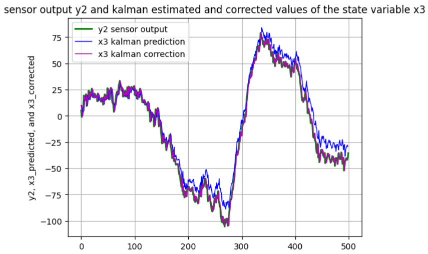

Figure 18: corrected and estimated x3 values tracking the sensor y2

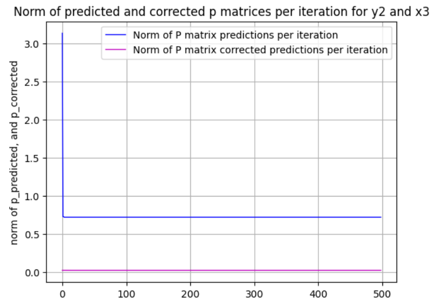

Figure 19: Norm of the estimated and corrected P matrices per iteration for state variable x3

The RMSE of the corrected and estimated state variable x_3 while using the sensor y_2 has been given in the following figure.

```
RMSE of Y2 and Estimated X2: 9.553812417928551 while RMSE of Y2 and Corrected X2: 0.07102592552303683
```

Figure 20: RMSE values for estimated and corrected x3 using output y2

(g) **using sensor y_3 for state variable x_1:**

By comparing the following figure to the plot of norm of p from parts (a) and (d) of this section, we can see that the magnitude of the norm has again had a slight decrease while using the output of the third sensor for correcting the values of the first state, rather than the second or first sensor.

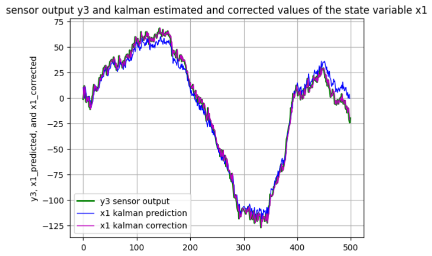

Figure 21: corrected and estimated x1 values tracking the sensor y3

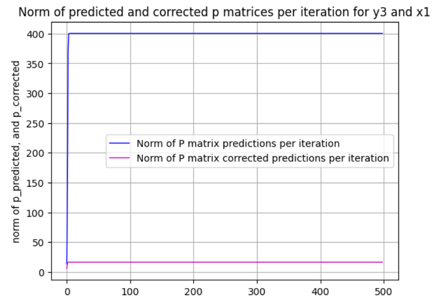

Figure 22: Norm of the estimated and corrected P matrices per iteration for state variable x1

The RMSE of the corrected and estimated state variable x_1 while using the sensor y_3 has been given in the following figure and it can be seen that both states have a lower RMSE than that of using eithr y_1 or y_2, meaning that **sensor y_3 is the best, between the three, in correcting the values of the estimated state x_1.**

```
RMSE of Y3 and Estimated X1: 7.154119154655084 while RMSE of Y3 and Corrected X1: 0.05775226303211331
```

Figure 23: RMSE values for estimated and corrected x1 using output y3

(h) **using sensor y_3 for state variable x_2:**
By comparing the following figure to the plot of norm of p from parts (b) and (e) of this section, we can see that the magnitude of the norm is almost the same as what it was for sensor y_2 and lower than what it was for sensor y_1, for correcting the values of the second state.

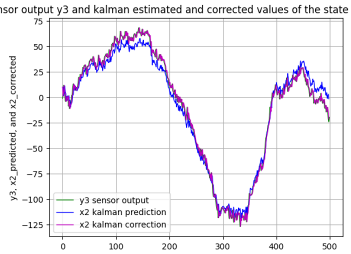

Figure 24: corrected and estimated x2 values tracking the sensor y3

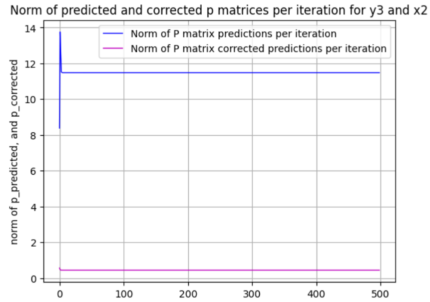

Figure 25: Norm of the estimated and corrected P matrices per iteration for state variable x2

The RMSE of the corrected and estimated state variable x_2 while using the sensor y_3 has been given in the following figure. From comparing the three RMSE calculations between the three sensors, it can be seen that both states have a lower RMSE while using sensor y_2 rather than using either y_1 or y_3, meaning that **sensor y_2 is the best, between the three, in correcting the values of the estimated state x_2.**

```
RMSE of Y3 and Estimated X2: 7.716543386496429 while RMSE of Y3 and Corrected X2: 0.0825895510294945
```

Figure 26: RMSE values for estimated and corrected x2 using output y3

(i) **using sensor y_3 for state variable x_3:**

By comparing the following figure to the plot of norm of p from parts (c) and (f) of this section, we can see that the magnitude of the norm is almost the same as what it was for sensor y_2 and lower than what it was for sensor y_1, for correcting the values of the third state.

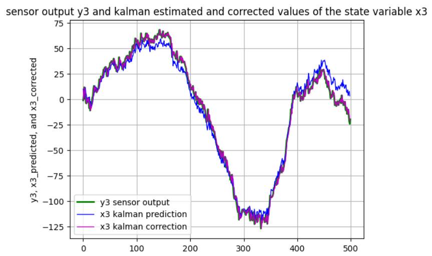

Figure 27: corrected and estimated x3 values tracking the sensor y3

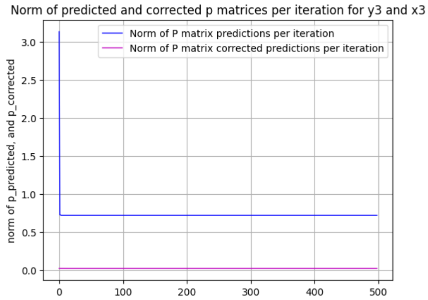

Figure 28: Norm of the estimated and corrected P matrices per iteration for state variable x3

The RMSE of the corrected and estimated state variable x_2 while using the sensor y_3 has been given in the following figure.

```
RMSE of Y3 and Estimated X2: 8.276129142520373 while RMSE of Y3 and Corrected X2: 0.061543558989310444
```

Figure 29: RMSE values for estimated and corrected x3 using output y3

From comparing the three RMSE calculations between the three sensors, it can be seen that for the prediction stage, y_1 has a lower RMSE than y_3 which has a lower RMSE than y_2. However for the correction state, while the RMSE measures are close, y_3 does better than y_2, which does better than y_1. Generally since the actual values of KF that will eventually be used are the correction values, it can be said that y_3 does the best in correcting the values of the third state.

Section 4: Analysis of the State Variables and Covariance Matrices for Fused Sensors

Here, we will again use the KF algorithm but for fused sensors, meaning that instead of the C vector for each y_i, we will use matrices of fused C vectors to implement the algorithm while using multiple sensors. Also, In each case the matrix of measurement noise or R, will be a matrix containing the previous R values of the single sensor case on the diagonal elements. So in the case of dual fusion of sensors, R will be a matrix of size 2x2, and for the fusion of all matrices, it will be of size 3x3. While for the former case C will be of size 2x3 and K will be of size 3x2, and for the latter case, C and K will be of size 3x3. Lastly, the Z_k element or the sensor output will change in the phrase used for correction of x_k, meaning that for the case of dual fusion, it will be a vector of size 2x1, while for the latter case, it will be of size 3x1. The C matrices used for each case have been reported for reference.

(a)Fusion of y_1 and y_2:

$$C12 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

By using the new matrices C, K, and R, and also the vector containing the first and second sensor's measurements, we run the iterative KF algorithm again. The plot of the Norm of P, the covariance matrix can be seen in figure 30.

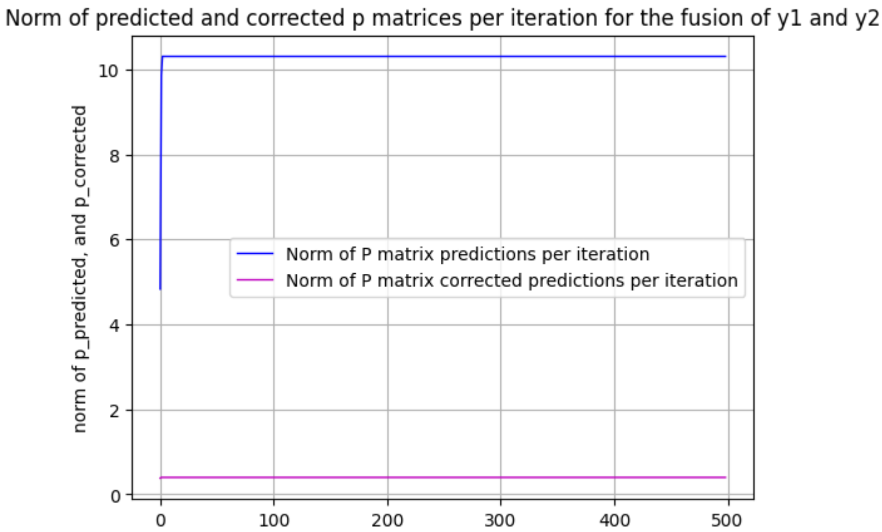

Figure 30: ||P|| plot for the fusion of y1 and y2 sensors

Also the table containing the RMSE for each of the corrected and estimated states can be seen in figure 31. Firstly the RMSE of the corrected estimation is generally lower than that of the estimated ones for all the states. Moreover, it can be seen that the the first and second states, for which the corresponding sensor outputs have been used, have a much smaller RMSE value in comparison to the third state, for which no measurement output has been used. **Meaning that this fused system optimizes the tracking of the first state better than the second one, and does the worst, by a large margin, in tracking the third state.**

RMSE of state i for sensors y1/y2	Value of RMSE
predicted state x1	2.3167999123515077
corrected state x1	3.556383761774429
predicted state x2	4.42876258438453
corrected state x2	4.542694997080404
predicted state x3	167.97112482223417
corrected state x3	188.10010310662477

Figure 31: RMSE table for the fusion of y1 and y2 sensors per state

(b)Fusion of y_1 and y_3:

$$C13 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

By using the new matrices C, K, and R, and also the vector containing the first and third sensor's measurements, we run the iterative KF algorithm again. The plot of the Norm of P, the covariance matrix can be seen in figure 32.

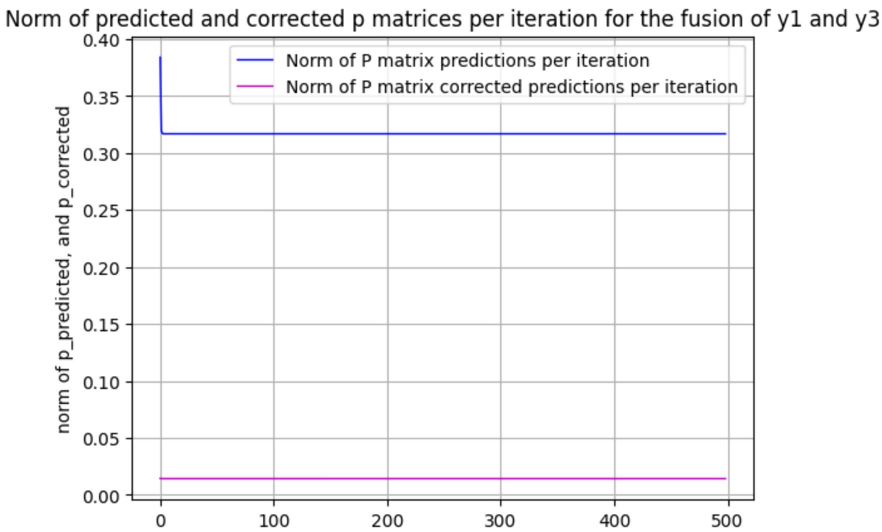

Figure 32: ||P|| plot for the fusion of y1 and y3 sensors

Also the table containing the RMSE for each of the corrected and estimated states can be seen in figure 33. Moreover, it can be seen that the the first and third states, for which the corresponding sensor outputs have been used, have a much smaller RMSE value in comparison to the second state, for which no measurement output has been used. **Meaning that this fused system optimizes the tracking of the third estimated state better than the first estimated state, while tracking the first corrected one better than the third (which is the actual RMSE of the output that we have in mind,) and does the worst, by a large margin, in tracking the second state.**

RMSE of state i for sensors y1/y3	Value of RMSE
predicted state x1	2.074557158372602
corrected state x1	3.8936525550809193
predicted state x2	49.17689226994097
corrected state x2	49.178438218945296
predicted state x3	1.334350668440222
corrected state x3	4.124044928156996

Figure 33: RMSE table for the fusion of y1 and y3 sensors per state

(c)Fusion of y_2 and y_3:

$$C23 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

By using the new matrices C, K, and R, and also the vector containing the second and third sensor's measurements, we run the iterative KF algorithm again. The plot of the Norm of P, the covariance matrix can be seen in figure 34.

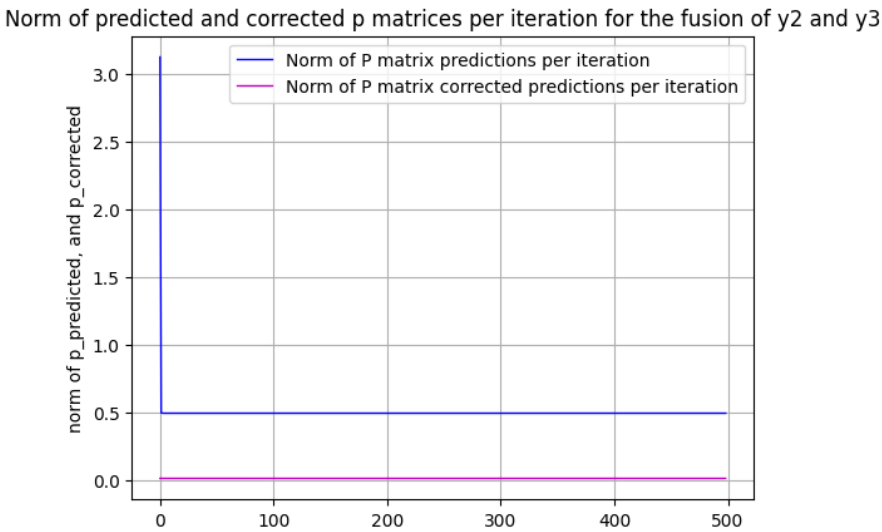

Figure 34: ||P|| plot for the fusion of y2 and y3 sensors

Also the table containing the RMSE for each of the corrected and estimated states can be seen in figure 35. Moreover, it can be seen that the the second and third states, for which the corresponding sensor outputs have been used, have a much smaller RMSE value in comparison to the first state, for which no measurement output has been used. **Meaning that this fused system optimizes the tracking of the second estimated state better than the third estimated state, while also tracking the second corrected one better than the third (meaning that in general it tracks the second state better than all the rest,) and does the worst, by a large margin, in tracking the first unmeasured state.**

RMSE of state i for sensors y2/y3	Value of RMSE
predicted state x1	64.27696018968977
corrected state x1	64.27127756688039
predicted state x2	1.7252742183978294
corrected state x2	3.376374099801475
predicted state x3	2.585281542700535
corrected state x3	4.091177078044747

Figure 35: RMSE table for the fusion of y2 and y3 sensors per state

(d)Fusion of y_1, y_2 and y_3:

$$C123 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By using the new matrices C, K, and R, and also the vector containing the first, second and third sensor's measurements, we run the iterative KF algorithm again. The plot of the Norm of P, the covariance matrix can be seen in figure 36.

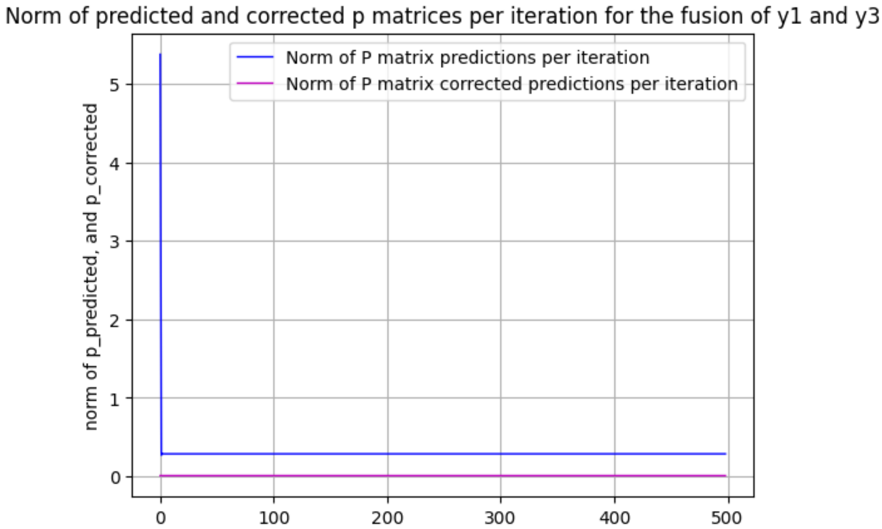

Figure 36: ||P|| plot for the fusion of y1, y2 and y3 sensors

Also the table containing the RMSE for each of the corrected and estimated states can be seen in figure 37.

RMSE of state i for sensors y1/y2/y3	Value of RMSE
predicted state x1	2.3692706296547916
corrected state x1	3.5834668670069685
predicted state x2	1.5151927752096783
corrected state x2	3.352913982986729
predicted state x3	1.3847765285955809
corrected state x3	4.076162749625683

Figure 37: RMSE table for the fusion of 1st, 2nd and 3rd sensors per state

Lastly, it can be seen that all the states, for which the corresponding sensor outputs have been used, have a rather small and comparable RMSE. **Meaning that this fused system optimizes the tracking of the third estimated state better than the second one, and sequentially these two do better than the first estimated state, while tracking the second corrected one better than the first, and then the first one better than the third(which is the actual RMSE of the output that we have in mind). However it should be noted that all these values are quite close to each other and have no significant difference meaning that this fused system lowers the RMSE of all the states significantly, and thus tracks all the states better than all the previously dually fused sensor systems**

Section 5: Conclusion: Comparison of All 8 Possible Combinations of Sensors

As per expected, the fusion of sensors helps reduce the average RMSE over the states, and their corrected and estimated values, almost reducing the error to 5 6 percent of its original value. It should be noted that I have made a mistake in the calculation of the corrected RMSE error of the first section, regarding the implementation of the KF filter on the single sensors for all the states, and was not able to correct the reporting of these values due to a lack of time. However, It can be reported that each sensor was the best at reducing the error for its corresponding state in the single sensor cases, and did not change the other states by much. Then in the cases regarding dual fusion, in each case the two states for which the corresponding sensors where entered into the KF correction phase, were tracked better and a had a much lower RMSE than the other unsupervised state. And lastly using all the sensors lowered all the RMSE values as expected, and even though the reductions happened to be less than that of the dual fusion for certain states of certain cases, but since this system manages to reduce all the error values and does not leave any of the states untracked, it can be perceived that **using all the sensors, yields the best result out of all eight possible scenarios.**